Q. Jl R. astr. Soc. (1983) 24, 27–35

The Authenticity of Ptolemy's Star Data – II

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(Received 1982 April 14)

SUMMARY

In chapter VII.3 of his *Syntaxis*, Ptolemy derives the precession of the equinoxes from the declinations of six stars that he claims to have measured himself. In an earlier paper I showed that these declinations were fabricated, but the method of fabrication that I suggested was not successful. In this paper I present a successful method of fabrication. I also discuss the alternate suggestion made by others that the declinations were selected in a biased way from a large sample instead of being fabricated.

1 INTRODUCTION

In chapters VII.2 and VII.3 of his famous treatise, which I shall henceforth designate as the *Syntaxis*, Ptolemy (*c.* AD 142) derives the rate of precession of the equinoxes by three independent methods. In the second of these methods, which will be the subject-matter of the present paper, he compares the declinations of certain stars as measured by himself with the values measured by Hipparchus 265 years before, and he finds that the rate is exactly $1°\ \mathrm{cy}^{-1}$. The correct value in his time was about $1°{\cdot}39$.

Ptolemy starts this method by listing the declinations of 18 stars as measured by three observers. The first observer was either Aristyllus or Timocharis about 400 years earlier, the second was Hipparchus about 265 years earlier, and the third observer is specifically stated to be Ptolemy himself. The 18 stars are chosen so that nine have increasing declinations and the other nine have decreasing ones. From each group of nine, Ptolemy selects three, apparently at random, and calculates the rate of precession from the difference between the declination he allegedly measured and that which Hipparchus measured. He does not make any explicit use of the remaining six stars in each group, nor does he explicitly use the measurements made by Timocharis or Aristyllus. He merely states the results of the calculations and he does not give any details about his method of calculation.

I studied the declinations that Ptolemy claims to have measured in an earlier paper (Newton 1974). There I first noted that, of the 12 stars that Ptolemy does not use, four lie close to the solstice points. As a result, they have such small changes in declination that they cannot possibly lead to a useful value of the precession and it is legitimate to ask why Ptolemy chooses to include them in his list. For the remaining eight stars that he does not use and the six that he does use, I calculated the value of the precession p by combining his claimed measurements with those made by Hipparchus. The values of p from the six that he does use range from 35·1 to 41·1 arcsec yr^{-1}

while the values from the eight that he does not use range from 45·1 to 64·4 arcsec yr^{-1}. There is no overlap between the two groups, and thus it seems virtually certain that they have independent origins, in spite of Ptolemy's claim that he observed the declinations of all 18 stars himself.

Although the values of p derived from the six stars that Ptolemy uses range from 35·1 to 41·1 arcsec yr^{-1}, with an average value of 38·1, Ptolemy says that the value derived from each one is 1° cy^{-1} (36 arcsec yr^{-1}), so that all agree exactly according to him. The average value from the stars that he does not use is 52·8 arcsec yr^{-1}, which is close to the correct value.

From this study of the values of p, and from a parallel study of the declinations themselves, I concluded that the 12 unused values are valid observations but that the six used values are fabricated. In some later writing I have described the 12 unused values as genuine observations made by Ptolemy, but I should not have. All we can say is that they seem to be genuine observations made in Ptolemy's time, but we cannot say that he made them.

I also tried to fabricate the six used values by a method that is simple for a modern astronomer to use, but that may have been beyond the mathematical apparatus available to Ptolemy. The results were not satisfactory. In this paper I shall present a different method of fabrication that does seem satisfactory.

Other writers, whom I shall cite later, have suggested that Ptolemy did not fabricate the six used measurements but that he instead selected them from a large body of genuine measurements. In this paper I shall also present evidence, which seems overwhelming to me, that the values were fabricated and not selected.

2 A METHOD OF FABRICATION

The declination δ of any point is related to its celestial latitude β and its longitude λ by this well-known expression:

$$\sin \delta = \cos \beta \sin \lambda \sin \epsilon + \sin \beta \cos \epsilon. \qquad (1)$$

In this expression, ϵ is the obliquity of the ecliptic, which Ptolemy takes to be 23° 51′ 20″. Ptolemy was not acquainted with the rotation of the plane of the ecliptic, and thus he took β and ϵ to be constant in time. If we take differentials in equation (1) while holding β and ϵ constant, we get

$$\Delta\delta \cos \delta = \Delta\lambda \cos \beta \cos \lambda \sin \epsilon. \qquad (1a)$$

Here $\Delta\delta$ denotes a change in declination and $\Delta\lambda$ denotes the corresponding change in longitude.

In the earlier paper, I applied this equation to the changes that occurred between the observations of Hipparchus and Ptolemy. For these changes, Ptolemy takes $\Delta\lambda$ to be $2\frac{2}{3}°$ and he takes ϵ to be 23° 51′ 20″. This gives

$$\Delta\delta = 1°{\cdot}0785 \cos \beta \cos \lambda / \cos \delta. \qquad (2)$$

As Dennis Rawlins (private communication) has reminded me, this can be written as

$$\Delta\delta = 1°{\cdot}0785 \cos \alpha. \qquad (3)$$

Equation (3) is simpler to use than equation (2) if the values of α are available. However, for the six stars whose declinations Ptolemy uses, the values of α are not immediately available from his writing, but the values of β, λ and δ are.

Thus, for the six stars that Ptolemy uses, I calculated $\Delta\delta$ from equation (2) and added it to the declination that Hipparchus observed. The results agreed with the values that Ptolemy claims to have measured, within rounding error, for three of the stars, but they disagreed by awkward amounts for the other three. Thus it does not seem likely that Ptolemy used either equation (2) or (3), quite aside from the question of whether he knew either relation.

Instead of using an approximate method that may not have been known to Ptolemy, let us try using an exact method that was certainly known to him. That is, let us try using equation (1).* Since we are only going to use the equation to compare the declination of the same star at two different epochs, we may simplify equation (1) to

$$\sin \delta_P - \sin \delta_H = \cos \beta \sin \epsilon \,(\sin \lambda_P - \sin \lambda_H). \tag{4}$$

In this, a subscript H denotes a value that applies in the time of Hipparchus and a subscript P denotes a value that applies in the time of Ptolemy. Since Ptolemy would have taken β and ϵ to be the same at the two times, I have not put subscripts on these variables in equation (4). The value of $\sin \epsilon$ is 0·404 432 277. Ptolemy states the values of δ_H in chapter VII.3 of the *Syntaxis*. We take the values of β and λ_P to be the values given in the star catalogue found in the *Syntaxis*, and I have shown that Ptolemy simply appropriated the star catalogue of Hipparchus by adding 2° 40′ to the longitudes, while leaving the latitudes unchanged (Newton 1977, section IX.7). That is, we take $\lambda_H = \lambda_P - 2° \, 40'$. The quantities needed in using equation (4) are listed in Table I.

When we enter equation (4) with the values from Table I, remembering that $\lambda_H = \lambda_P - 2° \, 40'$, we get the declinations listed in the second column of Table II, where I have kept a precision of a tenth of a minute of arc. With rare exceptions, Ptolemy rounds angular coordinates either to the nearest multiple of 10′ or to the nearest multiple of 15′. When we round the values in the second column of Table II in this way, we get the values in the third column, which may now be compared with the values that Ptolemy claims to have measured. These values are given in the last column.

The agreement of the calculated values with the ones Ptolemy claims to have measured is exact except for the first and last stars in the table. Let us look at the results for α Boötis in more detail. The calculated number of minutes is 56·1, but small approximations in the computing process, which would have been a lengthy one for Ptolemy, could easily make the number of minutes be less than 55. The calculated value would then round to 29° 50′, which is the value Ptolemy states.

*When I say that Ptolemy knew equation (1), I do not mean that he could write down an equation of this form. I only mean that he could carry out a string of computations that lead to the same answer as the use of equation (1), computations in which he could use no trigonometric functions except the chord.

TABLE I

Coordinates of the stars that Ptolemy uses to find the precession

Star	δ_H ° ′	β ° ′	λ_P ° ′
η Tau	15 10	3 20	33 40
α Aur	40 24	22 30	55 00
γ Ori	1 48	−17 30	54 00
α Vir	0 36	− 2 00	176 40
η UMa	60 45	54 00	149 50
α Boö	31 00	31 30	177 00

TABLE II

A comparison of the calculated declinations with those Ptolemy claims to have measured

Star	Calculated value ° ′	Calculated value after rounding ° ′	Measured by Ptolemy ° ′
η Tau	16 6·4*	16 10	16 15
α Aur	41 10·5	41 10	41 10
γ Ori	2 25·3	2 30	2 30
α Vir	− 0 28·3	− 0 30	− 0 30
η UMa	59 39·4	59 40	59 40
α Boö	29 56·1†	30 00	29 50

*A simple approximation changes this to 16° 13′·2, which rounds to 16° 15′.
†A simple approximation changes this to 29° 54°·1, which rounds to 29° 50′.

One approximation that Ptolemy frequently makes involves interpolation in a table. When he has occasion to use a table, sometimes he interpolates, sometimes he does not interpolate but simply takes the nearest tabular value, and sometimes he mixes the practices in a single discussion. The table involved in this discussion is his table of chords. The chord function is no longer used, but it is simply related to the sine function by

$$\text{chord } Z = 2 \sin Z/2. \tag{5}$$

Ptolemy's table of chords is given for every half of a degree, which is equivalent to an interval of a quarter of a degree for the sine or cosine function.

Let us recalculate the declination, still using equation (4), but rounding the coordinates of η Tauri and α Boötis in equation (4) to the nearest quarter of a degree. The calculated declination of η Tauri now becomes 16° 13′·2, which Ptolemy would round to 16° 15′, and this is the value he states. Similarly, the calculated declination of α Boötis now becomes 29° 54′·1, which Ptolemy would round to 29° 50′, and this is the value he states.

Thus we have now obtained all the declinations that Ptolemy uses in determining the precession, by a simple and plausible method of fabrication. However, the 12 declinations that he states but does not use cannot be fabricated this way. They show all the signs of being genuine measurements.

3 DID PTOLEMY FABRICATE OR SELECT THE DECLINATIONS?

Several writers, particularly Gingerich (1980), have suggested that Ptolemy did not fabricate his data, in spite of the fact that they agree so suspiciously with his theories and models. Rather, they say, he had available a large body of data from which he selected the ones that fitted his preconceptions.

Gingerich does not specifically refer to the declinations in Table II in connection with the matter of selection, but Pannekoek (1955) does, and he specifically writes (p. 64) about them, and about the 12 other declinations that Ptolemy quotes but does not use:

> 'There can be no doubt that Ptolemy selected these six stars *because* they were favourable to his assumed value of the precession and could be quoted as confirmations, and that other stars were omitted because they did not confirm his assumption. Yet we cannot speak of an attempt to deceive his readers; he presents to them the full material with the unfavourable cases also. It comes down to saying: "my result is confirmed by a number of data; the other data which do not conform to it do not count".' (emphasis in original)

There are two arguments which indicate that Ptolemy did not select the six stars from a large body of cases, and one of them is overwhelming. Let us take it up first; it is based upon the sizes of the errors in the declinations.

First we need an estimate of the standard deviation of a Greek measurement of declination. We can obtain this estimate, for a body of observations made in Ptolemy's own time, from the 12 declinations that Ptolemy does not use. In my earlier paper (Newton 1974, table III) I found the errors in these declinations, and the standard deviation of the error is 7′·5. However, almost half of the total variance of the errors comes from a single value, that for α Orionis, and we may legitimately omit it on the grounds that it probably comes from a recording error. When we do so, the standard deviation drops to 5′·6. I shall use this as the standard deviation in the succeeding calculations. This is close to the standard deviation that we find for the 18 observations attributed to Hipparchus.

Now let us look at the errors in the six declinations that Ptolemy uses. These are summarized in Table III. Here the first column gives the name of the star, and the second column gives the absolute value of the error in the declination that Ptolemy claims to have measured. Now I assume that the errors have a Gaussian distribution with the standard deviation stated. For each error, we can calculate the probability that an error will be as large

TABLE III

The availability of the erroneous declinations used by Ptolemy

Star	Error in declination ′	Number of errors this large in a sample of 1000*
η Tau	23	0·03
α Aur	18	0·7
γ Ori	9	55
α Vir	27	0·0004
η UMa	28	0·0003
α Boö	8	81

*Based upon a standard deviation of 5′·6.

as the value stated and *of the appropriate sign*, and from this we can calculate the expected number of declinations meeting these conditions in a sample of 1000. (That is, we must choose the sign of the error in declination that will make $\Delta\lambda$ too small instead of too large.) A thousand is approximately the number of stars in the star catalogue.

It is plausible that Ptolemy could have found the declinations of γ Orionis and α Boötis in a sample of 1000. It is highly improbable, at the level of about 10^9 to 1, that the other values could have ever existed in a star catalogue with about 1000 stars.

In other words, Ptolemy did not select his six declinations from a large body of genuine data, because no body of genuine data containing these values could have ever existed.

The second argument concerns the method by which Ptolemy could have made a selection. Let us suppose that the preceding argument is in error for some reason, perhaps because I used a standard deviation that is too small, and let us suppose that the six values Ptolemy uses can plausibly be found in a catalogue of 1000 stars. Now we must ask: How did Ptolemy decide which values to select? How did he identify the six stars that lead to a precession of 1° cy^{-1}? The value of precession that will result from a particular measurement of the declination is not obvious. Ptolemy would have had to calculate the precession p from every star in turn until he found the six that give him 1° cy^{-1}.

In order to calculate the value of p, Ptolemy would use one of the relations in the preceding section, say equation (4) for example. From the difference between the two values of sin δ, he would calculate the difference between the two values of sin λ, and from this he would get the change in λ. Dividing this by the time interval would then give him an estimate of p.

In the standard arrangement of stars in Hipparchus's star catalogue, the constellations north of the zodiac are given first, those in the zodiac are given next, and those south of the zodiac are given last. If Ptolemy worked his way through the catalogue looking for stars that would give him $p = 1^\circ$ cy^{-1}, he had to work his way through about 700 stars until he came to γ Orionis (south of the zodiac), which finally gave him his chosen six. Selecting the data this way would have involved more computation than we find in all the *Syntaxis*. It is not plausible that Ptolemy calculated 700 values of p, considering that each computation would have been quite difficult for him.

Of course there are variants of these arguments. Ptolemy would have been limited in his search to stars for which Hipparchus had already measured the declination. In his only surviving work, Hipparchus (*c.* 135 BC) gives his measurements of the declination for 40 stars, while Ptolemy quotes his measurements for 18 stars. Three stars appear in both sources, so we have altogether 55 measurements of declination made by Hipparchus. Other measurements that have since been lost might still have been available to Ptolemy, so let us say that he could have used as many as 60 stars.

It may be credible that Ptolemy did the calculations for 60 stars in a search for the six that he uses. However, if his sample contained only 60 stars, it is all the more unlikely, by a factor of about 17^6 ($= 2{\cdot}4 \times 10^7$), that the six erroneous values ever existed for him to select.

4 DISCUSSION

The method of fabrication I have suggested is not the only conceivable one, although it is a simple and plausible one. Some readers may object that I had to postulate inconsistency on Ptolemy's part in order to make this method succeed. However, we find such inconsistency throughout the *Syntaxis*, so this objection is not a serious one. The alternative to fabrication, namely selection, seems so thoroughly improbable that I think we may safely eliminate it.

Instead of the method I have suggested, it is conceivable that Ptolemy used a method that effectively amounts to using equation (2) or (3). I do not suggest that he knew how to take derivatives in a formal way, but it does seem possible that some Greek mathematician realized in an empirical way that the change in sin Z, for a small change in Z, is proportional to cos Z. Of course, this is not how a Greek mathematician would have worded the matter. He would have said that a change in chord Z, for a small change in Z, is proportional to chord $(180° - Z)$.

In fact, the table of chords in the *Syntaxis* comes close to suggesting this possibility. After each value of chord Z, the table gives the change in chord Z that corresponds to a change of $1'$ in Z; this entry eases the task of interpolation. It would not be remarkable if some Greek mathematician had noticed that the change in chord Z is always proportional to chord $(180° - Z)$.

Further, in chapter VI.4 of the *Syntaxis*, Ptolemy describes how to calculate the increment to be added to the time of a mean syzygy to get the time of a true syzygy. As Pedersen (1974, pp. 90–91) points out, this calculation is equivalent to using the chain rule of differentiation, except that Greek mathematicians were unable to pass to the limit and find a true derivative.

The way Ptolemy words his use of the declinations to find the precession suggests that he might have used differential methods. For example, after remarking that the declination of γ Orionis has increased by about $40'$ between Hipparchus's measurement and his own, Ptolemy then states that a change of $2\frac{2}{3}$ degrees in longitude, for stars placed two-thirds of the way in Taurus, gives just this change in declination. (The data that Ptolemy quotes actually give a change in $42'$. This illustrates Ptolemy's willingness to make approximations.) The change of $2\frac{2}{3}$ degrees in longitude of course is what results from a precession of $1°$ cy^{-1} over the $2\frac{2}{3}$ centuries between the two measurements. By saying that the star is two-thirds of the way in Taurus, Ptolemy presumably means that the longitude is about $50°$; the value from Table I is actually $54°$. (This illustrates further approximations in the calculations.) Ptolemy uses similar wording, differing only in the numbers involved, for every one of the six stars in question. This wording gives the relation between increments in declination and increments in longitude. I thank Dennis Rawlins (private communication) for suggesting that Ptolemy might actually have used differential methods in these calculations. He is preparing a paper that will investigate the question further.

Whether the wording is or is not considered as evidence for the use of differential methods, it gives further evidence for the fabrication rather than the selection of data. Suppose that Ptolemy had selected the data from a body of genuine observations, in spite of the arguments of the preceding section. Then he would have started with values of δ_P and δ_H. The natural thing to do with these declinations would be to calculate the change in longitude by using either equation (1a) or (4), depending upon whether or not differential methods were used. From the change in longitude, Ptolemy would then get the precession rate p by dividing by the time interval. Ptolemy's words do not correspond to this general approach.

Instead, they give an accurate description of the general procedure that would be used in fabricating the data, but without giving details about the numerical method. Specifically, in fabricating the data, Ptolemy would start from the postulated change in longitude, namely $2\frac{2}{3}$ degrees. He would then calculate the corresponding change in declination by using one of the relations in Section 2. This is exactly the set of calculations that he performs in the *Syntaxis*, according to his own words.

In other words, Ptolemy fabricated the six declinations that he used and rounded them to values that could plausibly be the results of measurement. Then he did not take the trouble to invert the calculations and to find the precession from the fabricated and rounded data. Instead, he simply described the calculations by which he fabricated the data, but changed the words to disguise the purpose for which the calculations had been made.

In the passage that I quoted in Section 2, Pannekoek (1955) says that Ptolemy does not attempt to deceive his readers; he presents the unfavourable cases along with the favourable data. I cannot follow this argument. What Ptolemy does is to make it appear that the six cases he uses are typical; he does not even hint that there are any unfavourable cases.

As we have seen, Ptolemy could have selected his six cases only by calculating enough cases to find these six. These six cases are so far from the mean that they must be on the tail of a distribution if they ever existed at all. This means that Ptolemy calculated six cases which lead to his adopted value of 1° cy^{-1} and that he calculated N cases which lead to a value near the correct one of $1^\circ{\cdot}39$ cy^{-1}. We cannot say precisely what N is, but it must be at least an order of magnitude greater than 6.

There are only two possibilities, fabrication or selection. If he fabricated his data, Ptolemy was deliberately deceiving his readers. On the other hand, if he selected his data, he knew that his six cases were not typical, and that the overwhelming body of evidence pointed to a result quite different from the one he claimed to prove. This would have been obvious to him; he would not have needed any sophisticated statistical methods to see the point. Nonetheless, he tried to make it appear that his six cases were typical even though he knew they were not.

In other words, whether Ptolemy fabricated his data or selected them, he did deliberately try to deceive his readers, and he succeeded for more than 1800 years.

ACKNOWLEDGMENTS

I thank Dennis Rawlins for the valuable suggestions that I have already noted, and I thank B.B.Holland for a careful reading of the typescript. This work was supported by the Department of the Navy under Contract N00024-81-C-5301 with the Applied Physics Laboratory of the Johns Hopkins University.

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