THE GREEK AULOS

BY

KATHLEEN SCHLESINGER

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PLATE I

AULETE PLAYING ON THE DOUBLE AULOI

FROM 'THE THRONE OF VENUS' 5TH CENTURY B.C.

The position of the Aulete with head bent over the Auloi, thus relaxing the muscles of the glottis, implies a melos with a low *tessitura* such as that of the Hypophrygian Harmonia

Museo Ludovici, Rome

THE GREEK AULOS

A STUDY OF ITS MECHANISM AND OF ITS RELATION TO THE MODAL SYSTEM OF ANCIENT GREEK MUSIC

followed by

A SURVEY OF THE GREEK HARMONIAI IN SURVIVAL OR REBIRTH IN FOLK-MUSIC

by

KATHLEEN SCHLESINGER

‘ἡ δὲ ἁρμονία . . . φαίνεται τε τὰ μέρη αὐτῆς καὶ τὰ μεγέθη καὶ αἱ ὑπεροχαὶ κατ’ ἀριθμὸν καὶ ἰσομετρίαν.’

ARIST. FR. 43 BEKKER

With an Introduction by

J. F. MOUNTFORD

PROFESSOR OF LATIN IN THE UNIVERSITY OF LIVERPOOL

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TO

ELSIE HAMILTON

AND OUR LONG AND HAPPY FRIENDSHIP

and to

THE MEMORY

OF THE BEST OF BROTHERS

HARRY ADRIAN BURGESS

PREFACE

FOR the last three centuries eminent scholars of many nations have attempted to fathom the mysteries of Greek music without reaching any general agreement. There would, therefore, seem to be some justification for the introduction at this point [1] of a new musical fact which bears directly upon the very foundations of the Greek Musical System.

The aim of the present work is not to supersede what has gone before, but to open out a new avenue of approach to this difficult subject. This study does not, moreover, attempt a comprehensive survey of the rise and development of Music in Ancient Greece, but it offers a new conception of the nature of Modality based upon ascertained evidence. I have, therefore, confined myself in this study to placing before the reader the new data with their implications, and to giving an account of such experimental tests and their results as may prove useful to other investigators in the field.

The basic principle which brings the Modes to birth is found embodied in pipes and flutes, in which the fingerholes have been placed at equal distances, a proceeding which seems to be instinctive and universal. A new world and a new language of Music stand revealed, in which surprising adventures and experiences in theory and practice abound ; but, it must be confessed, the perusal of this volume may put the enthusiasm and patience of the reader to a severe test.

I had for a long time been greatly attracted by the significance and reactions of harmonic overtones, and I was carrying out certain experiments on my long psaltery [2] when I came upon the discovery of the basic principle of Modality, which is described in Chapter i. That principle in operation revealed the cause of

[1] The latest authoritative pronouncement on the subject by R. P. Winnington-Ingram (Camb. Univ. Press, 1936), *Mode in Ancient Greek Music*, concludes a lucid and reasoned survey of available evidence on a somewhat despondent note admitting the failure of the quest, but adding the hope that one day the illuminating hypothesis will be struck which is to fuse all unrelated parts into a coherent whole.

[2] An instrument strung with eight steel strings of the same length, but of different thickness and weight, and tuned in unison—conditions which favour a rich polyphonic harmonic development, strengthened by the phenomenon of resonance.

what has sometimes been termed in writings on Greek music, 'the movable Mese', i.e. the Mese placed upon a different degree of the octave scale in each Harmonia or Species, viz. on the seventh degree in the Mixolydian; on the sixth in the Lydian; on the fifth in the Phrygian; on the fourth in the Dorian; on the third in the Hypolydian; and on the second for the Hypophrygian; while in the Hypodorian, the Mese was both Alpha and Omega. This new orientation inevitably led to a protracted investigation, lasting many years, into the behaviour in theory and practice of the Aulos and its mouthpieces, and later of the primitive and medieval flutes having equidistant fingerholes. The results of this investigation constitute the basis of the present work.

At the inception of this revolutionary idea, which shed a new light upon the Modal System of Ancient Greece, my joy was tempered by a faint realization of the immensity of the task thus imposed upon one who was so poorly equipped for the purpose. Only my intense belief in the Greek genius gave me the courage to proceed. In my eventual search through the Greek and Graeco-Roman sources, for a confirmation of this revealing modal process, I had the good fortune to enlist the help of Mrs. Elizabeth Johnson, B.A., who translated *viva voce* from the Greek and Latin, as literally as possible, in order to afford scope for discussion of passages of vital interest but now considered in the light of the new musical fact. These translations have been of great assistance to me in preparing the background. For some years we worked together at regular intervals translating the whole of Ptolemy's Harmonics, Meibomius and numerous other treatises and quotations. Needless to say, no such description of the basic principle of Modality was found in the sources, although Aristotle (in Fragment 43 Bekker) does name, in a terse statement, the factors responsible for the Harmonia. While to the initiated his few words constitute illuminating evidence, scholars have passed them by as meaningless. The passage is fully discussed at the beginning of Chapter v, which treats of the evidence brought forward in support of my thesis.

It is not suggested that the significance and implications of the Aulos Harmoniai altogether rule out the system founded upon the ditonal scale, described by the Graeco-Roman theorists, but that a corner of the veil, which had for centuries obscured the Modal System of the Greeks, has been lifted. The fact is thereby revealed that these two systems, the Modal of the Harmonists, and the non-modal ditonal, were in use contemporaneously in

Greece, a fact which is emphasized by some twelve Polemics directed by Aristoxenus against the Harmonists (the custodians of the Harmonia), which I have collected and discussed in Chapters ii and v. These bitter, and sometimes violent, tirades contain precious evidence of all the salient points in the system of the Harmoniai; they bear witness, moreover, to the great importance and influence of the teachings promulgated in the Schools of the Harmonists during the lifetime of Aristoxenus. The separate trails of these two systems are kept in view throughout the book. The culmination is reached when Sebastian Virdung and Martin Agricola exhibit graphic and textual evidence of an attempt that was under weigh early in the sixteenth century for fusing the two systems—the Keyboard scale in use by the Church musicians, and the Harmonia of flutes and shawms by town bands, and among the Folk. The process of transformation is discussed and illustrated in Chapter vii.

While there is, of course, a plentiful supply of *a priori* evidence of the existence of the Aulos Harmoniai, the evidence for the actual use by the Ancient Greeks of these modal sequences, with the ratios I have assigned to them—on ascertained facts—is set forth in detail in Chapter v. The most striking, positive testimony of all is, I consider, first the brief dictum of Aristotle, already mentioned, on the Harmonia as based upon a number and equal measure (ἰσομετρία), and secondly the Harmonic Canon of Florence, a unique document which reveals the sequence resulting from an aliquot division by a modal determinant, applied segment by segment, and to each degree by name in the Perfect Immutable System. To these may be added the identification of some half-dozen of Ptolemy's shades of the Genera—when arranged in sequence—with my statement of the modal ratios of the Tonos.

But one of the greatest tributes to the Greek genius offered by the Art of Music is, in my opinion, the subtle and original use made of the basic principle underlying the Modal Harmonia, in the inception of the scheme of Greek musical notation. Here it is found that the ingenious idea worked out in this scheme consists in the use of two progressions—both duly recognized as sequentially inevitable—on the one hand of the letters of the Greek Alphabet, and on the other of the arithmetical progression of ratios. How the juxtaposition of these two sequences occurs in the notation of the Tonoi (established in the Tables of Alypius) in such a manner that for every letter symbol there is—expressed or implied—a ratio number, is explained in Appendix No. 1, in which only a brief interpretation of the system could be included, for the adequate treatment of the questions involved would require a book to itself. The use I have made of the modal ratios in the interpretation of the Fragments of Greek Music in Chapter ix will, I hope, be found useful: they direct our attention to the characteristic modal features of the Harmonia displayed in the music with great emphasis as closes; but whether we should be justified in regarding this usage as consciously imposed by canons of composition is doubtful. These few relics of Greek Music —regarded as prototypes—form an apt introduction to our quest for the Harmonia in survival or rebirth in the Folk music of many distant lands. It is startling to find a native musician in Sumatra, for instance, every whit as susceptible as the Greeks, to the characteristic features of the Hypophrygian Harmonia, thus affording evidence of the inherent power of these universal modes. The emergence of the Hypophrygian mode in Sumatra is only one of many such examples recorded in Chapter ix. The origin of the Ecclesiastical Modes is traced in Appendix No. 2 to a radical change of mode from Dorian to Phrygian in the Perfect Immutable System of the Tonoi.

The possibilities of the adoption of the new language of Music, derived from the Ancient Greek Harmoniai, for use in modern composition—which have been considered in Appendix No. 3—have for some years been exploited by one modern composer, Elsie Hamilton, and performances of her compositions given in London since 1917 (invariably received with enthusiasm by the audience) may be remembered by some of the readers. The general adoption of this new language of music not merely entails a mastery of the novel intonation of the dialects, as well as a resigned acceptance of technical difficulties concerned with musical instruments; but unfortunately initial attempts sooner or later come up against economic barriers which, however, a considerable increase in adherents would tend to minimize. The crucial test will ultimately resolve itself into the decisive query: does the new language of music provide increased facilities for differentiated expression of more subtle psychological reactions and feelings, which are unobtainable with the older language of music?

Finally, future investigators in this domain will be well-advised to seek first of all the reed-blown pipe with equidistant fingerholes, for its verdict is final: MODALITY at discretion. If when tested the pipe should emit a scale resembling our modern major (i.e. the Greek Hypolydian Harmonia) let the inquirer not be led to false conclusions, but first try the effect of other mouthpieces—

the more primitive the better—with a longer stem extruding from the resonator. There are two kinds of scale which cannot be ascribed to a reed-blown pipe, bored according to the principle of equal measure: (1) the ditonal scale, (2) any one scale regarded as standard.

I do not forget what I owe to Mabel Goschen (Mrs. Gerard Cobb); for it was as a result of the many happy months spent with her in Dresden in the 'nineties, that our common interest and enthusiasm became centred in the instruments of the orchestra; it was, in fact, at her suggestion that I began to gather materials which eventually led to my work in the archaeology of Music.

During the many years spent in the preparation of this volume, so many kind services, rendered by colleagues and fellow-workers in this field of research, gifts of general literature, musical instruments, photographs, &c., have been lavished upon me, that I am glad to be able to recall and associate with me in this publication the names of the many who have encouraged and stimulated me in my work. Foremost of all my grateful thanks are due to my close friend, Elsie Hamilton (of Adelaide, South Australia), for her unvarying readiness to help on this work in every possible way, so that I have been able to devote these many years to the investigations and experiments upon which *The Greek Aulos* is based, and to the research for confirmatory evidence in the literary sources. As a composer, Elsie Hamilton has, besides, shown herself ever ready (as may be read in Appendix No. 3) to co-operate in the endeavour to make the new (old) language of Music, founded upon the Harmonia, a practical reality in the music of our own day, and of the immediate future—a project which finds itself checked ever and anon by economic barriers.

To Professor J. F. Mountford, I am deeply indebted for one of the rarest gifts from one worker to another, namely a disinterested and constant interest displayed over many years, not only in the somewhat revolutionary thesis of this volume, but extended generously also to the smallest details. He has willingly discussed difficult issues, and offered shrewd and pertinent criticism, valuable advice and encouragement. To my lasting regret, however, I realize that the disclosure, during our long correspondence, of new but unpublished ideas and data, may have caused him to postpone his own separate contributions to the study of Greek Music.

Warm appreciation and heartfelt thanks are due to my devoted and efficient secretary, Miss Annie Copperwaite, who has been

of the greatest assistance to me in typing the chapters ; copying diagrams and tables ; in the boring of innumerable flutes and pipes and finally in preparing the work for the press, when she helped very materially in carrying out the exceptionally heavy task of revising chapters, written over a long period of years, in correcting proofs and in making the index.

I am deeply indebted also to the Institute of Archaeology, in the University of Liverpool (and primarily to Professor John Garstang and Sir Robert Mond), who have granted me the distinction of holding their fellowship in the Archaeology of Music continuously since 1915. The confidence they have shown during this long period in the value of the work I had undertaken has been an unfailing source of strength and inspiration to me.

I also offer sincere thanks to Sir Robert Mond for his appreciative interest and for various contributions in aid of my investigations, of books, and musical instruments, including two interesting sets of panpipes from Sicily, tuned to the modal intervals of the Harmonia, and for a case containing some 50 flutes, obtained for me from Egypt, most of them precisely bored to give one of the Harmoniai (see Table XI).

To Mr. A. H. Fox Strangways I owe much, and notably my initiation into the mysteries of *cents*, some twenty years ago. On important questions of origins of scales and musical systems, our views tend to diverge fundamentally. I have, indeed, spent many hours on end with him in stimulating and enjoyable discussion, but on certain issues there is invariably a clash of opinions, without any hope of convincing the opponent. Nevertheless, the fact that both are so keen adds zest, so that in spite of all we remain friends.

I gladly take this opportunity of recording my indebtedness to the late Miss Maisie F. Grant (daughter of Mrs. Grant of Liverpool), a most enthusiastic, gifted and highly efficient student of all this lore concerning the Harmoniai. During her travels in many parts of the East, she did valuable work : measuring flutes —notably in the Cairo Museum—testing and comparing data with the modal monochord ; discussing scales, &c., with native musicians and Arabian professors, and carefully noting results. From her visit to South Africa she brought back many musical instruments ; she induced friends in India to collect flutes for her. All of these valuable specimens and data she bequeathed to me when her early death in Egypt deprived me of a valued collaborator.

My hearty thanks are due to many besides for gifts of instruments : the late Mrs. Ludwig Mond and Sir Robert Mond ; to

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I am also grateful for all the kind assistance given to me in the Reading Room at the British Museum by the Superintendents, past and present, especially to Mr. G. Barwick, and to Mr. A. J. Ellis, M.A., and also to Mr. F. G. Rendell, F.S.A. (assistant

[1] *The Music of Growth*, by Collum (Partridge, London, 1933), and extracts from the *Shoo King* (tr. by W. H. Medhurst, Sen.).

superintendent); and to Mr. A. H. Smith of the Graeco-Roman department.

I am glad to acknowledge here also how stimulating has been the effect of the generous appreciation of my work, bestowed upon me by Sir Percy C. Buck and Sir W. H. Hadow and others, on various occasions.

Nor do I forget finally how much I owe to staunch and valued friends for so readily proffering assistance of every kind: to my friend William Busch with whom, as a creative musician, I have enjoyed many stimulating and fruitful discussions on aspects of music of vital interest to my subject. To my friends Miss Zabelle Boyajian (for information on Armenian folk songs); to Miss Netta Peacock, and Mrs. Elizabeth Johnson, B.A., for their willing help in checking references at the British Museum. To Muriel Campbell with special reference to Anthropology, and to Jessie Cameron in respect of Palaeography; also to Mrs. Ethel Gillespie (Australia); to Miss Dorothy Fraser; and to Mr. and Mrs. R. Chatwin.

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INTRODUCTION

WHAT was the nature of Greek music and how did it differ from our own? A full and complete answer to these questions would be of enormous interest both to the student of Greek literature and philosophy and to the musician who concerns himself with the development of his art.

The curiosity of the Greek scholar is aroused especially by a passage in the third book of Plato's *Republic* (398*c*–399*e*), where the philosopher discusses the music suitable for his Ideal State. The education of the citizens of this State was not to consist of the acquirement of accomplishments, it was not to be even a merely intellectual process, but primarily a moral one. With this ethical end in view, Plato was prepared to include music as one of the studies of a young person. By a suitable training in music of the best and most fitting kinds, the child would be guided to virtue and his soul would be led in the right path. In a perfect world good music was to be no less important than good and moral literature. Some types of music current in Greece were regarded by Plato as unsuitable for use in his Commonwealth. The scales (ἁρμονίαι), such as the Mixolydian and the Syntonolydian, in which lamentations were composed, and those which, like the Ionian and the Lydian, were adapted to effeminate or convivial songs were rejected; and two scales only, the Dorian and the Phrygian, were left which would represent the noble endurance of a brave man in battle or the sobriety and moderation of a citizen at peace. This view of music, which is analogous to Plato's application of moral criteria to the judgement of literature, is something more than a young man's dogmatism; for it is reiterated in the *Laws* (812*b*), a work written when Plato had a still wider experience of men and affairs behind him. To the end of his life he held that music is in itself the representation or reproduction in another medium of goodness or badness in the soul, and that by hearing good music a child is brought into contact with a good soul and so through music is assisted on the path towards virtue.

Nor does this doctrine arise from some personal idiosyncrasy of Plato; for in his *Politics* (viii, 4–8), Aristotle expresses a similar view. While admitting that it is legitimate to make use of music simply as a relaxation, he is quite clear that music has a tendency to form the moral character and influence the very soul.

It is in rhythms and melodies [he says] that we have the most realistic imitations of anger and mildness as well as of courage and temperance and all their opposites and of moral qualities in general. This we can see from actual experience; for when listening to such imitations we suffer a change within our soul. But to acquire the habit of feeling pleasure or pain upon the occurrence of resemblances is clearly allied to having the same feelings in the presence of the original [p. 1340*a*].

A little later (p. 1342*a*) he says: 'Clearly we must make use of all scales (ἁρμονίαι), but not all in the same way; for education we must use the most ethical (ἠθικωτάταις) scales.'

We ourselves appreciate the transient effects of music on our emotions and we understand that some kinds of music are exciting and others soothing. But our likes and dislikes are determined by considerations of melodiousness or cacophony, or by the skill of the composer's harmonies and counterpoint, or by the almost intellectual pleasure in formal developments. We do not praise or condemn a musical work because of its possible effects on our own souls or characters, or even on those of our neighbours. Whether Plato and Aristotle were right in attributing such power to music need not concern us here; the essential point is that they associated with their various scales (ἁρμονίαι) distinctive categories of feeling. Their testimony in this matter is confirmed by many passages in the lyric, tragic, and comic poets, and most strikingly by the anecdote of the composer Philoxenus, who found it impossible to write a dithyramb in a scale other than the Phrygian usually associated with that type of poem. This well-attested sensitivity of the Greeks is not to be explained by supposing that in aesthetic appreciation they differed fundamentally from ourselves. The reason must be that their music was capable, in some way which modern music is not, of expressing clearly the varying shades of feeling.

The musician is attracted to the study of Greek music not merely because any manifestation of the art of organized sound is of interest to him; but largely because of a widespread impression that the music of Western Europe is in some obscure way derived from that of ancient Greece. There is a tradition that St. Ambrose of Milan, in the second half of the fourth century, took four modes 'from the Greeks', and made them the basis of ecclesiastical music; and that later Pope Gregory the Great added four more. This tradition does not account for the fact that the Church system has eight modes, whereas the ancient Greek system from which they are supposed to be taken had only seven; or for the fact that the interval sequences of the Church modes do not correspond with what we know of the Greek modes of the same names. The tradition, indeed, is not based on any solid authority; so far as St. Ambrose is concerned, it seems that his innovation was the introduction, not of 'modes' hitherto unknown to Western Europe, but of antiphonal singing from the Greek Church of Antioch; and Pope Gregory's services to Church music consisted in the systematization of the corpus of antiphons already existing, and the setting up of a school at Rome for the training of Church singers. Nevertheless, it is true that the Church modes, Dorian, Phrygian, Lydian, &c., do bear names which, by some channels and for some reason, must have been derived from Greece; and the tracing of that connexion, however tenuous it might prove to be, is a fascinating inquiry for musicologists. Furthermore, such common musical terms as 'tone', 'tetrachord', 'melody', 'harmony', 'diatonic', 'chromatic', 'enharmonic', and 'diapason', all testify to some kind of continuity; whether that continuity is simply a vague one of modified theory or whether it is a more solid one of practice is a question which cannot be settled without an adequate knowledge of Greek music itself.

What, then, did Greek music sound like? The answer to the question could in no case be a simple one; for the total impression made by a piece of music depends upon a number of factors. To commence with a point which is not of fundamental importance, we should need to know something of the timbre of ancient instruments and something of the principles of voice production which were admired by singers and listeners. The importance of differences of tonal quality can easily be appreciated by comparing the effects of performing the same piece of music on a harpsichord and on a modern piano, or by listening to the same piece on a recorder and on a modern concert flute. Were the ancient lyres and citharas and other stringed instruments which are depicted in vase paintings of the timbre of the harp, or were they more like the banjo and guitar? Was the aulos (*αὐλός*), which modern translators generally render by the word 'flute', really like a modern flute or was it more like the clarinet or oboe? We are so accustomed to the methods of voice production favoured by well-trained and sophisticated singers and approved by critics that we accept them as entirely natural; but one has not even to go outside Europe to realize from the singers of genuine folk-song that a shrill, nasal, and forced vocalization, which would be anathema in the concert hall of a city, can be acclaimed as a satisfying and artistic achievement. Would the songs sung by a Greek tragic chorus remind us of the choir of St. Paul's or of the peasant in the uplands of Andalusia? Much of the charm of our music lies in its form, in the interplay of contrasting and complementary themes, and in the development of a melodic or harmonic idea. We cannot assume, however, that even the separate stanzas of the shorter songs of the Greek lyric poets were set to a simple recurring melody; still less can we conjecture the form of the music of an elaborate dithyramb or the principles of composition which guided the musicians who competed in those contests of aulos and cithara playing which were features of the Greek games. Indeed, of all the possible features of Greek music, the only one which we can feel certain of being able to apprehend from the range of our own experience is the Greeks' neglect of harmony; for it is certain that their choruses sang in unison or at the octave, and that the instrumental accompaniment was in unison with the voice except for a few passing notes.

The points which have just been mentioned, however, do not include the most important question which needs to be answered about Greek music. Even when we have reconstructed as accurately as we can a Greek cithara and found that it has a full round tone, and discovered that the aulos had a quality approximating to that of an oboe, we can gain no useful conception of Greek music until we have acquired some definite information about the Greek scales. We need to know on what principles they were constructed, what intervals composed them, what tonic or focal point

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they possessed, how they were used in relation to one another, and, if possible, how they developed historically. We need a clear understanding not only of the Harmoniai (ἁρμονίαι) of which Plato and Aristotle speak, but also of the other scales, Octave Species (εἴδη τοῦ διὰ πασῶν), Tonoi (τόνοι), &c., which are mentioned by Greek writers.

Our modern pianoforte scale with its twelve equal semitones to the octave is the worst possible approach to the understanding of Greek music; for this scale is a comparatively recent compromise designed to secure as complete a freedom as possible of modulation from one key to another. The nature of this compromise is easily understood if we start from the fact that the physical basis of sound is a series of pulsations in the air. The rate at which these vibrations succeed each other determines the pitch of a note, so that the more vibrations per second there are, the higher in pitch is the note produced. A musical interval, therefore, can properly be defined only by expressing the ratio between the vibration frequencies of the two bounding notes. Now the interval called an octave is produced by two notes whose frequency ratio is 2 : 1; and a Perfect Fifth by notes whose frequency ratio is 3 : 2. Two notes which are seven octaves apart have a frequency ratio of $(2:1)^7$, which represents a relation of 1 to 128; and two notes which are twelve Perfect Fifths apart have a frequency ratio of $(3:2)^{12}$, which represents a relation of 1 to 129·745. The pianoforte scale ignores this difference between 128 and 129·745, and while keeping the octaves in true intonation adjusts the Fifths by flattening so that twelve of them coincide with the seven octaves. In an analogous way the lesser concord of the Third is also adjusted, with the result that while there are twelve equal semitones within the pianoforte octave, not a single note is in just intonation. There is no doubt that this 'tempered' scale has in some ways widened the boundaries of the art of music, but it has done so at the expense of some blunting of our ears. Though we enjoy the brightness of a Perfect Fifth played on the open strings of a violin, our ears have grown accustomed to accepting a 'tempered' Fifth as if it were Perfect.

In dealing with Greek music, therefore, we must be prepared to put out of our heads all conceptions derived from our modern scale. Nor is an acquaintance with our major and minor 'modes' likely to help us to understand the variety of scales used by the Greeks; for apart from the fact that our major and minor now involve false intonations, the differences between them—the flattening of the third and sixth degrees in the minor—are not as striking or as significant as their similarity in having a common Tonic and Dominant. The difficulty a modern musician has in distinguishing between scales of different structure from our own is shown by our reactions to Byzantine, Hindu, Arabian, and Chinese music. They all sound 'foreign' to us, despite the difference between them, much in the same way that French and German sound simply 'foreign' to a monoglot Englishman. To understand Greek music we must be ready, if necessary, to learn another musical language and train our ears to its separate dialects. How then are we to discover the nature of the Greek scales? The evidence is of various types and it is necessary to say something about them if the present book is to be placed in its true relation to earlier work.

Naturally one would turn first of all to the Greeks themselves to see what they have to tell us about their own music. In the poets and prose writers there are many allusions to the art of music; but with few exceptions they are of a vague and general nature and afford no clue to the fundamental principles. Here and there, as in the dialogues of Plato, there are more technical references; but they are of such a kind that a wide acquaintance with the musical theory of the Greeks would be necessary to understand their implications. Whereas Plato used musical analogies to elucidate his philosophical argument, we should have to work back from his philosophy to understand his musical illustration. However, there is still extant in Greek a corpus of theoretical writings on music which amount to not less than six hundred moderate sized pages. The most voluminous of these writings are: the considerable fragments of the treatise on the elements of music (called ἁρμονικὰ στοιχεῖα) in three books from the pen of Aristoxenus (flourished about 320 B.C.), the pupil of Aristotle; the three books on music (περὶ μουσικῆς) by Aristides Quintilianus, who wrote probably during the first or second century of the Roman Empire; the three books (τὰ ἁρμονικά) by Claudius Ptolemy, the great Alexandrian mathematician of the second century after Christ; and finally the incompete commentary on Ptolemy written by Porphyrius in the third century. Chief amongst the works of lesser extent—but not always of inferior importance—are: a number of discussions of acoustic and artistic questions in the corpus of *Problems* attributed to Aristotle; a series of simple acoustical propositions scientifically demonstrated by Euclid (fl. 300 B.C.) in his *Division of the monochord* (κατατομὴ κανόνος); a short but very lucid treatise (εἰσαγωγὴ ἁρμονική) formerly attributed to Euclid but written probably by a certain Cleonides of uncertain date; a section on concords in the mathematical work of Theon of Smyrna (fl. A.D. 120); and three short elementary treatises by Gaudentius, Nicomachus, and Bacchius who wrote between the end of the first and the end of the fourth centuries of our era. Mention must also be made of a work on music (περὶ μουσικῆς) attributed to the young Plutarch, in which matters of theory are mingled with snippets of musical history; and of Athenaeus (fl. A.D. 230) in the fourth and fourteenth books of whose *Deipnosophistae* there are many anecdotes relating to music and musicians.

From such a bulk of material it might be thought that little would remain unknown of Greek music and that all problems would be capable of easy solution. But such is very far from being the case for several reasons. In the first place, these theoretical writings are of such varying date that they range over not less than six centuries; and though a comparatively late writer may often be relying for his information upon a much earlier authority, it is not always easy to ascertain the parts which have such authority behind them; and in any case there is very little, if anything, which can be traced back to the writings of Lasus of Hermione (fl. 525 B.C.), Hippasus of Metapontum (fl. 500), Glaucus of Rhegium (fl. 450), Philolaus (fl. 440), Heraclides Ponticus (fl. 400), or Archytas (fl. 390). How can we decide whether a piece of theory penned in the first century of the Roman Empire is applicable to the music of Pindar and Aeschylus? Between the sixth century B.C. and the beginning of our era the art of music must inevitably have undergone some changes, and the innovations of practice must to some extent be reflected in the presentations of theory. The earlier Greek writers, for example, down to and including Plato and Aristotle, speak of Harmoniai, but in the theory books surprisingly little is said of such scales; in their place we read of Octave Species (*εἴδη τοῦ διὰ πασῶν*), Systems (*συστήματα*), Tonoi (*τόνοι*). What change of practice does such a change of nomenclatare conceal?

In the second place, the theoretical writers frequently offer us a great deal of elementary information which we do not need, and seem to omit just those points about which we require most guidance. Naturally they were writing for their own times and their own purposes; some, like Aristoxenus, were concerned with stressing their own point of view and deriding their opponents; others, like Cleonides, frankly reduced their subject to its smallest dimensions; and all of them were handicapped by the inevitable inability of a theorist to do more than give the osteology of the art.

A third and most vital inconvenience arises from the fact that the theorists fall into two schools which are not reconcilable: the Pythagoreans and the Aristoxenians; and the information we derive from a given writer is largely coloured by the doctrines of the school to which he adheres. According to tradition, Pythagoras (fl. 500) introduced from Egypt into Europe the knowledge of mathematical acoustics; whether that be true or not, it is certain that his immediate successors had some knowledge of those mathematical ratios which are connected with the physical basis of sound. They perceived that a string which played a certain note was just twice the length of a string, of the same tension, which would play the octave above; or again, if you placed a movable bridge or stop one-third of the way along a string playing bottom *Doh*, the remaining two-thirds would play the note *Soh* a Perfect Fifth above. Now the Pythagorean school of philosophy attributed great power and influence to Number and believed that the whole of the Universe could be reduced to a set of numerical relations. In the art of music, though a hasty observer might say that it was based on nothing more than pure sensations with no objective validity, the Pythagoreans found a clearly demonstrated set of ratios such as they were seeking; more than anything else, music seemed to bring them into close contact with that Number which they looked upon as the Ultimate Reality. Although this discovery gave to the art of music a philosophic sanction, it did not lead to a satisfactory, comprehensive, or even trustworthy theory of music; for fundamentally the Pythagoreans were more interested in their mathematics than in music, and they tended to reduce the divinity of the Muses to a mathematical proposition. They judged musical intervals not according to the pleasure they gave the ear, but according to whether their ratios could be reduced to one or other of their favourite formulae, such as $n : 2n$, $n : 3n$, $n : n + 1$. In their investigations they were interested in obtaining the 'right answer', and consequently we cannot feel certain that in their discussions of musical intervals they have not selected just those which suited their immediate need; and it is beyond doubt that they neglected the other scales in favour of the Dorian. From their pseudo-musical speculations there grew the doctrine of the Harmony of the Spheres according to which the sun, moon, and planets, as they went along their orbits, were supposed to make sounds which could be identified with the notes of the Dorian scale; and such guesses at the foundations of the cosmos did more than the legitimate mathematical investigations to withdraw the study of music from its proper track.

Sharply distinguished from this metaphysical theory of music is the system which was first enunciated by Aristoxenus. For him pure mathematics and physics had no attraction. He postulated that in music the ear is the sole and final arbiter and that a mathematical formula has little or nothing to do with music. In this he was absolutely wrong, so far as theory goes; and so far as the art of music is concerned, he was only partially right. Ears differ in sensitivity and one naturally asks what kind of ear is to be the sole criterion; is it to be the ear of a highly critical musician or of the average listener? To rely only upon the ear for the data of a system of musical theory is to use a rough-and-ready method; and Aristoxenus himself was content to speak vaguely of a 'semitone' without any precise definition of what he meant by the term. As an adjunct to this doctrine was his contention that the progression of sound from low notes to high could be regarded as a continuous line, at any point of which the voice could rest, and any section of which could be divided into any number of equal parts. Aristoxenus found it easy to assert that a tone is the difference between a Perfect Fifth and a Perfect Fourth and that a semitone is exactly half that interval. But the Pythagoreans would have told him, and modern acoustics would confirm them, that the tone which is the difference between a Perfect Fifth (ratio $3 : 2$) and a Perfect Fourth (ratio $4 : 3$) is represented by the ratio $9 : 8$ ($= \frac{3}{2} \div \frac{4}{3}$); and that since the ratio $9 : 8$ cannot be equally divided unless we use surds ($3 : 2\sqrt{2}$), there can be no interval which when taken twice will produce such a $9 : 8$ tone. This linear conception of intervals, however, lies at the very root of the Aristoxenian theory and proves to be a quite impenetrable barrier to a proper knowledge of the nature of Greek scales. It would be grotesque to suggest that this theory can be entirely neglected or to deny that from it we can infer much that is worth knowing; but for the fundamental question about the size of the intervals of the Greek scales it is too unscientific to be of real service.

Nevertheless, since the Aristoxenian theory, especially as expounded in the later treatises, has a superficial lucidity which has misled some of the most painstaking students of Greek music, it will be well to indicate here its major features. At its basis is a two-octave scale called the Greater Perfect System (*σύστημα τέλειον μεῖζον*), of which a very rough idea might be obtained by playing the white notes of a pianoforte from *A* to *a′*. This System consists of fifteen notes, each with a distinctive name, separated from each other by a tone (T) or a semitone (S), and grouped into tetrachords :

Proslambanomenos	Hypate Hypaton	Parhypate Hypaton	Lichanos Hypaton	Hypate Meson	Parhypate Meson	Lichanos Meson	Mese	Paramese	Trite Diezeug.	Paranete Diezeug.	Nete Diezeug.	Trite Hyperbol.	Paranete Hyperbol.	Nete Hyperbol.
(*A*	*B*	*C*	*D*	*E*	*F*	*G*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*a′*)

| T | S | T | T | S | T | T | T | S | T | T | S | T | T |

Tetrachord Hypaton (*B*–*E*) Tetrachord Meson (*E*–*a*) Tetrachord Diezeugmenon (*b*–*e*) Tetrachord Hyperbolaion (*e*–*a′*)

Each of the seven successive octave scales which could be found in this long scale was called an Octave Species, and to each (in the treatise of Bacchius) a distinctive name is given :

Mixolydian :	Hypate Hypaton–Paramese (*B*–*b*)
Lydian :	Parhypate Hypaton–Trite Diezeugmenon (*C*–*c*)
Phrygian :	Lichanos Hypaton–Paranete Diezeugmenon (*D*–*d*)
Dorian :	Hypate Meson–Nete Diezeugmenon (*E*–*e*)
Hypolydian :	Parhypate Meson–Trite Hyperbolaion (*F*–*f*)
Hypophrygian :	Lichanos Meson–Paranete Hyperbolaion (*G*–*g*)
Hypodorian :	Mese–Nete Hyperbolaion (*a*–*a′*)

Besides the four tetrachords of the Greater Perfect System there was another, called the tetrachord Synemmenon, of which Mese was the lowest note. The sequence from Proslambanomenos through the tetrachords Hypaton, Meson, and Synemmenon to the Nete Synemmenon was called the Lesser Perfect System (*σύστημα τέλειον ἔλαττον*) ; and the five tetrachords Hypaton, Meson, Synemmenon, Diezeugmenon, and Hyperbolaion, with the Proslambanomenos at the bottom formed the Perfect Immutable System (= P.I.S. ; *σύστημα τέλειον ἀμετάβολον*). This P.I.S. was not of one fixed pitch, though the relative positions of the intervals composing it were unalterable. According to the pitch of the Proslambanomenos the P.I.S. was called the Hypodorian Tonos, Dorian Tonos, Hyperdorian Tonos, &c. In other words, these Tonoi, of which fifteen were recognized, were transposition scales in the Aristoxenian theory ; and the range from the lowest note of the Hypodorian to the highest note of the Hyperlydian was a tone in excess of three octaves.

Superimposed, as it were, on this theory of the Systems and Tonoi is the theory of the *genera* (*γένη*), according to which each of the five tetrachords of the P.I.S. might have a diatonic, a chromatic, or an enharmonic form. If the figure 1 represents a tone, the *genera* can be represented thus :

High (σύντονον) Diatonic:	$\frac{1}{2}$	1	1
Soft (μαλακόν) Diatonic:	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{4}$
Tonic (τονιαῖον) Chromatic:	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$
Hemiolic (ἡμιόλιον) Chromatic:	$\frac{3}{8}$	$\frac{3}{8}$	$1\frac{3}{4}$
Soft (μαλακόν) Chromatic:	$\frac{1}{3}$	$\frac{1}{3}$	$1\frac{5}{6}$
Enharmonic:	$\frac{1}{4}$	$\frac{1}{4}$	2

The Aristoxenian theory thus outlined has a certain neatness to commend it and is easily apprehended; but, apart from the fact that it is unscientific in its description of intervals, it is in many respects unsatisfactory and raises a host of problems in itself. This is not the place to subject it to a thorough analysis; but three points may be mentioned. Firstly, the doctrine of the *genera*—which must be an attempt to bring the subtle intonations of the practical art within the scope of the theory—would imply that there was not only a diatonic Lydian octave species consisting of T T S T T T S, but also an enharmonic Lydian octave of $\frac{1}{4}$ 2 $\frac{1}{4}$ $\frac{1}{4}$ 2 1 $\frac{1}{4}$. Yet it seems incredible that such a scale, beginning and ending with a quartertone interval, could have had any existence except in theory; we can conceive neither how it originated nor how it entered into practice. Secondly, there is not in any Aristoxenian writer a single hint of the tonality of the Greater Perfect System as a whole; nor is anything said about the tonality of the various Octave Species. Yet it is quite as important for the understanding of a musical scale to know what is its focal point as it is to know the size of its intervals; for the intervals are musically meaningless except when related to a tonic, which need not be the first note of the scale. And thirdly, there is the tantalizing problem of relating the Octave Species to the Harmoniai of the earlier writers. The names are attached to the Species only in Bacchius; and his list contains names, like Hypodorian, which are never applied to any Harmonia and yet omits names like Syntonolydian.

Since the theoretical writers fail to give us all the information we desire, we are compelled to turn to other possible sources of evidence. Of these we may mention first the Greek musical notations, of which one was used for instruments and the other for the voice. Both notations are based ultimately on alphabetic signs; and in the treatise of Alypius (of uncertain date) we have an almost complete list of the signs and verbal descriptions of them for all three *genera* in the fifteen Tonoi. As the art of music develops, the need for a notation arises and we may well suppose that some kind of notation was in common use at least as early as the time of Pindar. Presumably the notations we possess contain within them a kind of kernel or nucleus around which there have been later accretions; and if we could decide what are the original signs which formed that nucleus, we should have some evidence for the state of music at the time when the notation was first evolved. The most generally accepted interpretation of the signs is due to the investigations of Fortlage and Bellermann, published independently in 1847, according to which the lowest note of the lowest Tonos is equated with the *F* below the bass stave, and the highest note of the highest Tonos with the *G* above the treble stave. The intervening signs are equated for each diatonic Tonos with the notes of a minor scale and the intervals are assumed to be approximately those used in modern music. Many writers have felt doubts about the validity of this interpretation; for in many of its details it is not homogeneous. But though various attempts have been made to solve the problems, none of the alternative interpretations has been commonly accepted. This much, however, can safely be said: no theory of Greek music is likely to be near the truth unless it provides a satisfactory account of the notation as we possess it and explains how it arose from an earlier and less complicated scheme.

A third type of evidence lies in the few fragments of music in Greek notation which we still possess. The most extensive are the Delphic Hymns which were composed during the second century B.C. and contain nearly two hundred bars of tolerably consecutive melody. The hymns to Nemesis, the Sun, and the Muse, attributed to a certain Mesomedes, were composed during the second century of our era. The other pieces are incomplete and very brief; only one of them, the fragment of a chorus from the *Orestes* of Euripides, has any definite claim to be the kind of music Plato and Aristotle might have heard. These fragments are not only of widely differing dates of composition, but their interpretation is dependent upon that of the Greek notation in which they are preserved. Whatever they may seem to teach us about the principles of musical composition or even about the tonality of the Greek scales, they cannot of themselves give us any information about the Greek intervals.

Finally there is the evidence which ancient instruments may provide. Not even a well-preserved lyre or cithara could be of any value to us; for the testimony of such instruments is entirely dependent upon the tension of their strings. With wind instruments, however, the case is somewhat different; for provided that they are not too seriously damaged it is possible to make facsimiles of them; and by good fortune the *Historia Plantarum* of Theophrastus (fl. 300 B.C.) and the *Historia Naturalis* of Pliny the Elder (fl. A.D. 70) contain valuable information about the reed mouthpieces which were a vital part of instruments of the aulos type. But the instruments preserved from antiquity are few in number (see Howard, 'The *Αὐλός* or Tibia' in *Harvard Studies in Class. Philol.*, Vol. iv). In the circumstances it is perhaps not surprising that little attention has hitherto been devoted to the records of Greek music lying dormant in these auloi. Howard, indeed, did make facsimiles of them; but his results are unsatisfactory because he did not probe deeply enough into the acoustic problems involved or define with the necessary precision the intervals of the scales. But it is just this line of investigation, scientifically conducted and correlated with the records of instruments not of Greek provenance, that is the foundation of the present book.

Before passing to a brief survey of modern studies of Greek music, it is worth while to point out that some kind of theoretic knowledge of the subject has never entirely died out in Europe. The fact that so many of the Greek treatises on music were written during the Roman Empire shows that the Greek theory was felt to be a sufficient account of music long after the Greek states had lost their political independence and the cultural hegemony of the world had passed from Athens to Alexandria and then to Rome. The Romans themselves were not moved by music as were the Greeks, and in the great Roman writers there are only the most casual and uninformative references to the art; and much of the serious music-making even in the Empire was due to professional musicians of Greek origin. However, there is a succinct account of the Aristoxenian theory in the *De Architectura* of Vitruvius, the architect who lived in the Augustan age. In the first half of the fifth century, Martianus Capella gave an outline of musical theory in the ninth book of his *De Nuptiis Philologiae*, and a century later Boethius composed a separate work on music in which the theory differs from what we find in the Greek writers more through misunderstanding than through deliberate intent. In the *Institutiones* of Cassiodorus, which contains a compendium of knowledge useful for the monks of the new foundation at Vivarium, there is a section on music which is clearly based on Greek authorities of whom Gaudentius, Alypius, and Ptolemy are mentioned by name. The writers on ecclesiastical music (contained in *Scriptores Ecclesiastici de Musica*, edited by Gerbert, 1784) had little or no knowledge directly of the Greek theorists, but relied to a great extent upon Boethius and upon one another; but a great deal of their theory is an attempt to adapt a garbled version of Greek theory to the facts of the music of their own times and their pages bristle with 'Phrygians', 'Dorians', 'Hypermixolydians' and 'netes diezeugmenon'. It was only gradually that an adequate theory for the ecclesiastical modes, based on the doctrine of the four Finals, was evolved, somewhere between the eighth and eleventh centuries; and though by the eleventh century the doctrine of Authentic and Plagal modes had established itself and much of the Latinized Greek jargon had been jettisoned, nevertheless the influence of Greek theory persisted in the attribution of names like Dorian and Lydian to the Church Modes.

The scholars of the Renascence were too busily occupied with their studies of the literary masterpieces of Greece and in the search for new manuscripts of standard authors to devote their attention to so subsidiary a subject as Greek music; and except for the Latin translations of Euclid and Cleonides by Georgius Valla (1498) no attempt was made to study Greek musical theory. In the sixteenth century there appeared only the incompetent Latin translation of Aristoxenus by Gogavinus (1542) and the Greek text of Euclid with a Latin translation by Joannes Pena (1557). The next century, however, saw the publication of all the remaining treatises of importance. Jan van Meurs published the Greek text of Aristoxenus, Alypius (without the Greek signs), Nicomachus, and Bacchius between 1606 and 1623. In 1652 M. Meibom, a better scholar than van Meurs though prone to prefer his own ingenuity to the evidence of his manuscripts, re-edited Aristoxenus, Bacchius, Euclid, Nicomachus, and Alypius (with the musical signs), and published Gaudentius and Aristides Quintilianus for the first time. The first edition of the hymns of Mesomedes was published by Vincenzo Gallilei in 1581. To John Wallis, Professor of Geometry in the University of Oxford, we owe the first editions of Ptolemy (1682) and Porphyrius (1699). Van Meurs, Meibom, and Wallis equipped their editions with notes and commentaries which, though restricted for the most part to the interpretation of the particular author under discussion, did nevertheless gradually extend knowledge of the subject as a whole. In particular, Wallis appended to his edition of Ptolemy a chapter (*De Veterum Harmonica ad Hodiernam comparata*) in which he skilfully gathered together such information as could be derived from the theorists and completely outdistanced the account which Athanasius Kircher published in his *Musurgia Universalis* (1650) before Ptolemy and Porphyrius were available. In the eighteenth century, knowledge of Greek music filtered through to purely musical works, and writers like Hawkins and Burney included chapters on the subject in their general histories of music.

In the nineteenth century by far the most important name connected with the study is that of Rudolf Westphal, professor of Greek at Moscow, who not only edited Aristoxenus and Plutarch's *de Musica*, but in a series of lengthy volumes attempted to fuse all our information derived from theorists, antiquarians, and classical writers into a coherent whole. With his wide erudition he was able to elucidate many matters of detail; but at the basis of his work was a belief that the Aristoxenian system of theory provided the essential key which, with a little manipulation, would unlock all the secret places. He does, indeed, make use of Ptolemy and the Pythagorean writers; but they are subsidiary to the charms of Aristoxenus, the difficulties of whose system were not apprehended by Westphal, much less squarely faced. His exposition was given an even wider currency by the publication of the two volumes of F. A. Gevaert's *Histoire et Théorie de la Musique de l'Antiquité* (1875); and later writers who have undertaken the difficult task of presenting a comprehensive account of Greek music, such as H. Riemann in his *Handbuch der Musikgeschichte* and M. Emmanuel in Lavignac's *Encyclopédie de la Musique*, have been influenced, if not dominated, by the example and methods of Westphal and Gevaert. Nevertheless, as has been suggested above, the Aristoxenian system cannot, by its very nature, tell us anything like the whole truth about the Greek scales. Dissatisfaction with the solutions of Westphal and his followers has tended to grow since the publication of the *Orestes* fragment of music in 1892 and of the Delphic Hymns in 1893; for these pieces were not easily accommodated to the Aristoxenian scheme. The interpretation of the notation, which was forced into support of Aristoxenus, has been assailed, notably by A. Greif (in *Revue des Études Grecques*, 1909) and by C. Torr (*On the Interpretation of Greek Music*, 1896); and further studies of difficult passages of Aristides Quintilianus and Plutarch, which were brushed aside by Westphal or treated in an unconvincing fashion, have emphasized the distrust of Aristoxenus. The present position is fairly summed up by Mr. Winnington-Ingram in his recent book, *Mode in Ancient Greek Music* (1936):

Not even the main course of development of Greek music, far less the full details of its modalities, can be established on the evidence. It is a result to give rise to pessimism; and the prospects of further advance in our knowledge are not bright. . . . Yet complete despondency is as unnecessary as it is ignoble. Every student of the subject must from time to time have the feeling that there is a certain amount of evidence, particularly concerning the earlier stages of Greek music, that is still unrelated together, and must hope that one day he will strike upon the true, the illuminating hypothesis which is to relate it.

So we come to the present book. Miss Schlesinger needs no commendation to any one who is aware of her established reputation as a historian of musical instruments or who has read her important chapter 'The Significance of Musical Instruments in the Evolution of Music' in the Introductory Volume of the *Oxford History of Music* (1929). The basis of her work is an investigation extending over nearly a quarter of a century into the capabilities of wind instruments, both of the type of the primitive flute and of the primitive pipe, to which latter category the aulos belongs. It is an especially important feature of the author's work that it has not been confined to paper calculations, but is firmly founded on a practical knowledge of the instruments concerned. A considerable number of pipes and flutes from various parts of the world have been collected and played upon; their scales have been carefully noted down, their intervals accurately measured; and the results have been repeatedly checked and compared with one another. Alongside of these actual specimens, facsimiles have been made of such of the remains of Greek auloi as are available for accurate measurement and of a number of ancient pipes found in Egypt; they have all been fitted with mouthpieces whose individual properties have also been subjected to rigorous tests. The striking result of these investigations is that a simple principle seems to have operated in the Greek aulos as it does in many primitive pipes: their fingerholes are placed equidistantly and the effective length of pipe and mouthpiece combined is a multiple of the distance between the holes. The practical and theoretical consequences of this principle of equal measurement are worked out in great detail in the following pages, and the reader is introduced to a widespread, natural, and almost inevitable musical language whose existence has hitherto been unsuspected by theorists. The author, however, has done more than experiment with actual pipes and facsimiles of pipes. She brings the results of her practical acoustics into relation with the written testimony of the Greeks; and in an appendix she demonstrates how the Greek musical notation was originally designed for the Harmoniai and expanded to meet the needs of later times. The line of investigation here pursued is in principle simple; in its working out it is full of surprises. Though this book upsets many of our previous notions of the intervals of the Greek scales, and especially of the way they were related to one another, there can be no doubt that all further study of Greek music will be indebted to Miss Schlesinger's pioneer work. It is, I venture to think, the most original and illuminating contribution yet made to a difficult and fascinating subject.

J. F. MOUNTFORD

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FORMULA NO. 1

(*a*) open pipes, (*b*) closed pipes

To find the v.f. from the length.

$$(a)\quad \frac{\textbf{340 m./s.}}{\textbf{2(L} + \Delta \textbf{ all.)}} = x, \textbf{ v.f. of sd-w.}$$

$$(b)\quad \frac{\textbf{340 m./s.}}{\textbf{4L} + \textbf{2}\Delta} = x, \textbf{ v.f. of sd-w.}$$

FORMULA NO. 2

(*a*) open pipes, (*b*) closed pipes

To find the length of sound-wave from the v.f.

$$(a)\quad \frac{\textbf{340 m./s.}}{\textbf{v.f.} \times \textbf{2}} = x = \textbf{L of } \tfrac{1}{2} \textbf{ sd-w.}$$

and L. of $\frac{1}{2}$ sd-w. – (Δ all.) = L. of flute.

$$(b)\quad \frac{\textbf{340 m./s.}}{\textbf{v.f.} \times \textbf{4}} = x = \textbf{L. of } \tfrac{1}{4} \textbf{ sd-w. or effective length of closed pipe, i.e.}$$

inclusive of $\frac{\Delta}{2}$, ∴

$$\textbf{L } \tfrac{1}{4} \textbf{ sd-w.} - \frac{\Delta}{2} = \textbf{L. of flute or pipe.}$$

FORMULA NO. 3

To find the position of Hole 1 or vent in a flute (useful also for the determination of the v.f. of cross-fingered notes).

$$\textbf{Length from emb.} - \left[\frac{\Delta}{2} + (\Delta - \textbf{D}) + (\Delta - \delta) + \textbf{i (I.D.) or more}\right]$$

$$= \textbf{C. of Hole 1 from emb.}$$

FORMULA NO. 4

To find the Standard Allowance at Hole 1 (which should agree with the allowance obtained by use of Formula No. 2, by subtracting the 'actual length' from the 'effective' or 'sound-wave length').

$$\textbf{2}[\Delta + (\Delta - \textbf{d}) + (\Delta - \delta) + \textbf{de}] = \textbf{Standard All.}$$

FORMULA NO. 5

To find the diameter allowance at exit.

$$[\Delta + (\Delta - \textbf{d}) + \textbf{de.}] = \Delta \textbf{ all. at exit; and Eff. } \Delta \textbf{ all.} = \textbf{2}(\Delta \textbf{ all. at exit.})$$

FORMULAE FOR AULOS MP.

FORMULA NO. 6

To find the v.f. of D-R. mp.'s proper note.

$$\frac{\textbf{340 m./s.}}{\textbf{4(V.L.} + \Delta)} = 4x = \text{v.f. of } \tfrac{1}{4} \text{ w.l.}$$

$$\text{and } \frac{x}{4} = \text{v.f. of } \tfrac{1}{1} \text{ w.l. of D-R. mp.}$$

(See Chap. iii, p. 125, Table V, etc.)
Tested and found correct.

FORMULA NO. 7

To find the *effective* length of D-R. mp. from the v.f.

$$\frac{\text{340 m./s.}}{\text{v.f.} \times 4} = x \text{ sd-w.l.}$$

$$\text{and } \frac{x}{4} = (\text{V.L.} + \Delta) \text{ of D-R. mp.}$$

FORMULA NO. 8

B-R. mp. proper note

To find the v.f. of B-R. mp.

$$\frac{\text{340 m./s.}}{8[\text{T.L.} + \Delta + (\Delta - \text{T.W.})]} = 8x = \text{v.f. of } \tfrac{1}{8} \text{ sd-w.}$$

$$\text{and } x = \text{v.f. of } \tfrac{1}{1} \text{ sd-w. of a B-R. mp.'s note.}$$

FORMULA NO. 9

To find the length of sound-wave from the v.f. of the B-R.'s proper note.

$$\frac{\text{340 m./s.}}{\text{v.f.} \times 8} = x \text{ length of whole sd-w.}$$

$$\text{and } \frac{x}{8} = \text{aggregate length of B-R. mp. } [(\text{T.L.} + \Delta + (\Delta - \text{T.W.})]$$

THE VIBRATION FREQUENCIES OF THE NOTES OF THE DORIAN HARMONIA OF MODAL DETERMINANT 22

CALCULATED FROM C = 128 AND 256.

(These are the notes of my tuned piano)

v.p.s.

C 11, 22	= 128	or 256	or thus	$\frac{C\ 11}{128}$	or $\frac{C\ 22}{256}$
$C\sharp$ 21	= 134·1	,, 268·2	,,	$\frac{C\ 21}{128}$	
D 20, 10	= 140·8	,, 281·6	,,	$\frac{D\ 20}{128}$	
$E\flat$ 19	= 148	,, 296	,,	$\frac{E\flat\ 19}{128}$	
E 18, 9	= 156·4	,, 312·8	,,	$\frac{E\ 18}{128}$	
F 17	= 165·6	,, 331·2	,,	$\frac{F\ 17}{128}$	
F 16	= 176	,, 352	,,	$\frac{F\ 16}{128}$	
G 15	= 187·7	,, 375·4	,,	$\frac{G\ 15}{128}$	
G 14	= 201·1	,, 402·2	,,	$\frac{G\ 14}{128}$	
A 13, 26	= 216·6	,, 433·2	,,	$\frac{A\ 13}{128}$	
B 12	= 117·3	,, 234·6	,,	$\frac{B\ 12}{128}$	

N.B.—The numerators are the ratio numbers (see note p. xlvii on the subject), the denominators express the octave, based on *C*, in which the ratios or v.f.s are taken.

Table of Reciprocals: Lengths and Vibration Frequencies

Effective Lengths converted by Formula i into Vibration Frequencies; or conversely, by Formula ii, Vibration Frequencies converted into Lengths—Lengths and Frequencies are therefore Reciprocals. (See Note p. xlvii.)

L	v.f.	L	v.f.	L	v.f.	L	v.f.
·100	= 212·5	·126	= 168·6	·151	= 140·6 = *D* 20	·176 (= **F 16**)	= 120·7
·101	= 210·3	·127	= 167·3	·152	= 139·8	·177	= 120
·102	= 208·3	·128	= 166 **F 17**	·153	= 138·8	·178	= 119·3
·103	= 206·3	·129	= 164·7	·154	= 137·9	·179	= 118·7
·104	= 204·3	·130	= 163·4	·155	= 137	·180	= 118
·105	= 202·3	·131	= 162·2	·156 (= **E 18**)	= 136·2	·181	= 117·4 = *B* 12
·106	= 200·4 **G 14**	·132	= 160·9	·157	= 135·3	·182	= 116·7
·107	= 198·5	·133	= 159·7	·158	= 134·4	·183	= 116
·108	= 196·7	·134	= 158·5	·159	= 133·6	·184	= 115·4
·109	= 194·9	·135	= 157·4	·160	= 132·8	·185	= 114·6
·110	= 193·1	·136	= 156·2 **E 18**	·161	= 131·9	·186	= 114·2
·111	= 191·4	·137	= 155	·162	= 130·7	·187	= 113·6
·112	= 189·6	·138	= 153·9	·163	= 130·3	·188 (= **G 15**)	= 113
·113	= 188	·139	= 152·8	·164	= 129·5	·189	= 112·4
·114	= 186·3 **G 15**	·140 (= *D* 20)	= 151·7	·165	= 128·7	·190	= 111·8
·115	= 184·7	·141	= 150·4	·166 (= **F 17**)	= 128 = *C* 11	·191	= 111·2
·116	= 183	·142	= 149·6	·167	= 127·2	·192	= 110·6
·117 (= *B* 12)	= 181·6	·143	= 148·6	·168	= 126·4	·193	= 110·1
·118	= 180	·144	= 147·5	·169	= 125·6	·194	= 109·5
·119	= 178·5	·145	= 146·5	·170	= 125	·195	= 108·8
·120	= 177·5	·146	= 145·5	·171	= 124·2	·196	= 108·4
·121	= 175·5 **F 16**	·147	= 144·5	·172	= 123·5	·197	= 107·8
·122	= 174·1	·148 (= *E* 19)	= 143·5	·173	= 122·8	·198	= 107·3
·123	= 172·7	·149	= 142·5	·174	= 122	·199	= 106·7
·124	= 171·3	·150	= 141·6	·175	= 121·4	·200	= 106·2
·125	= 170					·201 (= **G 14**)	= 105·6

N.B.—The octave in which the v.f. is given here is from 4 ft. g. to 8 ft. g.; it cannot be fixed without specifying the formula used, e.g. for open or closed pipes or for reed mouthpieces. (For an explanation of the nomenclature I have adopted for notes, e.g. $\frac{F\ 16}{256}$ or $\frac{F\ 16}{128}$, see statement on p. xlvii.)

CONCERNING THE MODAL RATIO NUMBER

THE use by the Author of the term Modal ratio number (or modal ratio) needs some explanation here, since it assumes knowledge by the reader of the theory and practical processes of the modal principle in operation (see Chap. i). The term is applied in a twofold sense of ' ratio ', and as the *number* of a note in the modal sequence. The division of a given length by the Modal Determinant results in a number of equal segments each bearing a number indicating its position and value in the ensuing sequence, which is expressed in full by a proper fraction, having the M.D. as constant denominator, and the order number of the segment as numerator. Once the M.D. or constant denominator has been indicated, the numerators used alone form ratios with the fundamental, as differentiated unit, e.g. 28/28, or with each other: each with the number on its left 27/28 as fractions of the length; with those on the right 27/26 as abstract ratios. The number used alone as a ' modal ratio ', therefore, has a specific meaning; and refers to the segment which bears that number in the aliquot division in question. In the case of a length of string on a monochord, the number enables a student to identify and produce instantly the corresponding sound in the modal sequence. Thus the term modal ratio, or ratio number, as herein applied, implies the possibility of forming inversely proportional ratios (to right or left), and provides each segment besides with a numerical nomenclature in its own modal sequence, e.g. 28, 27, 26 mentioned as modal ratios in a Tonos or Harmonia, not only signify two intervals $28/27 \times 27/26$ but also three notes in the modal sequence indicated, viz. 28, 27, 26. When the Harmonia is known, the exact theoretical vibration frequencies of these modal ratios used as notes may be easily computed.

CONCERNING TABLE I, p. xlvi

The reciprocity refers to the operation of the formulae 1 and 2 in which the optional use of length or v.f. as divisor results likewise in the consequent interchange of these two factors as quotients (see, for fuller explanation, pp. 225 and 317–18).

CONCERNING THE NOMENCLATURE $\frac{F\ 16}{256}$ *OR* 128 *USED ON p. xlvi*

The author has adopted as nomenclature the synthetic formula used throughout this work, of which *F* 16 is an example, as a terse statement of the following facts: the note, here *F* 16, has the name and ratio number it bears in the division by M.D. 22 on *C*—256 or 128 v.p.s., which produces the Genesis of the Dorian Harmonia on *C*. On consultation with the composer, Elsie Hamilton, this arrangement was considered the best for all purposes in theory and practice, and therefore the modal piano has been tuned in accordance with it.

The vibration frequency placed under the note $\frac{F\ 16}{256}$ thus indicates the octave in which the note occurs.

LIST OF ABBREVIATIONS OF TECHNICAL TERMS

Act. I.D.	= actual increment of distance on flute or pipe between the fingerholes
Act. L.	= actual length, visible length of flute or pipe in contradistinction to eff. L.
Adj. All.	= adjusted allowance, in regard to allowances for diameter in flutes
All.	= allowance in respect of diameter in flutes
B-R. mp.	= beating or single-reed mp.
C.H.	= centre of hole or fingerhole
Cent.	= cent of equal tempered semitone
Cum.	= cumulative
Δ	= diameter of the bore in flutes
Δ all.	= diameter allowance
Δ all. ex.	= diameter allowance at exit
Δ all. vt.	= diameter allowance at vent
D. or **d.**	= diameter of embouchure in flute
δ	= diameter of fingerhole in flute or pipe
D-R. mp.	= double-reed mouthpiece
de.	= depth of walls in flute
Eff. All.	= effective allowance = diameter allowance in respect of sound-wave, i.e., taken twice
Eff. I.D.	= effective increment of distance computed from sound-wave length, divided by the modal determinant
Eff. L.	= effective length = in relation to the sound-wave = length in relation to sound-wave inclusive of diameter allowance
Emb.	= embouchure
ex.	= exit
Exc.	= Excess
Ext. or **mp. Ext.**	= extrusion of mouthpiece reed from the resonator; an important factor in Modality on Aulos
F.	= fipple of flute embouchure
Fh.	= fingerhole of pipe or flute
Fl. All.	= floating allowance. (See Chap. vi, Allowance No. 9)
F.M.	= Folk Music
F.T.	= Folk Tune
G.C.S.	= Greater Complete System
gl.	= glottis
gl. N.	= glottis note (see Chap. iii). Note obtained on Aulos by releasing or tightening muscles of glottis
H.	= hole or fingerhole
Harm.	= Harmonia
Ho.	= Hypo.
H.S.	= Harmonic Series
Ict.	= Ictus; strongly forced or accented note on Aulos
I.D.	= increment of distance = length from centre to centre of fingerholes, in relation to multiple length, of flute or pipe
Inc.	= increment

Inc. All. No. 7	= incremental allowance No. 7 (see Chap. vi), calculated on allowance actually embodied on flute between exit and vent, divided by the modal determinant, e.g. on Sensa flute $\frac{\cdot 039}{18} = \cdot 00216$. The allowance is cumulative per fingerhole and remains latent until the length overtakes a nodal point
Kn.	= Knot in reed or bamboo
K. S.	= the author
L.	= length
M.D.	= modal determinant; the number in the aliquot division
mean.	= an average not exact, but the nearest useful, mainly used for I.D. or v.f.
m.m.	= millimetres
Mp.	= mouthpiece of Aulos or flute
Mp. Ext.	= mouthpiece extrusion or distance on stem or stalk of reed mp. between vibrator and resonator
M./s.	= metre per second
Norm.	= proper note of reed mp. produced with normal use of breath; its v.f. is calculable by formulae 6 to 9
N. or **N.P.**	= Node or nodal point with regard to Inc. All. No. 7.
Pent.	= Pentatonic
PH. or **Parh.**	= Parhypate
P.I.S.	= Perfect Immutable System of the Tonos
Pos.	= Position
Pr.	= proportional
Pr. All.	= proportional allowance computed by dividing the allowance by the ratio at a fingerhole
Pr. I.D.	= proportional increment of distance calculated on the actual length of flute or pipe divided by the modal determinant
Prosl.	= Proslambanomenos
R.	= resonator
S.	= second
S-R. mp.	= single or beating-reed mp.
Sd-w.	= sound-wave ($\frac{1}{2}$ or $\frac{1}{4}$ or 1)
Sd-w.l.	= sound-wave length
St. All.	= Standard Allowance No. 4, by Formula No. 4 (see Chap. vi), refers always to Hole 1 as vent. It is divided at each fingerhole by the ratio
Syn.	= Synemmenon
T.	= treated, applied to wheat and oat straws for D-R. mps. (See also contrary, U)
T.	= tongue of beating-reed
T.L.	= tongue length of beating-reed mp. measured from tip to base or hinge
T.W.	= tongue width of beating-reed mp., analogous to diameter of fingerhole; the wider the tongue the higher the note
U.	= untreated, applied to wheat or oat straws for D-R. mps.
vel.	= velocity, as of sound in air = 340 metres per second
v. or **vt:**	= vent: i.e. Hole 1 used as vent, left always uncovered as starting-point of modal sequence
v.f. }	= vibration frequency, or index of pitch
v.p.s. }	= vibrations per second
V.L.	= vibrating length of D-R. mp. determined by impact (not pressure) of lips on straw. (See Formulae Nos. 6 and 7)

ERRATA

p. 29, line 13 : for ‘ Hypophrygian ’ read ‘ Hyperphrygian.’
p. 86, fn. 1 end : for ‘ Table ix ’, read Table xi.
pp. 124 and 125, Tables iv and v, column 6 : for ‘ Formula viii ’ read Formula vi.
p. 126, Table vi, column ii : for ‘ No. of Fingernotes ’ read Fingerholes.
p. 171, Greek quotation, § 229, after W. & R. > : for comma read apostrophe before ἐν (’ἐν).
p. 210, fn. 1, § 2, line 2, ‘  ’ : the symbol of notation should be hemi-alpha, left limb.
p. 242 (4) : for ‘ Formula No. 4 ’, read No. 5.
p. 264 sqq., and passim : for ‘ Sarangdev ’, read Sārangdev.
p. 278, Fig. 55 : Each name of fret should be moved down one space, e.g. Sābbābā = 18/20 ; Wosta = 17/20, &c.
p. 281, line 6, under Fig. 57 : Binşir equation ($\frac{27}{21} \times \frac{4}{5} =$), for ‘ $\frac{64}{35}$ ’, read $\frac{36}{35}$ = *49 cents.*
p. 330, Pipe No. 4 : for $\frac{\text{‘ 340 m/s ’}}{\cdot 933}$, read $\frac{\text{340 m/s}}{\cdot 983}$.
p. 343, line 8 : for ‘ *gendér* ’, read *gendèr.*
p. 349, line 3 : for ‘ below ’, read Fig. 76.
p. 361, line 10 btm. : insert commas after 15/13 and (Rho and Mu)
p. 366 (lower half), bars 6–24 : for ‘Φ’ 27 (phi) read Θ 27 (theta).
p. 369, bar 164 : for ‘ Ϻ χι(ονος) ’, read Ϻ χι(όνος).
*p. 370, line 5 above * * * :* after Wosta, insert (Fig. 60).
p. 391, line 18 : for ‘ Idelsoln ’, read Idelsohn.
p. 438 (lower half), under THE MOUTHPIECES D-R. Mp. ‘ R.R.’ : for ‘ (Ʊ) ’ read (U).
p. 465, Hole 2 (addition) : add decimal point before 1012.
p. 520, line 14, after curtailed modes : add (Fig. 34).
p. 521, line 20 btm., after Lydian Tonoi : add (Fig. 101).
p. 521, fn. 1 end : add (p. xlvii).
p. 522, line 22 : for ‘ see Chap. ix, p. 25 ’, read p. 367.
p. 526, Fig. 103, left-hand division, top line of ratios :

for (Nete 24 Γ), (Diezeug,)' read Nete 22. Γ

p. 526, Fig. 103 : for ‘ Paranete ’, read Paramese.
28 M → 28 M

p. 528, in diagram || : for ‘ PM. K 28 ’, read PM. M 28.

p. 531, line 6 of contents : for ‘ Majra ’, read Mājra.
p. 537, middle : for ‘ Matna ’, read Matnā.

CHAPTER I

THE ORIGIN AND GENESIS OF THE HARMONIA ON THE AULOS

DEMONSTRATION OF THE MODAL PRINCIPLE IN OPERATION

INTRODUCTORY

Introductory. Outline of the Theory of the Harmoniai. Equal Measure in Acoustic Theory. Modal Determinants (= M.D.). The Mixolydian Harmonia. The Genesis of the Modal Material of the Mixolydian Mode. The Mixolydian Harmonia in the Diatonic Octave. The Lydian Harmonia. The Phrygian Harmonia. The Dorian Harmonia. The Hypolydian Harmonia. The Hypophrygian Harmonia. The Hypodorian Harmonia. The Bastard Hypodorian. The Aulos as Origin of the Harmoniai. The Modal System based upon the Operation of the Principle of Equal Measure. The Ethos of the Mode based upon the characteristic Features peculiar to each Harmonia. Professor H. S. Macran: the 'overlooked factor'

THIS chapter is intended to provide an exposition of the modal system which was the basis of ancient Greek music. Proof that the system here described is not an arbitrary figment, but represents the facts, will be afforded by the records of Auloi contained in Chapters ii and iii and by the documentary evidence examined in Chapters iv and v.

First, we must inquire what is a Mode. Modern musicians are generally agreed to define a Mode as an arrangement, sanctioned by convention, of tones and semitones within an octave of a 'standard' scale which itself is largely a convention. Such Modes may be primarily distinguished by the position of the semitones relative to the other intervals; or they may be designated, in reference to the white notes of the keyboard, by the octave containing the sequence of tones and semitones characteristic of each Mode, e.g. the *d* octave, the *c* octave. These descriptions are tolerably satisfactory for the Modes as represented by the Graeco-Roman theorists and for the Ecclesiastical Modes bearing the names of the ancient Greek Harmoniai with which they are supposed to have some obscure connexion. But these so-called Modes have no separate existence apart from the standard scale to which they are referred and upon whose origin they throw no light; they have no individual genesis and they display no characteristic intervals. It is not in this vague sense that the term 'Mode' is used in this work; and when we speak of the modal system of Greece we have in mind something far more subtle than an arrangement of tones and semitones whose precise values are generally taken for granted.

Although no detailed description of the ancient Greek Modes or Harmoniai (ἁρμονίαι) is known to exist, nevertheless some of their features are indicated in literary sources to which, in partial anticipation of later chapters, we must turn for a moment. From Aristotle [1] we learn that the Harmonia (ἡ ἁρμονία) is in some way connected with the operation of 'number and equal measure', and this brief description, as we shall see, is the key to the genesis of the Harmoniai. Furthermore, it is evident from several passages of other authors, as for instance some of the Problems of Ps-Aristotle,[2] that the note called Mese (μέση), in the character of an 'arche' (ἀρχή) or 'source', exercised some kind of causative or formative function in the generation of the Harmonia; that, in fact, the Mese conditioned the values or intonation of all the other notes in the scale. Ps-Euclid [3] states definitely that 'by means of the Mese, the values of the rest of the notes become known'. The Ancients,[4] moreover, according to Ar. Quin., used intervals other than tones and semitones in their Harmoniai; amongst such intervals are cited the Spondeiasmos and Eklysis of three dieses each, and the Ekbole of five dieses. Aristoxenus [5] also makes mention of the fact that the intervals separating the Tonoi used by the players of the Aulos are not always tones or semitones; and when discussing the divisions of the 5th, he allows that it may consist of four intervals of different magnitude (μεγέθη). These few passages provide a sufficient indication that the ancient musical system of the Greeks was not a haphazard series of notes or intervals, that it was founded on some definite rational principle, and that it was not limited to the intervals of modern music.

OUTLINE OF THE THEORY OF THE HARMONIAI

The theory of the ἁρμονίαι which is the keystone of the present work may now be described. The Harmonia among the Greeks was a sequence consisting of proportional, interrelated intervals within the octave; and the sequence itself was based upon natural law. Seven original Harmoniai may be found differentiated within the same octave, each having an individual and independent genesis from a common fundamental, believed to be *F*. Since these seven Harmoniai are found differentiated within the octave *F* to *F*, each having the characteristic order of intervals resulting from its own independent genesis, it follows that there must be for the second step, seven *G*'s according to modern nomenclature, all differing in intonation and bearing to the common Tonic *F* a different ratio. There will likewise be seven *A*'s, *B*'s, *C*'s, *D*'s, *E*'s fulfilling similar conditions. As a natural consequence of this, there will be seven different sequences starting from the same Tonic *F*, no two of which will begin with the same interval. The interval from the first to the second step is, therefore, indicative of the Harmonia. There is here no standard scale from which all

[1] Aristot., *ap.* Plut., *de Musica*, 1139*c* (φαίνεται τε τὰ μέρη αὐτῆς καὶ τὰ μεγέθη καὶ αἱ ὑπεροχαὶ κατ' ἀριθμὸν καὶ ἰσομετρίαν). Weil and Reinach in their edition of Plut., *de Mus.* (§ 229), emend ἰσομετρίαν to γεωμετρίαν and add <ἡρμόσθαι>. Cf. Bekker's ed. of Aristot., Vol. v, p. 1482, frag. 43.

[2] *Probl.*, xix, 20, 25, 36, 44. See later, Chap. iv, pp. 13–22.

[3] Ps-Eucl., *Intro.*, p. 18M. (= Cleonides, p. 202, von Jan).

[4] *de Mus.*, p. 21M., lines 3–6, and p. 28, middle.

[5] *Harm.*, pp. 37–8M.

the seven Harmoniai may be derived: each has an independent existence as the result of its own modal genesis. It is evident that this sevenfold differentiation, theoretically within the limits of the same octave, of each degree of that octave, is unobtainable with tones and semitones only. The implication of this proposition is, moreover, that if the second degree of one of the Modes be at the interval of a semitone from the Tonic, then in all the other Modes the magnitude of the interval separating these same degrees must be either greater or less than a semitone. Again, if the interval between the second and third degrees be a tone (9 : 8) in any one of the Modes, it cannot also be the same kind of tone in any of the other Modes: the magnitude of the interval between these degrees will again, therefore, be greater or less than a 9 : 8 tone in all the other six Modes.

If the use in pre-Aristoxenian days of such a Modal System can be established, it is difficult to see how the system could have been demonstrated upon any but a mathematical basis. How could seven different *G*'s, *A*'s, *B*'s, &c., be theoretically and exactly defined otherwise? It is easy to talk of dividing a Tone into four quarter-tones, or the semitone between *E* and *F* into the requisite seven dieses, but how would the limits of each interval be defined and fixed but by mathematical ratios? And by what canon would they be recognized by students? In practice, certainly the ear would suffice; but in a treatise the use of the ear as canon is impracticable. The primitive piper did not have to judge, he simply opened the holes of his instrument and the notes sounded; these at first, he accepted without demur or reasoning. If the Aulos imposes its scale upon the piper through the boring of the holes then we may say that the piper acquires the Mode through his ear as receptive agent and criterion. But when we theorists endeavour to discover what has taken place, we find that there is an exact mathematical basis underlying what the Aulos had to bestow: this basis is the natural law to which reference has been made. The teachers in the schools of the Harmonists, in the days of Aristoxenus, had obviously penetrated the mystery enveloping the boring of pipes (still obscure to a large extent at the present day), according to the testimony of Aristoxenus himself, in his polemic [1] on the use of ratios and rates of vibration in contradistinction to his own assertion of reliance on the musician's ear. Although the theoretical exposition of the basis of the Greek Modal System cannot be given without touching the fringe of the science of acoustics, it will be seen further on how the Modes came to birth quite naturally on the Aulos at the hands of the unsophisticated musician, ignorant of scales, Modes and theories, merely through the inevitable operation of natural law.

EQUAL MEASURE IN ACOUSTIC THEORY

The passage of Aristotle mentioned above has instructed us to look to the implications of equal measure (ἰσομετρία) as our guide in understanding the Harmonia. What, then, is the significance of equal measure; and how does it bring the Modes to birth? Equal measure in the domain of

[1] *Harm.*, p. 32M.

acoustics takes action as a formative principle in two opposite directions: (1) in the Harmonic Series, (2) in a series the reversal of (1), which I shall call the Modal Series.

(1) Equal measure implies the members of the Harmonic Series, infinite in number, which are recognized in their aggregate formative aspect as the physical basis of sound. If the fundamental note be *F* of 88 vibrations per second (v.p.s.), the second member of the series, *F* one octave higher, will have 88×2 ($= 176$) v.p.s.; the third member *C* a fifth higher 264 v.p.s., and so on. In practice, these constituents of sound, known as Harmonics or overtones, may, by concentrated attention, and under favourable conditions of resonance, be distinctly heard when the note of a string or pipe is sounded. Or again, these Harmonics may be induced to materialize from a string by lightly touching the Nodes at points one-half, one-third, &c., along the string and then plucking or bowing the string with decision. On a flute, harmonic overtones up to the 7th or 8th may be obtained by the practical device known as overblowing, i.e. doubling, trebling, quadrupling, &c., the force of breath pressure required to produce the fundamental note, either from the whole pipe with all holes closed or through any uncovered hole. All this is generally known, or may be ascertained from text-books. It has an important bearing, however, on the genesis of the Modal Series.

(2) The second aspect of this natural principle of equal measure applied to the length of string or pipe, implies a downward progression. The proportional steps familiar to us in the Harmonic Series are now generated segment by segment, through the division of the string by equal measure. Such an application of equal measure to the length of a string produces a series, identical with the Harmonic Series as to proportions, but reversed as to direction. As this second aspect of the principle of equal measure —the modal—may be more easily followed and checked on a string than on a pipe, it will suit our purpose better to use the former to demonstrate the operation of the law, and revert to the pipe later. Let *n* be a string of any given length, then

$$\frac{n}{8,\quad 9,\quad 10,\quad 11,\quad 12,\quad 13,\quad 14.}$$

represents the string divided seven separate times by equal measure, into 8, 9, 10, 11, &c., aliquot parts. This may be effected by measuring off these seven series of segments on the surface of the sound-box of the monochord in the manner shown in Fig. 1. The actual values in v.p.s. of such segments of a given string will shortly be given.

MODAL DETERMINANTS (= M.D.)

Such divisions of the string demonstrate the function of *Number* which, as we have seen, Aristotle associates with equal measure. The numbers 8–14, which were given by way of illustration, are, I believe, the individual Determinants of the seven original Modes; and by stopping the successive points of a string by means of a movable bridge at places determined by

FIG. 1.—Diagram illustrating the Sevenfold Monochord Division by Modal Determinants of the Seven Harmoniai

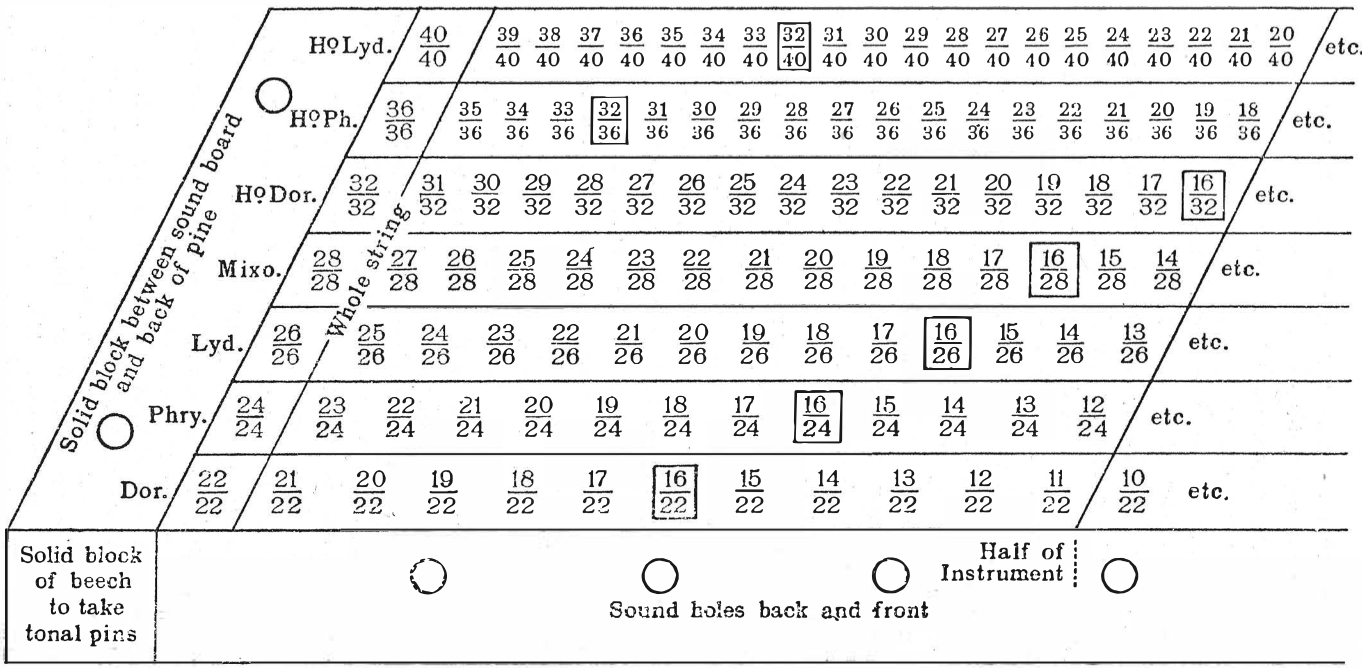
The segments are equal in the aliquot division by each Modal Determinant.

The lines dividing the numbered segments belonging to each Harmonia have been omitted in the diagram for the sake of greater clarity. On the monochord itself, these lines mark the path of the moveable bridge under the string.

N.B.—No. 16 marks the keynote of each Harmonia. There may be one (monochord) or more strings (canon) as desired. My monochord measures 1,000 mm. (1 metre) from the knife edge of the fixed bridges *ἀποψάλματα*.

The blocked portion—holding tuning pins at each end and on which rest the two fixed bridges—measures another 55 mm. Total length of box = 1,100 mm.

A conveniently shaped movable bridge is required



one or other of these numbers the germ of a Mode is produced in the form of all the available notes constituting its modal material. Such a Modal Determinant derives a special significance from the fact that, by virtue of its association with the operation of equal measure, it also denotes the member of the Harmonic Series bearing the same modal number. Most important of all is the rank borne by the Determinant number as member of the Harmonic Series; for that Harmonic is identical with the fundamental note given by the first segment or aliquot part of the string when plucked. That note, moreover, is the fundamental or Arche, of the modal series, and conditions the whole of the modal material. It is by imparting the characteristic essence which Arche possesses as member of the Harmonic Series, to the modal scale or Harmonia, that the Ethos or aesthetic quality of the Mode is made manifest. Thus it is Arche that wields the inherent formative power that was felt to be operating through the Modal System of the Greeks, for which reason we may call Arche the Causative or Creative Tone. The sequel will show whether the claim is justified or not.

The table opposite, showing the Archai of the seven original Modes in their characteristic positions in the Harmonic Series, which correspond, as already explained, with their own Modal Determinants, demonstrates (according to the claim made above) their identity with the seven Harmoniai mentioned by the theorists. It will be noticed in addition that these Archai taken on the *F* string are of the same pitch as the Mesai of the Tonoi of Alypius of the same name,[1] according to the generally accepted modern pitch valuation.

The first step in the genesis of a mode on a string is, therefore, the choice of a number—the Determinant[2]—which shall for the purposes of demonstration be 14. This fixes the identity of the Mode about to be generated as Mixolydian. There is sure to be a query about the apparently arbitrary selection of number 14 as Modal Determinant for this Harmonia. The justification for the selection of this and other numbers as Modal Determinants is the statement of several of the Theorists regarding the position of Mese (or of the Tonus Primus) on a specific degree of the Harmonia in the Perfect Immutable System (between Hypate Meson and Nete Diezeugmenon). Ptolemy, for instance (ii, 10), leaves no possible doubt

[1] It will doubtless be noticed that $n/15$ does not appear in the scheme. No Harmonia having Determinant 15 can be traced. The probable explanation of the absence of 15 as Modal Determinant is that when the Auloi were first grouped and codified, the beginning was made from 8, as in the scheme above in which the 7th Harmonia with Determinant 14 closes the significant cycle of seven Harmoniai on the minor 7th, which in the Harmonic Series appears an octave ahead of the major 7th or leading note. The fact that these seven Harmoniai are referred in the theoretical sources to planetary influences suggests that there may have been some esoteric reason for the exclusion of 15 as Determinant and for the limiting of the Harmoniai to seven.

[2] Towards the end of the chapter, where the part played by the reed-pipe in the creation of the Modes is briefly considered, it will be seen that the selection of the Determinant is a subconscious affair with the primitive pipemaker, whereas in using a string the selection must be made with intent.

on the subject: the Mixolydian Mese, he says, is on Paranete Diezeugmenon kata dynamin [1] (i.e. on the 7th degree—K. S.). Now 14 is the only number which, when used as Modal Determinant according to the coming description of procedure, will place Mese on the 7th degree above the Tonic (see Fig. 2 below).

A similar claim may be made for the number assigned as Modal Determinant for each of the other six original Harmoniai.

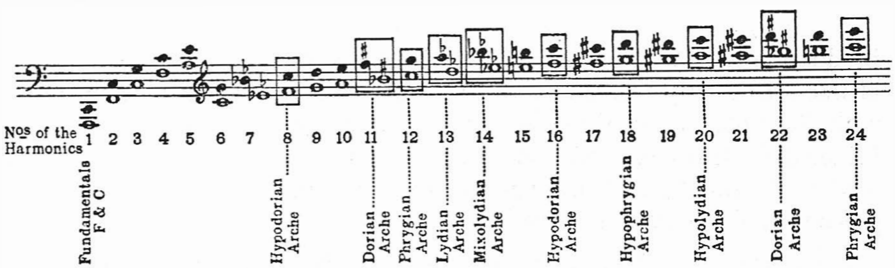
THE MIXOLYDIAN HARMONIA

The Mixolydian Harmonia has been selected for demonstration for several reasons, and primarily because 14 is the lowest Determinant that yields a complete Diatonic scale. If then we take 14 as the number which determines the aliquot division of the string (or pipe) a glance at the table will show that the 14th Harmonic on the *C* string is the notorious $\overset{\flat}{B}\flat$—

FIG. 2.—The Archai of the Seven Harmoniai in Position in the Harmonic Series on *F* and *C*

The Harmonic Series on *F* and *C*

Harmonics generated from the fundamentals *F* and *C*



N.B.—If each number denoting a Harmonia be used to divide a string into equal segments marked on the rule of a monochord, and the finger-tip be lightly placed on the string at 1/11, while the string is plucked or bowed, the *Dorian Arche* will be heard in a clear, ringing note; at the 1/12—with the same procedure the *Phrygian Arche* will speak; at 1/13, the *Lydian Arche*, and so on with the others.

very flat compared with the *B*♭ of the keyboard—known as the Harmonic 7th.[2] On the *F* string, that 7th is of course $\overset{\flat}{E}\flat$, the Mese of the Mixolydian Tonos of Alypius, but flatter than the *E* of the modern interpretation. The Arche of the Mixolydian Harmonia, an $\overset{\flat}{E}\flat$, then, is the Harmonic 7th. Let us realize what this implies. It means, as a practical fact, that

[1] Ps-Eucl., *Intro.*, p. 15M. (= Cleonides, p. 196, von Jan); Bacchius, pp. 18–19.

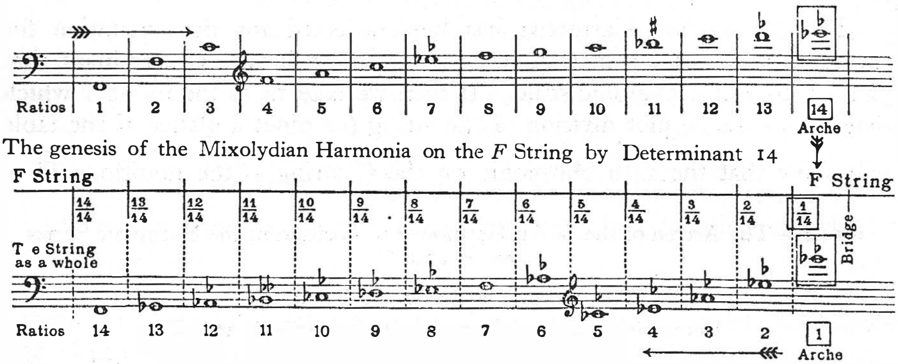
[2] This natural 7th has been condemned by the canons of modern music to have its flatness corrected in order that it may adapt itself to the intonation of our arbitrarily constructed scale, when it occurs, for instance, on the French Horn, the Trumpet or any other instrument whose compass of notes consists in some segment of the Harmonic Series.

F was probably the fundamental of the ancient Greek System; our modern fundamental *C*, more familiar for computations among musicians, has been used here in a few cases.

Arche, in generating on an *F* string the modal material of the Mixolydian Mode by means of Determinant 14, will impart its flatness as Harmonic 7th to every note of that modal material with the exception, of course, of the string-note or its octaves. This flatness will not be so perceptible in the descending genesis, in which all the intervals are commensurate, and

FIG. 3.—Genesis of the Mixolydian Modal Material resulting from the Aliquot Division by Determinant 14 on the *F* String

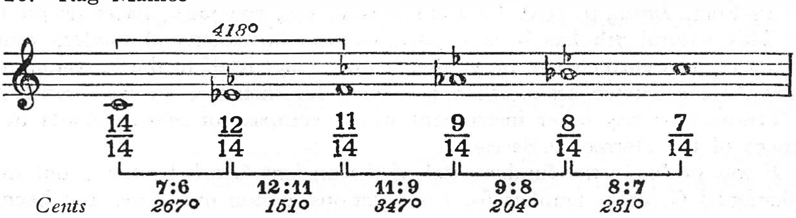
The finding of Arche in the Harmonic Series on *F*



in direct proportion to Arche, but the flatness is felt as a striking and characteristic Ethos in the ascending scale of the Harmonia which begins with the note of the whole string as Tonic. In this rising scale of the Mixolydian Harmonia all the notes give the impression of flatness in relation to the *F*, the starting-note of the scale, thus upsetting all our preconceived notions of Tonality.[1]

[1] An example of this modality may be found in *The Music of Hindostan*, by A. H. Fox Strangways (Clar. Press, Oxford, 1914), pp. 30–1. Ex. 29, a tune played upon the bamboo flute; see also p. 151, Hindostani Rāgs, No. 26, Mālkos, and Appendix ii (where, however, *E*♭ is given instead of $\overset{\flat}{E}$♭ as on p. 151). The string note in these examples is *C* not *F*, but the Mode is clearly the same as the ancient Greek Mixolydian, and has the same Determinant 14 with all its implications. His Ex. 29 is fortunately a Phonogram, and is therefore one of the exceptions to which the author refers when, on p. 17, he states that: 'Though many of these were in queer scales, no attempt has been made beyond an occasional superscript ♯ ♭ or ♮ (= an alteration of less than a semitone), where the effect was characteristic, to represent niceties of intonation.' The scale of Ex. 29 on pp. 30–1 is undoubtedly that of Rāg Mālkos given in the table, p. 151, No. 26, where Mālkos is thus stated.

(The ratios, and *cents* equivalent of the intervals, added by K. S.)
No. 26. Rāg Mālkos



THE GENESIS OF THE MODAL MATERIAL OF THE MIXOLYDIAN MODE

The genesis of the modal material of the Mixolydian Mode may now be demonstrated in relation to a string tuned to $F = 176$ v.p.s.

If n represents the vibration frequency of the string (176 v.p.s.), then, $n \times 14, 13, 12, 11, 10, 9, 8$, will give the frequencies of 1/14, 1/13, 1/12, 1/11, &c., of the string; and so 176 v.p.s. $\times$ 14 (= 2,464 v.p.s. = $\overset{\flat}{E}\flat$) will be the sound of 1/14 of the string, which is identical with the 14th Harmonic of the F string.

The FIRST segment (= 1/14) gives $\overset{\flat}{E}\flat$ (which is the Arche = 2,464 v.p.s.), three octaves and a Harmonic 7th above the fundamental (F = 176 v.p.s.) of the whole string.

TWO segments (= 2/14), by virtue of the ratio 2 : 1 to the first segment, give $\overset{\flat}{E}\flat$ an octave below Arche (= 1,232 v.p.s.).

THREE segments (= 3/14), by virtue of the ratio 3 : 1 to the first segment (or the ratio 3 : 2 to two segments), give $\overset{\flat}{A}\flat$ (= 821·3 v.p.s.), a 12th below Arche.

FOUR segments (= 4/14), by virtue of the ratio 4 : 1 to the 1st segment, give $\overset{\flat}{E}\flat$ (= 616 v.p.s.), two octaves below Arche.

FIVE segments (= 5/14), by virtue of the superparticular ratio 5 : 4 to four segments, give $\overset{\flat}{C}\flat$ (= 492·8 v.p.s.), the major 3rd below the 2nd octave of Arche.

SIX segments (= 6/14), by virtue of the superparticular ratio 6 : 4 to

With regard to Ex. 29 on pp. 30–1, the melody was played by a *Gond* on a bamboo flute, bāṉsrī, blown from the end (i.e. held almost vertically) without reed mouth-piece, and having 6 fingerholes covered by the left finger-tips and right knuckles. The author goes on to say : 'Assuming the C to be in tune the higher notes were all a little flat, the F most so, and the lower one a little sharp ; the C itself sharpened a good deal under the overblowing of the crescendo.'

From the scale given above it will be seen that this '*very flat F*' as 4th on the Tonic—which here is C—must obviously be very flat, owing to the ratio 14/11 = 418 *cents* (instead of 498) which it bears to the Tonic. In addition, the $E\flat$, $A\flat$, and $B\flat$ given in the Mālkos scale, are likewise flattened, as explained above, in accordance with the nature of the Mese or Arche as flattened Harmonic 7th in the Harmonic Series. The characteristic intonation of the scale has been accurately rendered by the flute, and has also been equally correctly interpreted by Mr. A. H. Fox Strangways.

The following facsimile reed-blown pipes from Ancient Egypt are also in the Mixolydian Mode of Ancient Greece (a Heptatonic scale, whereas Rāg Mālkos is pentatonic in modern India), viz.

Loret xxiii, xxiv, xxvi, xxvii, xxviii, xiii, xxv.

[See Records, Chapter x.]

The theoretical value of the notes may be computed by means of the following formula : when n = 176 v.p.s.

then $$\frac{n \times 14}{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14} =$$

the frequencies of the 14 segments of the string obtained by the aliquot division by Determinant 14.

four segments (= 3 : 2), give $\overset{\flat}{A}\flat$ (= 410·6 v.p.s.), the perfect 5th below the 2nd octave of Arche.

SEVEN segments (= 7/14, half the string), by virtue of the ratio 7 : 4 to four segments, give *F* (= 352 v.p.s.), the Harmonic 7th below the 2nd octave of Arche (and also the octave of the note of the whole string).

EIGHT segments (= 8/14), by virtue of ratio 8 : 4 (= 2 : 1), give $\overset{\flat}{E}\flat$ (= 308 v.p.s.), the 3rd octave below Arche.

NINE segments (= 9/14), by virtue of ratio 9 : 8, give $\overset{\flat}{D}\flat$ (= 273·7 v.p.s.), major tone below the 3rd octave of Arche.

TEN segments (= 10/14), by virtue of ratio 10 : 8 (= 5 : 4), give $\overset{\flat}{C}\flat$ (= 246·4 v.p.s.), a major 3rd below the 3rd octave of Arche.

ELEVEN segments (= 11/14), by virtue of ratio 11 : 8, give $\overset{\flat\flat}{B}\flat$ (= 224 v.p.s.), the Harmonic 4th, doubly flattened therefore, below the 3rd octave of Arche.

TWELVE segments (= 12/14), by virtue of ratio 12 : 8 (= 3 : 2), give $\overset{\flat}{A}\flat$ (= 205·3 v.p.s.), the perfect 5th below the 3rd octave of Arche.

THIRTEEN segments (= 13/14), by virtue of ratio 13 : 8, give *G*♭ (= 189·5 v.p.s.), the Harmonic 6th.

FOURTEEN segments (= 14/14 = the whole string), by virtue of ratio 14 : 8 (= 7 : 4) give ***F*** (= 176 v.p.s.), the Harmonic 7th below the 3rd octave of Arche.

This completes the genesis of the modal material of the Mixolydian Harmonia. The last number, 14, is the reflection of the number of Arche in the Harmonic Series. All progress is here effectually stopped at 14/14, since no lower note can be obtained than that given by the whole string, the original generator of the Harmonic Series for the whole Modal System.

The constitution of the Harmonia or modal octave calls for further explanation. The procedure just described, which effects the genesis of the Mode, has been explained upon a string [1] because it cannot be achieved

[1] A slight digression may be allowed here to show how near Ptolemy had come to the realization of the Modal Genesis ; though he makes no mention of the Arche or of Modal Determinant, he describes the process much as has been done above thus (ii, 13, p. 163, Wallis (1682) ; p. 69, Düring) : ' Having a canonion, applied to the strings, we divide the given length reckoned from the Apopsalma, which is at the higher end, right down to that mark which will be under the lowest of the notes, into segments that are equal and commensurate in size and we place numbers on them, beginning from the higher end, according to the number of the little segments there are, in order to have, from the common limit mentioned above, a series of numbers constituted in accordance with the ratios which are proper to each of the notes, and in order always to bring the Apopsalmata of the movable bridges easily to those positions which have been indicated by the Canonion,' &c. Thus Ptolemy describes half the process of the genesis, but he merely uses the division—which on his canon is into 120—as one would the inches or centimetres of a yard or metre measure with the results given in his Tables of the Genera. This description, which misses all the essentials of modality, is an obvious indication that Ptolemy was not versed in the lore of the Harmonists or in the theory underlying Modal Genesis.

in practice upon the Aulos; for the hole representing the position of Arche would have to be placed upon the little straw mouthpiece itself, and the 2nd and 3rd segments might likewise fall either upon it, or so close to it that no sound could be elicited. It is the ascending Harmonia alone that comes to birth upon the Aulos and in what manner will be seen before we proceed much further.

Mese is one of the lower octaves of Arche and whatever number indicates the Modal Determinant, Mese is indicated by 8 or a multiple. The Arche exercises a function similar to that of the fundamental in the Harmonic Series, in so far as it governs the ratios proceeding from itself, and consequently governs the exact pitch or intonation of each note of the series. Between Arche and the Tonic or note of the whole string or pipe there is a vital relationship which is inevitably reflected in the relationship of Mese to Tonic; and to these relationships the individual Ethos of the Harmoniai is in a large measure due. The practical result of these relationships are the characteristic modal intervals of each Harmonia. These intervals are expressed by the following ratios [1] consisting of 8 (or a multiple [2]) representing the Mese, and of the number of the Modal Determinant.

Harmonia	*Character of Interval*	*Ratio*	*Value*
Hypodorian . . .	Unison or octave	16/8	= *F* to *F*
Mixolydian . . .	Flat minor 7th (Harmonic 7th)	14/8	= *F* to *E*
Lydian	Flat major 6th	13/8	= *F* to *D*
Phrygian. . . .	Perfect 5th	24/16	= *F* to *C*
Dorian	Harmonic 4th	11/8	= *F* to *B*
Hypolydian . . .	Major 3rd	20/16	= *F* to *A*
Hypophrygian . .	Major tone	36/32	= *F* to *G*

In instrumental practice, Mese was the note from which all others were tuned. In this connexion, a much-quoted passage from the Problems of Ps-Aristotle will probably be recalled here; it receives full attention in Chapter iv.

The fact that the descending progression from the Arche has not yet

[1] The interval characteristic of each Harmonia is found duly indicated by implication in the Graeco-Roman sources. The ratio of the intervals, however, to which the distinctive modal essence or Ethos is due, are not expressly mentioned; they are implied, however, in the degrees of the modal Perfect Immutable System. See Ps-Eucl., *Intro.*, pp. 15–16M. (= Cleonides, pp. 196–8 von Jan); Arist. Quint., pp. 17–18M.; Bacchius, pp. 18–19M.; Ptol., ii, 11, pp. 136–8W.

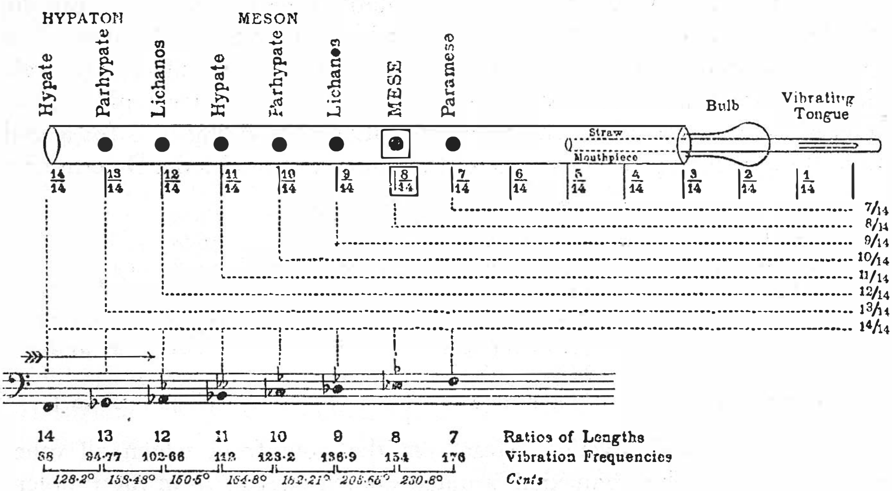
[2] Why may Mese be 8, 16 or 32? The use, more especially in the Perfect Immutable System, of one of these ratio numbers is not conditioned by pitch, but primarily by the Modal Determinant which falls to the Tonic. Modal Determinants from 11 to 14 have 8 as Mese; from 16 to 28, the Mese is 16 and from 32 to 64 = 32. Moreover, we must remember that the doubling of a Modal Determinant, or of the ratio number of Mese, is frequently only an indication that a modal segment has been halved in order to obtain an intermediate note in the Pyknon, as for instance in the Synemmenon Chromatic and Enharmonic, e.g.

Synem. Pyknon			Diez. Pyknon		
$\left.\begin{matrix}32\\16\end{matrix}\right\}$	$\left.\begin{matrix}30\\15\end{matrix}\right\}$	29	$\left.\begin{matrix}28\\14\end{matrix}\right\}$	27	$\left.\begin{matrix}26\\13\end{matrix}\right\}$

See Note on Ratio numbers in Abbreviations, &c.

been generally recognized by the ear as inherent in the phenomenon of sound, as is that of the ascending Harmonic Series, is of no importance to our subject. The ascending Harmonic Series cannot, on the other hand, produce Modes, but only non-modal species.[1] The Modes are the sole and original creation of the descending or reversed series, due to equal measure by a specified Determinant, dividing string or pipe into aliquot parts. The fundamental string note, common to all the Modes, takes the place of the modern Tonic, but with the added significance that it possesses real dynamic force as generator, in determining the value and character of Arche.

FIG. 4.—The Mixolydian Harmonia in the Diatonic Genus resulting from the Aliquot Division by Determinant 14



Note.—The characteristic Ethos : all the notes flattened in relation to the Tonic ; consequent on the characteristic interval of the Mode between Mese and Tonic, the flat Harmonic 7th of ratio 7/14, as Arche.

THE MIXOLYDIAN HARMONIA IN THE DIATONIC OCTAVE

From the genesis of the modal material, of which we have now obtained some idea, the Harmonia, or octave scale, of the Mode is derived ; it begins on the note of the whole string or 14/14 for the Mixolydian Mode, and extends to the octave above at 7/14, the half of the string ; its Mese will occur on the 7th degree of the scale, on the segment bearing the number of one of the octaves of Arche, viz. 8 in the division by Determinant 14. It is the invariable rule (in the system outlined here) in every Harmonia, that Mese is recognized by the ear as an octave of Arche, or on the monochord by ratios 8, 16 or 32 corresponding to those octaves. The Harmonia,

[1] The Harmonic Series forms an extended scale of infinite length within which numbers of octave sections may be selected, as for instance, from *D*, 9th Harmonic to the 18th ; from *F*, 11th Harmonic to the 22nd and so on ; these octave scales differ from one another in the same way as the octaves on the white notes of the keyboard. There is no question here of a Modal Genesis as in the Harmoniai, therefore, I call them non-modal species.

primarily considered as the Aulos scale, is thus a series of consecutive sounds beginning on the note of the whole pipe with holes closed, or where that note is unused, on the note given by the 1st hole (farthest from the mouthpiece) treated as a vent and always left uncovered. On the string the proportional sequence begins with the note of the whole string as Tonic. The Harmonia can be built up on a monochord from the Tonic by stopping, by means of a movable bridge, segment by segment, and plucking the *remaining* length of the string. The analogue in the Aulos is effected by uncovering each hole in turn, while blowing through the mouthpiece, when the part of the pipe below the centre of the open hole is cut off, and the remainder of the pipe from the centre of the open hole to the top of the mouthpiece alone speaks.

The 1st degree = the 14/14 of the string, the Tonic or Hypate = F.

The 2nd degree = 13/14 of the string, or the remainder when 1 segment has been stopped = $G\flat$ or on the Aulos the 1st fingerhole opened.[1]

The 3rd degree = 12/14 of the string, or the remainder when 2 segments have been stopped = $\overset{\flat}{A}\flat$ or on the Aulos, the 2nd fingerhole opened.

The 4th degree = 11/14 of the string, or the remainder when 3 segments have been stopped = $\overset{\flat\flat}{B}\flat$ or on the Aulos, the 3rd fingerhole opened.

The 5th degree = 10/14 of the string, or the remainder when 4 segments have been stopped = $\overset{\flat}{C}\flat$ or on the Aulos, the 4th fingerhole opened.

The 6th degree = 9/14 of the string, or the remainder when 5 segments have been stopped = $\overset{\flat}{D}\flat$ or on the Aulos, the 5th fingerhole opened.

The 7th degree = 8/14 of the string, Mese, or the remainder when 6 segments have been stopped = $\overset{\flat}{E}\flat$ or on the Aulos, the 6th fingerhole opened.

The 8th degree = 7/14 half the string = F or on the Aulos, the 7th fingerhole opened.

In the Mixolydian Harmonia, every note of the sequence, being based upon the $\overset{\flat}{E}\flat$ of the Arche, is also very flat in relation to the Tonic F (as indicated by the use of superscript accidentals), with the exception of the 13th ratio or segment, Parhypate Meson, on the 2nd degree of the scale; the $B\flat$ as 11th Harmonic is doubly flat, i.e. $\overset{\flat\flat}{B}\flat$, since the characteristics of the Modal Harmonic Series are also reversed in relation to the string-note. The Ethos of the Mixolydian Mode would merely be sweet and plaintive, but for the contrast afforded by the very high *tessitura* of the

[1] Opening the hole provides the outlet for the breath and cuts (or stops) the remaining length from hole to exit.

Mode, due to the position of the Mese upon the 7th degree of the scale, round which as real keynote, the melody circles. It was this feature, doubtless, which rendered it suitable for use in the Tragic Chorus, as an expression of poignant grief, which, as we know, is high-pitched in the East. The high *tessitura* involves increased tension of the muscles of the glottis, and increased compression in the breath-stream.

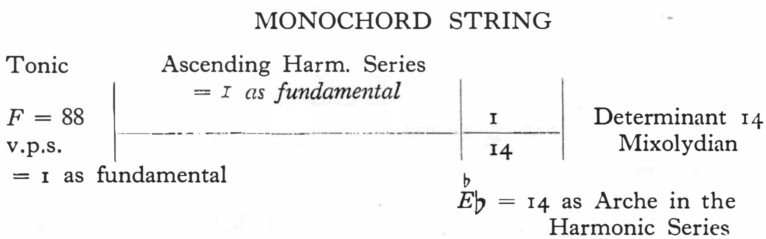
To return to the Harmonia in general, we find that it has a twofold origin in: (1) the Harmonic Series through the position Arche occupies therein; (2) in the reflection of (1), viz. the reversed Modal Series, produced by the aliquot division. The division of a string according to equal measure by any given Determinant produces members of both series; either (*a*) the Harmonic of the same number as the Determinant *from all segments alike*, by lightly touching the string at any one of the Nodes; or (*b*) the Modal Series of that Determinant from Arche to Tonic, in which all the segments *give different notes*, following the order and proportional intervals of the same Harmonic Series as in (1), but in descending order. The different result in (*b*) is due to the stopping of the string at each segment in turn, which has the virtual effect of converting the one string into many different lengths.

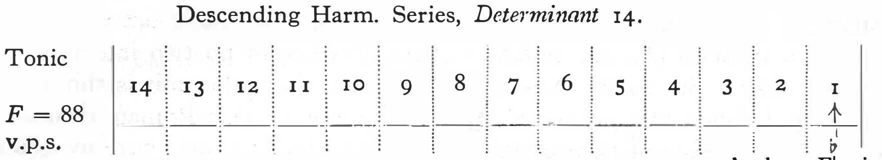
On the pipe, the different lengths implied by the uncovering of the holes are unified by the continuous flow of breath and analogously on the string by bowing.

In order to grasp the basis of the Modal System, *we must keep in view the double interchangeable function and significance of the Tonic or starting note and of the Mese as a reflection of Arche and as keynote.* This causative keynote is directly responsible for the following 7 modal features:

(1) The Determinant number of the Harmonia fixing the number of the aliquot parts of the division of the string.
(2) The distinctive order of the sequence of proportional intervals in the Harmonia from the first step upwards.
(3) The causative keynote's own position as Mese on its characteristic degree of the Harmonia.
(4) The limit of the genesis of modal material, which always ends with the *number* of the causative keynote in the Harmonic Series, and is the Tonic of the Modal Scale (e.g. 14 in the Mixolydian).
(5) The characteristic interval of the Mode which lies between the keynote or Creative Tone and the Tonic or initial note.
(6) The modal Tonality implicit in the Mode.
(7) The Ethos of the Mode.

Thus in the Mixolydian Mode the Tonic and Mese can both be represented by 1, just as by 14, because *F* as tonic or fundamental is No. 1 in the ascending Harmonic Series for the finding of Arche, and it is also 14 in the reversed Harmonic Series at the end of the Modal Genesis, as a whole string of 14 segments. The Arche (Mese) $\overset{\flat}{E}\flat$ is 14 as 14th Harmonic in the ascending series, and No. 1 as first segment in the descending series, or genesis of modal material.





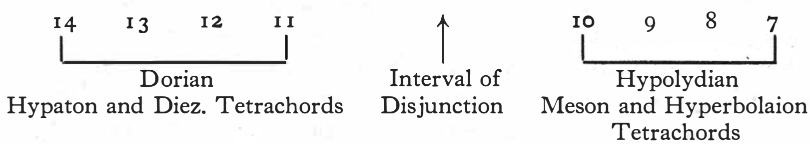
= 14/14 or the note of 14 increments of length.

Arche E♭ in the Katapyknosis, or Genesis of modal material, is now 1 as note of the first increment of length.

Therefore, the Tonic and Mese (as Arche), are both 1 and are also 14.

N.B.—In noting the ratios of the Harmonia, the denominator of the fractions of lengths *is constant* and therefore frequently omitted in this work, e.g. 14/14, 13, 12, 11.

A passage of Philolaus may be thought of interest in connexion with this twofold basis of the Harmonia; he says that '*ἁρμονία* positively comes into being from opposites; for *ἁρμονία* is the union of many ingredients and the connexion of varying purposes'.[1] The name Mixolydian [2] suggests a mixed origin and this may easily be traced through the ratios: 14, 13, 12, 11,[3] is the second Dorian tetrachord from Paramese to Nete Diezeugmenon; and 10, 9, 8, 7, is the Hypolydian Tritone.



The above was probably the original form of the ancient Mixolydian Harmonia; and the Diazeuxis did not necessarily consist of a 9/8 tone in the

[1] Philolaus, ed. A. Boeckh (1819), p. 61: '*ἁρμονία δὲ πάντως ἐξ ἐναντίων γίνεται. ἔστι γὰρ ἁρμονία πολυμιγέων ἕνωσις καὶ σύμφρασις διχᾶ φρονεόντων.*' Cf. Chap. iv. of present work.

[2] For the mixed origin, more fully discussed, together with allusions to its structural features explained through the Modal System, see Chap. iv, Mixolydian.

[3] Since the fractions expressing modal ratios or lengths of string have the Determinant number of the Mode as constant denominator, and as numerators the order numbers of the segments constituting the division into aliquot parts, *these numerators will often in the present work be used alone to indicate in brief form the modal ratios*, the common denominator for the Mode being understood. The numerators also form ratios with one another, as they occur in sequence, e.g. 14 : 13, 13 : 12, &c.

Modal System as it does in the Aristoxenian. The conjunct form of the scale as used in the Perfect Immutable System, viz.

Mese PM.
14 13 12 11 10 9 8 7

is discussed in the next chapter.

To return to the Harmonia once more, we find that the system involving a Mode is essentially an octave system, inasmuch as the characteristic intervals constituting the Modal unit extend to the whole octave (*τὸ διὰ πασῶν* = through all, [the notes or strings]), wherein no two intervals are exactly alike. But in the Aristoxenian system where the unit is the tetrachord and which was generally followed by the Graeco-Roman theorists, the octave consists of two tetrachords exactly similar in structure in which all the intervals are semitones or their multiples. The basis of the Harmonia, due to a division into aliquot parts of a string or pipe, is a section of the Harmonic Series reversed in direction as we have seen above; the sequence is not arbitrarily strung together, but directly conditioned by the equal segments taken in arithmetical order beginning with the Arche. All the degrees of the octave scale, or Harmonia, are in superparticular ratios to one another, and as these ratios are founded upon aliquot lengths of string or pipe, they are all commensurate with the Arche, their point of departure.

With the genesis of the Modes the theorist is mainly concerned, but the Harmonia is the affair of the practical musician. The Harmonia begins on Hypate Meson (or string-note) as ***highest number*** in the series, and at the point where the intervals are least in magnitude, and proceeds by proportioned steps which increase in magnitude until the octave of Hypate is reached at Nete, which forms the limit of the Harmonia. The order of the seven Harmoniai of pre-Aristoxenian days is known and the highest of these, the Mixolydian, I have identified as the one based upon the 14th member of the Harmonic Series: from its position as a species in the Perfect Immutable System according to Ps-Euclid,[1] Bacchius[2] and Ptolemy;[3] it may be identified also as the Mixolydian of Lamprocles (Plut., *op. cit.*, c. 16, § 156, p. 64). The rest of the Harmoniai may be found by descending the scale formed by the Harmonic Series from 14 to 8 to find their Mesai, a simple expedient in theory or even in practice given the sensitive ear of the Greek. As is right and proper, the order in which the Harmoniai have come down to posterity is that of their Mesai, which may be followed through the Tonoi of Alypius. Examples of the Mixolydian Harmonia embodied on the Aulos are to be found in the Records of Auloi Loret xxvii, xxvi, xxiii, xxviii, xxiv, xxv, xiii.

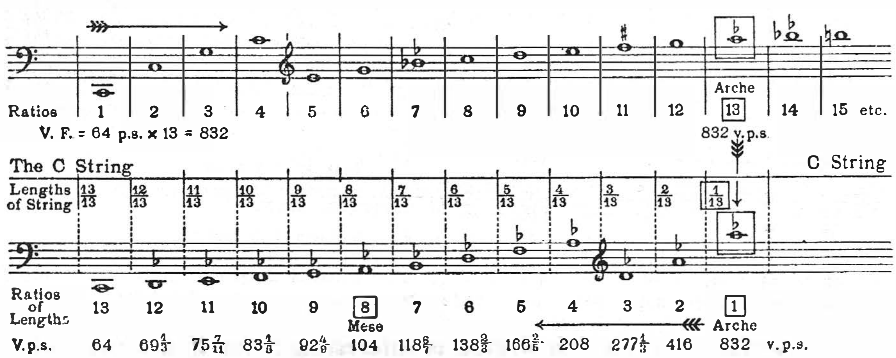
THE LYDIAN HARMONIA

After 14 in the descending order of the Harmonic Series comes 13, the number of the Lydian Harmonia. Although the original intention

[1] *Intro.*, p. 15M. [2] Bacchius, pp. 18–19M. [3] *Harm.*, ii, 10.

was to base the Harmonia on the Greek fundamental *F*, it is now considered advisable to use the modern standard *C* instead, in some of the modal sequences containing characteristic intervals, which also occur in the modern major and minor scales.[1] Based on *C*, the 13th Harmonic is the very flat *A*, nominally a major 6th above *C*, but in reality forming an interval intermediate between the major and the minor 6ths. The genesis of the Lydian Harmonia is shown in Fig. 5, based on *C*.

FIG. 5.—Genesis of the Lydian Harmonia by the Aliquot Division by Determinant 13
The Finding of Arche in the Harmonic Series
The Genesis of the Lydian Harmonia of M.D. 13 : Key Tonality of *A* Minor on the *C* String = 64 v.p.s.



The Arche, while retaining its characteristic essence as the flattened major 6th of the 13th Harmonic, now becomes 1 in the genesis which unfolds step by step, until 13/13 is reached on the *C* given by the string vibrating as a whole. Starting from the *C* as Tonic, the Lydian Harmonia ascends through the ratios (see Fig. 6).

From these diagrams of the Harmoniai it will be seen how the different steps are produced from the Aulos, bored according to equal measure by each Determinant in turn. From the drawing it is obvious that the Aulos is in quite a different category from the string; the Modal Genesis of material cannot take place upon the pipe, for the first three or four segments would occur on the little reed mouthpiece itself, on which no hole can be

[1] Since modern musicians make their computations from *C* (= 64 v.p.s.) a comparison has been worked out here upon that basis, viz.

$$\frac{64 \times 5}{3} = 106{\cdot}6 \text{ v.p.s.} \quad . \quad . \quad \text{maj. 6th on } C = 884{\cdot}4 \textit{ cents}$$

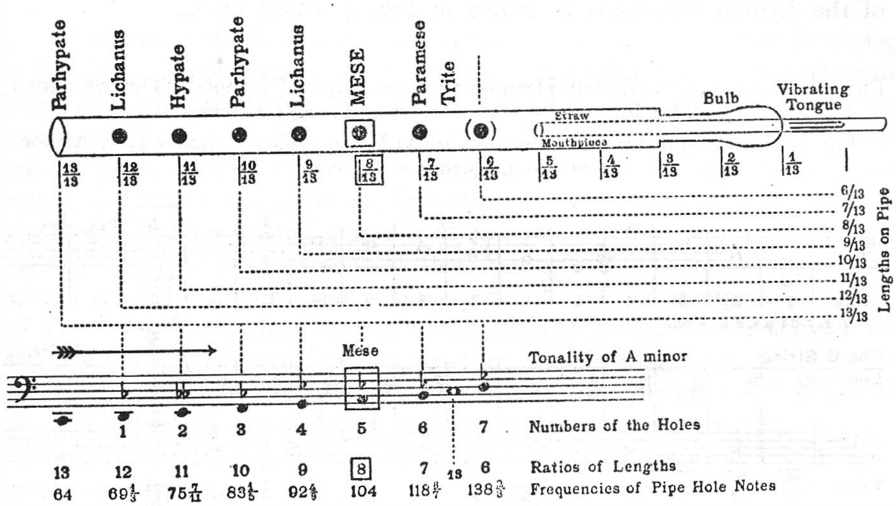
$$\frac{64 \times 8}{5} = 102{\cdot}4 \quad . \quad . \quad . \quad \text{min. 6th on } C = 813{\cdot}24 \textit{ cents}$$

$$\frac{64 \times 13}{8} = 104 \quad . \quad . \quad . \quad . \quad \text{Modal Harmonic 6th} = 840{\cdot}5 \textit{ cents}$$

Based on a *C* fundamental are the Phrygian (Figs. 7 and 8); the Hypolydian (Figs. 14 and 15) and the Hypophrygian Genesis (Fig. 16). The Hypophrygian Diatonic scale, however, has been based on *F*.

bored. The nomenclature of the Greek scale, *κατὰ δύναμιν*, shows the Mese in position, on the 6th degree for the Lydian Harmonia; the correspondence in staff notation shows the Key *A* minor to be inherent in the genesis of the Mode itself.

FIG. 6.—Lydian Harmonia (Diatonic Genus) resulting from the Aliquot Division by Determinant 13



The problem of the 8th degree is interesting; for if the next whole segment above the Paramese (i.e. the 6th) were taken, the octave would have to be overstepped, and the scale terminated by a septimal 3rd on the 2nd degree of the next octave. The primitive pipe-maker probably suffered it to be so at first, being unsophisticated, until he discovered that by halving the distance between the holes, he obtained the octave for which his ear was seeking. There are two other ways at least in which the pipe-maker may have achieved the desired result, and there is evidence that both were known in Antiquity: in the first, by half covering the top or 7th hole, the note was proportionately lowered in pitch, thus giving the same sound as a hole placed on the half segment;[1] in the second the result was obtained by decreasing the breath-pressure and slightly relaxing the muscles of the larynx, as implied by Aristoxenus: '*ὅτι δὲ χειϱουϱγίαν τὴν μὲν ἀπὸ τῶν χειϱῶν τὴν δ' ἀπὸ τῶν λοιπῶν μεϱῶν οἷς ἐπιτείνειν τε καὶ ἀνιέναι πέφυκε.*'[2] For an example of Folk Music in Lydian Harmonia, see Henebry, *Hdbk. of Irish Music* (Cork Univ. Press), 1928), 'Ring Promontary,' Nos. 99 and 95.

A third method of lowering the note of any hole is by cross-fingering, also described by Bhārātā as extensively used in Ancient India; the method is known to have been practised by the Greeks. The use of sliding bands is described in Chapter ii. In theory, the easiest way to obtain a note

[1] This device known in Ancient India, is described by Bhārātā, see Chap. iii.
[2] Macran, *Aristoxenus*, p. 132, Gk., and p. 196. Transl.: 'Nor is it because it submits to certain operations of the hands and of the other parts naturally adapted to raise and lower the pitch.'

intermediate between two segments is to double the aliquot number, i.e. $13 \times 2 = 26$ and to raise the relevant equivalents of the Diatonic scale in a similar manner.

	26	24	22	20	18	16	14	13
=	13	12	11	10	9	8	7	$6\frac{1}{2}$

It will be well to note here that to double the Determinant number, or to halve the distance on a pipe or string, is the legitimate natural process for converting the Diatonic into the Chromatic genus in the modern sense of the word.[1] A new note bearing an odd number may thus be introduced, theoretically and practically, between every two of the Diatonic genus, an operation which produces a perfectly graded harmonious sequence with intervals increasing in magnitude as the pitch rises. How satisfying to the ear the progression sounds no one would believe who had not heard it. Examples of the Lydian Harmonia embodied on Pipes will be found in the Records of Auloi Elgin and Loret xviii, xxiii, xxiv, xxx, and of the Japanese flute.

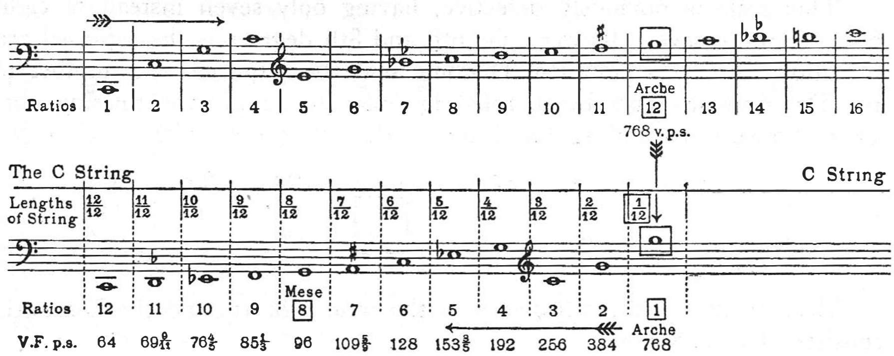
THE PHRYGIAN HARMONIA

FIG. 7.—Genesis of the Phrygian Harmonia by the Aliquot Division by Determinant 12

The Finding of the Mese in the Harmonic Series

Ratios Vibration Frequency = 64 p.s. × 12 = 768 v.p.s. Tonality of *G* minor

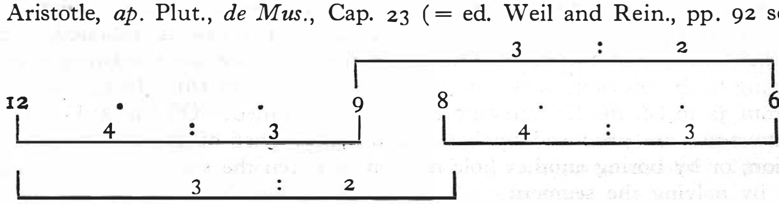
The Genesis of the Phrygian M.D. 12 on the *C* String = 64 v.p.s.



The Phrygian Mode is next in order with 12 as Determinant. This Harmonia at least would satisfy even the Aristotelian[2] and Aristoxenian

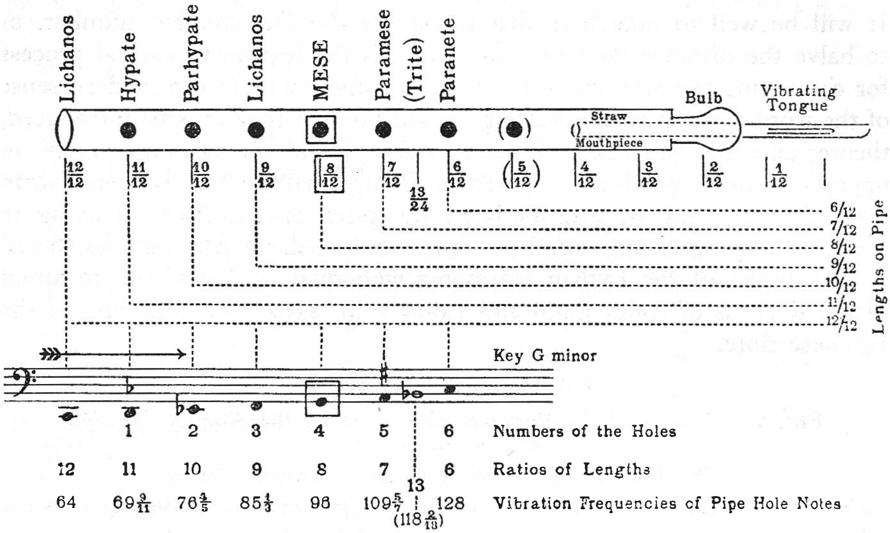
[1] This process has been described by Aristides Quintilianus (pp. 114–115M.) for the ratios of the Harmonic Series in connexion with the values of the semitone. By *modern sense* is meant a whole sequence of approximate modal semitones or halved increments.

[2] Aristotle, *ap.* Plut., *de Mus.*, Cap. 23 (= ed. Weil and Rein., pp. 92 sqq.)



canons, since it stands firmly balanced upon perfect 4ths and 5ths as the fixed notes (ἑστῶτες) of tetrachords.

FIG. 8.—Phrygian Harmonia (Diatonic Genus) [1] resulting from the Aliquot Division by Determinant 12 on *C*



This scale is obviously defective, having only seven instead of eight notes to the octave. Between the 6th and 8th degrees is the septimal 3rd of ratio 7 : 6. If a complete Diatonic scale be required, the number of the Determinant must be doubled in order to supply the missing note 13/24 between 14 and 12 for Trite on the 7th degree, viz.

Lich.	Hyp.	PH.	Lich.	Mese	PM	Tr.	PN.
24	22	20	18	16	14	13	12
12	11	10	9	8	7	↓	6
					septimal 3rd		

Mese occurs on the 5th degree at the ratio 8 or 16, and the Diazeuxis consists of a 9 : 8 tone.

The reader may feel that the addition of the intermediate note of ratio 13 (belonging to the next octave of generation) between the notes in the ratio 7 : 6 is arbitrary and demands some justification. The difficulty that

[1] By *Diatonic*, I understand the octave that develops between Harmonics 8 to 16 (or slightly overlapping above and below) in the Harmonic Series, whether ascending or descending (reversed), i.e. the first filling in of the octave by 7 different notes in sequence ; 15, the major 7th, being considered an alternative of 14, the minor 7th. In the next octave, from 16 to 32, the Chromatic compass is obtained, and it is seen that this occurs through the interpolation of a new note between every two belonging to the previous octave of genesis (i.e. from 8 to 16). In the next octave, viz. from 32 to 64, the Enharmonic compass is obtained. On the Aulos the intermediate notes are produced by half-closing the higher of the two fingerholes in question, or by boring another hole midway between the two ; on the monochord string by halving the segment.

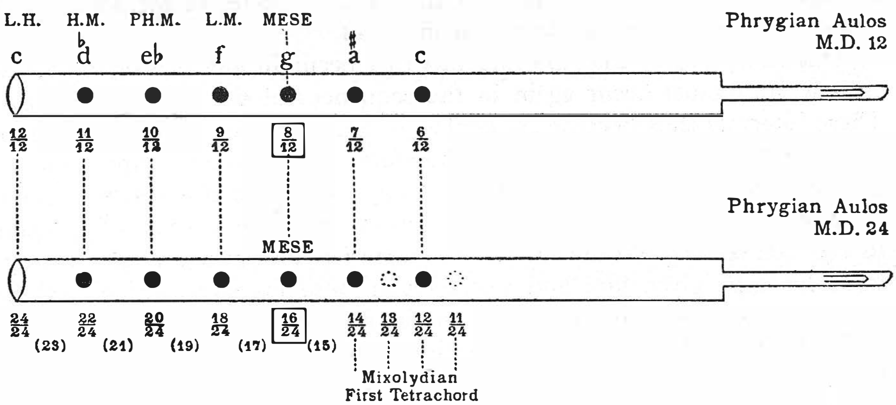
troubles the reader may arise from having to admit the possibility that a primitive piper should come upon the idea of making use of a division of ratios such as is suggested above.

If it be claimed that the Aulos taught the piper the Modes, why (one may ask) should he not have been content with the octave scale which the 6 out of 12 increments that his Phrygian pipe with six holes gave him? The answer is that he was probably quite content until, having fortuitously made a Mixolydian pipe based upon Determinant 14,[1] with perhaps only three holes, giving:

$$\text{exit } \frac{14}{14};\ \text{hole 1, } \frac{13}{14};\ \text{hole 2, } \frac{12}{14};\ \text{hole 3, } \frac{11}{14};$$

his ear became familiar with the sequence produced by ratios 14, 13, 12, 11. When, therefore, his Phrygian pipe skipped over from $\left.\begin{matrix}14\\7\end{matrix}\right\}$ to $\left.\begin{matrix}12\\6\end{matrix}\right\}$, he was worried, first of all by the disturbed rhythm, and secondly by the absence of the familiar note, indicated by ratio 13, which he missed between the notes of the last two holes on his Phrygian pipe (of ratio 7 : 6). In consequence he tried to get the intermediate missing note, either by half covering the hole that gave 6 or by boring another hole between the two illustrated in Fig. 9.

FIG. 9.—To illustrate the Filling In of the Septimal 3rd of ratio 7 : 6 on the Aulos



If the piper has had a Mixolydian Aulos, the sound of the note of ratio 13 between $\left.\begin{matrix}14\\7\end{matrix}\right\}\ \begin{matrix}13\\\downarrow\end{matrix}\ \left.\begin{matrix}12\\6\end{matrix}\right\}$ will come naturally to him on a Phrygian Aulos of M.D. 12 or 24.

There is evidence in the theoretical sources that this kind of division of ratios was actually practised by the Greeks. Among the formulae of tetrachords recorded by Ptolemy (*op. cit.*, pp. 170–2, W. 1682) (see also Tables appended to ii, 14 = pp. 70–3, Düring) there is, for instance, the Enharmonic of Didymus, in which the 16/15 of his Chromatic genus has undergone an analogous division to produce the Enharmonic Pyknon

[1] Like the Egyptian 'Maket 3' pipe with 3 holes, or Loret x, 3 holes; Loret xxvi, 3 holes; Loret xxviii, 6 holes; all of which give tetrachords of the Mixolydian Harmonia; see Chaps. ii and iii, Chap. x, Records.

32/31 × 31/30. Similarly in the Chromatic of Eratosthenes, the Chromatic Pyknon 20/19 × 19/18 becomes in his Enharmonic 40/39 × 39/38. Further evidence (dependent upon my interpretation of the Notation) that defective scales such as that of the Phrygian Harmonia of Determinant 12, and the Dorian scale of Determinant 11 (identified by me as the scale of Terpander), were actually treated as suggested above, is provided by the six ancient Harmoniai of Plato recorded through their musical notation by Aristides Quintilianus (p. 22M.; and cf. Chap. v). In the Phrygisti of Aristides Quintilianus the interpretation in ratios of the symbols of Notation is as follows:

FIG. 10.—The Filling In of the Septimal 3rd illustrated in Greek Notation

Φ	C	P	Π	I	Z	E	Δ	Ʊ
F	C	U	Ɔ	<	⊏	⊔	⊐	Z
24	21	20	19	16	28 = 14	27	26 = 13	24 = 12

Septimal 3rd

Ratios by K. S.

Further confirmation of the principle will be found in Chapter iv, where a suggestion about the evolution of the Perfect Immutable System through the natural development of the Modal Species of the seven Harmoniai on the Kithara is described in detail.

One must realize that the intervals that occur in any one of the seven ancient Harmoniai occur again in the sequences of the other Harmoniai. These intervals thus become as familiar in uninterrupted sequence as those of our modern scale do to us. Therefore, if a primitive piper finds a defective or transilient scale on his Aulos, that normally results from a Determinant number which is too low to produce a scale of 7 notes to the octave—e.g. M.D. 12, of the primitive Phrygian—he may accept what his pipe gives him and rest content until (as suggested above) he chances upon a Mixolydian pipe. In the first tetrachord of this Aulos he finds the note that divides the septimal 3rd of ratio 7 : 6 given out by holes 5 and 6 at the top of his Phrygian Aulos. This doubtless provides the natural explanation of the expansion of defective or (so-called) gapped scales into complete Diatonic scales of seven intervals to the octave, such as the Dorian Spondaic or scale of Terpander of M.D. 11, a scale of 6 notes ending with the same interval of the septimal 3rd, 7 : 6. Similarly, the ratio 15 is probably first brought to the piper's notice by a Hypodorian Aulos of M.D. 16, as the second step in its scale, and in the Hypophrygian of M.D. 18, as 3rd or 4th degree. It is the theorist, of course, who first discovers that the first tetrachord of the Hypodorian and Hypophrygian Harmoniai contains five notes instead of four, and who eventually deletes either 15 or 14.

When our attention is turned to the modal flute (not Aulos), however, the interval 15—13 is sometimes unmistakably found on a specimen having

M.D. 11, instead of the expected septimal 3rd, 7 : 6; and this, moreover, occurs when no spacing has been allowed for the intermediate ratio 14. An instance is the Inca flute and Bali i; the explanation of this phenomenon, however, applies to the flute, but not to the Aulos.

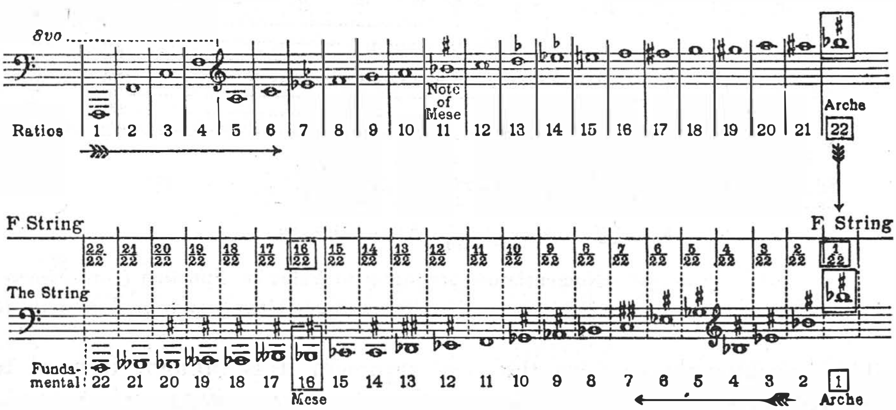
There is an inference that this Phrygian Harmonia was in use among the Ancient Greeks, for Gaudentius mentions that Pythagoras divided his monochord into 12 equal parts.[1] A scale having the formula $\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8} = \frac{3}{2}$ appears among those given by Claudius Ptolemy,[2] who calls it διατονικὸν ὁμαλόν; he does not relate the scale or divisions, as the present writer does, to the Phrygian Harmonia.

Examples of the Phrygian Harmonia embodied on Pipes will be found in the records of Auloi Elgin; of Loret xxi, xvi, xxxi; of Cairo G and of flutes No. 4, Mond; Sensa (short) B 12; Olympia and Corinth (see summary, end).

FIG. 11.—Genesis of the Dorian Harmonia through the Aliquot Division by Determinant 22 on the *F* String

The Finding of Arche in the Harmonic Series (on the *F* String)

The Genesis of the Dorian Harmonia on the *F* String by Determinant 22



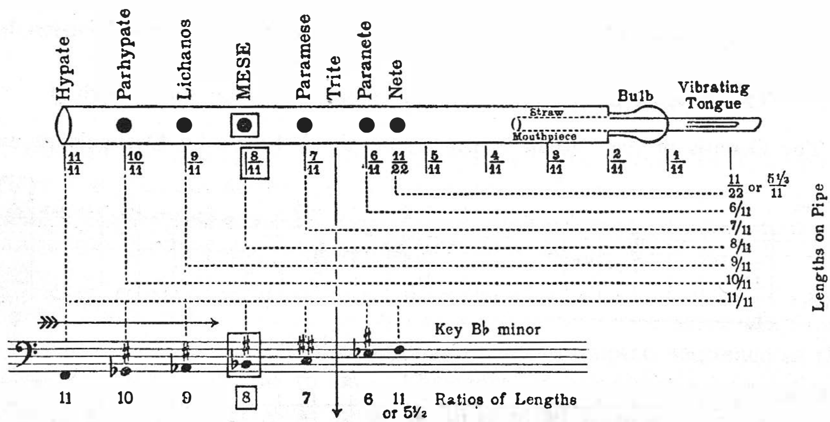
THE DORIAN HARMONIA

The next Harmonia in order, the Dorian, has as Determinant 11; its Arche falls upon the 11th Harmonic, known as the sharp Harmonic 4th. This is the simplest form of the Dorian Harmonia, such as would be produced, for instance, on an Aulos having five holes, when the octave would be lacking, or with the octave as Nete on a Kithara of seven strings. A description of such a scale on seven strings, with the range of an octave and the omission of Trite as 6th degree, leads to the identification of this Harmonia with the seven-toned octave-scale, persistently connected with the name of Terpander.

[1] *Harm. Intro.*, p. 14M., lines 28 sqq. [2] *Harm.*, ii, 16.

Figs. 12, 9 and 10 demonstrate that the omission of Trite between Paramese and Paranete, due to the natural occurrence of the septimal 3rd between the 4th and 5th holes, cannot be attributed to a sense of aesthetic eclecticism as Plutarch [1] would have us believe. Therefore, the earliest form of the Dorian Harmonia characterized by the omission of Trite was not in origin a defective scale, but an Aulos scale resulting from the aliquot division by Determinant 11 which includes the septimal 3rd undivided (ἀσύνθετος) between the superparticular ratios 7 and 6. One of the Elgin Pipes preserved in the British Museum was actually bored to give this scale (see Chapter v, and Chapters ii and iii (Aulos), and Chap. x, Records). In the Ps-Aristotelian Problem, Terpander is definitely connected with this so-called defective scale, and credited with having eliminated Trite.

FIG. 12.—The Dorian Harmonia (Diatonic Genus) resulting from the Aliquot Division by Determinant 11. Primitive Form connected with the name of Terpander



Nomenclature according to Value or Function (κατὰ δύναμιν)
The Terpandrian Harmonia with Trite omitted

This considerably weakens the very argument it is brought forward to strengthen, viz. that the Ancients called the octave *diapason* (διὰ πασῶν = through all), and not *diocto*, because there were but seven notes. How could it be *through all* if one note was known to have been eliminated? The chief value of this passage lies in the importance attributed therein to this scale as the standard scale of antiquity, the structural features of which might be considered to be responsible for the naming of the octave.

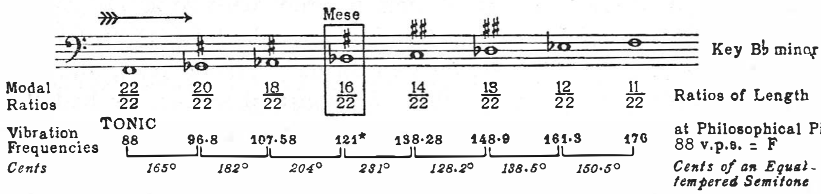
The true explanation of the matter would seem to be that it was the theorists later on who, when the complete octave obtained from a division by Determinant 22 was in use, discovered that Trite had been missing in the earlier scale; they then included that scale with other transilient or gapped scales under the generic term ὑπερβατόν.[2] This Aulos scale is not

[1] *de Mus.*, Cap 19 (ed. Weil and Rein., pp. 74–7); cf. also Ps-Aristotle, *Probl.* xix, 32.
[2] Cf. Aristoxenus, ed. Macran, p. 109, line 20 (and p. 177, tr.).

only known through its association with the name and practice of Terpander; it was perhaps still better known as the Spondiakos Tropos,[1] a liturgical Libation Mode. Plutarch raises points of great importance to the Modal System in connexion with the σπονδεῖον which have been reserved for a later discussion.

The complete octachordal Dorian Harmonia is found by means of

FIG. 13.—Dorian Harmonia, Complete Octave Scale on $F = 88$ v.p.s. resulting from the Aliquot Division by 22



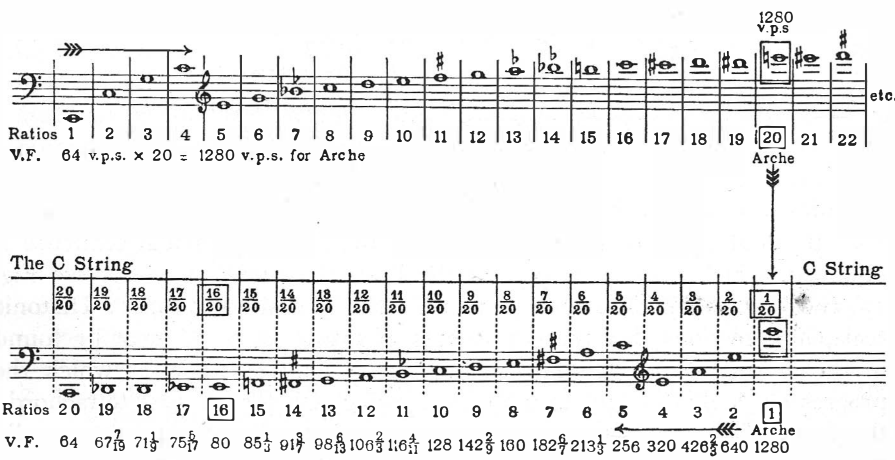
* The Mese of 121 v.p.s. is virtually $B\natural$ (of the 15th Harmonic of C) = 120 v.p.s., but the modal tonality, expressed in modern terms, demands as Mese $B\flat$.

N.B.—Note the characteristic Ethos: all the notes sharpened in relation to the Tonic, in consequence of the characteristic interval between Mese and Tonic, the sharp Harmonic Fourth of ratio 11/8.

Determinant 22; it now comprises both Paramese and Trite; the composition of the scale with the septimal tone 8/7 as Diazeuxis may be seen in the figure above. This is the standard disjunct octave scale, and it is this Harmonia—according to the present writer—and not the *E* to *E* species

FIG. 14.—Genesis of the Hypolydian Harmonia by the Aliquot Division by 20 on the *C* String = 64 v.p.s.

The Finding of the Arche in the Harmonic Series



of modern theorists on the white notes of the keyboard, which forms the basis of the Greater Complete System and of the Perfect Immutable System. This may be traced through Greek Notation in the tables of Alypius, and

[1] Plut., *de Mus.*, Cap. 11–12, 19 (= ed. Weil and Rein., pp. 46–52, and 74–7).

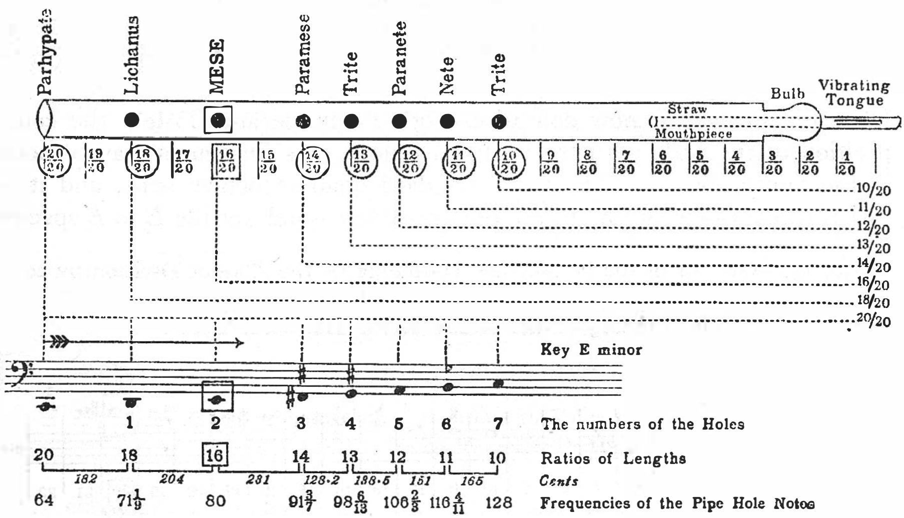
from the grouping together of certain of the formulae recorded by Ptolemy. (See Chap. v of present work, Fig. 40.)

Examples of the Dorian Harmonia embodied in Pipes will be found in the Records of Auloi Elgin, of Loret xxiii, xxxi, xxxii, xxxvi, xvi; of Cairo M, R, and G, and of Flutes Inca, Mond, Graeco-Roman No. 1; No. 3; No. 7; No. 9; Java i; Java ii; Bali i; Nauplia (see Chap ix, end).

THE HYPOLYDIAN HARMONIA

The Hypolydian Harmonia is the next in order with 20 as Determinant. It is evident that to take 10 as Determinant would lead no further than the Tritone 10, 9, 8, 7, followed by two thirds, 7 : 6 and 6 : 5, and would only yield 5 notes to the octave. Recourse must, therefore, be had to the doubling of the Determinant number, a process which, as explained above,

FIG. 15.—Hypolydian Harmonia (Diatonic Genus) resulting from the Aliquot Division by Determinant 20 on the Aulos



provides a new note between each of those obtained by the division by 10. By doubling the aliquot number 10, the following modal sequence is obtained, which consists of part of the Diatonic octave 16 to 10 (see Fig. 15) and part of the Chromatic 20 to 16. If it is desired to form a Diatonic scale, it is obvious that the equivalents of ratios 10, 9, 8, must be found in the Harmonia of Determinant 20, i.e. as 20, 18, 16, after which the progression yields 15, giving with 20 a perfect 4th for the first tetrachord; the form of the scale with the Tritone, viz. 20, 18, 16, 14, was equally well known; it was simply a matter of choice between Paramese (14) or Trite Synemmenon (15) as explained by Hucbald.[1] It may be added that

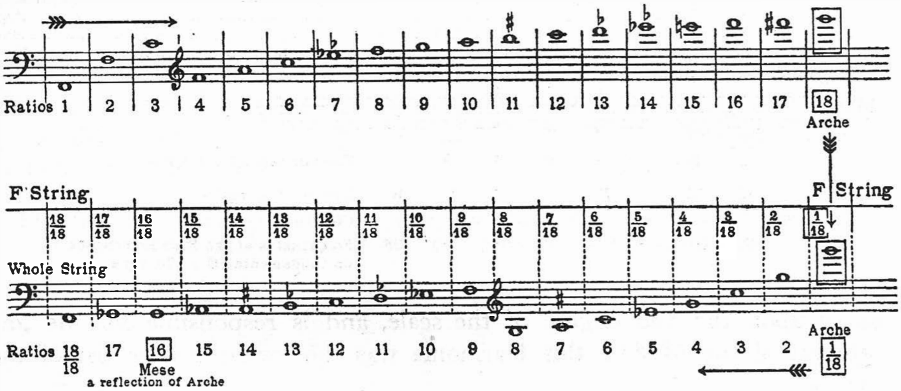
[1] Martin Gerbert, *Script. Eccles.*, Tome 1, pp. 113–14. Hucbald states that after the 7th note from Hypate Hypaton, i.e. Mese, the melos in the music of the Liturgy passed frequently through the Synemmenon, and he gives as example the Introit '*Statuit ei Dominus*'. Hucbald indicates in this passage the correct position

ratios 15 to 13 offer a large, and 14 to 13 a small, Diazeuxis; and that in the second tetrachord 13, 12, 11, 10, the ratios of the first Lydian Tetrachord may readily be recognized.

It has been mentioned that in the Harmoniai distinguished by the prefix *Hypo*, the first group of four ratios involves some new feature, while the second group corresponds to the first tetrachord of the Harmonia of related name. Examples of the Hypolydian Harmonia embodied in Pipes will be found among the records of the Elgin Aulos, of Loret xviii, xxiii, xxiv; of 'Cairo F.' and of Flutes No. 1, Graeco-Roman (from Hole 1), No. 8A, No. 8B.

FIG. 16.—Genesis of the Hypophrygian Harmonia resulting from the Aliquot Division by Modal Determinant 18

The Genesis of the Hypophrygian Modal Material on the *F* String by Determinant 18



THE HYPOPHRYGIAN HARMONIA

The Hypophrygian is the next Harmonia in the series; it has 18 for its Determinant. The Arche is found as 9th or 18th Harmonic, at a major tone above one of the octaves of the *F* of the string note. What has been said concerning the necessity for using a double Determinant for the Hypolydian holds good here also.

The numbers of the lengths of string forming the ratios of the modal scale are as follows:

18/18 $\boxed{16}$ 15 13 12 11 10 9

of the Hypolydian Modal Species, beginning on *F* as Parhypate Meson, of the *Authenti triti*, or its plagal. The syllable *sta*, on the 5th degree of the scale, is allotted, he states, to the 3rd note in the Synemmenon, i.e. to Paranete, adding that there is a semitone between the 2nd (Trite) and the 3rd intervals of the scale, viz.

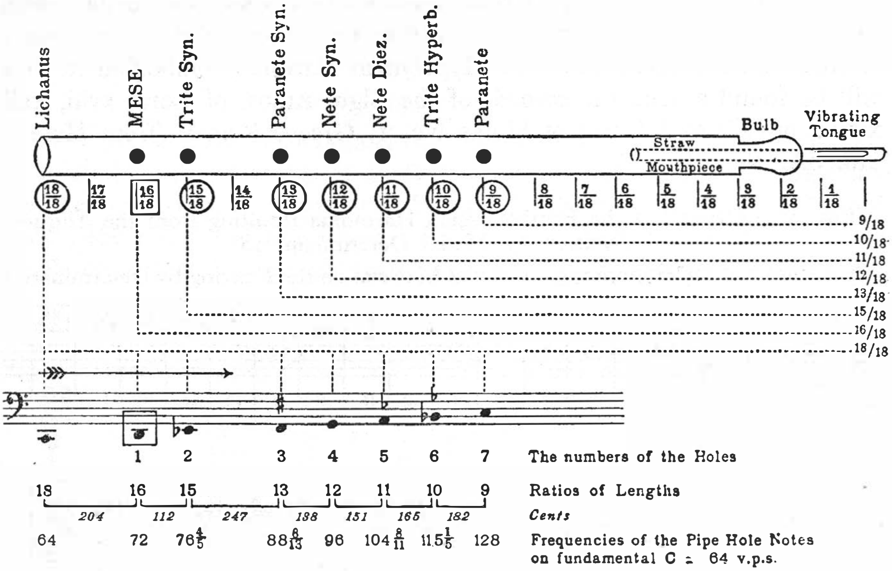
(1) Parhypate to Lichanus (Tone)
(2) Lichanus to Mese (Tone)
(3) Mese to Trite (Semitone)

or as steps, the 3rd to the 4th is a Semitone.

On p. 115 Hucbald gives a table of the notes of the P.I.S. in the correct notation of the Lydian Tonos accompanied by the letters of our alphabet from (*A*) Prosl. to (*E*) Nete Diez. and on p. 120 a table of Antiphons: in some instances he has noted the first phrase of the Antiphon in the Lydian Tonos—a valuable record of fragments of the music of the Liturgy in the ninth century.

The first tetrachord is new and characteristic of the Hypophrygian Harmonia, while the second is the first Phrygian tetrachord. Mese appears

Fig. 17.—The Hypophrygian Harmonia (Diatonic Genus) resulting from the Aliquot Division by Determinant 18

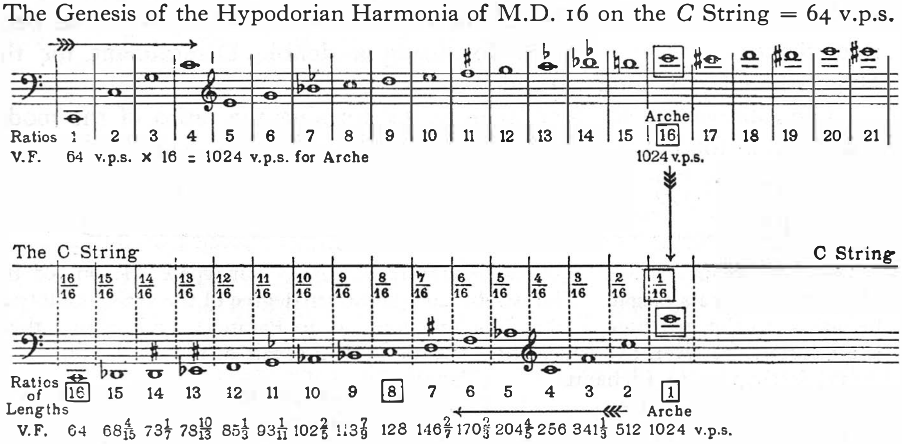


as 16 upon the 2nd degree of the scale, and is responsible for the low *tessitura* of the Mode; this Harmonia was the lowest on the list of the

Fig. 18.—Genesis of the Hypodorian Harmonia resulting from the Aliquot Division by Modal Determinant 16

The Finding of the Arche in the Harmonic Series

The Genesis of the Hypodorian Harmonia of M.D. 16 on the *C* String = 64 v.p.s.



Harmonists recorded by Aristoxenus (Cap. 37, p. 128, Macran). The interval of ratio 15 : 13, from Trite to Paranete Synemmenon, is a very characteristic modal one, which figures in many folk-songs, European and

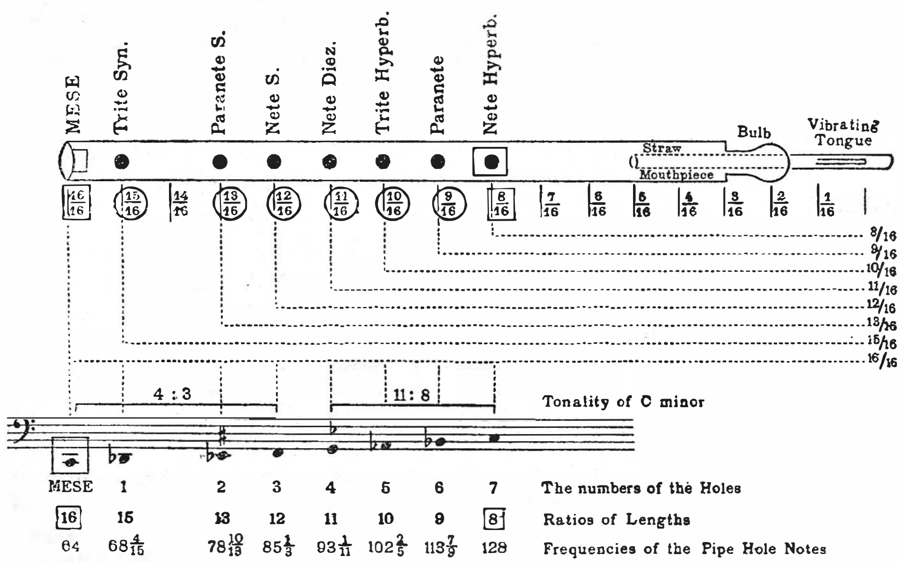
Oriental, and is frequently described somewhat libellously as 'an augmented second out of tune'. The Harmonia is also known as the Oriental Chromatic, and in Mohammedan countries generally has the first tetrachord repeated on the dominant, instead of the complementary Phrygian tetrachord belonging to the true Hypophrygian Harmonia. The interval of disjunction, of ratio 13 : 12, is considerably less than a tone, but more than a semitone. Examples of the Hypophrygian Harmonia embodied on Pipes are to be found on Records of Aulos Loret xv, and on those of the three Sensa flutes, A, B, C, and No. 7A. See also Chap. ix: the prototype Greek Fragments in this Harmonia.

The modal series now comes to a close with the Hypodorian, the lowest or highest of the Harmoniai; as highest it corresponds in the Tonoi to the Hypophrygian, which is in the same key but an octave higher.

THE HYPODORIAN HARMONIA

As the number of the Determinant is 16, Mese is both initial and final. This, of course, is one of the most harmonious of all the Modes, since the modal material generated presents no contrast or difficulties in intonation in relation to the string-note, which is an octave of Arche.

FIG. 19.—The Hypodorian Harmonia of Modal Determinant 16 (Diatonic Genus) resulting from the Aliquot Division by Determinant 16 on *C*



The order numbers of the lengths of string (i.e. the numerators) forming ratios are:

16/16 15 13 12 11 10 9 8

4 : 3 11 : 8

It will be noticed that the first tetrachord, characteristic of the Harmonia, constitutes the tetrachord Synemmenon in the Perfect Immutable System, while the second is found as the Hyperbolaion group (see Chap. v, Fig. 40, and Chap. iv, Fig. 33). The first tetrachord of the Hypodorian Harmonia has the ratio 4 : 3 of the perfect 4th, and contains, like the Hypophrygian, the arresting augmented second of ratio 15 : 13, while the second tetrachord is pure Dorian.

THE BASTARD HYPODORIAN

This modal Hypodorian has no connexion with the bastard species of the same name represented in the Perfect Immutable System by the octave from Proslambanomenos to Mese, and in modern equivalence by the *A*

FIG. 20.—The Seven Harmoniai expressed in Superparticular Ratios of Lengths of String or Pipe, taken within the Octave from *C* = 64 v.p.s. to *C* = 128 v.p.s.

Nomenclature according to Position *κατὰ θέσιν*	Hypate	Parhypate	Lichanos	MESE	Paramese	Trite	Paranete	Nete	
MIXOLYDIAN	$\frac{14}{14}$ 64	$\frac{14}{13}$ $68\frac{12}{13}$	$\frac{13}{12}$ $74\frac{2}{3}$	$\frac{12}{11}$ $81\frac{5}{11}$	$\frac{11}{10}$ $89\frac{2}{5}$	$\frac{10}{9}$ $99\frac{5}{9}$	$\frac{9}{8}$ 112	$\frac{8}{7}$ 128	v.f. by modal ratios on fundamental *C* = 64 v.p.s. for each Harmonia
LYDIAN	$\frac{13}{13}$ 64	$\frac{13}{12}$ $69\frac{1}{3}$	$\frac{12}{11}$ $75\frac{7}{11}$	$\frac{11}{10}$ $83\frac{1}{5}$	$\frac{10}{9}$ $92\frac{4}{9}$	$\frac{9}{8}$ 104	$\frac{8}{7}$ $118\frac{6}{7}$	$\frac{14}{13}$ 128	
PHRYGIAN	$\frac{12}{12}$ 64	$\frac{12}{11}$ $69\frac{9}{11}$	$\frac{11}{10}$ $76\frac{4}{5}$	$\frac{10}{9}$ $85\frac{1}{3}$	$\frac{9}{8}$ 96	$\frac{8}{7}$ $109\frac{5}{7}$		$\frac{7}{6}$ 128	
DORIAN	$\frac{11}{11}$ 64	$\frac{11}{10}$ $70\frac{2}{5}$	$\frac{10}{9}$ $78\frac{2}{9}$	$\frac{9}{8}$ 88	$\frac{8}{7}$ $100\frac{4}{7}$		$\frac{7}{6}$ $117\frac{1}{3}$	$\frac{12}{11}$ 128	
HYPOLYDIAN	$\frac{20}{20}$ 64	$\frac{20}{18}$ $71\frac{1}{9}$	$\frac{18}{16}$ 80	$\frac{16}{14}$ $91\frac{3}{7}$	$\frac{14}{13}$ $98\frac{6}{13}$	$\frac{13}{12}$ $106\frac{2}{3}$	$\frac{12}{11}$ $116\frac{4}{11}$	$\frac{11}{10}$ 128	
HYPOPHRYGIAN	$\frac{18}{18}$ 64	$\frac{18}{16}$ 72	$\frac{16}{15}$ $76\frac{4}{5}$	$\frac{15}{13}$ $88\frac{8}{13}$	$\frac{13}{12}$ 96	$\frac{12}{11}$ $104\frac{8}{11}$	$\frac{11}{10}$ $115\frac{1}{5}$	$\frac{10}{9}$ 128	
HYPODORIAN	$\frac{16}{16}$ 64	$\frac{16}{15}$ $68\frac{4}{15}$	$\frac{15}{13}$ $78\frac{10}{13}$	$\frac{13}{12}$ $85\frac{1}{3}$	$\frac{12}{11}$ $93\frac{1}{11}$	$\frac{11}{10}$ $102\frac{2}{5}$	$\frac{10}{9}$ $113\frac{7}{9}$	$\frac{9}{8}$ 128	
	C 64	*D* $71\frac{5}{6}$	*E* $80\frac{2}{3}$	*F* $85\frac{7}{16}$	*G* $95\frac{5}{10}$	*A* $107\frac{2}{3}$	*B* $120\frac{5}{6}$	*C* 128	Approximate degrees of Diatonic Keyboard scale with frequencies of Equal Temperament added below for comparison

to *A* octave of the keyboard without accidentals. In this sequence, stated in modal ratios [1]

16 14 13 12 11 10 9 8

16

Diazeuxis Dorian Hypaton Dorian Meson

the bastard Hypodorian, deprived of all individuality, is seen to consist merely of the first octave of the Perfect Immutable System from Proslambanomenos. As a logical consequence, the Harmonia would have to be regarded as thus constituted:

16 14 13 12 11 10 9 8

16

Phrygian (second Tetrachord) Dorian (first Tetrachord)

Such a scale is probably the Mixophrygian mentioned by Clement of Alexandria.[2] As this species would imply a mixture of Modes, which was a device of the decadent period, it must clearly be rejected, and the right of the Harmoniai, bearing the prefix *Hypo*, to their characteristic first tetrachord be emphasized once again. Examples of the Hypodorian Harmonia embodied on the Pipes are to be found in the Records of Aulos Loret xix, Loret xxvii, xxvi, xxi, and on the Records of Flutes Nos. 10 and 16. (See also Chap. ix.) This, then, completes the scheme of the seven original Harmoniai which constitute the Modal System of Ancient Greece (see also Plate of Harmoniai with their Notation, Chap. v, Fig. 37).

THE AULOS AS ORIGIN OF THE HARMONIAI

They may be studied in the table above. The Modal System of Ancient Greece is thus revealed as a profoundly rational system having a basis at once scientific yet natural. The inception of the system is extraordinarily simple in practice. The subtlety of the ideas and values, developed within the system in order to make of it an organic whole, may come as a surprise to some modern scholars and musicians when they begin to realize how immeasurably their estimate of Ancient Greek Music fell short of the reality.

A few words must now be devoted to the part played by the reed-blown pipe in the creation of the Modes; a fuller account will be found under the heading 'Aulos'. The Modes were derived quite naturally from the boring of the Aulos, *αὐλός*, or reed-blown pipe, in the most convenient manner for covering the holes with the fingers. This must have occurred in the remote days before the beginning of historic records in Greece, and elsewhere, for instance, during the epoch signified by the ascription of the invention of the art of playing the Aulos, *τὴν αὐλητικὴν τέχνην*, to Hyagnis, the Phrygian, by Plutarch,[3] and earlier still in Chaldea, India, Egypt, Persia.

The fact that Modes and scales have originated among primitives (with

[1] For the identification of the modal ratios with the nomenclature of the P.I.S., see Chap. iv, Fig. 33.

[2] *Stromata*, i, 16; p. 789, Migne Patrol.; Strabo, 572.

[3] *de Mus.*, Cap. 7 (ed. Weil and Rein., p. 34).

the possible exception of a certain vocal scale) through the boring of pipes and flutes, and the making of Panpipes, is one that will have to be reckoned with and accepted in the face of overwhelming evidence (see ' The Survival of the Harmonia in Folk Music ', Chaps. viii and ix). The influence of wind instruments on the musical systems of the world has never yet received adequate treatment or acknowledgement.[1]

The technical aspect of the genesis of the Modes on the Aulos is not difficult to understand. The Aulos consisted of a length of the hollow, jointed river reed (*κάλαμος*) converted into a pipe by boring through the joints, thus providing an unbroken column of air. Into the end of this pipe was inserted a corn-stalk (*καλάμη*), or later a mouthpiece called Syrinx (*σῦριγξ*). The pipemaker did not rest content with his mouthpiece until he could, by thrusting it more or less deeply into the reed pipe, blow a full-throated long and resonant note, which he was able to raise or lower by natural means, viz. by tightening or relaxing the muscles of the larynx in conjunction with a proportional increase or decrease of breath pressure. The pipe acted as resonator, while the little straw mouthpiece [2] was the note-giver.

During the best period of Greek Art the part of the straw mouthpiece not taken into the mouth was concealed within one or more of those beautifully shaped bulbs depicted on Greek vases, which are familiar to all Greek students. The simple little straw is the genius of the pipe and the most important part of it ; more important even, from the musician's point of view, than the holes (*κοιλίαι*) bored laterally and stopped by the fingers, although to us, in search of traces of the Modes, the holes give the clue to the mystery. The length of the column of air, which the breath of the player sets in vibration through the little straw mouthpiece, is reckoned from the end of the pipe serving as exit, to the tip of the vibrating tongue of the mouthpiece. When holes are bored in the side of the pipe at a distance from each other which is equal, and which at the same time forms an aliquot part of the total length of the pipe, plus that of the straw mouthpiece protruding from the pipe, then a Mode is born. The birth of the Mode occurs even when the pipe-maker is an untutored musician, unconscious of scales, Modes and theories. It has already been pointed out above that it is the Harmonia alone, and not the genesis of the modal material, that is obtainable on the pipe. The Modes thus are the creation of the reed-blown pipe (and, with certain reservations, of the flute) ; they are brought about through the unconscious participation of man, who by means of his innate feeling for proportion in the boring of holes is enabled to tap the natural law embodied in the pipe.

[1] ' The Influence of Wind Instruments on the Musical Systems of the World ', by Kathleen Schlesinger, *The Roy. Coll. of Music Mag.*, Vol. xii, Nos. 2 and 3, 1916, pp. 52–8 and 80–7 ; also by the same author, ' The Significance of Mus. Insts. in the Evolution of Music ', *Oxford Hist. of Music*, Intro. Vol., Oxf. Univ. Press, 1929, pp. 91 sqq.

[2] The mouthpiece, of two distinct categories, is described in Chap. ii on the ' Aulos ', to which reference should be made for technical details (and also to Chap. v, ' Polemics ').

In time our piper chances upon another Mode by varying the length of the pipe + mouthpiece in relation to the distance between the holes, of which the total length is a multiple. Or he may vary the distance between the holes in another specimen and thus enter upon a different aliquot division by a new Determinant. Or again, a change of mouthpiece, or even the pulling out of the mouthpiece, or pushing it in further into the resonator, may produce a different Harmonia (e.g. as in Loret Aulos xxiii), without change of fundamental. To the piper the Modes would appear as a collection of seven pipes of the same length from exit to the tip of the tongue of the reed or straw mouthpiece.

From a perusal of the section on the Polemics of Aristoxenus (Chap. ii, Aulos) it will be evident that in his day it was a recognized fact that the Modal System was derived from the boring of the Aulos.

THE MODAL SYSTEM BASED UPON THE OPERATION OF THE PRINCIPLE OF EQUAL MEASURE

In summing up, it may be urged that the Modal System—based upon the operation of the principle of equal measure upon a common fundamental length, by a series of Determinants, and considered with all its implications—constitutes a new musical fact in relation to the practical and theoretical aspects of Music; it is likewise seen to be a factor in the evolution of Music, the full significance of which has yet to be realized and taken into account. The history of the theory of Music offers no analogue for a concept so momentous, yet capable of being so simply and concisely resumed by the phrase 'number and equal measure', which carries with it implications of great subtlety and of far-reaching significance. It is found, moreover, that this concept stands for a principle implicit in the elements of Music; in the laws inherent in lengths of string and columns of air, so that the primitive musician stumbles unawares upon the means of embodying this principle in his earliest attempts at instrument-making. It is found that in practice equal measure in length produces musical intervals infinite in variety, generated in delicately proportioned sequences; that these intervals are capable of being measured with meticulous exactitude and computed by frequencies to the *n*th fraction of a vibration.

The principle of equal measure, considered apart from the modal incidence of Determinants, and as origin of a minor series, is of course not unknown to theorists, e.g. to Hugo Riemann,[1] who came across it in the Arabian *Messel* method of computing intervals; to Albert von Thimus[2] and Jean Marnold,[3] both of whom, on the verge of discovering the Modal System, were deterred therefrom by their unshakable faith in the pre-eminence of the modern system. Thus, in the operation of equal measure, these writers selected for use those notes and intervals which approximated

[1] *Studien z. Gesch. d. Notenschrift*, Leipzig, 1878, pp. 77 sqq.

[2] *Die Harmonikale Symbolik d. Altertums*, Köln, 1868.

[3] 'The Natural Foundations of Ancient Greek Music', *Intern. Musikgesellschaft*, Jhrg. x, 3 April, 1909.

to those of the modern Diatonic and Chromatic scales, discarding intervals such as 11/10 and 12/11, which they deemed incommensurate. Jean Marnold's article displays great ingenuity and an original point of view directed to the solution of many problems connected with Greek Music, which speculative students cannot well afford to miss. The two bulky volumes (unfortunately lacking an index) by von Thimus contain a wealth of material used by the author to support his thesis, and to bring his conception of the two progressions of the Harmonic Series, in opposite directions, into the domain of modern harmony.

THE ETHOS OF THE MODE BASED UPON THE CHARACTERISTIC FEATURES PECULIAR TO EACH HARMONIA

Our respect for the conception of the Modal System by the Ancient Greeks becomes more and more profound as the features of Modality emerge, viz. a sevenfold differentiation, within a common octave, of Modes characterized through the identity of the individual Determinant numbers which, under another aspect, function as *Archai*. Or if the Arche be considered as the primary stimulus in the genesis of the Harmonia, then the Determinant is the instrument of the Arche in action. The position of Mese on its own degree in the Modal Scale carries with it the implication of a highly significant interval between itself and the Tonic ; the implication of a Tonality implicit in the Mode, through the double incidence of Arche's number in the Harmonic Series, and of its reflection as the characteristic ratio of the Tonic, which is thus responsible for the sequence of proportional intervals constituting each Harmonia, and forming the impassable limit of genesis. Based upon all these cardinal points is the Ethos peculiar to the Harmonia, on which Plato, Aristotle and other classical authors insist. There is, furthermore, the light which the Modal Genesis throws upon the origin of the three Genera, whereby all vague and indefinite computations of the magnitude of the Enharmonic and Chromatic Dieses are once for all eliminated and replaced by a scientifically exact basis. Finally, there is the subtle distinction in essence between Modes and species to which attention will be devoted in Chapter iv.

PROFESSOR H. S. MACRAN : THE 'OVERLOOKED FACTOR'

It is perhaps not out of place to recall here Professor H. S. Macran's pessimistic negation of all hope of advance along the lines of archaeological research ; he says (Aristoxenus, Harm., 42, p. 81) :

> Many persons are under the delusion that to solve the problem of Ancient Greek Music means to bring to light some hitherto overlooked factor, the recognition of which will have the effect of making the old Greek Hymns as clear and convincing to our ears as the songs of Handel and Mozart. Very curious is this delusion, though not astonishing to any one who has reflected on the extraordinary ignorance of mankind about the most spontaneous and universally beloved of the Arts.

This pronouncement may perhaps signify that the 'Development of Greek Music', which Macran sketched in his 'Introduction to the Elements of Aristoxenus', satisfied him absolutely. Nevertheless, the 'overlooked factor ' of equal measure, recorded for us by Aristotle, has come to light after all, and should any fragments of the true modal music of the Golden Age of Greece be retrieved in the future, they may be read by means of the modal interpretation of Notation,[1] and may perhaps enable us to realize some of the beauty and power of modal music in Antiquity. Finally, strange as it may appear to modern musical thought and feeling, the Ancient Greeks did actually use in their Modal System all the unusual intervals formed by the Determinants identified in this chapter with the genesis of the seven Modes. The most characteristic of these intervals, from the modal point of view, occurs, as the first step in the Diatonic genus of the Harmonia, starting from the Tonic bearing the ratio number of the Determinant, which is also the constant denominator for the Mode, of the segments implied by equal measure. The intervals in question bear the following ratios:

$$\frac{14}{13}, \quad \frac{13}{12}, \quad \frac{12}{11}, \quad \frac{11}{10}, \quad \frac{10}{9}, \quad \frac{9}{8}, \quad \frac{8}{7}, \quad \frac{16}{15}.$$

They will all be found duly documented in the evidence presented in the following sections. The manner in which the modal Harmoniai evolved into the Greater Complete and Perfect Immutable Systems, and the various stages entailed in the evolution, form the subject of Chapter iv. It will certainly have to be conceded that the seven interrelated ancient Modes, born of equal measure, out of which the Modal System was evolved, do indeed constitute a new musical fact of fundamental importance, not only to the past history of Music, but we believe that it also holds the germ of the future development of the Art.[2]

[1] A brief exposition of the Modal System of Notation is given in the Appendix.

[2] For the use of the Modal System of the Harmoniai in modern music, see Appendix No. 3, on ' The Music of Elsie Hamilton '.

CHAPTER II

THE AULOS: ITS SIGNIFICANCE IN THE HISTORY OF GREEK MUSIC

The Aulos as Mode-bringer. The *a priori* Claim. The Importance of the Mouthpiece. The Tonic as Starting-note bears a different Ratio in each Mode (Arist. Quint.). Equidistant Fingerholes on Aulos or Flute cannot produce Equal Intervals. The Management of the Breath-stream in playing the Aulos. The Two Types of Mouthpiece. The Primitive Double-reed Mouthpiece preserves the Integrity of the Modal Scale. The Beating-reed Mouthpiece. The Influence of Tongue-length and Width on Pitch exhibited in the Beating-reed Mouthpiece. Fundamental Structural Change in the Harmonia brought about by the Unique Properties of the Beating-reed Mouthpiece. Significance of the Aulete's Attitude while playing the Aulos, illustrated on Vase Paintings at the British Museum. The Musical and Technical Significance of the Aulete's Two Movements, while playing, which are denoted by Aristotle, Aristoxenus and Plutarch by the opposites *ἀνασπᾶν* and *κατασπᾶν*. Polemic directed by Aristoxenus against the Aulos. The Effect of increased Pressure of Breath on Pitch and Harmonics. Aristotle on the Aulos and its Mouthpieces. Theophrastus on the Mouthpieces of the Aulos. Technical and Musical Possibilities of the Double Aulos. A Change of Mode on the Aulos. Ptolemy's Reference to the Beating-reed Mouthpiece of the Auloi. The Feats of Pronomus, the Theban. Macrobius on the Position of the Fingerhole. Find of Fragments of Auloi at Meroë by Professor John Garstang. The Feat of Midas of Agrigentum

THE AULOS AS MODE-BRINGER

THE Aulos will, in this chapter, be considered primarily as mode-bringer; secondly, as a surviving record of the use of the Modes by the Ancients and by the folk in the East and West; thirdly, in its fundamental relation to the practical acoustics of wind-instruments, in so far as this branch of the science is necessary for the understanding of the part played by the Aulos in the development of the practice and theory of music in Ancient Greece, and in the Ancient East generally. Knowledge of the acoustics of the Aulos is, above all, indispensable for the correct reading of the scales, of which surviving specimens of reed-blown pipes provide a record.

The most striking fact that emerges from a careful examination of facsimiles of the instrument itself is the extreme importance of the mouthpiece, and the significant light it throws upon certain passages in the literary sources hitherto considered obscure. The unsuspected properties and characteristics of each of two different types of mouthpiece—the single-reed and the double-reed—will be found to involve important acoustic implications.

The evidence provided by the Auloi for the Greek Modes would have

PLATE 2

CONTEST BETWEEN APOLLO AND MARSYAS

Marsyas playing on long slender Auloi, each fitted with two bulbs

Bas-relief from Mantineia, National Museum, Athens. By courtesy of the Director, M. Philadelpheus

been acquired more easily had scientists not completely overlooked the acoustic problems presented by the primitive prototypes of the modern oboe and clarinet mouthpieces; for the earlier types differ from the modern in features and properties, which are of sufficient importance to have influenced the development of music in Ancient Greece. Scientists appear to have confined their experimental work on reeds to the modern more sophisticated examples. This has had the inevitable result of obscuring the avenues of research, and of foredooming to failure all attempts (made on such a basis) to reconstruct the scale, pitch and tone quality of surviving specimens of the ancient wood-wind instruments.

THE *A PRIORI* CLAIM

Now, if it can be shown how Modes and species may be produced fortuitously upon the reed-pipes through the disposition of the fingerholes and how one Mode or another inevitably results from fingerholes disposed with due regard to proportion, then the *a priori* possibility will have been established that Modes such as are described in Chapter i could exist. If, moreover, it can be proved that aborigines in various parts of the world, and the untutored folk in East and West, do at the present day so dispose the fingerholes on their pipes and flutes as to produce these same Modes and species—facts which may be proved incontestably by measurements no less than by audible tests recorded by phonograph—then, leaving the purely presumptive, we reach the safer ground of conclusive facts. Man, as we shall see, has discovered the application of the law embodied in the pipe, and while still in ignorance of the nature of the law itself he has elected to use the sounds resulting therefrom in the making of music. But, for the major purpose of this book, there would still remain the difficult and onerous task of proving that the Ancient Greeks had in fact so disposed the fingerholes on their pipes as to produce the resulting sequences of intervals of specified ratios which have been, in this work, identified with the Harmoniai of Plato, Aristotle and other writers of the classical and Graeco-Roman periods. Evidence must, moreover, be produced to show that this fortuitous production of Modes had in due course passed into the domain of knowledge and practice and that it can be demonstrated to form the basis of the Greek Modal System. Such evidence will be examined in subsequent chapters.

THE IMPORTANCE OF THE MOUTHPIECE

In order to bring a critical judgement to bear upon these questions, the reader will be in need of definite information about the behaviour and structural significance of the two kinds of mouthpiece used for the Aulos, about their relation to the reed pipe, and about certain other agencies [1] required for the production of sound in the Aulos. Records—many of them dated—have been kept of the practical experiments and research work carried out by the present writer over a period of many years on

[1] For a reference to these agencies, see Aristoxenus, p. 42M. (. . . *χειρουργίαν τὴν μὲν ἀπὸ τῶν χειρῶν τὴν δ' ἀπὸ τῶν λοιπῶν μερῶν οἷς ἐπιτείνειν τε καὶ ἀνιέναι πέφυκε*).

reed-blown pipes and their mouthpieces and on flutes. Many of the pipes preserved in Museums and Collections (which have been carefully measured by M. Victor Loret [1] and others) have been reproduced in facsimile and tested. Since the frail mouthpieces of these pipes have perished with regrettably few exceptions,[2] the difficulties involved in the testing of these facsimiles, for the determination of their scale and modality, were at first greatly aggravated until the principles at the root of modality were found to be embodied in the interrelationship of mouthpiece and pipe. Thus it is a strange fact that the secret of modality has lain dormant and unsuspected for centuries in the reed-blown pipes. The onlooker, watching the fingers of the piper uncover hole after hole in the direction from exit to mouthpiece, and hearing the resulting succession of intervals rising in pitch, hardly realized the functions of the individual agents engaged in the production of sound: that, for instance, the distance from hole to hole has no equivalent in sound, but merely represents the excess ὑπεροχή, or difference in ratio between the notes of the two holes; the length of reed made ineffective by lifting the finger from a lower hole, perforce remains silent, while the remainder speaks. The speaking length of a pipe is determined —as the piper knows full well—by the path of the breath propelled through the mouthpiece, down the bore of the pipe as far as the nearest aperture through which it can escape. The speaking lengths are therefore measured from the vibrating tip of the mouthpiece to the centre of the nearest uncovered hole.

Obviously it is impossible to determine the pitch, scale, or modality of any pipe that lacks a mouthpiece which will play it. All computations of pitch or scale of reed-blown pipes preserved in museums, or represented in drawings or sculptures, are worthless, unless the pipes have been reproduced in facsimile from measurements accurate to the millimetre, and have been provided with a mouthpiece that speaks clearly and easily in the pipe with holes closed and open.

The main factors that inevitably produce the Modes on wind and stringed instruments are the total length of string (from nut to bridge)

[1] *Encyclopédie de la Musique*, ed. Albert Lavignac, Librairie Ch. Delagrave, Paris, 1913. Première Partie, 'Egypte', Fasc. 1, pp. 17–22.

[2] The only two known to the present writer are those of the Akhmim (the ancient Panopolis) pipes belonging to M. Gaston Maspero and to M. Victor Loret. The pipe and oboe mouthpiece belonging to the latter are described and illustrated in his pamphlet, 'Sur une ancienne flûte égyptienne découverte dans les ruines de Panopolis', par Victor Loret, *Soc. d'anthropologie de Lyon*, 1893. The mouthpieces of oboe type were made of the reed itself, and according to M. Loret's description, the pipe fitted into the cylindrical stem of the mouthpiece which was covered with a layer of resinous gum. A third surviving mouthpiece of straw is mentioned by the author but not described or illustrated. The illustrations of the mouthpiece given in the pamphlet (and in the *Encycl.*, fn. 1 above) are misleading and incorrect in one important feature, viz. the thread wound round the mouthpiece, which gives it the appearance of a modern oboe mouthpiece, is an addition by Victor Loret, for the sole purpose of keeping the fragments in place (they are complete). There was no such waxed thread on either of the original mouthpieces.

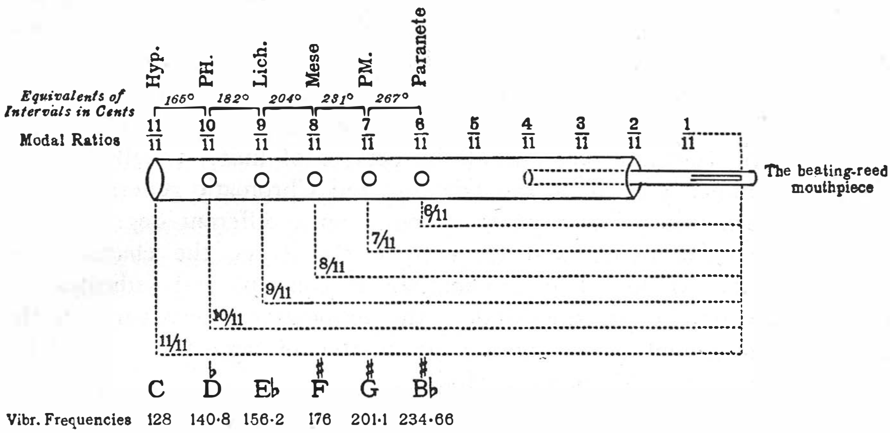
EGYPTIAN WALL-PAINTING FROM A TOMB IN THEBES, SHOWING MUSICIAN WITH DOUBLE PIPES

Note the pair of long slender reed-blown pipes : the reed mouthpieces are distinctively painted creamy yellow in contrast to the reddish-brown proper to the natural Egyptian reed used for the resonators. The wheat or barley stalks of the mouthpieces are shown to be of considerable length—as they intervene between the lips of the piper and the resonator—in addition to the vibrating portion in the player's mouth

British Museum. By courtesy of the Director

or of the column of air from the tip of the mouthpiece to the exit, divided into aliquot parts by the number Determinant of the Mode.

FIG. 21.—The Modal Scale on the Dorian Aulos
Terpander's Harmonia or the Spondeion



N.B.—It will be noticed that the increment between the exit and Hole 1 is the same as the other increments. As already stated, it is not necessary to make any allowance in respect of diameter in placing the first hole in reed-blown pipes. In the flute, however, diameter plays an important part in the determination of pitch (see Ch. VI and VII, *passim*).

The modal scale is obtained by starting with the whole string or column of air as Tonic or fundamental, and by stopping or silencing segment after segment, and playing the remainders until the octave is reached at the half-length of the string. What takes place on the Aulos, when hole after hole is uncovered in the direction from exit to mouthpiece, may be followed in Fig. 21.

THE TONIC AS STARTING-NOTE BEARS A DIFFERENT RATIO IN EACH MODE (*CF.* ARIST. QUINT.)

The Tonic represents the starting-point of the modal sequence, the note of the whole string or column of air expressed as a differentiated unit (e.g. 11/11 Dorian, 12/12 Phrygian, &c.). This conception of the Tonic probably underlies the remark of Aristides Quintilianus (p. 18M., lines 7 sqq.) when he states that the dynamic values (*δυνάμεις φθόγγου*) attached in each Harmonia to the same sign as starting-note (*σημεῖον πρῶτον*) determine the sequence of consecutive sounds that reveals the nature of each Harmonia.

Although this statement by Aristides seems to be quite clear in the light of the modal Harmonia, it appears to need a little explanation as it arises in the context. Aristides has been defining the species as they occur in the P.I.S.; then he proceeds to consider the basis of the species as Modes, all starting from the note which is expressed by the same sign (*σημεῖον πρῶτον*), i.e. in Alypius this sign is *Ω* (Omega); 'and even for those people'—he seems to say—'who put the same sign first', i.e. who use them as Modes, 'the nature of the Harmonia is made clear from the succession of consecutive sounds'.

This seems to be an implication of the difference between species and Modes and also of what is common to both, namely the succession of sounds or ratios. Evidently Aristides recognizes the fact that to some people the species, from their position as octaves in the P.I.S., are more easily identified; whereas others, who use the Harmoniai as Modes, find that the same sign (*Ω*) used as starting-note (or Tonic)—but each time differentiated as to dynamis or value (δυνάμεις φθόγγου)—reveals the nature of each Harmonia through the resulting sequence of ratios or intervals.[1]

The keynote is the ἀρχή, the root or beginning of the Mode, represented in the Harmonia by one of the octaves of unity, usually bearing the ratio number 8 or 16 in the Diatonic and Chromatic genera, and 32 in the Enharmonic. The keynote is found on a different degree of the scale in each Harmonia, and forms with the Tonic the characteristic interval of the Mode. The denominator is constant and indicates the Determinant number of the Mode; the numerators, decreasing in the direction low to high pitch, express the number of segments sounded for any given note, e.g. Mixolydian Mode.

Mixolydian Mode	14/14		13/14		12/14
Value in Cents of E.T. Semitones	C	*128·2*	‡ d♭	*138·48*	e♭
	7/6 Septimal 3rd = *267 cents.*				

When the Mode is known, the numerators may be used alone to indicate the ratios of the intervals, e.g. for the above-mentioned intervals.

EQUIDISTANT FINGERHOLES ON AULOS OR FLUTE CANNOT PRODUCE EQUAL INTERVALS

The natural instinct of man for proportion, which leads him to place the holes of a pipe at equidistant intervals and so to embody unconsciously the Modes in his pipes, involves much more than a predilection for equal or complementary measurements in spacing and ornamentation; and it is a confusion of terms to call the scales of pipes having holes bored at equal distances 'decorative scales',[2] if this description is accompanied by the

[1] See *Mode in Ancient Greek Music*, by R. P. Winnington-Ingram, Camb. Univ. Press, 1936, p. 56, where the passage comes under discussion, but with the omission of the point at issue, viz. the Mode as distinguished from the species by *the same sign taken as starting-note*, whereas each species starts on a different note in the P.I.S., and is expressed in notation by a different sign.

[2] Thorvald Kornerup, *Akustische Gesetze f. d. Akkord- u. Skala-Bildung*, Copenhagen, 1930, p. 14. E. M. v. Hornbostel, 'Tonsysteme', *Hdb. d. Physik*, 1928, Bd. viii, Cap. 9, pp. 441–2. Curt Sachs, 'Die griechische Instrumental-Notenschrift', *Zts. f. Musikwissenschaft*, 1924, März, Heft 6, Jahrg. 6, p. 296. Siegfried F. Nadel, 'Marimba–Musik', 62, *Mitt. d. Phonogrammarchivs-Kommission*, Jan., 1930. Vienna and Leipzig, Holder-Pichler-Tempsky, *A.-G.*, 1931, pp. 32 and 36—contains photographs of instruments and 7 pages of music.

For further reference see present work, Chap. i, Chap. viii, and Chap. ix, 'The Survival of the Harmonia in Folk Music', *Abhandlungen z. Vergl., Musikwissenschaft*, von A. J. Ellis, J. P. N. Land, C. Stumpf, O. Abraham and E. M v. Hornbostel.

implication that equal distances produce equal intervals.[1] This, of course, is pure fallacy, born of insufficient acquaintance with either theory or practice of these scales. The intervals of such a pipe cannot possibly be equal.

In such statements 'the equal distance between holes' has obliterated the senior partner in the modal proportion ; for the Determinant number is missing, which alone can give to each equal segment its true proportional value corresponding to the sound actually produced. The fact that the holes are equidistant, for instance spaced at 27 millimetres from centre to centre, in one or more pipes, is meaningless unless the multiple length, of which it is an aliquot part, is known ; and for this the mouthpiece is indispensable. The distance of 27 mm. is merely the modal unit, an integral factor in the proportion. In a scale resulting from a pipe of a total length (including mouthpiece) of *ten* such increments of 27 mm. the scheme of intervals produced places the equal measurement fallacy in its true perspective.

It will be noticed at once on glancing at the diagram (Fig. 22) that the symmetrical or decorative disposition of the holes upon a pipe bears no relation whatever to the sounds produced through them, nor indeed to the magnitude of the intervals ; moreover, half a dozen pipes, all having holes at the same equal distance may give entirely different sequences of intervals, according to the combined length of pipe and mouthpiece. Again, one such pipe may produce as many as three different sequences of intervals, if the mouthpiece be drawn out to the extent of one, two, or three more increments of distance. Finally, by changing the mouthpiece, any of the sequences may be played in a different key. Examination of the schemes A, B, C, in Fig. 22, reveals the fact that equal distance between the holes of a pipe results in anything but equal intervals ; that, in fact,

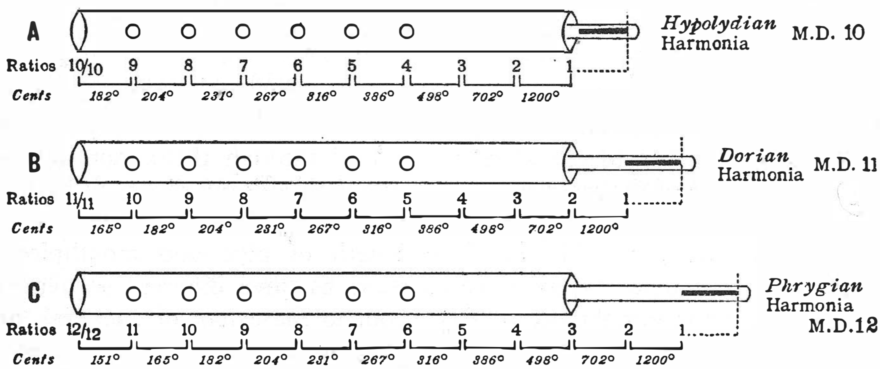
[1] In support of his epithet for the scales resulting from what he calls a secondary formation (i.e. from the aliquot division of string or column of air that produces our Modal Scale), Kornerup quotes Professor E. M. von Hornbostel thus : 'The striving after equal steps (intervals) is satisfied optically (*wird optisch Genüge getan*) = they make the distances between the fingerholes equal.' Removed from its context, this quotation is misleading and hardly does justice to the results of Hornbostel's research in this domain, further particulars of which are given in Chap. viii.)

Hornbostel's table (*op. cit.*, p. 442) shows the sequence of possible intervals resulting from equidistant holes, for instance, in the ancient Egyptian pipes measured by M. Victor Loret. But the important part played by the mouthpiece has escaped his notice, consequently his computations—apparently based upon theory rather than actual practical tests with exact facsimiles—do not correspond with theory or practice : the intervals assigned by him to the fingerholes in sequence are too small by one, two or three places in the table. This must obviously be so, since his calculations are based upon the numbers of increments of distance contained by the resonator alone, to which the mouthpiece must add one, two or three more increments. (See further in Chap. viii.)

Dr. Curt Sachs has also fallen foul of the equidistant borings of flutes and reed-blown pipes in his article on Greek Instrumental Notation when he states that : 'all (wind) instruments with fingerholes gave approximately equal intervals, because the fingerholes were bored at approximately equal distances. . . .'

it gives no indication whatever of the magnitude of the intervals to be heard as the fingers uncover the holes, one by one. The spacing of the intervals carries with it no direct correspondence in sound, it signifies merely that the path of the breath in the form of a sound-wave is increased or diminished by one or more increments of length, and the musical significance of the increment depends upon its number in the arithmetical progression of equal increments that start theoretically from the vibrating tip of the mouthpiece. For example, if the holes successively uncovered correspond to the 4th and 5th of these increments of distance, the interval heard will be that of a major 3rd, while the uncovering of holes corresponding to the 7th and 8th increments will sound the septimal or $\frac{8}{7}$ tone.

FIG. 22.—Modes (Harmoniai) resulting from the Disposition of Equidistant Fingerholes



A, B and C are 3 resonators equal in length. They have 6 fingerholes at equal distances. The increment of distance is the same for A, B and C.

The extrusion of the mouthpiece is of one I.D. in A
„ „ „ „ „ two I.D. in B
„ „ „ „ „ three I.D. in C.

N.B.—As Modal Determinants, the denominators are constant. The Numerators form the ratios of the intervals.

If the same mouthpiece be used for the Aulos in the three positions, A, B, and C, and should give the same fundamental, then the Aulos in A, B, and C plays Modes (Harmoniai); if the fundamental changes, A, B, C, give Species.

The factors which in combination produce notes of exact pitch in wind instruments are (1) length, which is determined by the diameter of the bore (subject to certain modifications more important in the flute than in the reed-blown pipe); (2) the mouthpiece, which is responsible, as a formant, for the velocity of the vibratory impulse (productive of pitch), as well as for the form of the sound-wave (productive of quality), and of its amplitude (productive of dynamic intensity and variation); and (3) the breath or wind, intimately and of necessity associated with (2) in making the pipe speak. Breath, when propelled by way of the mouthpiece through the pipe, is instinctively controlled in respect of force—resulting in volume—and of compression—resulting in density; these two factors combined with length are responsible for relative pitch. In instruments in which the amount of compression of the breath-stream varies with the other factors, some contrivance is required to contract the aperture through which the wind is fed to the pipe. In the flute and in the brass-wind the function is carried out through the muscular tension of the lips, which is automatically increased as the width of the aperture between them is reduced to produce a more incisive air reed or breath-stream.

THE MANAGEMENT OF THE BREATH-STREAM IN PLAYING THE AULOS

To ensure that a Mode shall come to birth in sound, correctly and in tune, other subconscious or instinctive proportional impulses come into play in the piper himself: reference is here made to the little-recognized function of the glottis, as the agent instrumental in compressing the air, when playing wind instruments vibrated by means of a reed. The amount of compression of breath, together with the voluminal capacity of the glottis, is instinctively controlled by sets of muscles. These are relaxed or contracted in exact proportion to the factor of length required to produce a note of given pitch, a function readily understood by singers, and in a measure verifiable by all. Closely allied to this activity of the glottis is that of breath pressure, concerning the true relation of which to pitch, misconception still exists among writers on musical instruments. It is incorrect to state, for instance, that increase of breath pressure alone can be responsible for raising the pitch of any note obtained through one of the fingerholes of pipes and flutes. To say, for example, that the compass of a pipe can be extended by producing three different notes from each hole, merely through increase of breath pressure, is only a half-truth. Once the length of the column of air to be set in vibration has been defined by the opening of a fingerhole, and the note has been produced, an increase of breath pressure alone results in greater dynamic intensity and volume of sound; but the pitch does not change unless the factor of length is altered, either by restricting the opening of the hole to half or a quarter of its area—an expedient which adds to the length of the sound-wave and lowers the pitch—or by opening another hole. The normal pitch of the note obtained from any fingerhole may, however, be raised or lowered by varying the degree of compression of the breath, obtained by tightening or relaxing the muscles of the glottis without interference with the factor of length.

Moreover, if in a modal pipe the length of the air column be shortened by opening fingerholes in succession in the direction of the mouthpiece while the breath is kept at an even pressure,[1] and the function of the glottis

[1] The reason for this becomes clear upon drawing a parallel from organ pipes. Here the length is fixed for each pipe to give one note of accurately determined pitch, and one only, in conjunction with a specially designed and fitted mouthpiece. In the reed-blown pipe, on the other hand, each lateral hole opened in turn converts the one pipe into several of different lengths.

In the organ the wind compression is fixed, the same for all the pipes. The mouthpiece of the organ pipe, whether flue or reed, is specially designed for it; set and tuned on each pipe for the note required of it at the degree of compression selected.

Of the three factors: (*a*) wind compression, (*b*) length, (*c*) mouthpiece, which suspended, the result in sound is a meaningless succession of small intervals [1] approximating to semitones which replace the modal sequence proper to the scheme of boring adopted. Attention has elsewhere [2] been drawn by

combine in the organ to determine pitch, the first alone is invariable for all pitches, whereas length and mouthpiece are inalterably fixed for each pitch.

In the reed-blown pipe, relative or proportional length is the fixed factor determined once for all by the dimensions of reed-pipe plus mouthpiece, and by the boring of the fingerholes—the length being measured for the latter from the tip of the mouthpiece to the centre of the hole. The mouthpiece is—within limits—the same for all the holes.

There remains, therefore, as variable factor in the determination of pitch for any given length, wind compression (distinct from force in blowing or pressure.)

Compression can only be obtained by a narrowing of the aperture through which the volume of air is driven, e.g. in the reed-pipe through the agency of the larynx. In the organ the amount of compression is the same for pipes of all pitches —length and mouthpiece being the determinant factors—but in the reed-blown pipe the agent of compression is free, and the amount of compression is intuitively determined by the player's manipulation of the muscles of his glottis. These are set by him instinctively for each note, together with the necessary volume (i.e. pressure of breath) of air required by the degree of compression, taken in conjunction with the length of the air column governed by the fingerholes. Any excess over the requisite proportion of volume of breath produces, as already indicated, amplitude in the sound-waves generated by the mouthpiece in the stationary column within the pipe, the effect in sound being an increase in dynamic intensity. This effect is unobtainable in the organ. What then constitutes the difference in this particular between the organ and the reed-pipe blown by the agency of the human larynx? It is this: in the organ the compression of air and the volume introduced by blowing are set for the whole organ of many pipes at different pitch, and for many registers of characteristic tone-quality: wind is the common factor upon which all computations of length and mouthpiece have been based for each pipe.

In the reed-blown pipe the adjustment of volume to compression, and of both to length, is the affair of the player, who, according to the degree of his musicianship carries this through with more or less success.

In the organ, increase of volume in blowing is distributed, as required, over all the pipes brought into play: in the reed-blown pipe, on the other hand, any excess in supply over density for a given note may produce an increase in dynamic intensity, for the note of one hole only.

It is, therefore, not correct to state that increased force in blowing alone causes variation in pitch.

[1] See A. A. Howard, 'The Aulos or Tibia', *Harvard Studies in Class. Phil.*, iv, 1893, pp. 47–60, Pompei Nos. 76891, 76893, 76894, &c. The succession of semitones from the equidistant fingerholes of Modal Auloi, as given by A. A. Howard and Victor Loret, can only occur when modern clarinet or oboe mouthpieces are used with the conventional even breath-stream. Such a sequence from a Modal Aulos is an unconscious falsification of the scale embodied in the instrument. See 'Sur une ancienne Flûte Egyptienne', par Victor Loret, *Société d'Anthropologie de Lyon*, 1893, p. 15 (illustration of mouthpiece, p. 12).

[2] See 'The Significance of Musical Instruments in the Evolution of Music', by Kathleen Schlesinger, in the Introductory Volume of *The Oxford History of Music*, edited by Percy C. Buck, M.A., Mus. Doc. Oxon, &c.; Gen. Editor, Sir W. H. Hadow (Oxford Univ. Press, 1929, p. 90).

'The Influence of Wind Instruments on the Musical Systems of the World', by Kathleen Schlesinger, *The Royal College of Music Mag.*, Vol. 12, Nos. 2 and 3, 1916, pp. 52–7 and 79–87.

the present writer to the importance of wind instruments in their threefold capacity as (*a*) law-givers, (*b*) recorders,[1] (*c*) servants of man.

(*a*) Law-givers are wind-instruments, so termed because they embody a natural law in such a manner that, when exploited by man according to his inborn instinct for proportion, the instrument imposes upon him, absolutely and inevitably, a definite scale or sequence of intervals of which he had no preconceived notion. Every law-giver is also a recorder, but every recorder is not a law-giver. The simplest form of law-giver is that of the end-blown vertical flute (or *nay*) with lateral holes, and of its aggregate form (without fingerholes), the Syrinx or Panpipe, with which we are not concerned here. The most important law-giver is undoubtedly the reed-blown pipe with fingerholes disposed at equal distances—when measured from centre to centre—along the surface of the pipe, for it has revealed to man the Modes, which he would scarcely have discovered for himself, even when experimenting with strings. Moreover, through the Greek genius the Modes were developed into the only musical system of antiquity concerning which we possess definite information.

THE TWO TYPES OF MOUTHPIECE

Since it is impossible, as stated above, in any consideration of the Aulos as bringer of the Modes, to make abstraction of the mouthpiece, which must always be included in the measurements of length, our investigation must now be directed to the characteristics of the two types of mouthpiece known to have been used upon the Aulos in Ancient Greece.

These two types of mouthpiece are the prototypes of those in use upon the modern oboe and clarinet, i.e. the double reed and the single or beating-reed. Both mouthpieces possess in common the property of converting a cylindrical pipe into what is known as a closed pipe, the pitch of which is approximately an octave lower than that of an open pipe of the same length. The open and closed pipes differ also in their production of Harmonics: the open pipe overblows the octave, and within limits, the next few overtones of the Harmonic Series, whereas the closed pipe overblows only the uneven numbers of the series, such as the 3rd, 5th, 7th Harmonics, but never the 2nd Harmonic which gives the octave. This, at least, is the accepted theory; but the little beating-reed mouthpiece made from a cylindrical length of straw (in which a narrow tongue is cut) certainly produces the octave among its extraordinary Harmonics, and frequently sounds two or even three different overtones simultaneously. The primitive forms of mouthpiece in use upon the Aulos obey without interference the laws governing the vibrations of reeds and have, therefore, little in common with their modern more sophisticated descendants. It is thus necessary, in order to form a correct estimate of the characteristic behaviour and possibilities of the Aulos, to understand the nature of its mouthpiece. The more primitive and simpler type of the two, both to make and to play, was undoubtedly the stalk of wheat or oat used as a double-reed

[1] The word, of course, is here used in the sense of preserving a record; no connexion is implied with the fipple-flute of that name.

vibrator. Picked while still soft, the shoot, cut between two knots, was slightly flattened at one end by careful pressure and stroking. When held between the lips at from 1 to 2 inches from the tip and blown with an even, gentle breath, the common wheat stalk—known by the young folk of the countryside as a ' squeaker '—was now ready for use as a musical instrument, and emitted a soft, sweet note.

The idea of boring lateral fingerholes in the pipe may have originated through the amateurish borings of the weevil or other denizen of the fields. The hole eaten through one wall of the stalk, perhaps covered at first during the blowing, and thereafter just as fortuitously uncovered, may, through the rise in pitch of the note, have suggested to the primitive mind the practical possibilities latent in length as applied to columns of air in wheat-stalks and reeds. It is thus possible that the way was prepared for the discovery of the Modal Scale by experiments suggested by the depredations of the humble weevil. The wheat or oat stalk, which later became the mouthpiece of the reed-pipe, was at first in all probability a musical instrument with fingerholes complete in itself. For although no early writer, foreshadowing Theophrastus, has recorded these facts, they are all in the way of evolution.[1]

[1] For what else is the oaten pipe of the shepherd immortalized by Shakespeare ?

' When shepherds pipe on oaten straws '.
Love's Labour's Lost, v, 2, 913.

' Playing on pipes of corn and versing love to amorous Phillida.'
Midsummer Night's Dream, ii, 2, 8.

' Pypes of grene corne '.
Chaucer (played by shepherds),
Hous of Fame, lib. iii, l. 134 (l. 1224 of poem).

' Silvestrem tenui musam meditaris avena '.
(You practise your woodland muse on a thin oaten straw.)
Virgil, *Ecl.* i, 2.

Readers may inquire whether there is evidence that the oaten pipe was actually used as an instrument. I have received from Finland two specimens of the little modal oaten pipe, reproductions of originals in the Museum at Helsingfors, presented by Miss Carita Stenbeck. The mouthpiece is of the beating or single-reed type, cut in the oat straw itself, and from the tip to the base near a natural knot = ·02, while the whole pipe from exit to tip of tongue measures ·124. The holes are very roughly cut out (not burnt) ; equidistance seems to be indicated though not carefully carried out. The scale from exit is in tune :

from exit	H 1	H 2	H 3	H 4
$\frac{A\ 13}{256}$	$\frac{C\ 11}{512}$	$\frac{D\ 10}{512}$	$\frac{E\ 9}{512}$	$\frac{F\ 8}{512}$

i.e. from Hole 1, the tetrachord of the Dorian Harmonia, tested March, 1937.

N.B.—The tongue of the second specimen was unfortunately damaged in transit.

' A pipe made from a straw stalk ' (Strohelm) is mentioned, but with condescension, among the instruments in use in his day by Sebastian Virdung (in *Musica Getutscht*, Basel, 1511, fol. Diij *vo.*) together with other small pipes, such as

THE PRIMITIVE DOUBLE-REED MOUTHPIECE PRESERVES THE INTEGRITY OF THE MODAL SCALE

The double-reed mouthpiece, freshly picked from the cornfield, yielded a spontaneous response and did not need the coaxing and taming which the more ungovernable, if more durable, mouthpiece, made later on from the same reed as the Aulos itself, required before it would speak. This primitive mouthpiece of wheat or oat straw, however, was and is still available to all: dry stalks some months or years old may be handled after soaking for an hour or two; they may then be gently flattened at one end to a distance of one to two inches; no constriction or binding is necessary. To obtain the lowest fundamental from the mouthpiece will, as declared by the ancient writers, need perseverance and much coaxing. The straw is held vertically and it will be found that when—after duly relaxing the muscles of the glottis—the straw speaks freely with a full round tone, the true fundamental of the straw has been discovered. This note will remain constant for years, as proved by dated tests, but, of course, after lying by, the soaking and coaxing have to be repeated before use.[1] In order to stabilize the pitch, it is a good practice to press the mouthpiece gently but firmly against the lower lip so that the powerful vibrations are felt. Science should be able to evolve a formula for preparing a straw mouthpiece to give a note of definite pitch when sounded alone, and another equally constant formula for the mouthpiece when wedded to the modal pipe. But the property of this simple double-reed[2] straw mouthpiece, which entitles it to be considered as law-giver and Mode-bringer, is that it preserves the integrity of the notes of the scale, according to the ratio proper to each fingerhole, without being affected by variations in the breath-pressure; it is the distance at which the lips close upon the reed mouthpiece which is the determinant of pitch in the note of the D-R mouthpiece (see Chap. iii). Thus, it may be stated that to increase the breath-pressure at any uncovered fingerhole does not alter the pitch of the note in course of production; if the double-reed mouth-piece speaks at all through a fingerhole, the intonation is always the same. But when the mouthpiece is out of practice, or does not fit the pipe, the

decoy pipes used by fowlers, pipes made from the bark of trees and from the quills of bird's feathers. (For Reprint, see Chap. vii.)

Moreover, the children of the countryside, and notably in Beds., make *Squeakers* of oat and corn stalks, with beating reed mouthpiece all in one, then pierce them with lateral holes roughly made. Baron Alexander Kraus informed me that such oaten pipes are still in use in rural districts in Italy at the present day.

[1] The primitive double-reed mouthpiece when treated as mentioned above is more musical and more tractable than the simple untreated mouthpiece, the latter responds accurately as regards pitch in accordance with the formula, and is as reliable for tests and experiments; it has the additional advantage of always being ready.

[2] Double because the lateral pressure of the lips induces a simultaneous vibration in the two parallel walls of the cylindrical straw, although they may not actually be divided into two separate blades, which was a later development of the double-reed mouthpiece.

fundamental is not always immediately available, and the pipe may make difficulties, or refuse to speak on the higher notes, thus justifying the strictures of Theophrastus and Aristoxenus.

In testing the mouthpiece alone, increased breath-pressure does not alter pitch ; in order to change pitch, the distance at which the lips press against the straw must be altered to produce a different vibrating length (V.L.) and the muscles of the glottis must function in co-operation. It is a help in tests to keep the thumbnail against the measured length to prevent any change in the V.L. By squeezing the vibrating reed near the tip, moreover, Harmonics are produced as Aristotle truly observes in the following passage concerning the perfect Auloi (τελεῖοι) 'for it is clear', he says, 'that if anyone pinches the Zeuge (ζεύγη) the sound becomes sharper and more delicate.'[1] The rest of the quotation is used further on in another context. Therefore, with a pipe blown through a double-reed mouthpiece of the simple type here described, once the fundamental has been found and the reed speaks readily and freely, the true notes of the Mode embodied in the fingerholes will be heard and no others ; the sequence of intervals is always the same. A different mouthpiece, however, may give the same sequence upon a different fundamental, i.e. in a different key. Another highly significant property characteristic of the double-reed mouthpiece alone is that of speaking freely—when happy in collaboration with its resonator pipe and in good practice—over a compass of a whole octave. This property is denied to the single-reed mouthpiece, which only possesses a natural compass of a tetrachord or at most of a fifth. (By what device the beating-reed enlarged its range will be seen hereafter.) This characteristic of the double-reed carried with it the implication of being, in conjunction with the Aulos pipe, the creator of the Harmonia, or octave modal scale ; and secondly the further somewhat paradoxical implication that surviving specimens of pipes having 3 or 4 fingerholes can no longer be regarded as necessarily indicative of an earlier stage in musical development, nor indeed in the facture of the Aulos, nor yet in the art of auletics.

Further, this property of the Aulos, when fitted with a double-reed mouthpiece, of conferring the gift of the sevenfold Harmoniai, may suggest an explanation of the fact recorded by Nicomachus, for example (pp. 7 and 33M.), that each of the seven original Modes was considered by the Ancients (and by the Chaldeans more particularly) to represent the manifestation of some force or quality inherent in one of the planets. The sphere of influence belonging to each planet was defined by Nicomachus in terms of the position which it occupied in the modal octave of the Greater Complete System—the standard nomenclature of his day.

The pipe fitted with a double-reed mouthpiece is thus an infinitely precious document, providing incontestable evidence of the origin of the

[1] Aristotle, *de audibilibus*, p. 804*a* = in Porph., *Comm. in Cl. Ptol.* (ed. Düring), p. 75.

Although the present writer possesses no aptitude for playing wind instruments, she has been able to verify Aristotle's assertion which applies equally well to double and single-reed primitive mouthpieces.

full octave modal scales or Harmoniai, founded upon a mathematical basis of proportional ratios, inevitable in their succession from a given fundamental, and having as first cause the number determinant of the Mode as divisor of the total length. The definition of the Harmonia by Aristotle will here be recalled : (The Harmonia) ' its parts, magnitudes and excesses appear according to number and equal measure.' [1]

Experimental pipes made as described above could, of course, only be considered evidence of an *a priori* nature, but the Records, Chap. x, giving measurements and the ratios of resulting scales of actual surviving specimens, show that the Aulos was in truth a law-giver and bringer of the Modes in Antiquity and that it still continues, among primitives as well as among the folk at the present day, to act in the same capacity.

It is because none of the properties which is the apanage of a Mode-bringer can equally well be claimed for the Aulos played by a single-reed mouthpiece that this species of Aulos has to be placed in a different category : that of recorder only.

THE BEATING-REED MOUTHPIECE

The pipe fitted with a beating-reed mouthpiece, on the other hand, is the instrument of the musician and creative artist who, being already familiar with the Modes, is able to make use with advantage of the more elastic sonorities and greater beauty of tone conferred by this type of mouthpiece, while consciously preserving the purity of the pipe's modality. The beating-reed vibrator is by no means as simple a device as the double-reed. For this type of mouthpiece, in its primitive form, a straw of fine satin-like texture, firm and straight, and terminating at one end in a natural knot is selected. The inner diameter of the bore should be fairly even, especially near the knot. In our own day the cutting of the stalk for the mouthpiece should be done by means of a safety-razor blade in order to make a successful incision through the silica of the covering of the straw. It is best to use a straw not less than 14 or 15 cm. long when testing for modality, but a shorter one will often suffice to play the pipe. Holding the straw firmly in a horizontal position, and the blade directed at a very acute angle, an incision is made of the width desired for the tongue at a distance of ·003 or ·004 from the knot. After having with care and delicacy raised the narrow strip—from 1 to 3 mm. in width, the blade is let down till it lies flat and almost parallel against the stalk ; it is then gently guided to form a tongue of even width for a length of 2 to 4 cm., according to the range of pitch required, or corresponding to the increment of distance for any individual pipe. The length of the straw stem in which the tongue is cut has no ascertainable influence on pitch, considered apart from the dimensions and elasticity of the tongue itself. The factor of length in the stem of the mouthpiece is mainly useful for the facilities it affords of modulating into a different Mode by pushing in or pulling out the mouthpiece to the extent of one or more increments of distance. This operation is discussed further on in connexion with modality and bulbs.

[1] Arist., *ap.* Plut., *de Mus.*, Cap. 23 (Weil and Reinach, p. 226).

When the tongue has been detached and the length selected, the edges of the aperture against which it beats may require trimming if jagged, but this is a delicate operation as the merest hair-breadth left uncovered by the tongue prevents it from speaking and renders it useless. If the mouthpiece refuses to speak when cut—or even to emit a sob—the reluctance may be overcome by (1) gently stroking the base of the tongue with the finger-nail; (2) scraping away pith from the under-surface of the tongue—with care; (3) clearing a slight obstruction in the bore of the straw; (4) by alternate blowing and suction from the exit of the straw, causing the tongue to vibrate with the former and to emit a note when the breath is withdrawn. If the straw tapers towards the knot, a tone of richer quality may be obtained by cutting off the latter and replacing it by sealing-wax, an expedient which may be used to convert a lemonade straw into a mouthpiece in the absence of a direct supply of straws from the cornfields. The mouthpiece must fit tightly into the resonator: this impermeable condition may be secured by winding strips of damped tissue paper tightly round the stalk at the proper distance from the base of the tongue, and pressing each layer in winding, so that a kind of papier mâché results. The theoretical length of a modal pipe which is divisible by the determinant number characteristic of the Harmonia, must—under perfect conditions—be an exact multiple of the increment of distance measured from centre to centre of the fingerholes. As the theoretical length includes that of the mouthpiece, in so far as it extrudes from the resonator, this latter must be taken into account.

THE INFLUENCE OF TONGUE-LENGTH AND WIDTH ON PITCH INVESTED IN THE BEATING-REED MOUTHPIECE

Slight discrepancies may, with this type of single-reed mouthpiece, be overcome in the blowing, but the accurate test of the modal sequence fails when carried out note by note, each detached and tested by the monochord, unless the mouthpiece be drawn out to the correct distance. It is in the blowing that the degree of the piper's musicianship is at its height. With the rise in pitch his breath becomes more compressed through the action of the glottis, and the pressure or volume of the air-stream, propelled through the mouthpiece, is proportioned intuitively as the sound rises and falls. The musician is able by means of the resilience and responsiveness of this type of mouthpiece to express his emotion and rapture through the changing sonorities, the subtle inflections and gradations of tone. The piper is the master of mouthpiece and resonator; he can make the pipe speak and give utterance to the music that surges up within him. But, alas! the delightful straw mouthpiece has its waywardness and failings, which the creative artist and improvisator is often able to control. The plight of Midas of Agrigentum,[1] whose victory at the Pythian games was

[1] Midas of Agrigentum won the prize twice at the Pythian Games, and also at the Panathenaea, so that Pindar's Pythian Ode xii may be referred either to Ol, 71; 3 (494 B.C.) or to Ol, 72; 3 (490 B.C.). At this time Aulos-players contended with their instruments alone ψιλῇ αὐλήσει. This feat of Midas is discussed in some detail later on where the Greek Scholium is quoted.

celebrated by Pindar, is described in the first scholium to his xiith Pythian Ode thus: 'The little tongue being accidentally broken by cleaving to the roof of his mouth, he played on the reeds, καλάμοις, alone in the manner of a Syrinx, and so delighted the audience and won the victory.' The meaning of the scholium is clear: Midas pulling out the broken mouthpiece blew across the top of the reed resonator, using the different technique of the Panpipe and its harmonic possibilities, but at the same time with the extended compass afforded by the fingerholes of the Aulos. The implications of this feat, for which Midas was celebrated as victor at the Games, are of interest and receive full consideration in due course later on.

Hundreds of these mouthpieces have been cut by the writer, measured and tested for pitch, quality of tone, elasticity, &c.; the particulars have been collected with a view to coming to some conclusion on the influence of the *length* and *width* of the tongue on the pitch of the central and lowest notes given by the mouthpiece; on the influence of the interrelationship of the diameter of the bore of the straw with (1) the length of the tongue, (2) with its width.

The general inferences will be discussed now, leaving a few examples of more detailed analysis to follow later with reference to tables in Chap. iii, which contain a selection from the most significant of the records.

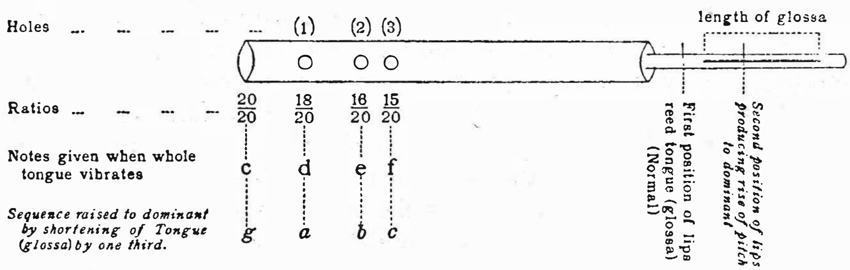
Repeated tests have clearly shown that the length of the tongue in this type of mouthpiece exercises a relative and regional influence on the range of the notes which are obtainable, not only from the mouthpiece itself, but from the whole instrument; but these notes are seldom identical in pitch owing to the incidence of the laws of resonance operating between the pipe and its mouthpiece. The length of tongue is of course limited first of all by the dimensions of the cavity of the piper's mouth, since the base of the *glossa* must be well covered by the lips. An analysis of the records determines the regional pitch of mouthpieces having a tongue-length (= T.L.) of ·04 with a width of ·002 (2 mm.) to lie between *B* and *E*, a 4th above, ex. R 1; B 12; Kn 11. The variations within the range may be attributed to the diameter of the bore and also to the clearance under the tongue which, when clean and complete, tends to lower the pitch and improve the quality of tone. The regional pitch of mouthpieces having a tongue length of ·02 and a width of ·0025 was found to lie in the octave above $C = 256$ v.p.s.: ex. R 12; B 6; B 5; B 2; B 4. Further, in mouthpieces having the same tongue-length, a narrow tongue (from 1 to 2 mm.), in conjunction with a narrow diameter of the straw under the tongue, accounts for a lower pitch. It may thus be concluded that the factor of length as a determinant of pitch in the primitive beating-reed mouthpiece, has no significance whatever as regards the total length of the straw from exit to tip of tongue. The length of the *glossa* combined with its width, is the significant factor responsible for the frequency of the pulsations, which are communicated to the air within the straw, by the vibrations of the *glossa* beating against the walls of the aperture.

FUNDAMENTAL STRUCTURAL CHANGE IN THE HARMONIA BROUGHT ABOUT BY THE UNIQUE PROPERTIES OF THE BEATING-REED MOUTHPIECE

It has been found in comparing mouthpieces that, other dimensions being equal, the ratio between the length of *glossa* in any two mouthpieces produces a rise or fall in pitch in the same ratio. Ex. B 14 and B 15 and B 11 and R 10. This finding carries with it an unsuspected implication of the greatest importance, i.e. that a rise in pitch may be obtained for the whole pipe by the simple expedient of shortening the vibrating tongue of the mouthpiece by a movement of the lips. The harmonious and unanimous collaboration of mouthpiece and resonator is such, moreover, that the rise of pitch operates in the whole pipe, changing the fundamental note of exit and of vent-hole, so that the series of notes issuing from the fingerholes, as they are uncovered one by one, is transposed according to the self-same ratio, into a higher key. It may be added that the piper, while familiarizing himself with his instrument, which frequently demands much coaxing and ingenuity in order to yield its best, can hardly fail at an early stage, to discover this property of the mouthpiece.

FIG. 24.—Momentous Significance of Shortening the Vibrating Tongue of the Mouthpiece by one-third of its length, on the Hypolydian Aulos of M.D. 20 with 3 Fingerholes

Birth of our Major Scale of duplicated tetrachords



N.B.—The pipe is sounded first with all holes closed, giving *C*; then the holes are uncovered in turn, in the direction of exit to mouthpiece. As the lips move upwards on the *glossa*, shortening it at the second position by one-third, the fingers close the holes and the pipe sounds *g* as fundamental, the octave of the modal scale being reached at *c'*, as the third hole is uncovered.

What, then, are the implications of the power latent in the beating-reed mouthpiece to raise the pitch of the pipe by shortening the length of the vibrating tongue? If the tongue of the mouthpiece be shortened by one-third of its length, the rise in pitch of the pipe's fundamental, through the application of what may be termed the modal principle of length applied to instruments, is a perfect 5th. This apparently simple property of the primitive beating-reed mouthpiece is of paramount importance for understanding the origin and development of our modern musical system. The momentous significance of such a manipulation of the mouthpiece by the lips of the piper may be realized by examining the result produced on a Hypolydian Aulos having three fingerholes, of a reduction by one-third of the length of the mouthpiece tongue.

Fig. 24 shows that the result on this simple pipe with three fingerholes is our major scale, consisting of two tetrachords, similar in structure ; [1] the one on the Tonic, the second on the Dominant with a tone of disjunction between them.

SIGNIFICANCE OF THE AULETE'S ATTITUDE WHILE PLAYING THE AULOS, ILLUSTRATED ON VASE PAINTINGS AT THE BRITISH MUSEUM

It is obvious that the shorter the tongue of the mouthpiece, the less responsive it proves to changes of pitch through breath pressure and compression through the action of the glottis.

The expedient of shortening the tongue in order to raise the pitch of the whole instrument by an interval of definite ratio cannot take effect with the same facility upon the primitive double-reed mouthpiece, and not at all, for obvious reasons, on the beating-reed, cut arghool fashion with the base or hinge of the tongue towards the knot ; for the same reason no harmonic compass is obtainable on the arghool, or on the Aulos fitted with an arghool mouthpiece. It is important to note at this juncture that most passages in the literary or theoretical sources, which mention the rise in pitch obtained by certain movements or devices, can only apply to the mouthpiece of beating-reed type, with one exception which will be duly noted.

If the Aulos had fingerholes sufficient for an octave, it would mean the power to modulate into higher keys ; but it has already been pointed out that the compass of this type of mouthpiece is a 4th, occasionally a 5th, but not more without audible straining and loss of beauty of tone. This fact explains why so many of the ancient Egyptian surviving specimens have three or four fingerholes only, at a time when the evidence of the harp-stringing reveals the use of a diatonic octave scale.[2]

The beating-reed mouthpiece, cut as directed above, will give (without difficulty in most cases) an additional harmonic compass by overblowing the 12th, and rising with the opening of each fingerhole, for the whole sequence with reedy quality diminished : the high Harmonics obtainable from the mouthpiece alone are as pure as the highest notes of the nightingale.

[1] The difference between the modal Hypolydian and the modern major in just intonation is shown by the following ratios. Identical structure would demand the transposition of the first two ratios of the first tetrachord :

Major scale in just intonation. $\frac{9}{8} \times \frac{10}{9} \times \frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} \times \frac{16}{15} = 2$

Modal Hypolydian on the Aulos in Fig. 24. $\frac{10}{9} \times \frac{9}{8} \times \frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} \times \frac{16}{15} = 2$

[2] The vertical flute might, like the Aulos, have been the fortuitous agent of the reduplication of the modal tetrachord in the Hypolydian mode (which was the origin of our major scale), if the fundamental was unmanageable, as in many of these flutes, the compass was played on the octave harmonic instead.

It is evident that with this type of mouthpiece no speakerhole is necessary in order to produce the harmonic compass readily. It is obtained by merely pressing the little tongue near the tip, but not pinching, and breathing suitably. [1]

Another device for extending the compass of the instrument without increasing the number of fingerholes consists in half or third stopping the holes, which gives excellent results with the double- as well as with the single-reed mouthpiece.

Before proceeding further, reference must be made here to the use of the word σῦριγξ [2] (Syrinx) in connexion with the Aulos. This word is interpreted by Liddell and Scott (8th edition) to mean the mouthpiece of the Aulos. If this translation be adopted, and it has been adopted by the present writer, it eliminates all difficulties and ambiguities in the text of the passages where the word is used of the Aulos. The translation of σῦριγξ as 'mouthpiece' can only apply, however, to the mouthpiece of the beating- or single-reed type.

MUSICAL SIGNIFICANCE AND TECHNICAL EXPLANATION OF THE AULETE'S TWO MOVEMENTS WHILE PLAYING, WHICH ARE DENOTED BY THE OPPOSITES ἀνασπᾶν AND κατασπᾶν BY ARISTOTLE, ARISTOXENUS AND PLUTARCH

How, it may be inquired, does the piper shorten the vibrating tongue while playing, i.e. with the mouthpiece in his mouth, his lips at the start covering the base or hinge of the tongue, and with his hands and fingers busy with manipulation of the pipe? How does this subtle control of the *glossa* strike an observer? As the piper can accomplish his purpose in two ways, the spectator describes the visible movement as a *drawing up* of the mouthpiece (Syrinx)—or rather of the lips upon the mouthpiece—with an upward movement of the head, or else by a *drawing down* when the piper actually does pull the whole Aulos down in front of him to bring his lips higher up on the *glossa* without moving his head. This is the explanation of the use of two diametrically opposite terms κατασπᾶν and ἀνασπᾶν used to account for the same result, i.e. a rise in pitch for the whole instrument. These two apparently contradictory movements, bringing about the same result, are not merely idiosyncrasies peculiar to one or another piper: each of these movements has a musical significance—they are not alternatives. The explanation is based upon the function of the glottis which has already received attention. When the spectator noticed the upward movement, the piper was not only advancing the position of his lips upon the *glossa*; he chose this way of accomplishing his purpose because he was at that time using notes of a high *tessitura* (see Plate No. 8), which could only be raised to a still higher pitch by straining at the muscles controlling the glottis, and thus compressing the air according

[1] An example of this overblowing on an Aulos without speaker-hole will be found described in Chap. iii under 'Maket 3', *Harmonic Compass with Mouthpieces* (See Chap. x, Record Maket 3).

[2] See Plut., *Non posse suaviter*, p. 1096B, and p. 1139A; also Macran, *Aristoxenus*, Notes, p. 244, and Weil and Rein., *Plut. de Mus.*, p. 82, § 196.

BLACK FIGURES ON WHITE

The piper bending low over his long slender bass pipes, with the muscles of the glottis well relaxed, is playing in a Harmonia with a low *tessitura* such as the Hypophrygian

British Museum. By courtesy of the Director

to the requisite proportion—a purely subconscious function. When the piper, on the other hand, appeared to be only drawing down the whole instrument, together with the mouthpiece, it was because the musician was then using the lower register of the pipe and he therefore needed the bent position of the head in order to relax the muscles of the glottis, and so bring the density of the breath-stream into line with the altered conditions brought about by the shortening of the *glossa*, as now applied to the same length of stationary air-column determined by the opening of the fingerholes.

The rise in pitch, consequent upon the shortening of the tongue of the mouthpiece, furnishes a clear demonstration of the fact that length in the resonator of the Aulos is in no sense a determinant of pitch. The same length of the stationary column of air in the resonator sounds, at the bidding of the piper, imposing his will through the mouthpiece, at one moment *C* as fundamental, at the next *g* and according to the same ratio for the notes given through the fingerholes. This property of the single- or beating-reed mouthpiece, hitherto undetected, may be considered a new musical fact that throws a light on the development of the scale. This property characteristic of the beating-reed alone, sharply differentiates the two classes of Auloi in Ancient Greece ; this same property and the fact that the harmonic octave can be obtained on this type of mouthpiece suggests the necessity for the reconsideration of the question of the production of the octave on wood-wind instruments. It would seem that the crux may lie with the mouthpiece itself.

The explanation of apparently simple movements of the pipe, indicated by the use of the Greek words ἀνασπᾶν and κατασπᾶν, thus draws attention to the subtle processes of adjustment, which the altered rate of pulsation, caused by the shortening of the *glossa*, imposes upon the length of a stationary column of air within the pipe, usually regarded as predetermined by the distance of the centre of any given open fingerhole from the tip of the *glossa*. That subtle change of values in the dimensions of length may be applied in a double sense : (1) to the spatial conception of the term in a stationary condition of the air within a pipe, or (2) to the element of duration in time which the living breath of the player superimposes upon the spatial length. The results of these differences in values is at once appreciated by the ear as sounds of different pitch. The difference in pitch caused by the shortening of the vibrating tongue of the mouthpiece, with its resultant quickened pulse, awakens a new response in a latent sound-wave, the length of which had until then been considered fixed. What, then, is the new factor introduced ? It is not, of course, increased breath-pressure but intensified compression, an active process initiated in the glottis—a condensation of air into a smaller space—i.e. diminished length or width.

Although the fundamental note of the pipe can be varied at will—within limits—by the piper through his beating-reed mouthpiece, the proportional relationship to the fundamental of the notes obtained from the holes remains unaltered. Proportional length does not seem to be subject to the laws of resonance,[1] so that the same relative ratios persist in the sequence of intervals, whatever the change of pitch imposed upon the fundamental by the shortening of the *glossa*. (See also further on.)

It will be seen further that when the pipes are played in pairs, and one of these as a drone, yet another factor arises to complicate the difficult subject of resonance. When these facts are considered—not scientifically, but as they strike one in practice—our deductions will be formed from the perceptions of pitch by the ear. These, again, are based upon the interval of time that elapses between the successive periodical impacts upon the ear of the sound-waves emitted by the pipe. For this, science provides an explanation, but of the part played by the spatial length of the pipe as a whole, or as modified by the opening of fingerholes, and of the coercion of that spatial length by the time-length induced by the pulsations of the beating-reed, (1) at its normal length, and (2) when shortened, a clear exposition would be welcome.

Although the flute owes its sound production to the same law of adjustment, but more limited in extent than in the reed-blown pipe, the flute does not possess the power of transposing the tonality of the whole instrument. The writer knows of no exact analogous property in a wind instrument; for although the slide trombone, for instance, reproduces the notes of the Harmonic Series on different fundamentals by the use of the slide, the change of tonality is in each key directly referable to the change in the spatial length, i.e. of the lengthening of the tube of the trombone by means of the slide. In the reed pipe the length remains the same, whether the fundamental be *C* or *G*: it is the change in the vibrating length of the *glossa* that is the responsible agent. On the string the analogue might be the rise in pitch of the fundamental note of the string induced by retuning or screwing up the peg, when every note produced upon the same string by stopping would then be of higher pitch. This suggestion was, however, open to serious objection among the Ancient Greeks from the artistic point of view; we may in this connexion recall the prohibition recorded by Plutarch[2] as having been laid upon any change of Harmonia, rhythm, or *tasis* (i.e. tension or tuning) in the Nomos, from the days of Terpander until the decadence set in.

Besides these simple methods of shortening the *glossa* and thus raising the pitch, i.e. changing the key—the movements for which are denoted in the Greek sources by the two contradictory terms *ἀνασπᾶν* and *κατασπᾶν*—there are others more spectacular, the significance of which has proved a puzzle. In one of these the Aulete stood with head bent over his pipe or pipes, held vertically down in front of him, and when the melody rose to the upper tetrachord, he swept up his Auloi horizontally in front of him, throwing back his head with a gesture of exaltation. This picture

[1] See Sedley Taylor, *Sound and Music* (Macmillan & Co., London, 1883), pp. 80, § 40, and p. 127.

[2] *de Mus.* (ed. Weil and Rein., pp. 26 and 28, § 67: 'οὐ γὰρ ἐξῆν . . . οὐδὲ μεταφέρειν τὰς ἁρμονίας οὐδὲ τοὺς ῥυθμούς. ἐν γὰρ τοῖς νόμοις ἑκάστῳ διετήρουν τὴν οἰκείαν τάσιν.'

of the artist seized with inspiration is depicted in the vase paintings and sculptured bas-reliefs, as for instance in the figures below.

The second of the methods of shortening the *glossa*, in a manner calculated to impress the spectator with a sense of wonder, may best be described first in the words of Aristoxenus, which occur in the course of his famous polemic against the Aulos and the Harmonists, beginning thus: 'Now some find the goal of the science of Harmonic in the notation of melodies, declaring this to be the ultimate limit of the apprehension of any given melody. Others again find it in the theory of the Auloi,[1] and in the ability to tell the method by which the pipe-scales are produced and their provenance.'

I give Macran's translation of the Polemic of Aristoxenus in the left-hand column, and my comments in the right.

POLEMIC DIRECTED BY ARISTOXENUS AGAINST THE AULOS

No less preposterous is the above-mentioned theory [a] concerning clarinets. Nay, rather there is no error so fatal and so preposterous as to base the natural laws of harmony on any instrument.[b] The essence and order [c] of harmony depend [d] not upon any of the properties of instruments. It is not because the clarinet has fingerholes [e] and bores and the like, nor is it because it submits to certain operations of the hands and of the other parts [f] naturally adapted

[a] Quoted above with Greek text in fn.[1].

[b] τὴν τοῦ ἡρμοσμένου φύσιν.

[c] τάξιν, i.e. the order in which the numbers forming the ratios of the harmonic progression, or more simply, of the Harmonia, follow one another from the ἀρχή or fundamental.

[d] δι' οὐδὲν 'depend', or as Meibom renders, 'propter quod'; it is not a question of dependence but of embodiment; dependence follows embodiment.

[e] Aristoxenus is here quibbling again. Of course it is not because the Aulos has fingerholes, but because of the position of these fingerholes in their proportional relationship to length.

[f] The other parts: 'τὴν δ'ἀπὸ τῶν λοιπῶν μερῶν οἷς ἐπιτείνειν τε καὶ ἀνιέναι πέφυκε,' i.e. the breathing apparatus, and more especially the muscles of the glottis.

[1] *Harm.*, p. 39M. (p. 130, Macran): 'οἱ δὲ τὴν περὶ τοὺς αὐλοὺς θεωρίαν καὶ τὸ ἔχειν εἰπεῖν τίνα τρόπον ἕκαστα τῶν αὐλουμένων καὶ πόθεν γίγνεται.'

Macran's commentary on the passage, p. 270 (*b*)—'the equally absurd theory, which basing the law of harmony on the construction of clarinets, reduces musical science to the knowledge of instruments and their construction'—adds nothing to the context. The harmonic basis does not lie in the construction but in the embodiment (unconscious) of the laws of the Harmonia, devolving upon all applications of aliquot divisions of length. See Chaps. i, iv and v, and App. 'Notation'.

to raise and lower the pitch,[g] that the 4th and the 5th and the octave are concords, or that each of the other intervals possesses its proper magnitude.[h]

For even with all these conditions present, players on the clarinet fail for the most part to attain the exact order[j] of melody, and whatever small success attends them is due to the employment of agencies external to the instrument, as in the well-known expedients [1] of drawing the two clarinets apart and bringing them alongside,[k] and of raising and lowering the pitch by changing the pressure of the breath.[l]

Plainly then, one is as much justified in attributing [k] their failures as their success to the essential nature of the clarinet. But this would not have been so if there was anything gained [m] by basing harmony on the nature of an instrument. In that case, as an immediate consequence of tracing melody up to its original in the nature of the clarinet, we should have found it there fixed, unerring and correct.[n] But as a fact neither clarinets, nor any other instrument, will supply a foundation for the principles of harmony.[p] There is a certain marvellous order which belongs to the nature of harmony in general; [p] in this order every instrument, to the best of its ability, participates under the direction of that faculty of sense perception (*αἰσθήσεως*) on which they, as well as everything else in music,

[g] See *ante*.

[h] No one in his senses would assert that the existence of the concords is due to the fingerholes and other properties of the clarinets; but these fingerholes and the like determine which of the concords or other intervals may be obtained on any particular Aulos, and most certainly whether each is possessed ' of its proper magnitude '.

[j] *τοῦ ἡρμοσμένου τάξεως* : the onus passes from the instrument to the player. This confusion between the laws embodied in the instrument, whether played upon or not, and the skill or absence of it on the part of the piper, is unworthy of the musicianship that Aristoxenus arrogates to himself.

[k] This is pure sensationalism! The movements specified bring about a change of key in the whole instrument; the significance and effect of the movements on the mouthpiece are explained a little further on, together with other references of similar import.

[l] Merely affects single notes.

[k] Both expedients are relative and are contingent upon the cut and proportions of the mouthpiece.

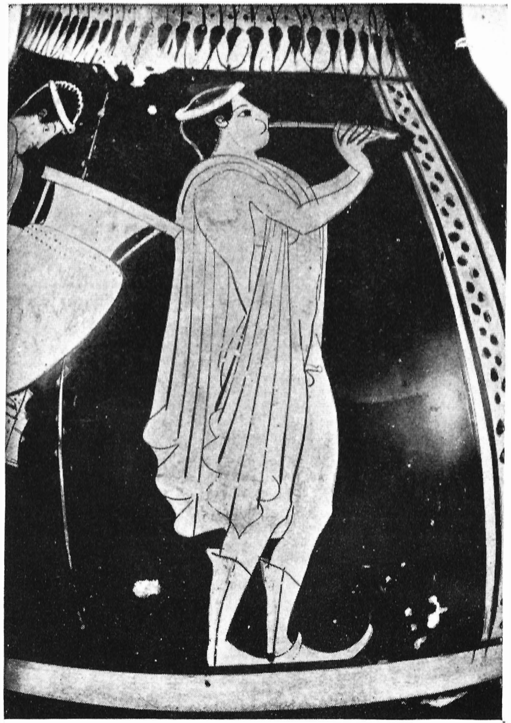
[m] Quibbling again.

[n] And this is exactly what is found embodied in the very nature of the Aulos, wherein what is fixed through the fingerholes remains as a record, and persists as long as the instrument itself. What the piper makes of it is a different matter. This melos, ' fixed, unerring and correct ', is the prerogative of the Aulos with double-reed mouthpiece which alone ensures constant results.

[p] *τὴν τοῦ ἡρμοσμένου φύσιν.* The

[1] See also Plut., ' Non posse suaviter ', Cap. xiii, 1096B, and Didot, Vol. ii, p. 1339.

PLATE 5

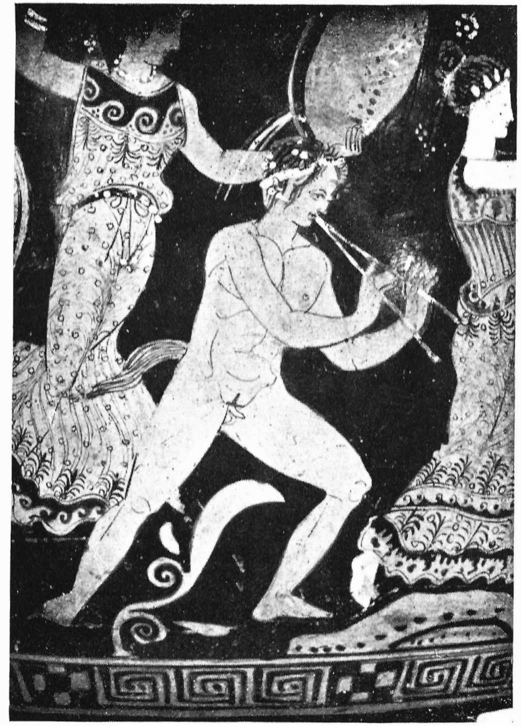


E. 351

The Piper has thrown back his head in order to bring his lips higher up on the *glossa* of the Aulos ; a movement that results in a rise in pitch

British Museum

PLATE 6



F. 75 Late 4th C. B.C.

REVEL WITH MAENADS

The Piper has spread his pipes out at an angle in order to raise the pitch by pressure of his lips on the vibrating tongues of the mouthpieces

British Museum

By courtesy of the Director

finally depend.[r] To suppose because one sees day by day the fingerholes the same[s] and the strings at the same tension, that one will find in these harmony with its permanence and eternally immutable order[s]—this is sheer folly.[s] For as there is no harmony in the strings save that which the cunning of the hand confers upon them, so there is none in the fingerholes save what has been introduced by the same agency.[t] That no instrument is self-tuned,[u] and that the harmonizing of it is the prerogative of the sense perception, is obvious and requires no proof. It is strange that supporters[v] of this absurd theory can cling to it in the face of the fact that clarinets are perpetually in a state of change; and of course what is played on the instrument varies with the variation in the agencies employed in its production. It is surely clear then that on no consideration can melody be based on clarinets; for, firstly, an instrument will not supply a founda-

Aulos embodies the natural order of Harmonic or of the Harmonia. Of this marvellous order of harmony, τὸ ἡρμοσμένον, Aristoxenus supplies no explanation whatever; this in a polemic of such a nature is a serious omission. If he had been able to give the ratios in their order, the whole challenge would have been settled at once.

[r] The faculty of sense perception enables the musician to obtain the best results from his Aulos, but if the order of harmony were not embodied in the pipe, through the position and proportions connected with the fingerholes, the faculty of sense perception would be helpless.

[s] This is exactly what a modal disposition of the fingerholes provides. The Harmonia is to be found there with its eternally immutable order.

[s] The folly is not where Aristoxenus would place it.

[t] This, of course, to borrow the phraseology of Aristoxenus, is a preposterous error, which those who have read Chaps. i and v will detect for themselves.

[u] There can be no question of *tuning* the Aulos; what lies embodied in it through the position of the fingerholes in relation to the proportions of the whole instrument, can only be realized to the full by the piper through a mouthpiece properly adapted and in place in the instrument; his sense perception and musicianship direct his operations, but can do nothing of themselves to bring forth a Harmonia that has not been embodied in the Aulos. That alone is the true foundation of the Harmonia.

[v] We note that the supporters of the theory cling to it, in spite of any aberrations or imperfections of the

BAS-RELIEF FROM HERCULANEUM. REVELS BY BACCHUS WITH FAUN AND BACCHANTE

The Faun is depicted as a skilled Aulete, playing on the double pipes spread out at an angle ; the straining of the muscles of the glottis denotes the high *tessitura* of the melos

Museo Nazionale, Napoli

tion for the order of harmony ; [w] and secondly, even if it were supposed that harmony [w] should be based on some instrument, the choice should not have fallen on the clarinet, an instrument especially liable to aberrations,[x] resulting from the manufacture and manipulation of it, and from its own peculiar nature.

instrument, or of the piper who plays upon it.

[w] τὴν τοῦ ἡρμοσμένου τάξιν, see above. The foundation of the Harmonia as already stated, and of the science of Harmonic founded upon it, is fixed in the modal Aulos, and remains an unchangeable record for the life of the instrument. What the piper makes of the law, how he breaks or modifies it, does not alter the foundation of the Harmonia seated in the Aulos.

[x] The Aulos nevertheless deserves the strictures of Aristoxenus in a certain measure, for its waywardness and frequent intractability, and for the absence at times of that ' sweet reasonableness ' dear to the heart of the piper. But in those golden days the piper was first composer, and then executant ; it was expected of him that he should be able to improvise on all occasions.

This long and impassioned polemic of Aristoxenus is directed first of all against the ' fatal and preposterous error ' of basing the natural laws of harmony (i.e. of the Harmonia)[1] on any instrument ; the text makes it clear that it is to the Aulos [2] that the indictment is more especially directed —but the real grievance is the strong support given to this theory by an important section of the musical world, the rival school of the Harmonists.

It has been considered advisable to reproduce in full this long polemic, despite its many repetitions, for the following reasons :

(1) To draw attention to the status of the Aulos and its scales among musicians during the lifetime of Aristoxenus and his predecessors.

[1] The fact may be recalled here that the *natural law* to which reference has been made in the polemic, i.e. that which is known as the physical basis of sound and hearing—the Harmonic Series in ascending and descending progressions, with all their implications in resonance, modality and tonality—form the basis of the Harmonia itself. For the Harmonia's very existence depends upon the peculiar quality and function of its Arche in the ascending Harmonic Series from the fundamental, while its modality is consequent on the leadership of the Arche as the One, the generator, the Hegemon, in its descending course back to the fundamental from which the original Harmonic Series started.

[2] We should be able to judge of the extent of Aristoxenus' knowledge of the structure and theory of the Aulos and its mouthpieces if the treatise ' περὶ αὐλῶν τρήσεως ', attributed to him by Didymus, according to Athenaeus (xiv, 634), were extant.

(2) To suggest that the Harmonists and others claimed, or acknowledged the claim, that the Harmonia originated on the Aulos.

(3) That the Aulos-scales or Harmoniai were related through their keynotes by intervals other than tones and semitones, which Aristoxenus —not distinguishing between them—evaluates approximately at three-quarter tones.

It is evident that Aristoxenus had no practical acquaintance either with the structure or with the technique of the Aulos; although he had undoubtedly observed the agencies employed by the Auletes in the production of sound and had noted the results of their movements.

The foundation for the order of harmony ἡ τοῦ ἡρμοσμένου τάξις, i.e. the sequence of superparticular ratios, is of course proportion, *embodied*, not supplied by, the boring of fingerholes at equal distances; and with the total length, from exit to tip of mouthpiece, as multiple of the increment of length from centre to centre of the fingerholes. Once the fingerholes have been correctly bored, ' Harmony with its permanence and eternally immutable order ' is inevitably installed as a record in the Aulos.

Since the expedient of drawing the pair of Auloi apart and bringing them alongside again, with the resulting rise in the pitch of the whole instrument, is used as an illustration in the argument pressed by Aristoxenus, it is clear that the polemic, as it stands, can only apply to the Aulos played with a beating-reed mouthpiece. The question of skill, or the lack of it, in the performer has no bearing whatever on the embodiment of the principles of harmony in the Aulos. With a double-reed mouthpiece, the Aulete would bring out the order of the Harmonia, the foundation or original of melody, ' fixed, unerring and correct ' time after time unfailingly, whereas with the beating-reed mouthpiece, the purity of the Harmonia, though inherent in the pipe itself, was at the mercy of the artist or virtuoso in the course of its transmission.

The significance of quotations from other theorists may now be investigated. Plutarch mentions certain problems concerning the Auloi, the rhythms and the Harmoniai, which interested Aristotle, Theophrastus, Dicearchus and Hieronymus, ' such as why the narrower of the equal Auloi, τῶν ἴσων αὐλῶν, plays lower [and the wider plays higher]? '[1] τῶν ἴσων αὐλῶν refers to the Auloi of approximately equal length played in pairs, while *narrow* and *wide* refer to the bore. It will be noticed, moreover, that it is to the calibre of the bore and not to the length that the pitch of the instrument is referred. It is well to bear in mind the fact that pitch cannot be computed from the length in the reed-blown pipe, more especially when the resonator alone is measured. Pitch is the affair of the reed mouthpiece in the first place, and unlike the flute, the Aulos resonator has no absolute pitch to be determined by the formula based upon the velocity of sound in air at a given standard temperature, and subject to the law of diameters in the form of (so-called) end-correction. In fact,

[1] Non posse suaviter, Cap. xiii, p. 1096B. The text is sometimes emended, to read ὁ στενώτερος <ὀξύτερον, ὁ δ' εὐρύτερος> βαρύτερον φθέγγεται (cf. Plut., *de Mus.*, ed. Weil and Rein., § 196, note, ' The narrower plays higher, the wider plays lower ').

the influence of diameter upon pitch (which implies additional length and gravity, i.e. the greater the diameter, the lower the pitch) works inversely in the reed-blown pipe, as stated in Plutarch : 'the narrower the bore in relation to length, the lower the pitch, and the greater the number of harmonic overtones perceptible in the timbre of the instrument. From the buzzing tone of long narrow pipes comes the frequent allusions to their droning (see Plate No. 4) : the episode in the Acharnians of Aristophanes (line 866), for instance, in which he describes a band of Theban Auletes as 'Bumble-bee pipers', *βομβαύλιοι*, a comic compound of *βομβέω* and *αὐλός*, with a play on *βομβυλιός*, 'the buzzing insect'.[1]

THE EFFECT OF INCREASED PRESSURE OF BREATH ON PITCH AND HARMONICS

Plutarch then asks :

> Why is it that through the drawing up of the Syrinx [the mouthpiece of the Aulos—K. S.], *ἀνασπωμένης τῆς σύριγγος*, the Aulos is sharpened in all its notes, and through the letting down [or reclining, *κλινομένης*] of the Syrinx the pitch is lowered again ? And when one Aulos is brought near to the other it sounds lower, and when it is drawn away it sounds higher . . . ?

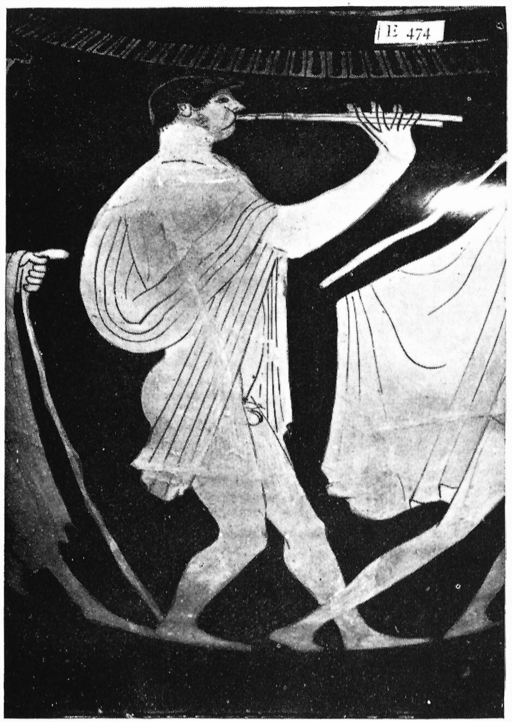
Plutarch uses the expressions *τῆς σύριγγος ἀνασπωμένης* to characterize the movements of the piper who, in order to produce a rise in pitch (or change of key) for all the notes of his Aulos, appears to be drawing up the Syrinx or beating-reed mouthpiece into his mouth ; and then when he indicates the return to the lower compass, Plutarch uses the expression *τῆς σύριγγος κλινομένης*. In this passage both rise and fall of pitch are characterized as effects of the movement of the Syrinx up and down, as it appears to the onlooker. In a similar passage by Aristotle (*de Audibilibus*, 804*a*), the Syrinx is *drawn down* (*κἂν κατασπάσῃ τις*) to make the pitch rise, or else the Zeuge are pinched. Aristoxenus also uses *κατασπᾶν* in referring to the highest limit, from its lowest note, of the compass of the Aulos with the Syrinx pulled down (21M.).

An explanation has already been given above of a similar feat, viz. the somewhat sensational expedient, mentioned by Aristoxenus, of the Aulete who holds the pair of pipes side by side in front of him, with the mouthpieces lying normally in his mouth, and speaking at their normal pitch. He then draws the pipes aside at an obtuse angle,[2] so that the mouthpieces are gripped one at each corner of his mouth by the lips, a movement that automatically shortens the reed-tongue, and raises the pitch of the whole instrument by an interval, the ratio of which is in exact proportion to that of the amount by which the reed-tongue has been shortened. The expedient was already a familiar one in the days of Aristoxenus, who quotes it in his polemic against the Aulos as 'well known'. But Aristoxenus wrongly

[1] See Kathleen Schlesinger, 'Researches into the Origin of the Organs of the Ancients', *Intern. Mus. Ges.*, Jahrg. ii, Heft 2, Jan.–March, 1901, pp. 198–200 ; see also Plato, *Crito* 54D ; Jowett's translation, 'This, dear Crito, is the voice which I seem to hear murmuring in my ears like the sound of the flutes (Auloi) in the ears of the mystic ; that voice, I say, is humming in my ears.'

[2] See Plate No. 6.

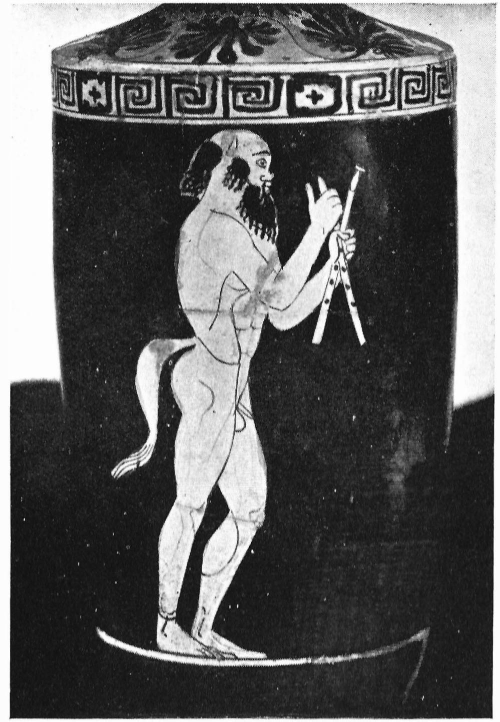
PLATE 8



Aulete with head thrown back and muscles of the glottis tense, as though playing in a Harmonia having a high *tessitura*, such as the Mixolydian or the Lydian. There are two bulbs visible; the beating-reed mouthpiece is in his mouth, his lips are firmly compressed

British Museum

PLATE 9



E. 583. Early 5th C. B.C.

Silenos changing the bulbs on his Auloi. He has taken one off the left-hand pipe and is fitting one on the right-hand pipe

British Museum

By courtesy of the Director

attributes the rise and fall of pitch to a change in breath-pressure; for this would not be effective for more than one note and could not be said to raise the pitch of the whole instrument. The change in the pressure of breath is only contingent upon the modification in the length of the reed-tongue which is the definite, deliberate causative of the higher pitch. An illustration of the device described by Aristoxenus is provided by a fine bas-relief from Herculaneum depicting Bacchic revels in which a faun is seen playing upon double Auloi, drawn out at the corners of his mouth so that the two pipes now form an angle. The unknown sculptor was an adept in at least two Arts, and shows an intimate knowledge of the Aulos and of the devices used by Auletes to raise and lower the pitch of the pipes. The sculptor was likewise a student, not only of the anatomy of the human body, but also of the psychological reactions of the larynx and more especially of the glottis in the production of music of a high *tessitura*: the muscles of the glottis have been depicted as strained to the utmost, a musical necessity borne out by the fact that only one bulb is visible on each Aulos, a fact which emphasizes the shortness of the reed mouthpiece necessary for the high pitch and *tessitura* of the melos he was playing. A copy of this bas-relief in the British Museum ('Maenad and Satyrs', Roma Vecchia) justifies this eulogy, by the very absence of the muscular response of the body to the musical urge in the mere copy.

In order to test the correctness or otherwise of Aristoxenus' statement, 'The Lady Maket' Aulos with three holes was tested to find out whether increased breath-pressure, once the note from a fingerhole has been formed, can raise the pitch. The test gave a negative result: the power of the note is intensified, the tone becomes richer in Harmonics; but to raise the pitch of the note, the muscles of the glottis had to be brought into play and tightened (Test A, Lady Maket 3, June 5, 1931).[1]

But with the straw mouthpiece alone, away from its resonator, it is a different matter altogether; every slight increase of breath-pressure raises the pitch proportionately, but the factor of length in the air-column of the resonator is absent, and that is the crux of the matter. Other tests undertaken during the same year are of interest in this connexion. With the beating-reed mouthpiece, R 2 played alone (without resonator), *G* of 96 v.p.s. was sounded with a clear, strong note; then immediately without a break, merely by increasing the tension of the muscles of the glottis, the octave sounded, pure and in tune at 192 v.p.s. It is worth noting that these straws are cylindrical in bore and, being played by the tongue of the mouthpiece, might be expected to react as closed pipes, giving only the Harmonics of uneven numbers, yet the octave was actually available (Test B on R 2, June 4, 1931). As in the case of the test on the mouthpiece of Lady Maket 3, the single-reed mouthpiece R 4 likewise confirmed the ruling that increased breath-pressure *does* produce a rise in pitch in the note of the mouthpiece, but only for the reason that the factors of length and resonance (i.e. through the fingerholes) are not brought into play (Test C on R 4, June 4, 1931). The octave was also sounded through

[1] See Chap. x, Records.

the exit of the pipe with a single-reed mouthpiece, K 10, inserted into Lady Maket 4 (with four holes), with an extrusion of ·075 ; with normal use of mouthpiece the note given was $F = 88$ v.p.s., then with tongue of mouthpiece shortened to half, the octave $F = 176$ v.p.s. sounded clear and true. It was found possible, moreover, with two good specimens of mouthpiece, that give Harmonics easily, to place both in the mouth at once, and to obtain Harmonics simultaneously from both. (The present writer, however, could not control the Harmonics at will; but it is probable that expert players could do so easily.)

Re Tests. The dimensions in metrical measurements of the straw mouthpieces are as follows:

FIG. 25.—Tests on B.-R. Mouthpieces for Regional Pitch ; Octave Shift on Tongue of Mouthpiece and Harmonics

Record Mark	Length of Straw	Diameter	Length of Tongue	Width of Tongue	Regional Pitch	Remarks
R 2	·147	·0025	·04	·002	$\frac{G}{64}$ to $\frac{C}{128}$	
R 4	·149	·0035	·04	·002	$\frac{E}{128}$ to $\frac{B}{128}$	
K 10	·152		·04	·002	$\frac{F}{64}$ and $\frac{F}{128}$	WITH octave shift on tongue
R 7	·103	·0025	·03	·002	$\frac{B\ 12}{64}$	Harmonics $\frac{B}{128}$
R 12	·114	·003	·02	·002	$\frac{D}{256}$	$\frac{D}{512}$ and $\frac{A}{864}$

ARISTOTLE ON THE AULOS AND ITS MOUTHPIECES

The following passage from Aristotle contains a reference to each of the two kinds of reed mouthpieces used for the Aulos : ‘ If one pinches the Zeuge, the sound becomes sharper and more delicate, as also if one draws down (*κἂν κατασπάσῃ τις τὰς σύρριγγας κἂν δ'ἐπιλάβῃ*) the Syrinxes, and if one grasps or holds [the Syrinx] the volume of the sound becomes very full through the pressure of the breath, as in the notes from thicker strings.’ [1]

[1] (See also Porphyry in J. Wallis, *Opera Mathem.*, Vol. iii, p. 246, where the whole of *de Audib.* is given.) *de Audibilibus*, 804*a* = Porph. Comm. in *Cl. Ptol.*, ed. Düring, p. 75 ; Didot, Vol. vi, p. 661. An English translation is contained in the volume ‘ *De Coloribus* ’, &c., by T. Loveday and E. S. Forster, Oxford (Clar. Press, 1913). The translation of this passage is open to serious objection ; ‘ if you

In the opinion of the present writer, Zeuge here refers to the double-reed mouthpiece of the Aulos in general, whether the instrument be used singly or in pairs, and the text describes the manner in which the Harmonics are produced with a primitive double-reed mouthpiece. This could not apply to the single-reed mouthpiece, the Syrinx, which as already indicated is undoubtedly the beating-reed for the obvious reason that it is the only kind of mouthpiece which can do the things with which the Aulos is credited. When the Syrinx appears to be drawn downwards, the vibrating tongue is automatically shortened by the lips, and the sound rises in pitch. Aristotle thus draws attention to the two expedients used by Auletes for raising the pitch, or changing the key, of the whole instrument: (1) by overblowing as a result of pinching the double-reed at the tip; or (2) in the case of the Syrinx mouthpiece, of raising the pitch of the Aulos by a definite interval, e.g. 4th or 5th, determined by a proportional shortening of the little vibrating tongue of the single- or beating-reed type of mouthpiece. Both expedients, it will be noticed, are used to transpose the scale given through the fingerholes of the resonator. As a contrast Aristotle then explains the effect of closing all the fingerholes, whereby the pitch drops down to the fundamental tone of the instrument, and the volume of sound becomes far greater, like the notes produced by thicker strings. This applies to both kinds of instrument, with double- and with single-reed mouthpieces, but the shortening of the tongue cannot be carried out on the double-reed mouthpiece,[1] nor on the single-reed cut in arghool fashion, i.e. with the base of the vibrating tongue at the knot end of the reed.

THEOPHRASTUS ON THE MOUTHPIECES OF THE AULOS

Theophrastus,[2] writing as a naturalist, rather than as musician, gives a precise account of the growth and cutting of reeds for the mouthpiece of the Aulos from the species called *Kalamos Zeugites*. The best Zeuge were cut from the middle joint of the whole reed.

And few turn out well in the making [he adds]; until the time of Antigenidas[3] [the celebrated Aulete who flourished during the reign of Alexander the Great],

---

compress the *mouthpiece*' (for Zeuge) obliterates the distinction between the two kinds of reed mouthpieces; and to render ἐπιλάβῃ by 'if one stops up the exits' makes nonsense, for there would be no sound if the exits were stopped up: it should be fingerholes. The actual meaning is, however, clear, as will be seen further on, in our text. The Preface to *de Audibilibus* by the translators states that, 'This tract appears to be a fragment of a larger work. It is certainly not Aristotle's, and has been ascribed with some likelihood to Strato. Prantl's text in the Teubner edition (1881) has been used. Mr. W. D. Ross's advice has again been invaluable to us.' T. L. and E. S. F. See Ing. Düring's edition, *Ptolemaios und Porphyrios* (German translation).

[1] It is actually possible to shorten the vibrating portion of the double-reed mouthpiece, and to obtain a rise in pitch thereby, but the change cannot be effected with ease and musically, as with the beating reed; it must, in fact, be regarded merely as a theoretical possibility, unused in practical music.

[2] *Hist.*, Pl. iv, 11, 4.

[3] See also Plut., *de Mus.*, ed. Weil and Rein., § 198, pp. 82–5.

when they played the Aulos naturally (ἀπλάστως), the time to cut the reeds was when Arcturus rose in the month Boedromion [September]. For those thus cut did not become useful until many years afterwards and needed much preliminary practice with the Aulos to get the proper pitch (προκαταύλησις), but the outlet of the tongues is well closed in practice (συμμύειν δὲ τὸ στόμα τῶν γλωττῶν).

This observation undoubtedly refers to the action of the double-reed, alternately opening and closing the aperture between the two blades of the mouthpiece in the act of vibration. As the manipulation of the reed mouthpiece takes place out of sight within the mouth of the piper, the naturalist may be excused for imagining that the sound was produced by the closing only of the tongues. This is made clear from the elder Pliny,[1] paraphrasing Theophrastus, who states that 'the *Tibiae* had to be tamed by much practice, and taught to sing with the ligulae pressing themselves together'.

Practical experience with both kinds of mouthpiece makes it certain that Theophrastus and Pliny are here referring to the double-reed variety. Theophrastus then continues :

But when they changed to elaborate playing (εἰς τὴν πλάσιν μετέβησαν) the cutting also was changed. They cut nowadays in the month Skirophorion and Hekatombaion, at the solstice or a little later. And they say that they become useful in three years, and need but little preliminary playing upon, and they say that the tongues (γλώττας)[2] curb the beatings (κατασπάσματα) and this is necessary for those who play in the elaborate style.

It is not improbable that this beating-reed was new to Theophrastus; the change in the music to elaborate playing, for which the mouthpiece here described was being used, implies the single-reed : the instrument of the virtuoso, the improvisatore, not the older instrument used for the Spondaic hymns and the auletic nomos which had the mouthpiece with a double-reed, and was in use up to the time of Antigenidas. All this is in accordance with historical facts. Theophrastus does not seem to have used the word Syrinx for the beating-reed mouthpiece.

We learn in addition that :

the best sections of the reed for making mouthpieces are the middle ones, those cut near the root are very hard [shrill—K. S.] and are suitable for the left-hand pipe ; the tongues cut near the tip are very soft and are suitable for the right-hand pipe. And the tongues cut from the same section harmonize.

Elaborate playing on the Aulos demands great elasticity in the tongue of the mouthpiece ; this may be increased by scraping and thinning the back of the tongue, a delicate operation which has the effect of lowering the pitch of the mouthpiece and greatly increasing the beauty and sonority of

[1] *Nat. Hist.*, 16, 36.

[2] Gevaert, *Probl. Mus. d'Aristote*, pp. 348–9, note 4, translates this passage thus : 'et les languettes se prêtent aux intonations abaissées ce qui est indispensable pour les aulètes qui pratiquent le style figuré'. If κατασπάσματα refers to the action of the lips on the reed tongue, brought about by the pulling down of the Syrinx, then the effect on the intonation of the Aulos would be the opposite of what Gevaert imagines, i.e. the pitch would be raised, not lowered.

the tone. Theophrastus does not touch directly upon the elasticity of the *glossa*, but the implication is there.

The question of the exact function and potentialities of the double Auloi now arises, primarily from the important viewpoint of the instruments' own essential nature, which may be considered in relation to length and diameter, mouthpiece, the boring of fingerholes, and various external agencies and contrivances. A survey of the vase paintings at the British Museum and elsewhere immediately impresses one with the fact that nearly all the Auloi played in pairs are of equal length, as Plutarch [1] stated in the passage quoted above. But length, as already indicated, is no criterion of absolute pitch in reed-blown pipes: they are but resonators. The initiation of pitch is solely the affair of the mouthpiece. For obvious reasons, not all the possibilities conferred upon the Monaulos by either of its two types of mouthpiece can be brought into play on the double Aulos. The production of Harmonics, which requires a highly skilled manipulation of the reed tongue by the lips, is certainly possible as a feat on both beating-reeds simultaneously, but the Harmonics frequently come at their own sweet will; to control them from both mouthpieces simultaneously, if possible at all, would require an expert piper and a high degree of musicianship.

After all that has been said about the balance of compression through the glottis, and of the volume or pressure of breath during propulsion through the mouthpiece, which are proportional for each hole uncovered, it is surprising to find that these subconscious activities of the larynx may be exercised under still more complicated conditions. It is not only possible to play simultaneously two beating-reed mouthpieces, having tongues of different length, width, elasticity and, therefore, pitch, but as many as four or five, carefully disposed in the mouth, can be operated simultaneously by the same breath-pressure and compression, but with a fourfold differentiation in pitch and expenditure of breath.[2] The experiment at first trial produces a spontaneous polyphony, which is always interesting, and not inharmonious. With practice and musicianship the possibilities are amazing.

TECHNICAL AND MUSICAL POSSIBILITIES OF THE DOUBLE AULOS

This slight digression was necessary to enable us to make a practical estimate of the kind of music that could be produced on the double pipes. It is evident that unless means are taken to prevent it, both pipes will sound simultaneously, so long as the breath is propelled through the two mouthpieces. By implication, therefore, the two pipes, each having three or four fingerholes, could not be used successively to produce a range of an octave, unless the two pipes could be induced to speak separately. The obvious expedient is to stop the exit of one or both pipes with wax or pitch to ensure silence when the holes are covered. In such a case the division

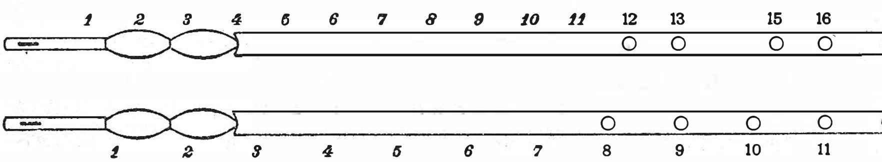
[1] *Non posse suaviter*, p. 1096B; *τῶν ἴσων αὐλῶν.*

[2] The implications from the behaviour of these choral mp.s form an obvious confirmation of the acoustic theories advanced above, *passim.*

by the Modal Determinant takes effect from the centre of Hole 1, used as vent-hole when the pipe is speaking, and the distance of the first hole from the exit is incommensurate and irrelevant. Numerous examples of such boring seem to occur in the vase paintings in which the piper's fingers appear to cover holes near the middle of the pipes. This, however, is not the only solution. With the mouthpiece of the beating-reed type, cut as described, it is possible, by a swift movement, to withdraw one of the pipes from the lips just sufficiently to prevent its speaking; if this be done dexterously (and it is only necessary to uncover the base of the tongue), the little piece of straw would pass unnoticed between the bulbs and the lips and none but an expert would be the wiser. The expedient has, in fact, passed the test, and it may, of course, be used at will concurrently with the blocking of the exits.

With regard to the Modal System, the implication of such a potentiality is of far-reaching importance. The single pipe played by means of a beating-reed had, in emulation of the full octave range conferred by the double-reed mouthpiece, deprived the Modal Scale of its second tetrachord, and substituted for it a reduplication of the first on the dominant, thus foreshadowing, or perhaps even initiating, the structure of our modern major scale. The double pipes played by means of the same beating-reed mouthpiece, in the manner suggested above, were thus able to restore to the Aulos its pure modality and full range of an octave.

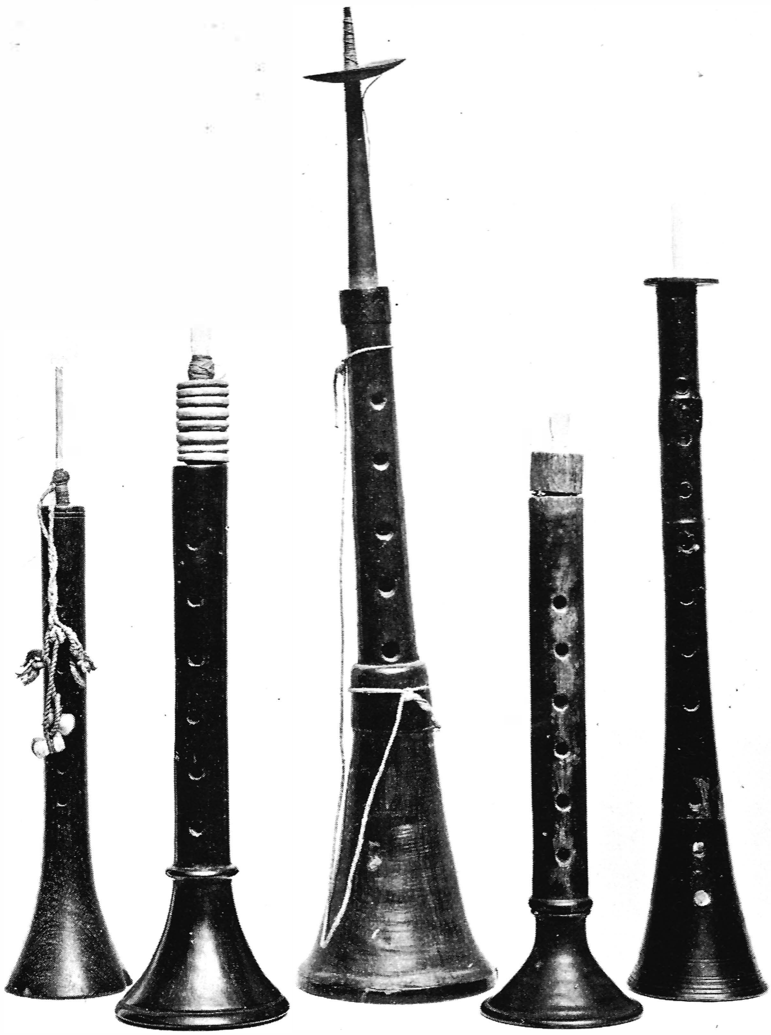
FIG. 26.—The Double Auloi bored to give the Hypodorian Modal Octave of Modal Determinant 16



N.B.—Both exits are stopped.

The following suggestion has also been made, notably by A. H. Fox Strangways, with regard to the practical use of the second of the pair with exit free: viz. that when the fingerholes were covered, the note of the exit would sound as a drone. Experiments in this direction yielded astonishing results due to laws governing resonance and forced vibrations, which belong to the domain of acoustics. Supporters of the theory of the drone as an influence predisposing to harmony would find ground for satisfaction in the pursuit of such experiments. The drone note, sometimes descending an octave to an unexpectedly low note, is ever changing, dragged into a rude but agreeable harmony with its fellow-pipe, partly through the coercion known as forced vibration, which seems to be ever struggling to keep intact the proportional ratios of the Harmonic Series. The net result of this behaviour is a qualification of the concept of a drone pedal bass. It is probable that an expert piper using a heavier and less

PLATE 10



1 4 2 5 3

PRIMITIVE OBOES

(1) From N. Egypt; (2) S. Africa; (3) N.W. India; (4 & 5) Mexico, Tarascan Chirimías: in the latter, note the grooved cylinder attachments (tall and short) supporting the mouthpiece

elastic beating-reed, cut from a piece of reed instead of from a straw or oat stalk, would be able to stabilize the drone note, as in the bagpipe.

In our inquiry into the technical capabilities of the Aulos and of its two kinds of mouthpiece, modality has been relegated into the background. Spatial length as a determinant of pitch has been proved to be capricious in its incidence on the Aulos, owing to the interference of the laws of resonance. When, however, the reactions of the Aulos to the laws of acoustics are considered from the point of view of modality, fresh light is shed on the function of length, a hint of which was revealed through the significance of the dimensions of the tongue of the beating-reed mouthpiece. The whole significance of length in the Aulos, revealed through modality, is *proportion* which, of course, affects pitch always in relation to a given fundamental.

In order to embody modality in the Aulos, it is essential (as already stated) that the extent to which the stalk of the mouthpiece extrudes from the pipe be added to the length of the latter before division by the Modal Determinant takes place. This division produces the modal increment of distance (I.D.), always found exhibited in a modal pipe between the finger-holes measured from centre to centre. As a consequence, since the I.D. is an aliquot part of the total length, it is obvious that to add or deduct one or more of these increments must automatically change the determinant number, and consequently the Mode of the Aulos.

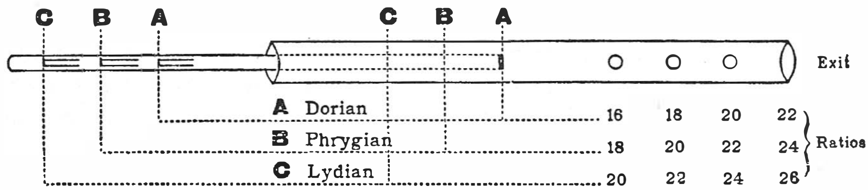
A CHANGE OF MODE ON THE AULOS

It is at this point that the fundamental difference between strings and pipes is realized in their embodiment of the Modes and species, with the resultant effect on tonality. On the monochord *C* string, for instance, with the rule divided by Determinant 24 (Phrygian), to keep the same I.D., and simultaneously to change the Modal Determinant to 22, is only possible when the second Mode (Dorian) is taken as a species of the first, that is, on a different Tonic, *D*, but with the same Mese, *G*. To change the Mode on the same string without retuning means a change of both determinant and increment of distance.

On the pipe, since the length does not affect the pitch, but only the modality—while it is played with the same mouthpiece—it is obvious that to preserve intact the increment of distance, while changing the Determinant (effected by changing the extrusion of the mouthpiece), produces a different Mode on the same fundamental note given by the whole pipe, or by the first hole used as vent. Thus it happens that through the unique proportional properties of length in reed-blown pipes, the same pipe, blown by means of the same mouthpiece, the little tongue of which determines the fundamental Tonic, more than one Mode can be produced, merely by drawing the mouthpiece out, or pushing it in, to the extent of one or more increments of distance. (See Fig. 27.)

There are very clear indications that this method of changing the Mode of the Aulos was practised in Ancient Greece. Although there is no verbal explanation of the manner in which this change was effected, there are numerous graphic illustrations extant. In the analysis of our record of beating-reed mouthpieces, it was shown that length in the stem of the mouthpiece has no bearing on pitch. Long stems afforded the necessary means of changing the Mode by varying the number of increments of distance through the extrusion of the mouthpiece from the pipe. The

FIG. 27.—Suggested Feat of Pronomus, the Theban. Three Modes played upon the same Aulos by changing the Extrusion of the Mouthpiece



N.B.—The same mouthpiece shown in normal position A, and pulled out to B and C to produce the three modal tetrachords through the same three holes and exit.

visible extrusion of a length of straw, though tolerated by the Egyptians,[1] offended the aesthetic sense of the Greeks, who concealed the stalk by means of the graceful olive-shaped bulbs, one, two or three in number, which occur in many of the vase-paintings. These bulbs [2] are made hollow and tubular at each end to fit into the resonator, or into each other, so as to form a continuous passage for the stem of the mouthpiece. Adhesion

[1] See Plate No. 3, the piper playing on double pipes, long and slender. The straw of the mouthpieces is shown in the wall-painting from Thebes differentiated by pale yellow colour from the reddish-brown pipes.

[2] Plate No. 10 represents a group of primitive oboes. No. 1, from N. Egypt, presented by H. A. Burgess; No. 2, in centre, from S. Africa, presented by Miss M. F. Grant; No. 3, from N.W. India, presented by Ferosa (Mrs. P. A. Narielwala of Bombay); Nos. 4 and 5 are Chirimías of the Tarascan Indians from the Sierras of Mexico in the neighbourhood of Uruapan (State of Michoacán). Specimens of the Chirimías are rare and very difficult to obtain, and much coaxing is needed to induce the Indians to part with them. I am indebted for the possession of these two instruments to the generosity of Miss Marian Storm (author of *Little-known Mexico*, Hutchinson, and of *The Life of St. Rose*), an enthusiastic admirer of the strange, wild, exultant music of the Chirimías played by Indians of San Lorenzo in the Sierras. Let us consider the instruments: the fingerholes are equidistant and thus the scale is one of the Greek Harmoniai; the movable cylindrical attachment with the spiral grooves varies in height according to the Mode; it fits into the resonator and receives and conceals the actual reed vibrator of the mouthpiece (*bocal*), which in these Tarascan Chirimías consists of two blades of palm bound with thread to a goose quill. These cylindrical end-pieces originally served the same purpose as the olive-shaped bulbs of the Greek Aulos, to conceal the bare end of the corn-stalk of the beating-reed or double-reed mouthpiece, which extrudes from the resonator.

The shorter of the two end-pieces on No. 5 belongs to an ancient traditional instrument, which was acquired from a very old Tarascan musician. The traditions go back to the prehistoric Maya culture.

More remarkable still is the form of the Aztec musician's Chirimía (cf. Plate No. 10 and 11) at the embouchure end of which are two globular bulbs—reminiscent of a Greek or Etruscan Aulos—through which is threaded the slender stalk of a beating-reed mouthpiece. It will be noticed that the other three primitive oboes have no end-piece or bulb, but only a flat circular rest.

is secured by rows of waxed thread. To add or take off a bulb was, therefore, a visible symbol of a change of Mode. In connexion with this decorative symbol, the significance of which had not been realized until the discovery of modality, it may be added that the most successful and satisfying mouthpieces are those in which the length of the tongue is equal to one I.D. of the pipe which it is to play.

If the length of the resonator be not an exact multiple of the I.D., the complement is supplied by the stalk of the mouthpiece. It would seem that the Greek word ὑφόλμιον applies to these bulbs. The etymology of the word suggests that the Hypholmion was a support for the Holmos (ὅλμος), a shallow cup-shaped device fitted on to the topmost bulb which concealed the entry of the actual reed-tongue into the mouth of the piper. The Holmos, for which there is no reference so early as the use of the word Hypholmion suggests, could only have been used on the single Aulos (Monaulos), whereas the bulbs were fitted to both Auloi of a pair. Two of the bulbs—one in a damaged condition—were found with the Elgin Auloi and are preserved in the Graeco-Roman Department at the British Museum; but there is no trace of a Holmos. The indented lines made by the waxen thread are still visible on the rim of one of the bulbs. Both Hypholmion and Holmos may be seen on the pipes found at Pompeii which are preserved in the Naples Museum. [1]

PTOLEMY'S REFERENCE TO THE B.R. MP. OF THE AULOS

Ptolemy has a passage [2] dealing with the beating-reed mouthpiece of the Aulos, which is of considerable interest and reads as follows: Ptolemy has been speaking of the influence of length on pitch in strings and adds:

> And in Auloi the sounds are acuter which come from the holes nearer the Hypholmion, that is the beating thing [(τοῦ πλήττοντος), i.e. the Hypholmion is the bulb concealing the beating thing or single-reed mouthpiece]. For the tube of the windpipe is like a natural Aulos, only differing in this: that in the Auloi the place of the beating thing remains the same [*puta, in ore flantis*, Wallis]; the place of the beaten thing [*in foraminibus*, Wallis] is transferred nearer or further from the beating thing by the device of the fingerholes. But in the windpipe, the place of the beaten thing remains the same. But the place of the beating thing is transferred nearer or further from the beaten thing through our faculties leading it by way of our innate musical perceptions, and finding and taking those places in the windpipe, as it were with the bridge underlying the string from which the distances to the outer air make differences in sounds proportional to their excesses.[3]

This is, as far as is known to the present writer, the only passage in the Greek sources in which any attempt is made so to define the action of the mouthpiece of the Aulos that no possible doubt is left as to its identity.

[1] 'The Pompeian Pipes' from the Mus. Naz., Napoli; photograph by Brogi (see Pl. 12); see also T. Lea Southgate, 'Ancient Flutes from Egypt', *Jrnl. of Hellenic Studies*, Vol. xxxv, 1915, pp. 12 sqq.

[2] *Harm.*, i, 3; Wallis, 1682, p. 14; Düring, p. 9.

[3] Ptol., Lib. 1, Cap. iii; see also Ingemar Düring, *Ptolemaios und Porphyrios über die Musik* (German translation), Göteborg, 1934, pp. 26–7 (9) and comment, p. 151. Porph., 63, 3; ed. Düring.

In a later chapter (Lib. ii, Chap. xii) Ptolemy makes the following statement :[1]

> We will first briefly discuss the imperfection of the monochord, which until now has been regarded as the only instrument devised that could enable us to determine the harmonic ratios computed theoretically, belonging to sequences which run through the whole scale, and to compare them easily with our aural perceptions. This instrument appears to have fallen into disuse at the present time in the practice of music as well as for the demonstration of the harmonic ratios ; and for the reason that it did not seem adequate for either of these propositions ; while the canonists used (the monochord) only for their theoretical calculations, and the lyres and kitharas for the practice of music. And in these instruments, the emmelic intervals are constituted in accordance with the appropriate ratio : but on these instruments (which would be preferable for both kinds of exposition) it is not demonstrated—since not even on Auloi and Syrinxes is there precision on such a matter—that they have differences of pitch resulting from differences of length.[2]

What Plato said of the Aulos after specifying the instruments for which there will be no use in the state (*Rep.*, p. 399C.) may now be recalled : ‘ Has not the Aulos a great number of notes, and are not the scales which admit of *all the modes* (τὰ παναρμόνια) simply imitations of the Aulos ? ’

A glance at the diagram (Fig. 27) reveals the significance of the bulbs. The end of the pipe and each of the holes can sound three notes differing in intonation and belonging to as many Harmoniai, but of course, after manipulation of the mouthpiece. As the bulbs cannot be removed during the playing, this contrivance does not, in the manner of the expedients mentioned below, provide an increased compass, so much as a potential change of modality on the same Aulos, in a different melody. The bulbs could be used equally well with either of the two types of mouthpiece (see Plate No. 13).

These means of increasing the compass are concerned with the technique of a skilled musician ; and with certain structural improvements in the Aulos. The desire for an extended compass on the Aulos suggested the means for obtaining it.

THE FEATS OF PRONOMUS THE THEBAN ON THE AULOS

The fact that three different notes—approximately called quarter-tones—can be obtained through each fingerhole is generally considered to imply partial obturation of the holes, so that if the hole sounding *C* had been covered, then the gradual uncovering of hole *D* by three movements of the finger, produced in turn $\overset{\sharp}{C}$, *C*♯ and the fully opened hole *D*.

In the development of the Aulos the name Pronomus is especially prominent. The perpetuation of his musical feats and inventions by the

[1] Ptol., ii, xii. Düring, p. 66.

[2] I am indebted for the translation of this passage to Professor J. F. Mountford. Ptolemy recognizes the fact that difference of pitch results from difference of length, but cannot give a precise explanation of the matter in relation to the wind instruments. Ptolemy, however, in an earlier chapter (1, 8), recognizes among the determinants of pitch, not only length but also width (πλάτος, diameter of bore—K. S.) besides ‘ the blowing in of the breath ’. See Chap. iii, pp. 130–131.

PLATE 11

AZTEC MUSICIAN WITH CHIRIMÍA FITTED WITH TWO BULBS

By courtesy of Miss Marian Storm

erection of a statue in his honour, testifies to the importance attached to them by his compatriots. The facts are recorded in the following passages from Pausanias and Athenaeus. No description is given of the means whereby the results were achieved by him ; but the fact recorded that he was the first to devise an Aulos suitable for every Harmonia seems to point to his having invented the device of sliding bands with their extra, secondary fingerholes. These were later on fitted with short additional tubes (Bombyxes) mentioned by other writers such as Theophrastus and Arcadius.

The account given by Athenaeus [1] is as follows :

> Of old, beauty was thought of in Music and everything in this Art had its fitting honour, wherefore, there were separate Auloi for each Harmonia, and each Aulete at the Games had Auloi adapted to each Harmonia. But Pronomus, the Theban, first played [all] the Harmoniai from the same Aulos ; but now at random, without ratio they meddle with music [see Fig. 27].

Of the results of the various attempts to increase the compass of the Aulos for modal use, Pausanias [2] relates the following :

> And there is a statue of Pronomus, the man who fluted most attractively to the multitude. For a long time the Auletes possessed three kinds of Auloi, and on some they played the Dorian Aulema, and different ones were made for the Phrygian Harmonia and the one called Lydian was played upon other Auloi [i.e. Modes, not Species]. And Pronomus was the one who first devised Auloi suitable for every kind of Harmonia and was the first to play so many different melodies on the same Auloi [3] (double Auloi). And by the appearance of his face and the movement of his whole body, he charmed the audience exceedingly. And by him was made the song for the entrance into Delos of the Chalcidians, this statue, therefore, the Thebans dedicated and also one of Epaminondas, the son of Polymnis.

Proclus,[4] the neo-Platonic author (A.D. 410–85), records the fact concerning the Aulos of many Modes (i.e. with sliding bands) and the facilities it afforded to musicians thus :

> And the reason is the intricacy of this instrument (I mean the Aulos), which has rendered the art used by it elusive. For the Panharmonia and the Polycordia are imitations of the Auloi. For each fingerhole (*ἕκαστον γὰρ τρύπημα*) in Auloi emits at least three sounds, they say, and if the secondary holes (*παρατρυπήματα*) are opened [i.e. those covered by the sliding bands—K. S.] it emits more, and it is not necessary to receive the whole of music in your education, but only what is sufficient.

Various methods of transposing the Aulos into a higher Tonos have been discussed, e.g. by pushing in the mouthpiece to a shorter extrusion, and the use of devices affecting the structure of the instrument remains to be

[1] Athenaeus, *Deipn.*, xiv, p. 631e.

[2] Paus., ix, 12. Passages translated by E. J.

[3] The idea of using three positions of letters of the alphabet for the instrumental musical notation, (1) the upright normal letter, (2) the same recumbent, (3) the same reversed and facing (1), is considered by some authorities to have been suggested by this device of the early Greek Auletes.

[4] *Comm. in Alcib.*, Cap. 68, ed. Kreutzer, pp. 196 and 197, line 11. (Translated by E. J.)

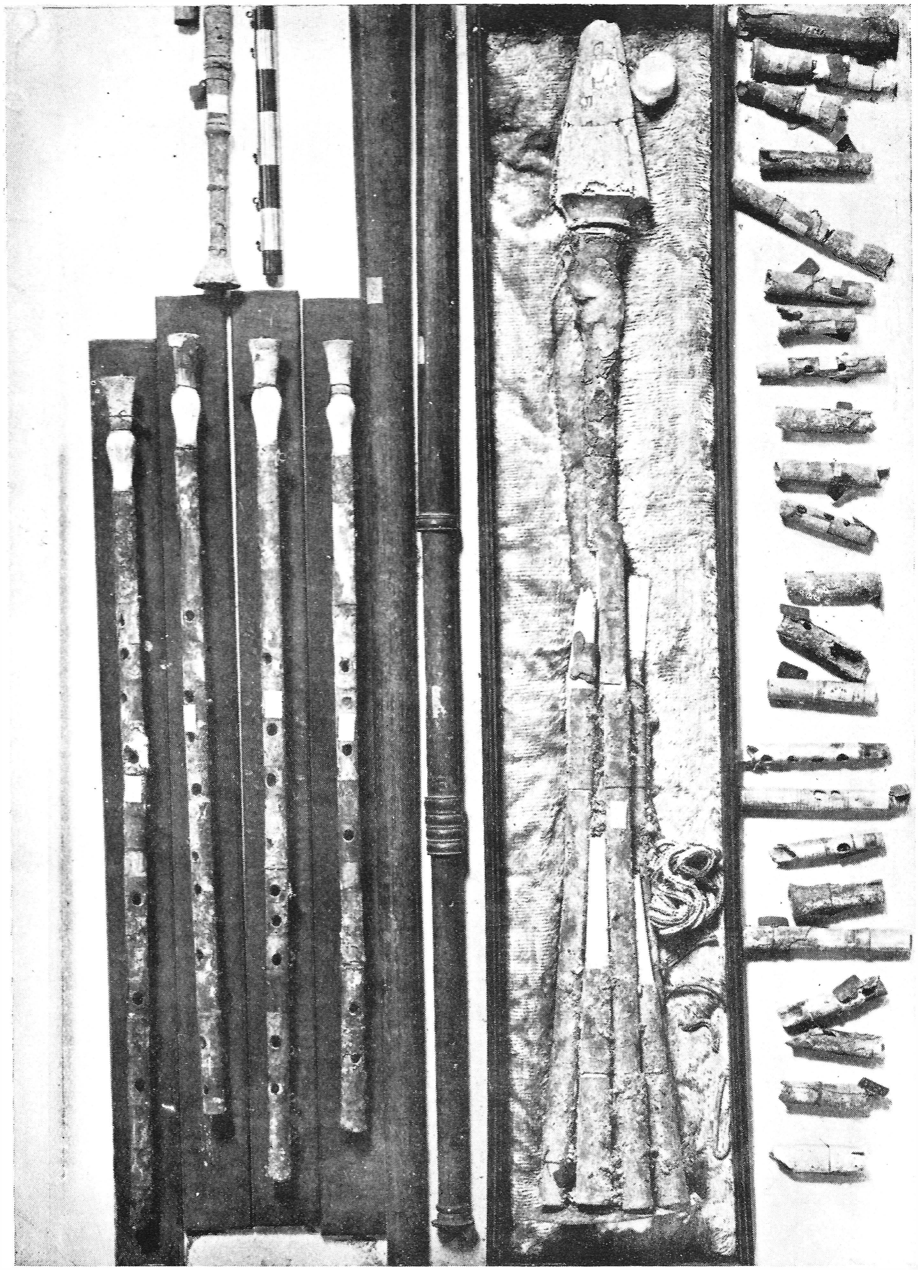
considered.[1] The simplest means of increasing the range of the Aulos was to add to the number of fingerholes, and to devise some contrivance for temporarily covering over those not required. Bands of silver, having one or more holes of the same diameter and bored in the same relative position as those on the pipe, served for alternative use during the course of the melos. The metal bands provided with little rings or horns (*κέρατα*) allowed the piper to turn them at will round the bone pipe, while a narrow fixed ring prevented their slipping down the Aulos. Arcadius,[2] the Grammarian (second century A.D.), alludes to these contrivances:

> But at each breath, he says, the accents (*πνεύματα*) are put on, not unskilfully or unmusically, just as those searching for the holes on Auloi to close and open them when they wish, have contrived or invented them with little horns or movable bombyxes (*κέρασι τίσιν ἢ βόμβυξιν ὑφολ.μίους*) above and below, and turning within and without. In this case, as in that, signs for the breath are made like horns, meaning one and the same scheme in each. This one, as in an Aulos, in that turning inwards and outwards, shows us how to close and open the breath, for it shows how to close and hold it when we are using the *spiritus lenis*, and it lets it go and opens it whenever breathing out, we are compelled to speak with an aspirate.

Bombyxes were possibly so named from the resemblance of the Aulos, provided with additional tubes, to a caterpillar. The structure of the Bombyxes, or additional tubes, inserted into the fingerholes to lower the notes by a ratio proportional to the length of the Bombyx, may be realized from the photograph in spite of the exuberant fancy of the sculptor. (See also Chap. iii). The additional tubes of appropriate lengths for the Harmonia were probably inserted before performance. The Aulete is represented raising his Auloi and throwing back his head in exultation, and in order to obtain the high notes required for his triumphal Ode.

[1] Tradition credits Pronomus, the Theban, with the performance of certain feats upon the Aulos which suggest phases in the evolution of the instrument. To obtain three notes from each fingerhole (as described above) was a simple feat carried out by partial obturation of the fingerholes, which could take place equally well on the primitive Aulos played by a double-reed or a beating-reed mouthpiece. This manipulation was mainly used for changing the genus from Diatonic to Chromatic or Enharmonic. It was further claimed that Pronomus, excelling other Auletes, was able to play three Harmoniai on the same Aulos (or double Auloi). This may easily be done by changing the extrusion of the mouthpiece as illustrated in Fig. 27, whereby the change is one of species, since the I.D. remained the same for all three sequences. Neither of these feats required any structural alteration in the Aulos. A third claim—not directly attributed to Pronomus—is implied in the use by Auletes of a contrivance containing secondary holes, pierced in silver sliding bands. These holes—of different diameters, and placed at different distances from embouchure—correspond with those on the Aulos resonator. The bands, turned by means of little horns or rings, opened or closed certain series of holes at will. This means a change of Mode through the operation of the bands. For the implication here is that more than one aliquot division had been made, i.e. by the Modal Determinants of the Harmoniai in question; so that instead of one I.D. on the Aulos, there were virtually two or three alternatives, according to which set of holes was opened in succession. (See in this connexion the photograph by Brogi [Plate No. 12] of the Pompeian Auloi at Naples and the drawing of the Candia Aulos by Prof. J. L. Myres, Fig. 29.)

[2] *De Accentibus*, ed. by E. H. Barker, 1820, p. 188, line 12.



THE POMPEIAN AULOI, WITH REVOLVING BANDS, AND (ON THE RIGHT) AULOI SURMOUNTED BY A MOUTHPIECE SUGGESTIVE OF A MODERN CLARINET OR FLAGEOLET

Museo Nazionale, Napoli

An examination of Mr. A. A. Howard's well-known pamphlet on 'The Αὐλός or Tibia' (*Harvard Studies in Class. Phil.*, Vol. iv, 1893, pp. 1–60) yields much valuable material, of a general nature, derived from the literary sources, and from an investigation—as thorough as was practicable—of the actual museum specimens [1] preserved in the Museo Nazionale at Naples.

The four Pompeian Auloi, Nos. 76891–2–3–4, belong, as may be detected from the Brogi photograph No. 12489 (see Plate No. 12), to the period deplored as decadent by Plato, and after him by Plutarch, which was marked by the insistent popular demand for greater facilities for the production of more and more notes, and for modal modulations. This predilection was satisfied on the Aulos by means of such devices, among others, as sliding metal bands—the working principle of which is described further on.

Owing to the delicate state of these relics, it was not possible to lay bare the ivory tube of the Aulos, in order to gain precise information as to the number and position of any supplementary fingerholes concealed under the metal bands. The lengths have been measured from the exit, or lower end of the Aulos, to the nearest edge of hole or band, and the diameters of the holes are carefully noted. In pipes Nos. 76891 and 2, the increments of distance, taken strictly in order from centre to centre of the fingerholes, are irregular and suggest that each Aulos was bored to produce two Harmoniai, each with its own I.D.: a conclusion borne out by the number of fingerholes from 11 to 13. The piper set the bands in order—opening or closing the holes as required for the selected Harmonia before playing. A glance at the measured drawing of the Aulos in the Candia Museum—which had 24 holes of different calibre in 14 bands—made by Professor John L. Myres, if read with the explanation given further on, makes the matter clear. The drawing exhibits conditions similar to those existing on the Pompeian pipes.

It is unfortunately impossible to diagnose the Harmoniai embodied in these Pompeian pipes from the particulars supplied: exact reproductions of the Auloi would have to be made on the spot and tested with a suitable mouthpiece, in which the vibrating reed is left unfettered and free to control the resonator, instead of the clarinet mouthpiece used by Mr. Howard. The miscellaneous small fragments of pipes (to the right of the photograph) reveal (in No. 8 from bottom) two holes of different calibre in one band; in the next fragment above there are four holes in alignment at equal distances, with a fifth hole bored diagonally above the last hole; No. 13 shows two holes in the ivory pipe with a third of half the diameter on a band, which would flatten the note of the ivory pipe by a half increment, if superimposed over it.

With reference to Mr. Howard's ingenious suggestion that the term Syrinx, applied in the sources, to some movable part of the Aulos was the speakerhole, used in the clarinet to facilitate the playing of Harmonics, the choice for the student lies between this and the application (by K. S.)

[1] See Photograph No. 12 (by Brogi, No. 12489), Pompeian Auloi, Mus. Naz., Napoli.

of Syrinx to the beating-reed mouthpiece (distinguished from Zeuge for the double-reed mouthpiece) on the grounds that it is the only kind of mouthpiece which can produce the puzzling results (of raising the pitch), identical for two diametrically opposed movements and operations of the Syrinx, ascribed to it by Aristotle, Aristoxenus and Plutarch. The fact that it is unnecessary to postulate a speakerhole for these pipes is emphasized by Mr. Howard's production of the 3rd and 5th Harmonics from all the holes *without any speakerhole.* A fact that is confirmed by my own experience with beating-reed mouthpieces.

Mr. Howard includes in his paper the measurements of the Elgin Auloi at the British Museum, which were supplied to him by Mr. A. H. Smith of the Graeco-Roman Department. The measurements tally with mine except for those of the diameters of bore and fingerholes, for which I was not allowed callipers. The scale given by Mr. Smith for Aulos B (the straight one) corresponds exactly with the result I obtained, except for the note from Hole 5, viz.

ELGIN AULOS B (STRAIGHT)

A. H. Smith	*exit*	*H* 1	2	3	4	5	6
	A	*c*	*d*	*e*	*f*♯	*a*	*b*

K. S. with 6 different double-reed mouthpieces :

A 13	*c* 11	*d* 10	*e* 9	*f* 8	*g*♯ 14	*b* 12

This scale from Hole 1 will be recognized as that of the Dorian Harmonia of M.D. 11 (see *Records of Elgin Aulos*, Chap. x, Plate No. 17. The Elgin Aulos, Brit. Mus.).

The expedient of cross-fingering about to be described was widely practised in the Ancient East and was described in the treatises of Bhārātā [1] and Sarangdev [2] of Hindostan, and in the sixteenth century by Martin Agricola.[3]

Cross-fingering (*doigté fourchu*), another device for lowering the pitch of single notes, is operated by closing one or more holes, normally left open, below the open one that is speaking ; this has the effect of lowering the pitch of the latter by a quarter-tone or more, according to the number of holes closed, but at the expense of the quality of the tone. This expedient

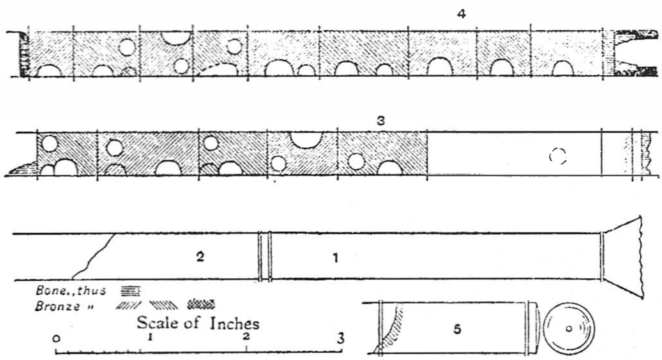
[1] Bhārātā (fifth century A.D.) author of the *Nātyashāstra* Treatise on the Drama, including chapters on Music and Musical Instruments used in the Drama—mentions several kinds or schemes of cross-fingering (*op. cit.*, Chap. xxx, 6–9, and of partial obturation of the holes). The chapter treating of Music translated by Joanny Grosset, Lyons, 1896.

[2] In Sarangdeva's *Samgīta Ratnākara* a list is given of 15 flutes of bamboo with measurements (see vi, pp. 424–51, and for cross-fingering, pp. 447–8, and 457–61). For 37 and 37A see ' Inde ', par Joanny Grosset, *Encycl. de la Musique et Dict. du Conservatoire*, Fasc. 12, pp. 353–4, ed. by Albert Lavignac (Paris, Ch. Delagrave). See K. S., *The Modal Flute*, Chap. vii.

[3] *Musica instrumentalis deudsch.*, Wittemberg, 1528 and 1545. Reprint by *Gesellschaft. f. Musikforschung*, Jahrg. 24, Bd. 20 ; Leipz., *Breitk. u. Härtel.*, 1896, pp. 159–75. See K. S., Chap. vii.

and its effect are described by Quintilian [1] who, in a discourse on pronunciation, shows a critical appreciation of the difference in tone quality between the open or clear notes of the Aulos and those obtained by cross-fingering.

FIG. 29.—Plan of Fragments of an Aulos in the Candia Museum. Drawn by Professor J. L. Myres from the actual Fragments of the Specimen (to scale)



(Reproduced by kind permission of Professor J. L. Myres)

Fragments of an Aulos showing 24 holes in 14 bands are preserved in the Museum at Candia, measured drawings of which were made from the instrument in 1893 by Professor John L. Myres, to whom the writer is greatly indebted for the communication of the diagram.

MACROBIUS ON THE POSITION OF THE FINGERHOLE

The pipe is clearly a modal one, showing part of an aliquot division in the placing of fingerholes in the pipe, with their duplicates in the sliding bands, sufficient for at least two Modes. It will be noticed that the holes are of different diameters, all of narrower calibre than that of the bore. The ancients understood the acoustic principle which permitted of substituting a hole of smaller diameter, placed correspondingly nearer the mouthpiece, for the larger one in the correct theoretical position. This principle is indicated in the following passage by the Grammarian, Macrobius [2] (fourth century A.D.): 'Nec secus probamus in tibiis de quarum foraminibus vicinis inflantis ori sonus acutus emittitur, de longinquis autem et termino proximis gravior; item acutior per patentiora foramina, gravior per angusta.'

The rise in pitch brought about by making a fingerhole of a diameter wider than the norm by *n* millimetres renders it possible to place the fingerhole farther from the mouthpiece by the same *n* millimetres: the converse is equally true, i.e. if the fingerhole be made of a diameter smaller by *n* millimetres than the normal one, then the correct position of the hole to give the next note of the modal series, will be found by the same number

[1] *Inst. Orator.*, I, 11, 6 and 7 [2] *Comm. in Somn. Scip.*, ii, 4, 5.

of millimetres nearer to the mouthpiece.[1] The effect of a divergence from the normal diameter of a fingerhole is of greater moment on the modal flute than on the reed-blown pipe.

This application of an acoustic law has been largely utilized in the Candia pipe for the greater convenience in the disposition of the extra holes; the significance of this principle is made clear later on, and the formulae concerned are given in Chap. iii.

FIND OF FRAGMENTS OF AULOI AT MEROË BY PROFESSOR JOHN GARSTANG (LIV. UNIV.)

There are other examples of Auloi provided with sliding bands,[2] e.g. a pair of pipes in the Cyprus Museum in fragmentary condition is, according to Professor J. L. Myres, of the same construction as the one in Candia. At the British Museum there are the two Maenad pipes in the Castellani Collection, and one from Halicarnassus. An important find of fragments of Auloi, by Professor John Garstang, in the ruins of the Ancient Royal City of Meroë in 1914, is preserved in the University Institute of Archaeology in Liverpool. The fragments belong to the type of Aulos made of fine ivory fitted with bronze revolving rings, as described above on the Bombyx Auloi, for the extension of the compass of notes, and to enable the Aulete to play in different Modes on the same instrument. The lugs, to which the little horns for turning the revolving rings were attached, are to be seen as triangular protuberances on Nos. 3, 4 and 5. Dr. Southgate, who has described the fragments, mentions the fact that the bores of the ivory tubes of these Auloi, found at Meroë, were tested with callipers and found to be perfectly cylindrical, with diameters of from ·011 to ·016; Dr. Southgate suggests that as the pipe was made with separate joints,

[1] The apparent contradiction as regards the effect on pitch of an increase in diameter (1) *lowering* on the bore of a pipe, and (2) *raising* on that of a fingerhole, is due to the following acoustic property: width of diameter in (1), the bore, adds an equivalent length to the half sound-wave produced from the end of a flute. The whole of the bore is involved here, and consequently likewise the pitch of the note of each fingerhole, provided its diameter is the same as that of the bore. In (2), where the fingerhole has a lesser diameter than that of the bore, the effect of any departure from the norm is in inverse proportion for this reason: it results from the deduction in millimetres of the diameter of the fingerhole from that of the bore, *ergo* the larger the fingerhole, the less is the amount of length to be added to the half sound-wave and, therefore, the higher the pitch of the note produced through that fingerhole. Thus, the sound-wave, instead of effecting an exit through the diameter of the bore, finds its length determined by the opening of a fingerhole. The diameter of this fingerhole, being less than that of the bore, must be subtracted from the latter in order to compute its effect upon the length of the sound-wave (i.e. $\Delta - \delta = x$), to which this difference must be added. Moreover, the normal diameter of the holes has already been taken into account in the placing of Hole 1: the correct intonation of each note of the sequence given by the equidistant holes is dependent upon the diameter of the fingerholes remaining constant, therefore, a lesser diameter than the normal takes effect as added length, flattening the note of the hole in question, unless the effect be compensated by placing this hole of smaller diameter nearer the mouthpiece by the amount of the reduction in this diameter.

[2] Relief in Vatican Museum, No. 535; Helbig's Wandgemälde, Nos. 56, 60, 730, 765, &c.

these may have been fitted together in a different order at times, thus varying the disposition of the fingerholes and consequently of the intervals. This, of course, is not an impossible hypothesis, but one must look below the surface for the implications. The distance from centre to centre between fingerholes only influences pitch as a length in direct proportion to the total length, from the centre of the lower of the two holes concerned, to the tip of the mouthpiece. In any case, the suggestion would not apply to the modal pipes having equidistant fingerholes. The Auloi are Greek in origin, and may have been the work of a fine craftsman of Corinth or Alexandria, the cities most famous for their flutes; they were probably brought to Meroë by visiting musicians from Greece.

Three types of single pipes were known to the Greeks; those first described belong to the Monaulos class, (I). To the same class belong the Pompeian Auloi in the Muzeo Nazionale of Naples, which are shown in our *plate* (Plate No. 12), No. 76894 (Brogi); one of the specimens has the maximum number of fingerholes, viz. 15 or 16. The Maenad pipes and the pipe from Halicarnassus are plagiauloi (Class ii). The πλαγίαυλος, so called by the Alexandrian Greeks, was said to have been invented by Osiris;[1] while Pollux[2] attributes the invention to the Libyans, and Pliny[3] to Midas of Phrygia. The *plagiaulos*, *photinx* (φῶτιγξ), or *tibia obliqua*, was held like the modern concert flute, but was played by means of a reed. The two in the British Museum are single pipes stopped at the end; the mouthpiece that projects from the side is decorated with figures of the Maenads. A round hole at the top of the mouthpiece leads by means of a slanting tubular passage into the main bore of the instrument and serves to hold the reed vibrator. There is in the British Museum a statue of Midas, the Phrygian Aulete, playing upon the plagiaulos, the invention of which is ascribed to him.

The third type of single pipe is the *Syrinx monocalamus*,[4] the *nay* of Egypt, a bamboo or reed pipe, blown across the end like the Syrinx, but having lateral fingerholes.

THE FEAT OF MIDAS OF AGRIGENTUM

Pindar's Twelfth Pythian Ode celebrates Midas of Agrigentum for his Solo-playing on the Aulos. The Scholiast says: 'The little tongue having been accidentally broken by cleaving to the roof of his mouth, Midas played on the pipes alone in the manner of the Syrinx; the audience was amazed and delighted at the sound and in this way he won the victory.'[5] It may be inferred that this performance by Midas

[1] Eust., 1157, 43; v. Sturz, *D. Mac.*, p. 82; Plut., Vol. 2, p. 961E; Athen., 175E, 182D; Theocritus, 20, 29; Bion, 3, 7.

[2] iv, 74.

[3] vii, 204.

[4] Athen., 184A. Many references to Monaulos in Ed. Lugd., M.D.LVI, Lib. iv, Cap. xxiv, pp. 222–6.

[5] *Schol. vet. in Pindari carm.* (ed. A. B. Drachmann, 1910), Pyth. xii: ἀνακλασθείσης τῆς γλωσσίδος ἀκουσίως καὶ προσκολληθείσης τῷ οὐρανίσκῳ μόνοις τοῖς καλάμοις τρόπῳ σύριγγος αὐλῆσαι, τοὺς δὲ θεατὰς ξενισθέντας τῷ ἤχῳ τερφθῆναι, καὶ οὕτω νικῆσαι αὐτόν.'

was considered a somewhat sensational feat, and that the Syrinx monocalamus was not popularly known in Greece. The implications of the Scholium are of interest. Midas was threatened with catastrophe as competitor on the Aulos at the Games, by the breaking of the tongue of the mouthpiece as he began to play—time and opportunity were lacking for the adjustment of another mouthpiece. It was not presence of mind alone that saved the situation, but his skill on the Syrinx monocalamus. His compass on the double Auloi was a modal octave with an eventual harmonic extension: for experiment reveals the fact that it is possible to obtain Harmonics simultaneously on both single-reed mouthpieces, but only a skilled musician, accustomed to play these delicate reed mouthpieces, could gauge their melodic and harmonic possibilities. One very important fact emerges from experiments in Harmonics on these mouthpieces: that in spite of the cylindrical bore of mouthpiece and resonator, the octave harmonic, as well as the octave obtained by shortening the tongue by half, both come out clearly. But alas! as Aristoxenus hints in his polemic, the instrument is subject to aberrations, gives sudden surprises, and the story of Midas emphasizes its fragility.

In what, then, did this feat of Midas consist? Given the boring on his Auloi of a full modal octave (as suggested above), this would require the use of both Auloi, but only one of these could be played or held as a Syrinx at one time. The Scholion does, in fact, state that Midas played on the reeds alone (*μόνοις τοῖς καλάμοις*) but if this is not an erroneous report, he must have taken up and played upon the two Auloi alternately. Moreover, the removal of the mouthpieces altogether changed the nature of the instrument and of the blowing, for the Aulos, played as a Syrinx, became to all intents and purposes an open pipe. Since the proportions of a modal pipe are calculated on the combined length of the resonator and the mouthpiece, it follows that to remove the mouthpiece alters the Mode. But a further change has been effected; for the acoustic conditions are now entirely altered from those of a reed-blown pipe to those existing in a flute: there is now a need for so-called end-correction owing to the lengthening influence of diameter. If Midas had entered the contest with a Dorian Aulos, with 22 as Modal Determinant, the loss of the mouthpiece might easily change it to Hypolydian (20) or more probably to Hypophrygian (18). As it is an easier matter for a skilled piper to overblow on a long Syrinx of narrow bore than to play the fundamental notes, the first overblown tetrachord sounds an octave higher, and the second, overblown a twelfth through the same holes, completes the octave by the repetition of the first tetrachord on the dominant. It was evidently the good fortune of Midas to be playing upon a well-proportioned Aulos, with a tongue-length corresponding to the modal increment of distance, and a mouthpiece extrusion which left the resonator an exact multiple of that increment. If the Aulos happened to have become Hypolydian by the loss of the mouthpiece, then the scale, from three holes with its harmonic register, became the prototype of our modern major scale (see Fig. 24). Plutarch [1] records that Tele-

[1] *de Mus.*, Cap. 21, ed. Weil and Rein., § 196, pp. 82–3.

phanes [1] of Megara had such a dislike for the Syrinx that he would never allow his flute-makers to fit one to any of his Auloi, and this aversion for the Syrinx was the principal reason for his abstention from competing at the Pythian Games. From the detailed description of the Nomos Pythicos by Strabo,[2] introduced by the famed Sacadas of Argos (*c.* 580 B.C.), the inference is that the obnoxious Syrinx in question was a mouthpiece having a coarse, stiff tongue cut from the *Kalamos Zeugites* (*καλάμος ζευγίτης*), which was eminently suitable for producing sounds suggesting the cries of the expiring Typhon.

[1] Telephanes was a contemporary of Demosthenes (b. *c.* 385 B.C.).
[2] Strabo, ix, 3, 10.

CHAPTER III

THE AULOS, PART II: THE AULOS IN ANCIENT AND MODERN THEORY; ITS MOUTHPIECES AND MODALITY

The Reactions of Reed-blown Pipes. The Incidence of Length in Wind Instruments in the Determination of Pitch. The Incidence of Diameter in the Mouthpiece. The Closed Pipe; Fundamental Resonance and the Determination of Pitch. Part played by the Mouthpiece of the Aulos. The Double-reed Mouthpiece. The Proper Note of the Mouthpiece. Significant and Unique Property of the Double-reed Mouthpiece. The Struggle for Mastery between Mouthpiece and Resonator. The *modus operandi* of the Proportional Law productive of the Harmonia on the Aulos. Puzzling Properties of the Aulos and its Mouthpiece. Unsuspected Factor in the Interior of the Aulos. The Arche dominates the Inner Reactions of the Aulos Resonator. The Plan of the Harmonia in the Interior of the Aulos. How the Modality of an Aulos may be judged. Bulbs on an Aulos are a sign of a change of Harmonia. Main Points concerning the Double-reed Mouthpiece. The Beating-reed Mouthpiece. The double Movement of the Tongue of the Beating-reed Mouthpiece. Implications of the Feat of Midas of Agrigentum. Plato on Empiricism in Aulos Music. Momentous Significance of Shortening the Tongue of the Beating-reed Mouthpiece recalled. Change to Elaborate Playing on the Aulos. Performance of Aulos Loret xxiii with Beating-reed Tongue of Mouthpiece shortened by One-third. Piper persuaded by Pythagoras to change the Modality of his Aulos from Phrygian to Dorian Spondaic. Examples of the Performance of Reed Mouthpieces under various Conditions. Concerning the Octave Relation in Auloi (Porphyry's Comm. on Ptol., i, 8). Notes on Debatable Points in the Quotation from Porphyry. The Ethos of the Harmonia

THE REACTIONS OF REED-BLOWN PIPES

THIS chapter will be concerned mainly with an investigation into the reactions of reed-blown pipes [1] to the laws embodied in their structure, and to the musical impulses conveyed by the breath of the player. Although it may be easy enough to produce sounds on a pipe, the subtle processes involved are not always understood or consciously used by the player himself. Such an inquiry is not only of some importance for the science of acoustics, but it will also be found that the tests and experiments here described will throw much light on the nature of the Greek Modes. The scale given by a pipe in its simplest form, through the position selected for the fingerholes and quite independently of what further extension of compass may be contributed by individual players, is of paramount importance in the elucidation of all questions bearing upon the foundations of our musical system, in its evolution under the auspices of Ancient Greece from still more ancient systems of the East, of which

[1] The investigation into flutes is postponed to Chaps. vi and vii.

little authentic written information is available at present. For this reason the utmost enlightenment that can be obtained from the instruments must be welcomed. A stringed instrument belonging to a bygone age may be discovered,[1] but valuable as this find is archaeologically, it has little to reveal of the music to which it gave utterance thousands of years ago; it bears no record of the scale to which it was tuned. The discovery of a flute or pipe, on the other hand, is generally but erroneously considered of much slighter archaeological and artistic interest. Yet as long as a flute or pipe survives destruction, its fingerholes bear an imperishable record of the scale for which it was made and which may be correctly read after centuries of silence.

The discovery of a reed-blown pipe appears at first sight to be less promising as a record, and opens an avenue to controversy, seeing that its mouthpiece has invariably perished. Nevertheless, if it be a modal pipe (recognized as such by having its fingerholes bored at equal distances), it will yield up its secret on closer acquaintance. As soon as a mouthpiece is inserted into the embouchure which will play the pipe easily and freely in its whole range, the Aulos will give a true account of itself and it will be found more reliable even than the flute.

LENGTH AS A DETERMINANT OF PITCH

Length as a determinant of pitch is of greater and more abstruse significance in wind than in stringed instruments, and involves many subtle problems, the solution of which is of some importance to our subject. Pitch is the result of the interaction of four main factors, which considered from the point of view of practical acoustics, are as follows:

(1) Length (actual and effective), of string or air column.
(2) Weight of string, or volume of air in pipes.
(3) Tension [2] of the string, or compression of air in pipes.

[1] As, for instance, the prototypes of the Greek Kithara found by Mr. Leonard Woolley during the excavations at Ur of the Chaldees. See also *The Music of the Sumerians, Babylonians and Assyrians*, by Canon Galpin, Camb. Univ. Press, 1937, Chap. ix (end) by K. S.; and also Review in 'Music & Letters,' April, 1938, pp. 209–13, and in July, 1938 and October, 1938, *Correspondence*.

[2] Tension is force applied to an elastic substance in order to increase its elasticity and resistance and to decrease its density or weight for a given unit of length. In a string this is achieved by the use of tuning-pins. The analogue in wind instruments, and more especially in the Aulos, is the compression of air induced by narrowing the aperture through which the air-stream is being driven (as in the fipple of a flageolet or of the flue-pipe of an organ) and by reducing the capacity of the air-chamber behind the aperture, while increasing the volume or pressure of air—a proceeding that results in a rise of pitch. Such an increase of volume or pressure takes place first of all in the larynx and may be experienced in singing, and also in playing the Aulos while passing from a note of low pitch to a higher one (as already described in Chap. ii).

The whole process is accentuated in overblowing when compression, with all its attendant subsidiary operations, increased proportionally in duple or triple ratio as required, produces the harmonic compass. The process initiated in the larynx is continued in the mouth, when the act of stiffening the lips while reducing the

(4) Resonance as a factor in the determination in reed-blown pipes, not only of timbre but also of pitch and tonality.

The combination of these four elements, according to acoustic laws (into which we cannot enter here) determines pitch. Any given pitch may, however, be varied by changing the proportions of one or other of the components, a fact understood as a general principle by Ptolemy.[1]

THE INCIDENCE OF LENGTH IN WIND INSTRUMENTS, IN THE DETERMINATION OF PITCH

In computations of vibration frequency, length which is the basic factor of pitch, is generally regarded as the simplest of the four. In the evaluation of pitch in wind instruments, this factor requires some explanation of a more practical nature than is obtainable from a treatise on acoustics.

It is a well-known fact amongst acousticians that two open pipes of identical length but of different diameters do not, if played without a reed, give the same note. The difference is due to the fact that a wider bore has the effect of lowering pitch and acts as if the open pipe had additional length. But it must be stated at the outset that in reed-blown pipes diameter is disregarded in the determination (*a*) of the fundamental pitch or key of the Aulos as a whole; and (*b*) of the modality embodied in any particular specimen. In both of these contingencies the instrument is free from the influence of diameter for the following reasons. Although the influence of diameter on the pipe resonator is, in a restricted sense, operative as regards pitch, this is only true of the resonator tested without mouthpiece, and is therefore negligible. The addition of the indispensable reed mouthpiece converts the open pipe reactions of the resonator into those of a stopped pipe, and at the same time effectively puts an end to any correspondence between length and pitch. Many illustrations of this fact are provided in this section. Moreover, the mouthpiece acting with sovereign power, selectively, upon the fundamental pitch of the resonator, evokes the particular harmonic resonance that is best suited to reinforce its own proper note. The net practical result of these relations between mouthpiece and resonator in the Aulos is that the pipe-maker is able to dispose the fingerholes at equal distances along the pipe *from the exit*. He thus avoids the distortion of the Modal Scale that would result from similar spacing on a flute, in which the incidence of diameter is of signal importance.[2]

aperture is accompanied by a muscular contraction of the cheeks which proportionally reduces the capacity of the mouth to the quantum necessary to produce the note of requisite pitch. The main part of the process is, of course, subconscious, but highly significant musically, as will be realized when the three distinct kinds of note obtainable from an Aulos mouthpiece are described. Indications of what is taking place may be observed in the throat of a flute-player, in which the contraction of the muscles controlling the glottis (or air-chamber) is visible as he passes from a low note to a higher one.

[1] *Harm.*, Lib. 1, Cap. viii, p. 33 sq.; Lib. 1, Cap. xi, pp. 50–4; Wallis, ed. 1687; Düring, Cap. viii, p. 17; Cap. xi, p. 25.

[2] A flagrant case of such distortion is illustrated by the Record of Flutes, e.g. Java No. 5, Inca No. 12, Chap. x.

INCIDENCE OF DIAMETER IN THE MOUTHPIECE

In the reed mouthpiece itself, whether of the double-reed or beating-reed type, diameter comes into its own in stabilizing the subtleties of intonation. The piper may be regarded as the overlord, pronouncing through the mouthpiece—within its technical limits—the last word in the determination of the modality and tonality of the Aulos.

The primitive piper accepts at first without question the Mode bestowed by his Aulos, but when he has had time to become acquainted with the possibilities and limitations of his instruments, and also with the Modes they have brought him, he then assumes control. We cannot afford to pass over any item of practical knowledge concerning the structure or playing of this remarkable instrument, if we would understand the significant vogue of the Aulos, and the part it played in the artistic and social life of Greece and of the ancient world generally. How, for instance, the Aulos came to win for itself privileges equal to those accorded to the Kithara at the Pythian Games, in spite of Apollo's valuation of the two instruments, signified by the mythical contest between the god and Marsyas (see Plate No. 2).

With flutes, diameter is an imperious tyrant, exacting its dues to the uttermost millimetre. Through the reciprocal workings of pitch and length on the factor of the velocity per second of sound-waves in air, diameter makes its legitimate demands known in aggregate form: this means that the length of the half-sound-wave corresponding to the pitch of the fundamental note emitted from the exit of the flute, is greater than the actual length of the flute. This excess, which frequently amounts to an inch or more, is due to the lengthening influence of diameter in the bore. To theorists and acousticians falls the task of apportioning, amongst the various participants in the reactions of a flute, the sum total of what is known as allowance in respect of diameter. In computations involving length, this allowance must be added to *actual length*, and the aggregate multiplied by 2 (or 4) for open (or closed) pipes gives the *effective length* of the whole sound-wave. It is this excess of the effective length of the half-sound-wave over the actual length of the flute that causes distortion of the modal sequence in flutes—but not in reed-blown pipes—in which the fingerholes have been equally spaced from the *exit* in order to produce a Mode (q.v. Flutes, Chap. vi). Unless the amount of the excess can be correctly computed, and allowance made for it in placing the first hole above the exit, the Mode is distorted, and the only remedy is to use the first hole as vent, with a lower determinant and so change the Mode. It will be realized from these few hints (which anticipate the treatment of this subject given under 'Flutes') how thorny is the path of the primitive flute-maker, and how necessary it is to go carefully over the whole ground.

It is because the Aulos is exempt from the necessity of making allowance in respect of diameter that it is privileged to be regarded as Mode-bringer. Observant students will probably have noticed that in many of the Auloi—especially in those used in pairs—the first hole is sometimes placed at a great distance from the exit. How is it, they may inquire, that this peculiarity does not, as in the flute, indicate the influence of diameter? The answer to this legitimate question is that the longer reeds had been deliberately selected on account of their sonority and greater beauty of tone with which the wealth of harmonic overtones, possessed by long pipes of relatively narrow bore, endows those reeds. It follows that the position of the first hole on the long reed is a matter of agreement between the convenient increment of distance from fingerhole to fingerhole, balanced by the determinant of the Harmonia selected, and the extrusion at which the mouthpiece will play; all of which sounds very complicated, but is nevertheless instinctively negotiated by a piper familiar with the Harmoniai.

THE OPEN PIPE

We can now consider a few fundamental data concerning the relations between length and pitch in pipes generally.

On examining a pipe open at both ends with a view to ascertaining the pitch of its potential resonance note, the proceeding is as follows: the pipe is held in a vertical position in front of the player, the breath is directed in a thin stream, through a narrow aperture between the lips, so that it impinges on the sharp further edge, e.g. as in playing the primitive end-blown flute (not as a *nay*),[1] the Panpipe, or in testing the resonator of the Aulos.

If to twice the actual length of an open pipe we add twice the internal diameter, we have the effective length of the complete sound-wave; its vibration frequency is equal to half that of the actual length.

In order to find the equivalent vibration frequency (= v.f.) of the fundamental of an open pipe, the velocity of a sound-wave in air at moderate temperature (340 metres per second, i.e. *actual* length) must be divided by the effective length of the complete sound-wave = $2(L + \Delta)$ and this involves an operation in inverse proportion. The quotient then gives the vibration frequency of the simple or half-wave[2] length and must be divided by two before it can represent the vibration frequency of the note or complete sound-wave.

The formula for the determination of the pitch of the fundamental

[1] In this instrument, used at the present day in Northern Africa and in many other Eastern Countries, the breath-stream is directed not across the end, but propelled obliquely *into* the bore of the flute, thus reducing the actual length of the instrument (see Chap. x, Table ix).

[2] The v.f. of the half-wave (= s.v.f.)—the *onde simple* of the French and Belgian schools—is the standard mainly adopted for scientific work in French-speaking countries; it corresponds to the *actual length* of an open pipe as used in this work or to the quarter-wave length (*demi-onde simple*) of a closed or stopped pipe. In England and in Germany—unless otherwise stated—the complete sound-wave length is the standard. Its vibration frequency (termed by the French school 'vibration double' [v.d.]) is thus half that of the actual length or s.v.f., i.e. the vibration frequency of the simple or half-wave. It must be noted that the frequency of the half-sound-wave is a pure convention, existing only on paper, for it cannot be heard.

resonance of an open pipe may be stated thus (in these formulae, (*a*) refers the formula to an open and (*b*) to a closed pipe):

FORMULA I(*a*)

If L = length in metres per second.
and Δ = internal diameter of bore, then:

$$\frac{340}{2(L+\Delta)} = 2\,x \text{ No. of v.f. of half-sound-wave.}$$

and x = v.f. of complete sound-wave or fundamental note of pipe.

Conversely the length of an open pipe of a given diameter required to produce a note of definite pitch may be computed thus:

FORMULA II(*a*)

$$\frac{340}{\text{v.f.} \times 2} = 2\,x \text{ effective length of sound-wave.}$$

So that x = effective length of half-sound-wave,
and $x - \Delta$ = length of pipe.

THE CLOSED PIPE: FUNDAMENTAL RESONANCE AND DETERMINATION OF PITCH

The fundamental resonance for a closed pipe, such as a stopped organ or Panpipe, is ascertained by taking the internal length of the pipe from end to end, with the addition of half the diameter; this is equivalent to the quarter of the length of the sound-wave and must, therefore, be multiplied by 4 for the effective length. The Aulos with its D-R. or B-R. mouthpiece in position has the reactions of a closed pipe. By fundamental resonance is always meant, in these pages, the note obtained by blowing across the top of a pipe or mouthpiece straw; it represents the pitch value of a given length + diameter. The fundamental resonance is the latent power of the pipe which does not come into play as an actual note, but rather as a qualitative influence, as a factor in the final settlement of the tonality of the Aulos, and as an index of the tonality of its harmonic constituents.

By fundamental *note* of a pipe, on the other hand, is meant the lowest note obtained from the exit, with all fingerholes closed, or from the first hole, used as vent, and always left uncovered.

FORMULA I(*b*) (closed pipe) [1]

the converse, by analogy with Formula i(*a*)

The formula for the determination of the pitch of a stopped pipe from its actual length may, therefore, be thus expressed: when L indicates the internal length in metres.

Formula i(*b*):

$$\frac{340 \text{ m./s.}}{4\left(L + \frac{\Delta}{2}\right) (\text{or } L \times 4 + \Delta \times 2)} = 4x.$$

So that x = v.p.s. of sound-wave.

[1] Formulae i and ii(*b*) apply to the Aulos and to its mouthpieces as *closed* pipes.

These formulae suffice as preliminaries to our investigation of the structural laws which determine the characteristic features of the Aulos, its reactions and behaviour, when in practical use in the Art of Music.

THE PART PLAYED BY THE MOUTHPIECE OF THE AULOS

Since the reed-blown pipe does *not* depend upon the operation of the rule for the determination of pitch from the actual, or from the effective, length of the pipe, as described above, it is not possible to compute the vibration frequency of any note produced on the Aulos without possessing a specification of the mouthpiece to be used, and of the Mode to be obtained by its insertion into the resonator at a fixed amount of extrusion. The influence of diameter in cylindrical pipes has no significance in the determination of exact pitch in a reed-blown pipe; its influence is only effective in the mouthpiece, and indirectly on the resonator through the mouthpiece. Modality, however, not being essentially an affair of absolute, but rather of relative pitch, may be established on the basis of a general law of proportion, which remains unaffected by the dimensions of length or diameter in individual pipes. In fact, it may be laid down that the law producing modality [1] on a reed-blown pipe is independent of the actual length and diameter of individual pipes.

Since repeated tests have proved that the actual pitch of reed-blown pipes cannot be computed by means of Formula II, the implication is that the influence of the mouthpiece is paramount, and overmasters that of the length of the resonator. No excuse is needed, therefore, for the amount of attention and space which it is proposed to devote to these small but important adjuncts of the Aulos, or for our treatment of these mouthpieces as independent instruments, possessing a determinant influence on the whole instrument, as regards pitch, ethos, modality and tonality, and in a certain definite sense also on the music and musical system of Ancient Greece. It is not claimed that anything approaching an exhaustive study of the behaviour and properties of primitive reed instruments has been achieved; the ground has merely been broken. Nothing short of an intimate experience of the subtle charm and unexpected properties and qualities of these instruments enables the student to realize the hold they had on the Greeks, and how they were primarily responsible for the birth of the Harmoniai, and for what modern musicianship regards as exotic subtleties of intonation.[2]

We will now proceed to what perhaps constitutes a new and original contribution to the study of reed instruments and mouthpieces. The facts have been elicited during experiments carried out over a period of many

[1] The basic law of Modality and the law in operation have been fully described in Chapter II. Once the mouthpiece has been made that will play the pipe freely and with ease, the determination of the Mode on any Modal Aulos is a simple matter explained later.

[2] 'Aristoxenus and the Intervals of Greek Music', by R. P. Winnington-Ingram, *Class. Quarterly*, xxvi, 1932, pp. 195 ff., and 'Further Notes on Aristoxenus and Musical Intervals', by Kathleen Schlesinger, *Class. Quarterly*, xxvii, April, 1933, pp. 88–96.

years, with innumerable mouthpieces of several varieties belonging to the two types: double- and beating-reeds. Information on the laws governing the reactions of these simple mouthpieces is not available in the text-books consulted; and without specific knowledge on the subject, no adequate estimate of the nature of the practical music of Ancient Greece would be possible.

THE DOUBLE-REED MOUTHPIECE

The D-R. mouthpiece may now, as the more primitive of the two types, be studied first as an independent little instrument. The type of double-reed mouthpiece in use on the primitive Greek Aulos was obviously not the highly sophisticated oboe reed of the present day, but the simple ripe wheat or oat stalk, plucked from the cornfield. For the purpose of these tests and experiments, I have with great success used matured dry stalks in preference to those treated by soaking, pressing and flattening, because they were always ready for use, and proved to be of equal efficacy. Cylindrical stalks, having round (or nearly so) apertures at exit and embouchure, are the best; they are in a category by themselves, and react with great regularity and precision to the formula I have evolved. The straw in its dry state does not seem to be inferior musically; fine, resonant and very powerful tones are obtainable from straws of relatively wide calibre and of somewhat loose, thin texture, a result probably due to elasticity. For practical use, the treated straw, described in Chapter ii, probably has some advantages which compensate for the time and trouble spent upon them; but I find that with reasonable care the untreated straws last for years without deterioration.

Unfortunately, most of the surviving specimens of the resonators of ancient reed-blown pipes have a bore of small calibre, and it is often found that a particularly desirable mouthpiece from the point of view of its proper note, tone quality and readiness in speaking, is too large to fit into the pipe.

THE PROPER NOTE OF THE MOUTHPIECE

By this is meant any note produced by the mouthpiece used alone. It is found that with this type of mouthpiece there is a choice of a number of notes of definite pitch which may be obtained, for experimental purposes, by means of a simple calculation of a selected vibrating length (V.L.), which is measured from the upper end (or embouchure) down the length of the stalk, and thus determines the pitch of the note of the mouthpiece. This vibrating length is taken into the piper's mouth and a Node is induced by the impact of the lips (aided on a new mouthpiece by a slight pressure of the thumb-nail), at the exact point which is to ensure the correct intonation of the desired note. If the response does not come at once, a strong *ictus* breath, used many times in rapid succession, will make the mouthpiece speak, at first, maybe, with many rumblings. After this a soft normal breath, without forcing, produces the note known in this work as the Norm. It is the note that corresponds invariably in pitch with the theoretical note indicated by the formula (to be given later) for the Vibration Length selected.

This normal note sounds again and again, staccato and unchanged, as long as the breathing is normal and unforced and the vibration length kept stable.[1]

Before giving the formula, I must mention that I distinguish three different kinds of note, obtainable at the same V.L. from reed-mouthpieces of both types, D-R. and B-R.

(1) The *norm*, invariably corresponding with the formula, when a normal breath is used, i.e. when not forcing the tone, or making use of the glottis action for the purpose. This note is of a pitch determinable by measurement of the V.L., but it is not necessarily the most useful note from the musical point of view. Obviously then, the norm belongs to theory rather than to practice.

(2) The *ictus* note, obtained on the same V.L. as the norm, by forcing with emphasis, when the note bursts forth with power, sforzando, and sometimes higher in pitch. It is an individual note, at present beyond the tyranny of formulae.

(3) The *glottis* notes, likewise obtained on the same V.L. as the norm: (*a*) by relaxing the muscles controlling the glottis, when the norm may drop a fourth, fifth or even an octave; (*b*) by tension of the muscles (as when singing a high note), which sends the pitch up and imparts a brilliant quality. It must be borne in mind, however, that such a note, produced by extra tension, is proper to a relatively short length of the air column, and is therefore not suitable as fundamental note from exit or vent. Such use would imply a power to rise at least five or possibly more steps higher up the Modal Scale, a power beyond the limit of these little mouthpieces except by use of the harmonic register.[2] Musically, the mouthpiece when used in the Aulos gives the best results with the *ictus* and *glottis* types of note, and 3(*a*) must always be preferred, if a sequence of seven degrees in the Modal Scale is to be obtained for the purpose of tests. The reason for this has been explained in Chapter ii; for as the fingerholes are opened one by one, and the column of air shortened degree by degree, greater compression of breath is required to produce the requisite rise in pitch from the mouthpiece at the same V.L. This cannot be carried out artistically and successfully, except from the easy, free and full note of lowest pitch at which the mouthpiece is capable of speaking by the help of the glottis action.

In order to select a suitable and satisfactory note for the mouthpiece, a V.L. is chosen which, with the addition of the diameter of the straw (entering into the computation as added length), taken four times, corresponds with the effective length given in the table, in which length and v.f. are reciprocals in the computation by the velocity of sound in air (p. xlvi).

[1] Should the pitch of the note change while the test is being carried out, the V.L. should be checked, as the lips may have shifted their position and altered the length.

[2] See further on, for the discussion of the *speaker*-hole in connexion with the harmonic register on the Aulos and A. A. Howard's interpretation of 'Syrinx' as 'speaker-hole' (*Harv. Stud. in Class. Phil.*, Vol. iv.).

Should the straw have become flattened at the embouchure by use or by treatment, the width of the narrowed aperture is measured instead of the diameter (the edges of the straw not included). The most useful vibration lengths are, according to my experience, ·058 to ·063 for $\frac{F\ 17}{256 \text{ or } 128}$ [1] $= 332$ v.p.s., and ·075 to ·080 for $\frac{C\ 11}{256 \text{ or } 128}$.

I have experienced great difficulty in getting a D-R. mouthpiece to give the norm G 15 $= 375{\cdot}5$ v.p.s. at a V.L. of ·049 to ·051. Out of a dozen or more tried at a sitting, only one D-R. mouthpiece responded, and then grudgingly to the V.L. ·051 on G, whereas as soon as I touched the Node at ·060 to ·064, the F 17 spoke instantly, and the C 11 likewise at ·075 to ·080. It would be interesting to hear of the experience of others on this point. Is it a case of the personal equation or of a real difficulty? The length of straw designed to form a D-R. mouthpiece reacts as an open pipe only when blown across the top, for the purpose of ascertaining its fundamental resonance, the pitch of which is dependent upon the length of the straw + the diameter, both multiplied by two. But as soon as the mouthpiece is taken into the mouth, and the Node of vibration determined by a slight pressure of the lips, the mouthpiece behaves as a stopped pipe and speaks at the pitch proper to the V.L. + diameter taken 4 times for the quarter-wave length, or 16 times for the whole-wave length; the v.f. of the norm is, therefore, 4 times lower. This value applies only to the mouthpiece used alone. The formula which has been used in drawing up the table containing the reciprocals of effective length of the norm and its v.f. is, therefore, as follows:

THE DETERMINATION OF PITCH FROM LENGTH ON A D-R. MOUTHPIECE

FORMULA VI

$$\frac{340}{4(\text{V.L.} + \Delta)} \text{ m./s.} = 4x, \text{ and } x\ [2] = \text{v.f. of complete sound-wave.}$$

This survey of the D-R. mouthpiece and its possibilities, when speaking alone, impresses one strongly with its highly individual nature. But one wonders what happens when the mouthpiece is inserted into a resonator possessing a fundamental resonance of its own: will the mouthpiece be the master or will there be co-operation on an equal footing?

[1] Consult note on my nomenclature at beginning of book.

[2] Or alternatively $$\frac{340}{16(\text{V.L.} + \Delta)} = x \text{ v.p.s.}$$

As an example of the working out of the formula we may cite D–R. mp. No. 'O4'. The length of straw = ·134; Δ = ·004; V.L. ·060; so

$$4({\cdot}06 + {\cdot}004) = {\cdot}256,$$

the effective length, then:

$$\frac{340}{{\cdot}256 \times 4}\text{m./s.} = \frac{340}{{\cdot}1024} = 1328 = 4x, \text{ and } x = 332 \text{ v.f.}$$

SIGNIFICANT AND UNIQUE PROPERTY OF THE DOUBLE-REED MOUTHPIECE

Not every D-R. mouthpiece will play in a given pipe, nor at a given extrusion, and therefore Mode. There is, however, a tremendously significant property to be placed to the credit of the D-R. mouthpiece—which cannot be claimed for the B-R. mouthpiece—viz. that once happily installed in a modal Aulos, the effect of the mouthpiece remains constant in tonality and modality ; and moreover, the intervals of the sequence, hole by hole, as played by the Aulos, also remain constant over a period of years. This statement applies, for instance, to the Elgin Aulos, the first D-R. mouthpiece of which, tested in 1925, still plays the pipe on $C = 128$ v.p.s. in October, 1933, with every note issuing from the fingerholes in tune, according to the intervals of the sequence. The author possesses six D-R. mouthpieces that give the same results on $C = 128$ v.p.s., while a seventh at a different extrusion, and therefore in a different Harmonia, plays from the same Aulos on $A\ 27/128 = 208{\cdot}6$ v.p.s., giving forth intervals of different ratios from the fingerholes, all equally true. The significance of the Aulos with D-R. Mp. which establishes it as Mode-bringer will be still better appreciated further on, when the performance of Aulos Loret xxiii in two different Harmoniai, played by the same mouthpiece at a different extrusion is examined.

THE STRUGGLE FOR MASTERY BETWEEN MOUTHPIECE AND RESONATOR

We have now established on a secure basis, by means of two formulae, the reciprocal relations of length and pitch in a double-reed mouthpiece used alone, but the additional factors governing the fundamental note of mouthpiece + Aulos resonator still remain unsettled : to what is due the change of the proper note of the mouthpiece when inserted into the resonator ?

The proper note of the mouthpiece for use in a modal Aulos should always be, as already mentioned, the lowest glottis note obtainable from mouthpiece and resonator combined ; the glottis note, although produced on the V.L. of the norm, is not subject to computation by length, but is governed by the laws of resonance, which still remain somewhat obscure. One of these laws concerns the power inherent in a sounding note of calling forth, in order to reinforce itself, a note of the same V.F. in a body capable of periodical vibration, such as a string, or a musical instrument : this is known as sympathetic vibration or resonance. There is, in addition, the less obvious power inherent in the sounding note of a mouthpiece of drawing forth from a resonator a strong component harmonic, such as a 5th, or a 4th or Harmonic 7th, imposing upon the resonator its own identity, which predominates, so that the mouthpiece still remains the more powerful of the two ; or else the proper glottis note prevails in octave relation, but always at a lower pitch than that of the resonator taken alone, on account of the transformation of the latter, through the addition of the mouthpiece, into a closed pipe sounding an octave lower.

The nature of the struggle[1] for mastery that goes on in the Aulos is due to the latent power of resonance operating in both mouthpiece and resonator. The result of the struggle is manifest in the final stability of the fundamental of the instrument, once a satisfactory mouthpiece has established itself as dictator. The pitch of this fundamental remains constant so long as the mouthpiece is not shifted, i.e. the amount of extrusion, and therefore the Mode is not altered. The opening of the fingerholes then produces notes in accordance with proportional length from the centre of the hole to the tip of the mouthpiece, but always in relation to the fundamental note of the Aulos; and the resonator pipe reinforces each one; for the opening of a fingerhole virtually creates a new resonator of the same pitch as the note of the fingerhole, or harmonically related to it.

It is important in this connexion to remember that the distance from hole to hole from the exit upwards, as the fingers are lifted one by one, does not represent length convertible into pitch but the ratio of one segment to another in the modal sequence from the Tonic. The interval produced, for instance, from Hole 1 to Hole 2 may be $\frac{11}{10}$ with one mouthpiece in position and $\frac{12}{11}$ or $\frac{10}{9}$ with the same mouthpiece, or another, at a different extrusion. The whole significance of fingerholes in the Aulos is based on proportion, calculated in the following manner: the length from the centre of Hole 2 to the tip of the mouthpiece, which has produced the interval $\frac{11}{10}$ from Hole 1 used as a vent (or Tonic), is not reckoned *qua* length but as the multiple of the increment of distance between Holes 1 and 2, which here contains 10 of the increments. The uncovering of Hole 3 will shorten that length by one more increment, leaving 9 of these and producing a note in the ratio of $\frac{9}{11}$ to that of Hole 1. Of course, the length would be the same whether measured by rule or by increments, but the reciprocal pitch equivalents of the length by Formula i would probably be wide of the mark:[2] whereas if calculated by the ratio of the increments on the V.F. of the fundamental, they will be found correct theoretically for the modal degree in question, and for the actual note of the practical test.

We now have to face new difficulties, for the resonator holds a check over the mouthpiece in the matter of calibre; and many an excellent mouthpiece has to be rejected because its external diameter is too great to enter the bore of the resonator.[3] When this difficulty has been safely negotiated by the provision of a mouthpiece of smaller calibre, modality may still hold the mouthpiece in check (as seen in fn. [3]), owing to the length of extrusion required by the Harmonia in order to complete the multiple of the increment of distance required by the Modal Determinant.

[1] This struggle is sometimes audible, as when after playing two or three consecutive notes, the Aulos drops down a 4th, hesitates between that and its higher octave and then plunges back into the sequence controlled by the holes.

[2] Instances of actual correspondence in absolute pitch are quite exceptional. See Records of Performance of Auloi.

[3] The Aulos numbered Loret xxvii (Chap. x) furnishes an example of this: when mouthpiece xxvii(*a*), which gave a fine tone in the resonator at Ext. = ·088 had to be rejected because the Harmonia required an extrusion of only ·065 and the mouthpiece could not be pushed in so far.

It is clear, for instance, that with an extrusion of ·055, the mouthpiece could not make use of either of its best norms, viz. *F* 17 (= 332 v.p.s.) with a V.L. of ·060, or *C* 11 (= 256 v.p.s.) with a V.L. of ·080. The alternative is either to fit a different mouthpiece, or to play in another Mode, having a higher determinant number, by increasing the extrusion by one or even two increments.

How this was achieved and concealed in practice is revealed by the vase paintings reproduced in Chapter ii, in which the Aulete is seen adding a bulb over the mouthpiece of his Aulos, to conceal the extra length of the shank. If the increased extrusion leaves the fundamental note of the Aulos unchanged, the result is a different Harmonia ; but if with a mouthpiece at longer extrusion, the pitch of the fundamental happened to fall to that of a lower degree of the original Mode, the lengthened extrusion would bring about a change of species. This is the exception.

An illustration of such a change of Mode, brought about by pulling out the mouthpiece to the length required for the additional increment of distance (= ·028) demanded by the new Mode, based upon a higher Determinant, may be quoted here from our record of Aulos Loret xxiii (q.v.). D-R. mouthpiece N. 32 was used in the pipe and at an extrusion of ·099 and a V.L. of ·060 (= *F* 17 as norm), the Aulos played the first tetrachord of the Lydian Harmonia of Determinant 13, on its proper Tonic, $\frac{A\ 13}{128}$ (= 216·6 v.p.s.) from Hole 1 as vent with all notes in tune, viz.

D-R. MOUTHPIECE N. 32 AT AN EXTRUSION OF 099. SCHEME I

v.f.	216·6 ;		234·6 ;		256		281·6	
Holes	1		2		3		4	
Ratios of length	$\frac{13}{13}$		$\frac{12}{13}$		$\frac{11}{13}$		$\frac{10}{13}$	(*Dated Nov. 4, 1933*)
Intervals in cents		*138·5°*		*150·5°*		*165°*		

Diminished 4th $\frac{13}{10}$ = *454° cents*

The mouthpiece, untreated, had not been used for several months, but it played at once with a firm, rich tone on $\frac{A\ 13}{128}$, as recorded, on April 24, 1933, testifying to the stability of its intonation. Then, without pausing, the mouthpiece was pulled out to extrusion ·127, thus adding the length of one more increment, so that they now totalled 14, the Modal Determinant of the Mixolydian Harmonia. The result was eminently satisfactory ; for in spite of the added length, the Aulos still played on the same fundamental $\frac{A\ 13}{128}$ = 216·6 v.p.s., a fact that deserves some attention. Here we have an increment of distance added, i.e. increased length, that on a string would have meant a different species taken in the same Mode, but on a lower Tonic, bearing the ratio number of the new Modal Determinant.

On the Aulos, however, by virtue of the unique conditions characterizing the behaviour and reactions of the mouthpiece, the pitch of the lengthened Aulos remains the same, but the fingerholes now give out notes of different intonation, corresponding to the ratios of the new Modal Determinant 14, as seen in the second scheme below.

D-R. MOUTHPIECE N. 32 AT AN EXTRUSION OF ·127 ON $\frac{A\ 13}{128}$ = 216·6 V.P.S.

SCHEME II

v.f.	216·6 ;	233·3 ;	252·7	274·7 v.p.s.[1]
Holes	1	2	3	4
Ratios of length	$\frac{14}{14}$	$\frac{13}{14}$	$\frac{12}{14}$	$\frac{11}{14}$
Intervals in cents	*128°*	*138·5°*	*150·5°*	

Diminished 4th $\frac{11}{14}$ = *417·4° cents*

For a comparison of the actual practical results given above for Loret xxiii with mouthpiece N. 32, with the theoretical v.f. worked out by formula (see below).

LYDIAN HARMONIA

$$\frac{340\ \text{m./s.}}{\underbrace{\text{L}(=\cdot 265)+\text{Ext.}(=\cdot 099)}_{=\cdot 364}} = 4x \text{ and } x = 233{\cdot}5 \text{ v.f., i.e. } \frac{B\ 12}{128} \text{ v.p.s.}$$

and

MIXOLYDIAN HARMONIA

$$\frac{340}{\underbrace{\text{L}(=\cdot 265)+\text{Ext.}(=\cdot 127)}_{=\cdot 392}} = 4\,x \text{ v.f.}\left(= 217 \text{ v.p.s., i.e. } \frac{A\ 13}{128}\right).$$

shows that the vibration frequencies of Aulos xxiii played in the Mixolydian Harmonia, at extrusion ·127, agree, note for note, with those derived from the working out of the formula, but with this difference : the theoretical working out by lengths, through the addition of an increment of distance, makes the change one of *Species*, whereas in practice it is one of *Mode.* This correspondence of practice with the theoretical formula must be regarded as purely accidental ; it is the exception. The strange phenomenon is thus witnessed of an Aulos played in two Harmoniai in quick succession,

[1] The v.f. given here is not the theoretical one calculated by formula, but that of the actual sounds. The fundamental $\frac{A\ 13}{128}$ corresponds with that note on my modal piano, tuned to the Dorian Harmonia of Determinant 22, calculated thus :

$$\frac{128 \text{ v.p.s.} \times 22}{13} = 216{\cdot}6 \text{ v.p.s.}$$

Each of the ratios is computed from the fundamental $\frac{216{\cdot}6 \times 13}{12,\ 11,\ 10}$.

by the same mouthpiece, drawn out to a different extrusion, when added length does not result in lower pitch, but produces instead the same fundamental note.

The change of modality is significant. The added increment reacts proportionally on the intervals of the modal sequence obtained through the fingerholes; the denominator of the fundamental note of the Aulos is changed, without alteration of pitch, from $\frac{13}{13}$ to $\frac{14}{14}$ and with it the ratios of the whole sequence.

When it is realized that this change of modality and of ethos is effected by the same mouthpiece merely through the addition of an increment, adding a few millimetres (28) to the length, without change of Tonic, it is clear that latent in this little D-R. mouthpiece is to be found the secret of modality. The length of the column of air from centre to centre of the fingerholes remains as before, the same for each, but the one added increment of length possesses, through proportion, the power to change the ratios and vibration frequencies of all the notes obtained through the fingerholes. Herein lies one of the differences between the modal reactions of strings and of pipes. On strings a change of Mode on a common Tonic involves a different I.D., consequent on a higher or lower Determinant, whereas in the Aulos, the I.D. remains of the same length, determined by the fingerholes, in a primitive instrument.[1]

In Ancient Greece such an Aulos, when played, would have had two bulbs with an extrusion of the mouthpiece to ·099, and we might have seen the Aulete adding a third bulb after pulling out the shank of the mouthpiece to ·127.[2]

The modern oboe, needless to say, possesses none of the characteristic features of the modal Aulos, for the intervals provided by the boring of its holes are irrational, and not proportionally related to each other or to the fundamental; its mouthpiece, moreover, has undergone radical changes.

MODUS OPERANDI OF THE PROPORTIONAL LAW PRODUCTIVE OF THE HARMONIA ON THE AULOS

Thus, the primitive mouthpiece of the Aulos was endowed with the power of changing the Harmonia for which the holes had originally been bored. This unique feat was accomplished, as we have seen, by merely pulling the mouthpiece out, or pushing it further into the Aulos, not at random, however, but to the extent of one or more increments. A variation in the number of increments of distance means a change of Determinant and, therefore, of Harmonia.

[1] Towards the close of Chap. ii, there is a description of certain devices calculated to counteract the disadvantages of fixed fingerholes, in order to make it possible to obtain more than one Mode on the same Aulos, and to modulate from one to another. Allusion is here made to the sliding bands and to the bombyces or additional tubes.

[2] A bulb, found with the Elgin Aulos, which may be seen in the Graeco-Roman Department at the British Museum, measures about ·045 (not quite 2 in.). These bulbs were probably made of various lengths according to the I.D. of the Aulos (see photograph No. 17 of Elgin Aulos, Chap. x).

Now, how does this affect the basis of our thesis? Does it invalidate the claim that the Aulos was a Mode-creator? Let us see.

A deliberate change of Harmonia is the privilege of cultured musicians, not of the primitive piper; the latter proceeds empirically, feeling his way in the dark as he goes, guided by intuition or instinct; it may take him months, years, or half a lifetime to learn his lesson; but beyond all doubt the lesson was learnt: that the Harmoniai duly came to birth on the Aulos, and were cherished by the Greeks and by other nations in the Ancient East. There is much to follow concerning this power to change the Harmonia, by merely altering the extrusion of the mouthpiece, which implies a period far removed from early beginnings. The Harmoniai had, in fact, at such a time, already been practised and adopted as the language of Music. Since the Harmoniai are based upon a natural law of proportion embodied in all pipes and flutes alike, the same for all races and all ages, it is legitimate to supplement the evidence from Ancient Greek sources about early pipe-playing by what we can learn from the pipe-making among the folk in our own Europe, or from primitives in distant lands. What they are able to accomplish in our own time with the same crude means at their disposal, and to preserve unchanged when handed down from one generation to another, justifies us in deducing similar practices among the ancient nations. This reasoning is more especially applicable to the Greeks who have, in addition, provided evidence of the survival of the Aulos scales, at successive stages in the development of Music by their race, in their civilization, and in others derived from it.

Is it claimed or proven, first of all, that the genesis of a Harmonia can only occur when all the increments of distance are absolutely exact? To this an unqualified negative must be returned. The increments may and do vary by an excess, negative or positive, by as much as 5 or 8 mm., especially when there are six fingerholes and only one shows an abnormal excess, as is the case in the Elgin Aulos 1. It would, moreover, be a likely happening that the unsophisticated piper, in his early gropings in modality on the Aulos, would frequently have an excess, negative or positive, in the amount of extrusion he gave to the mouthpiece. It must have caused the primitive piper a pleasurable excitement when he found that by drawing out his mouthpiece a little further from the Aulos, he could get—instead of his Dorian Harmonia, by whatever name he called it—the Phrygian sequence played by a friend on his own pipe. Such a discovery probably stimulated curiosity as to the reason for this, and no doubt led to further revelations. The fact that the piper did frequently hit upon the right amount of extrusion for his mouthpiece while experimenting, need not cause surprise, for the lay of the fingers on the equidistant holes would subconsciously predispose him to adopt the I.D.

It is, however, a fact worth noting that this excess in question may occur without distortion of the Harmonia, provided that it is appreciably less than the half increment. The result of such slight excesses is loss of freedom and ease in the production of tone; loss of stability and tone quality; and in addition to these penalties there may occur, with pipes having six or seven holes, a slight flattening in the two highest notes, which does not, however, amount to a distortion of the Mode. A practical proof of this assumption has been afforded by numerous experiments tested by means of a carefully and accurately marked monochord for all seven of the original Harmoniai. A case in point is the first interval of the Elgin Aulos with an I.D. of ·040, whereas the mean is ·032. This Aulos was recently tested again on this point with D-R. mouthpiece Cl. 18; at first the $C = 128$ v.p.s. was difficult to obtain as fundamental for Hole 1. On investigation, it was found that the V.L. used had been much too long, viz. ·075 instead of ·060;[1] as soon as this had been rectified, the whole sequence was played in perfect tune on $C = 128$ v.p.s. as in previous tests. Particular notice paid to the intonation of Hole 2, at a ratio from Hole 1 of $\frac{11}{10}$, testified that this was invariably played in perfect tune in spite of the excessively large I.D. between Holes 1 and 2. For further confirmation of the practical bearings of the basic law of the Harmonia, examination of the Records is invited.

PUZZLING PROPERTIES OF THE AULOS AND ITS MOUTHPIECES

We have seen that the length from exit or vent to the tip of the mouthpiece must be a multiple of the I.D. in order to produce the Harmonia, and that a shift of the mouthpiece by adding or removing one or more I.D. effects a change of Harmonia. According to the number of increments, the fundamental note from exit or vent may thus be represented on paper by a differentiated unit, such as $\frac{11}{11}$, $\frac{12}{12}$, &c., while the pitch of the fundamental may be unaffected by the altered length (e.g. as recorded under Aulos Loret xxiii played by D-R. mouthpiece N. 32). Does this differentiated fundamental then exist only on paper to be allotted retrospectively? Is it a practical reality or merely an abstract notion? The fact that when Hole 1 is uncovered the vibrating column of air within the pipe may, by one shift of the mouthpiece, lose $\frac{1}{10}$ of its length, at another shift $\frac{1}{11}$ or $\frac{1}{12}$, in spite of the fixed position of the fingerholes and of the I.D., and that, consequently, three or more notes of different pitch may at times issue from the same hole, while remaining stable and constant as members of one modal sequence or another, seems to demand some explanation which is not available in text-books.

The crux of these puzzling properties of the Aulos and its mouthpiece is evidently connected with the I.D., since absolute length as determinant of pitch is excluded, a fact which points at once to proportion as the alternative principle.

We must here recall (cf. Chap. i) the results of an aliquot division of string or pipe by a determinant number, which may be made in two different ways:

(1) A very slight exciting cause at a nodal point—or end of any one of the equal segments—produces the harmonic overtone (or partial) bearing

[1] It may be noted that a V.L. of ·060 in a D.R-mp. with Δ of ·004 or ·005 gives as proper note F 17, i.e. it induces a proportioned resonance on a C fundamental of a 4th or 5th.

the same number in the Harmonic Series as the Determinant of the aliquot division.

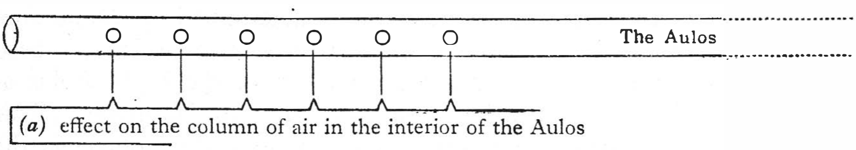
(2) A definite stop (made on a string by finger or movable bridge, on a pipe by the uncovering of a fingerhole) will produce the note of the reversed Harmonic Series, that bears the same numeral as the number of increments of distance between that hole and the tip of the mouthpiece. That same numeral also figures as the numerator of a fraction of the whole length, the denominator of which consists of the Determinant of the aliquot division. In No. 1 each aliquot division produces but one and the same Harmonic by slight impact at any of the nodal points.

In No. 2, each segment, as we know, produces a different member of the reversed series. The Harmonics may be considered as always latent, ready to spring into being in response to a stimulus; they may exist as constituents of the fundamental of the Aulos, influencing its tone-quality and resonance; or they may be produced as active notes and integral parts of the compass extended to its harmonic register.

UNSUSPECTED FACTOR IN THE INTERIOR OF THE AULOS

These preliminary remarks may prove helpful in grasping the implications of a suggestion about to be made, of the presence in the Aulos itself of an unseen and unsuspected factor, which preserves and co-ordinates the increments of distance and fixes the proportional value of each of these. When the fingers cover the holes, the soft pads of the fingertips or the joints only penetrate to a very short distance below the surface, and a nodal point, coinciding with the centres of the fingerholes, is formed by the compression in the column of air under the fingers. Thus, inside the Aulos there exists a concrete plan of the position of the fingerholes, and therefore of the increments of distance separating the centres from hole to hole. This internal concrete plan becomes active as soon as the piper breathes into the Aulos and the vibration at the centres of the holes may be felt as it tingles against the fingers. The line followed by the stationary column of air within the Aulos thus assumes a shape somewhat resembling the rough diagram below at (*a*):

The Plan of the Harmonia in the Interior of the Aulos



The indentations represent the centres of the covered fingerholes in the interior of the Aulos, and denote the position and nature of the stimulus provided, with the effect that the column of air is induced to vibrate in segments in response to the breath of the piper propelled through the mouthpiece and the resonator.

THE ARCHE DOMINATES THE INNER REACTIONS OF THE AULOS RESONATOR

By analogy with strings, in our argument, Nodes must surely be formed at the internal centres of the fingerholes, while the fingers cover them. The slight impact at one of the Nodes calls forth a faint response from the Harmonic related to the aliquot, as described in No. 1 above; this Harmonic remains latent as a factor of resonance in the fundamental produced by blowing, with all holes closed; but it is more significant still in its function of *Arche* or first cause of the Harmonia. The Arche is a note not otherwise obtainable on the Aulos, for reasons already given, except as Mese, a lower octave of Arche.

The opening of the fingerholes in turn reduces the total length each time by one I.D. but leaves the remaining internal Nodes and their latent effect unchanged.

It is when Hole 1 is uncovered after playing the fundamental that the astonishing transformation takes place, from the partial vibration of the pipe that produces the murmuring overtones as a consequence of a slight impact, to the complete vibration of the whole column of air as a full-voiced note. What is not realized is that the transformation from a latent murmur to an actual note likewise involves the reversal of the Harmonic Series which, starting from the Arche in the well-known progression, proceeds downwards on the return journey towards the fundamental, from which it originally derived. Thus it is that in the pipe the nodal point, always present on the inner surface, exercises a twofold function alternatively, according as the holes are open or closed. The influence of the fingerholes may be thus briefly stated: when closed, one constituent Harmonic, the same for all the closed holes; when open, the full-voiced note from each open hole that is proper to its position in the modal sequence, founded upon the reversed Harmonic Series of the M.D. Change in the extrusion of the mouthpiece involves the following alterations in the inner reactions of the Aulos: (1) the length of the column of air; (2) the determinant number of the aliquot division; (3) the ratio of the fingerholes; (4) the constituent Harmonic induced at the nodal points, which functions as Arche of the Harmonia while the fingerholes are closed.

When all holes are closed the pulses, initiated by a slight stimulus at the nodal points, contribute together one single powerful harmonic constituent to the fundamental tone; its identity is determined by the number of increments contained between the exit (or the centre of Hole 1 used as vent) and the tip of the mouthpiece; viz. 11 in the Elgin Aulos at Extrusion ·108, i.e. the sharp Harmonic 4th or 11th Harmonic. This member of the Harmonic Series—the same for all nodal points—is the Arche of the Dorian Harmonia, which remains as a latent force in the pipe, manifested only through one of its lower octaves, as Mese when Hole 4 is uncovered. The pitch of Arche is, as already seen, directly due to the mouthpiece as determinant factor in the tonality of the Aulos. The opening of Hole 2, at 10 increments from the tip of the mouthpiece, produces as actual note the 10th in the reversed Harmonic Series of which

Arche is number one. The third hole at 9 increments, the fourth at 8, &c., all follow the same process of genesis.

When the mouthpiece is pushed in to the extent of one I.D. (·032), reducing the extrusion in Elgin 1 from ·108 to ·076, one of two things occurs: either the fundamental note of the vent remains at the same pitch, but with a change in the tone quality due to the 10th Harmonic (major 3rd) which now takes the place of the 11th as principal constituent Harmonic of the timbre. The alternative is that the fundamental and the constituent Harmonic both change with the shift of the mouthpiece. It will thus be realized how it may come about as a consequence of the function of the fingerholes when closed, that the timbre of the Aulos may change with the Harmonia.

THE PLAN OF THE HARMONIA IN THE INTERIOR OF THE AULOS

Is this suggestion of the existence in the interior of the Aulos of a display of the grouping of the fingerholes at equal distances, with all its implications, a reality, or is it merely a figment of the imagination—a paper theory? Let it be banished from our minds for the time being, while we endeavour to discover an alternative explanation for the results obtained from the tests.

Consider, for instance, Elgin Aulos 1, which played, with six different D-R. mouthpieces, at an extrusion of ·108, and an I.D. (mean) of ·032, the sequence of the Dorian Harmonia of Modal Determinant 11, beginning at Hole 1 used as vent, on $C = 128$ v.p.s. The length from the centre of Hole 1 to the tip of the mouthpiece $= ·244 + ·108 = ·352$. With no visible or implied guidance for the vibrating column of air, it seems reasonable to assume that the first increment of distance (at ·040),[1] which becomes effective as Hole 2 is uncovered, should set the pace and form the obvious proportional basis for the Modal Genesis and scale, by providing the Modal Determinant—now clearly 9.

But this sequence on paper bears no resemblance whatever to the actual one obtained on the Elgin Aulos with any of its mouthpieces at extrusion ·108. Yet the total length as multiple [$·040 \times 9 = ·360$; or $·039 \times 9 = ·351$], with a slight defect of only 8 mm. to be spread over the 9 increments, is unexceptionable. Why is this? There must be a flaw somewhere to produce such a hitch between theory and practice.

The first I.D. is certainly in excess of the mean, but in playing Hole 2,

[1] $\left[\frac{·352}{·040} = 8·8 \text{ or } 9\text{; and better still } ·352 \div ·039 = 9\ (·039 \times 9 = ·351)\right]$.

This Modal Determinant should give the sequence of the Hypophrygian Harmonia thus:

Holes	1		2		3		4		5		6
Ratios	$\frac{9}{9}$		$\frac{8}{9}$		$\frac{7}{9}$		$\frac{6}{9}$		$\frac{5}{9}$		$\frac{4}{9}$
Cents		*204*		*231*		*267*		*316*		*386*	

the column of air, stimulated into vibration by the breath of the piper, is not cognizant of any of the other increments. How, then, may one account for the fact that the Aulos nevertheless behaves emphatically in favour of an I.D. at ·032, obtainable only as a mean of all the increments; and that it plays the sequence for which Determinant 11 can alone be responsible?

In the face of such uncompromising results, it is evident that the action of the reed mouthpiece—which is induced by some physical stimulus—must be allowed to point the way. The Aulos unequivocally elects to play the sequence based upon the mean I.D., instead of being influenced by the first I.D. from Hole 1 to 2, with which it is brought into actual vibratory contact; it is clear, therefore, that some physical stimulus has been received on the way down BEFORE it reached the first I.D., or else the Elgin Aulos 1 under conditions mentioned above would certainly have played the sequence of the Hypophrygian Harmonia produced by Determinant 9.

Further, if the suggested influence of the covered holes at the inner surface of the pipe, with all its implications, be allowed, then it is found that as the breath travels from the mouthpiece down the bore of the Aulos, setting the column of air in vibration, it stimulates the fingerhole centres on its way, creating Nodes and segments in the column of air, corresponding in length with the increments of distance on the outer surface. The result is that the aliquot division initiated inside the pipe from the mouthpiece end takes the mean as I.D., and that the Modal Determinant is fixed already during the first quarter of the sound-wave's complete journey, which covers four times the distance between the tip of the mouthpiece and the outlet at Hole 1 in the Elgin Aulos.

Since the properties of the mouthpiece of the Aulos clearly rule out length as a basis of computation by the usual Formula No. vi for the intonation of the notes produced through the fingerholes, it was seen that the reed-blown pipe must be subject to some other law. This was found to be based upon the proportion that exists between the equal distance from centre to centre of the fingerholes and the total length of Aulos + mouthpiece. This could be easily measured; and one by one the implications were revealed as forming part of a new musical fact, confirmed by practical experiments which bore out the theory. So far, so good. But the *modus operandi* of the proportional law on the reed-blown pipe had until now remained a mystery, to which the suggestion here offered may provide a clue.

The arguments in favour of the new suggestion seem to be conclusive, and must hold the field until disproved. They provide an explanation of the intimate connexion that exists between the Harmonic Series—recognized in its ascending form as the physical basis of sound—and the series reversed in direction, but identical in the order and magnitude of the intervals of the sequence, introduced in this work as the origin of the Modes, known in Ancient Greece as Harmoniai.

In the Aulos this intimate connexion between the two forms of the

Harmonic Series is thus seen to be preserved as an imperishable record so long as the instrument survives.

HOW THE MODALITY OF AN AULOS MAY BE JUDGED

In order to judge at a glance whether a specimen pipe is a modal Aulos, one must examine the distances between the lateral holes, measured from centre to centre ; should these be perceptibly *unequal*, the pipe is not a modal one. Disregarding the distance from the exit to the centre of the first hole, if these successive distances, when accurately measured, differ only by some 5 mm., or something less, the pipe has modality ; it gives one of the 7 Harmoniai, or one of its derivatives. If the distance from the exit to the centre of Hole 1 is found to measure one or more increments, the matter is easily readjusted, but the distance is only too frequently incommensurate and intended to be disregarded.

Unfortunately specimens in museums and collections will lack a mouthpiece, since these delicate, essential parts of the pipe are the first to perish. The first step, therefore, is to provide the pipe with a mouthpiece that will play the fundamental very freely, for in the mouthpiece lies the crux of the problem, and without it the modality cannot be ascertained. All attempts to judge the scale of a pipe from a photograph, or from a description, even when accompanied by the requisite measurements, must be regarded as hypothetical and approximate, and not as data on which a theory may be based. For, however well the theory appeared to work out, it might be entirely upset by the mouthpiece ; and in any case, the omission of the extrusion length from the calculation would falsify the result and be misleading.

The method of procedure is as follows : it may be taken as fixed by experiment that the ideal conditions under which the Aulos functions demand : (*a*) that the V.L. of a D-R. mouthpiece, or the vibrating tongue of a B-R. mouthpiece, should be equal to one I.D. or more—*not less*. The V.L. is naturally limited by the capacity of the receptacle, i.e. the mouth cavity ;

(*b*) that the mouthpiece should protrude from the reed pipe beyond the base of the vibrating tongue of the B-R. mouthpiece or the Node of vibration on the D-R. mouthpiece by a minimum of $1\frac{1}{2}$ cm. (·015) ;

(*c*) therefore the extrusion of the mouthpiece should not measure less than ·045 or ·05 for a B-R. mp. and *c.* ·075 for the D-R. mp.

To determine the Mode, find the modal theoretical length (θ.L) of the pipe + mouthpiece in position, i.e. at the correct extrusion (= ext.) i.e. the amount by which the reed mouthpiece projects beyond the point at which it leaves the pipe, to the tip of the vibrating tongue ; divide this total length by the increment of distance, or the mean if the distances are not exactly equal to a millimetre or two : the result is the Modal Determinant of the Harmonia.

Obviously the theoretical length of a Modal Aulos cannot be computed in the absence of a mouthpiece, which actually plays the pipe with ease at a definite extrusion, and on a good, low glottis note. Nor can it be asserted that an Aulos may only be played with one mouthpiece at a given extrusion, and therefore that the fingerholes indicate one Harmonia only.[1]

It is assumed, of course, that the mouthpiece has been satisfactorily tried in the pipe and is playing on a low glottis note. If after playing one or two notes the mouthpiece should refuse to continue, a different V.L. may be tried; or the extrusion—and Harmonia—may be changed.

BULBS ON AN AULOS ARE A SIGN OF A CHANGE OF HARMONIA

It will be realized that the correct amount of extrusion of the shank of the straw mouthpiece is of great importance in maintaining the modal sequence in tune, and in the theoretical determination of modality. This extrusion, however, is a variable factor in one sense, for upon it depends the modality of the Aulos; therefore, any change in extrusion, when the length has once been adjusted, has results which must be duly considered. Any modification of this extrusion should maintain the modal relations of the pipe, i.e. as a multiple of the I.D., otherwise distortion of the modal sequence of intervals from the lateral holes will ensue. This being so, extrusion of the mouthpiece may be considered as a means of changing the Mode of the Aulos and the bulbs as an outward sign of such changes of modality. Thus, when a piper is seen removing the third bulb from his Aulos, this means that he is about to shorten the extrusion of the mouthpiece and that his Modal Determinant will be a lower number, e.g. 11 instead of 12, Dorian instead of Phrygian; 13 instead of 14, Lydian instead of Mixolydian, &c. Bulbs may be used with both types of mouthpiece.

TEN MAIN POINTS CONCERNING DOUBLE-REED MOUTHPIECES

The main points elicited during our survey of the reactions of the D-R. mouthpiece, used alone or when inserted into the Aulos resonator, are the following:

(1) The determination of pitch in a D-R. mouthpiece is based upon the V.L. plus the diameter of the straw, taken as for a stopped pipe, four times, and used as a divisor of the 340 metres that represent the velocity per second of sound (as actual length) in air at a moderate temperature.

(2) In the determination of the pitch of a D-R. mouthpiece, the length of the stalk of the straw has no influence.

(3) A consideration of the resonance note of the whole mouthpiece may, however, be useful in improving the tone of the mouthpiece.

(4) The determination of pitch in the combined Aulos and mouthpiece is more particularly the affair of the mouthpiece, the proper note of which governs that of the composite instrument by accommodation through resonance.

(5) The fundamental note of the Aulos, whether from exit or from Hole 1 as vent, once the relations between mouthpiece and resonator have been satisfactorily settled, *remains constant* for years (judged from experience),

[1] See records of Elgin Aulos, Loret xxiii and xxvii, &c.; also Pythagoras and the Piper of the Spondaic Hymn and the Phrygian Melos. Iamblicus, *Vita Pythag.*, Chap. xxv.

480–450 B.C.

Piper holding two Auloi in left hand; one pipe is ready; two bulbs and the reed mouthpiece are visible; the right hand is fixing a bulb on the other pipe (suggestive of the Feat of Pronomus the Theban)

British Museum. By courtesy of the Director

so long as the same mouthpiece is used at the same extrusion and V.L., i.e. for the same Harmonia. The fundamental changes only when those factors are in any way altered.

(6) The intervals of the modal sequence obtained by opening the fingerholes in succession, from the exit upwards, are fixed by exact ratios dependent upon the *Modal Determinant* of the Aulos in any particular combination; thus the pitch of the fingerholes is relative, based by virtue of these ratios upon that of the fundamental.[1] The intervals remain constant so long as the conditions are unchanged.

(7) It may thus be said that the D-R. mouthpiece preserves the integrity of the notes of the Modal Scale, according to the ratio proper to each hole with its fundamental (cf. Chap. ii). But this statement applies only to fingerholes in relation to the Determinant in command at the time, not to the hole *per se*, independently of the Harmonia in which it is being used. Pitch and ratio may be checked, note by note, in practice upon the string of a modal monochord, and in theory by means of a simple calculation. Multiply the v.f. of the common Tonic or fundamental by the ratio of each interval (an improper fraction having as numerator the Modal Determinant, and as denominator the ratio number of the fingerhole). Thus, for Holes 1 and 2 in an Aulos playing in the Dorian Spondaic, Determinant 11, on $C = 128$ v.p.s. from exit: $\frac{128 \times 11}{10}$ = v.f. Hole 1; $\frac{128 \times 11}{9}$ = v.f. Hole 2.

(8) In the Aulos played with a D-R. mouthpiece we have the '*bringer*' and *custodian* of the *Harmoniai*, an instrument upon which absolute reliance can be placed, so long as the essential conditions of structure and playing are fulfilled.

(9) One more property peculiar to the Aulos of this type must be recalled here, which is of supreme importance to the thesis of the work, viz. that with the D-R. mouthpiece a complete octave scale, i.e. the ἁρμονία, can be obtained without difficulty, whereas this feat lies beyond the scope of the B-R. mouthpiece, which can only quite exceptionally, and at the cost of great straining of the glottis action, exceed a range of 4 or 5 notes in sequence. This, moreover, entails a loss of the peculiar beauty of timbre, otherwise characteristic of this type of mouthpiece. As examples, the Elgin Aulos, Loret xxvii and xxviii, may be cited for which the individual records should be consulted.

(10) Finally, the Aulos, within certain technical limits, may accommodate one, two, or even three related Harmoniai, i.e. related through the arithmetical progression of their Modal Determinants. It is not, of course, suggested that the theoretical data discussed in this section formed part of

[1] The statement of Aristides Quintilianus (p. 18M.) in his discussion on the Harmoniai of the Ancients may be recalled here. He calls attention to the common Tonic or fundamental and its implications thus: 'Accordingly it is clear that if one takes the same sign first (i.e. the same starting-note) and calls it at different times by the different value of the note (δύναμις φθόγγου), the nature of the Harmoniai is made manifest from the sequence of consecutive sounds.' He also mentions the fact that the scales have different names according to their species. See also Chap. ii.

the primitive piper's equipment; or that a knowledge of the theory underlying the birth of the Harmoniai upon his Aulos would have been helpful to him in playing it. All that was necessary for him as a practical musician, he acquired by empirical methods, intuitively, and guided by an inherent feeling for proportion, lost only when the primitive becomes sophisticated. It may be suggested here that the great discovery made by Pronomus of Thebes—of obtaining 3 notes from each hole on his Aulos—consisted in pulling out his mouthpiece twice.

THE BEATING-REED MOUTHPIECE

The era of the D-R. mouthpiece may confidently be defined in each ancient civilization as beginning with the birth of the Modes on the reed-blown pipe in remote antiquity. It was signalized during many centuries by the use of the Aulos in a simple, dignified style of music of great beauty, at once impressive and elevating. The passing of this golden age in music on both Kithara and Aulos was deplored by Plato, Theophrastus, Plutarch and others.

With equal certainty the wane of the popularity of the Aulos, played with a D-R. mouthpiece as the instrument of the virtuoso, may be accepted as coinciding with the rise of an elaborate technique, and of music of many notes, mentioned by Plutarch in connexion with Lasos of Hermione [1] (born about 548 B.C.), 'who was obsessed by the multiplicity of notes on the Auloi'. An attribution that can only apply to the B-R. mouthpiece. This decadence, regarded as a falling away from the ancient purity and beauty of music, was heralded, according to Theophrastus,[2] by the celebrated Aulete, Antigenidas, who flourished during the reign of Alexander the Great (born 356 B.C.). The naturalist, as might be expected, had occasion to take notice of the new tendency in Auletics on account of the difference brought about thereby in the growing and cutting of reeds for instruments and mouthpieces (*v. ante*, Chap. ii).

The law-abiding and somewhat pedantic D-R. mouthpiece, invaluable as a record and custodian of the purity of the *ἁρμονία*—testifying to its claim as generator of the Modes—was justly venerated, and its era was acclaimed as golden, not only in Auletics, but also in the history of music. Invaluable as a standard of purity in intonation and as an instrument of reference, the Aulos with D-R. mouthpiece survived long after its limitations as the instrument of the artist came to be reluctantly conceded. It was readily abandoned by the virtuoso, already familiar with the *ἁρμονία*, in favour of the Aulos with the B-R. mouthpiece, or may we say Syrinx, which eventually won the day.

The B-R. mouthpiece, by virtue of its resilience, responsiveness, its sonorities, and more especially of the exhilarating effect of its powerful constituent harmonic overtones,[3] endeared the instrument to the creative musician, inspiring him to an ecstasy which frequently electrified his audience.

[1] Plut., *de Mus.* (ed. Weil and Reinach, Paris), pp. 112–15, § 293.

[2] *Hist. Plant.*, iv, 11, 4–5; *Theophrastus*, xvi, 170, *ap.* Pliny, *Nat. Hist.*, xvi, 36, 66; Dinse, *De Antigenide Thebano Musico*, p. 53; Plutarch, *op. cit.*, p. 84, § 198.

[3] The 5th Harmonic (major 3rd) is generally the first to ring out, followed by the 7th, the octave, and a riot of others in a kind of natural polyphony. Musicians

The description of the making of a B-R. mouthpiece of primitive type—the only form which in itself satisfies a sense of beauty of tone—has already been given in Chapter ii. My early experiments with the B-R., dating back many years, aroused the keenest interest ; amazement at the varying, rich sonorities, at the depth and volume of tone elicited from these tiny, delicate instruments, was evoked, not only in the present writer, but in many others, experts on questions of pitch and in the acoustics of music, who heard them played for the first time. On one memorable occasion, in my study, one of these experts (Mr. D. J. Blaikley) who had been the first to test the capabilities of the famous Lady Maket pipes, discovered in Egypt by Professor W. Flinders Petrie, was examining my facsimiles of these pipes.[1] I told him I desired his opinion of a *new instrument* which I played in concealment. At the first booming note, 8 foot *C*, he was struck with the beautiful tone, ' like a bassoon '. On turning to the instrument, he was amazed to see the tiny straw mouthpiece No. 65, some 5 inches in length (·130), with a vibrating tongue ·031 in length and no wider than 1½ mm. He had used the stiff arghool type of B-R. and the modern oboe reed for his tests. This completely upset all preconceived notions concerning vibrators, and was for me an indication of the direction in which investigations should be carried out.

My earliest B-R. mouthpieces had been cut with tongues of from ·003 to ·004 in width, equal to the diameter of the straw, and only about ·02 in length ; the tone was unpleasant in the extreme, shrill and harsh, in fact a mere squawk. Experiments soon revealed the fact that the fine, narrow tongue, scraped on the under-side to increase resilience, produced results beyond all expectations. Nevertheless, it was not easy to determine, unaided, the factors responsible for the pitch of the proper note of the mouthpiece. Comparisons with specimens equal in length of straw and tongue, but of varying width of tongue, provided certain indications of regional pitch (cf. records of ' B ' and ' R ' mouthpieces) ; and the difference in diameter of these slender straws seemed at first too slight to be of importance, although at times the diameter did appear to have the casting vote. At that stage of research, the three kinds of note which it is possible to obtain from the mouthpiece (of both types) had indeed been experienced and put down to instability, and assigned to caprice on the part of the Aulos and its mouthpiece. So far these results seemed to endorse the strictures of Aristoxenus.[2] Later, however, a discovery of real importance provided the solution of the puzzle, and better-equipped investigators would probably have reached the conclusions more quickly.

with ears sensitive to harmonic overtones would find it repay them to make a few of these little instruments, and to verify for themselves these assertions ; a very little practice would enable them to follow the play of the harmonic development, which is a far more intimate experience with pipes than with strings.

[1] The two precious originals are no longer in England. No purchaser could be found for them, and funds were needed for the continuance of the excavations, so they passed into the possession of the German Museum of Musical Instruments at Charlottenburg.

[2] *Harm.*, Macr., pp. 196–7 (end of polemic on Aulos) ; see also Chap. ii.

THE DOUBLE MOVEMENT OF THE TONGUE OF THE B-R. MOUTHPIECE

It was found eventually (through tests of D-R. mouthpieces) that the middle note—always promptly discarded on account of its inability to rise in ratio with the successive opening of the fingerholes, except by forcing and loss of beauty of tone—was in reality the norm, based upon ascertainable laws now to be discussed.

The length of the straw shank—as in the D-R. mouthpiece—exercises only an indirect influence on pitch, through resonance, upon the proper note of the mouthpiece.

The determinants of pitch are first and foremost the *length* of the *vibrating* tongue; secondly, the *diameter of the straw*; and, thirdly, the *width of the tongue.* The mouthpiece, with its natural knot or sealed end, reacts as a stopped pipe, and therefore all factors bear only upon a quarter-wave length. The tongue as vibrator, however, has a double movement of its own, opening and closing the entrance of the column of air within the pipe; it therefore produces at each closure of the aperture (which corresponds to the length × by the width of the vibrating tongue), a pulse complete in itself consisting of rhythmical, periodical recurrences of condensation and expansion, i.e. a complete pulsation.

The length of the tongue of the B-R. is an index of the half-pulsation only, as the stroke or beat falls; it is a factor which, with the implications of length due to diameter, must be multiplied by two. The implications are introduced by diameter: that of the straw first of all. Then, by analogy with the lateral holes of a flute, the width of the tongue exercises an augmentative, lengthening influence on the internal diameter of the straw, adding to this the amount of the difference between them. It will now be seen why the narrower the width of tongue, the lower the pitch of the note, just as a smaller fingerhole on a flute gives a lower note. There are thus three separate factors in the B-R. mouthpiece to be multiplied by two in order to find the length of the pulse produced by the double movement of the beating tongue, e.g. on mouthpiece E 10 (B-R.) with a Tongue Length (= T.L.) of ·041, a Tongue Width (= T.W.) of ·002 and a diameter of ·003 (= Δ).

BY FORMULA NO. 8

$$\left.\begin{array}{l} \cdot082\ \mathrm{T} \times 2 \\ \cdot006\ \Delta \times 2 \\ \underline{\cdot002\ (\Delta - \mathrm{T.W.}) \times 2} \end{array}\right\}\text{Actual length of pulse from tongue.}$$

$$\underline{\underline{\cdot090}} = \text{actual length of pulse of tongue.}$$

The actual length of pulse must now be taken 4 times, since the mouthpiece reacts as a stopped pipe, the length of which = a quarter of the wave-length of the note: ·090 × 4 = ·360, which is converted into v.f. thus:

$\frac{340}{\cdot360} = 944$ v.f. $= 8x$, and $x = 118$ v.p.s., the v.f. of the note of the mouthpiece.

The proper note of mouthpiece E 10 is actually by repeated tests fixed at $\frac{B\,12}{64} = 117{\cdot}3$ v.p.s. The glottis note sinks to $\frac{\dot{G}\,15}{64} = 93{\cdot}8$ v.p.s.

FORMULA VIII. DETERMINATION OF PITCH OF PROPER NOTE OF THE B-R. MOUTHPIECE

$$\text{(A.)} \qquad \frac{340 \text{ m./s.}}{8[\text{T.L.} + \Delta + (\Delta - \text{T.W.})]} = 8x\,[1]$$

when x = v.f. of norm of proper note of mouthpiece.

The glottis note is in the epitritic ratio, two octaves below the resonance fundamental.

No explanation can be offered at present of the fact that the normal note of the mouthpiece lies three octaves lower than theory requires, but so it is. That is why, after the formula has been worked out, x requires to be divided by 8 (= 3 octaves) in order to coincide with the note actually produced by the mouthpiece with normal blowing.

In the meanwhile, I suggest the following explanation which may prove to be the true one: the wave-length, which is the cause of the astonishingly low pitch of the proper note of this type of small and delicately proportioned B-R. mouthpiece, may perhaps be attributed to two factors:

First, the effective length of the pulse from the tongue of the beating-reed with its two movements—both of which are taken into account as added length ($\times$ 2); and, secondly, to the acoustic fact that the B-R. mouthpiece itself has the properties of a closed pipe; therefore, the actual length of the pulse from the tongue is only a quarter of the wave-length of the note emitted, and must be multiplied by 4. These two factors are thus together responsible for the multiplication by 8 of the divisor in the formula.

Since length and vibration frequency operate in inverse proportion, it follows that the aggregate length of sound-wave taken 8 times will produce a frequency per second 8 times lower, i.e. 3 octaves lower.

In many mouthpieces, tested before the discovery of Formula viii, the result of repeated tests was accepted and registered in the records. But now in light of Formula viii, which has worked out correctly with scarcely any exceptions, the dissentients were re-examined, and it was found in all cases that scraping the fibre away (with the most tender care) from the under-surface of the tongue, nearly always resulted in a norm coinciding with that of the formula. Occasionally it was found that the clearance under the tongue, near the root, was at fault, and had been imperfectly carried out, and that a little pith remained, impeding the free vibration of the tongue, so that the calculated length did not represent the effective length, a defect easily corrected. The safest way of scraping the tongue is to lay the mouthpiece down with the tongue resting on a flat, firm surface; then, while holding it steady with a fingernail, the under-side is gently

[1] Or as an alternative

$$\text{(B.)} \quad \frac{340 \text{ m./s.}}{64[\text{T} + \Delta + (\Delta - \text{T.W.})]} = x \text{ v.f.}$$

scraped with a sharp blade. This treatment gives the mouthpiece the resilience requisite for effective vibratory movement. An ocular demonstration of the difference before and after scraping is obtained if the straw be alternately vibrated from the exit by blowing and by suction—the latter producing the normal note—while the eye critically examines the periodic oscillations of the tongue.

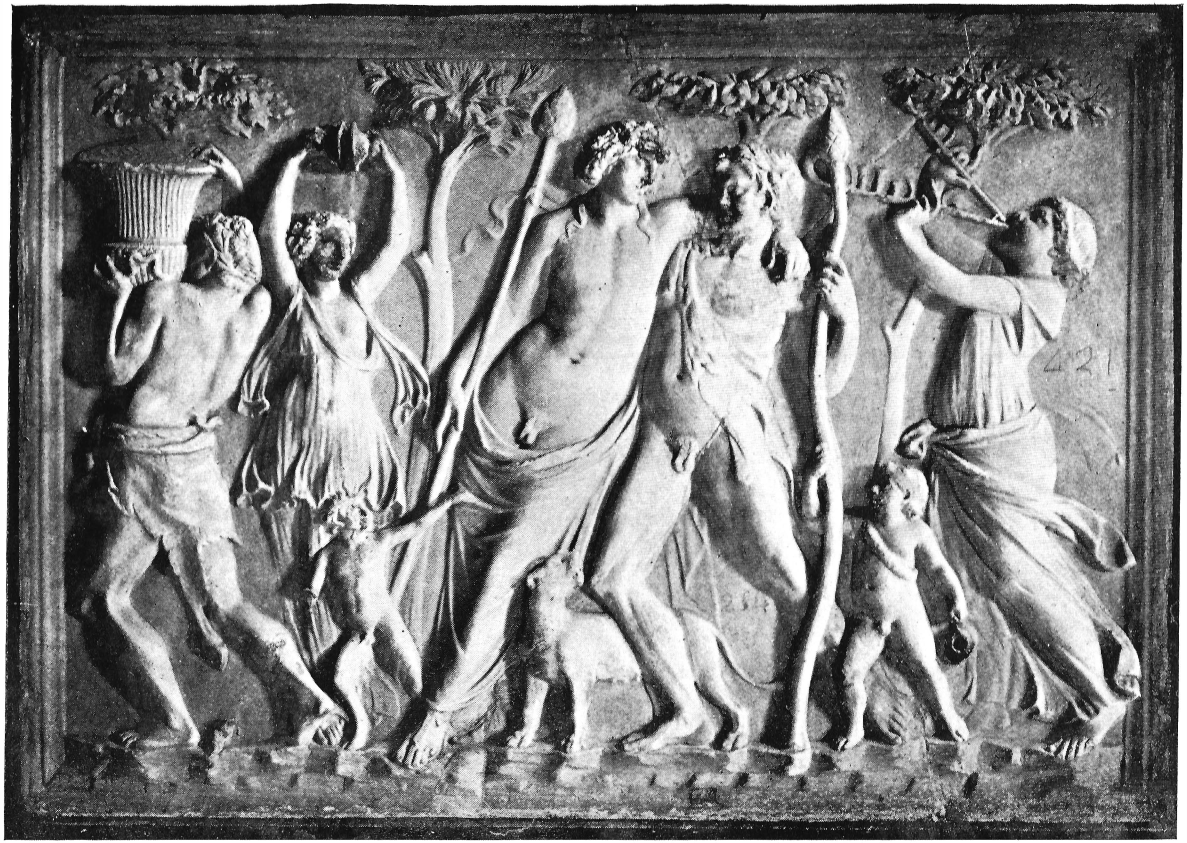
IMPLICATIONS OF THE FEAT OF MIDAS OF AGRIGENTUM

Compared with the D-R. mouthpiece, the B-R. is by far the more responsive, and the easier to manipulate; the slightest, almost unconscious glottis action will produce a delicate shade of intonation, higher or lower at will. The tone quality on finely cut mouthpieces with a tongue of from ·03 to ·04 in length, and ·001 to ·002 in width, produces a tone quality of extraordinary beauty and richness; the harmonic texture of the note, definitely perceptible to a practised ear, is easily varied by extra pressure of breath, while the compression and length of tongue remain unchanged. The little instrument must have been a joy to the creative musician, and a source of inspiration; its chief drawbacks are its fragility and the difficulty the piper experiences in preventing the little tongue, weighted by the moisture from the breath, from becoming suddenly inarticulate, through loss of its elasticity. This would, of course, be a serious handicap in modern music; but the shortness and simplicity of the phrases, and the spontaneous nature of the musical improvisation, which seem to have characterized the music of the second of the Ancient styles, rendered this drawback less serious. The accident that befell Midas of Agrigentum at the Games, when the tongue of the Syrinx of the Aulos broke, through cleaving to the roof of his mouth, is an example of such ill-luck, and also of the improvisational nature of the music.

From the Scholium to Pindar, ***Pyth.*** xii, we learn in addition that already in Pindar's day, the Aulos was played by professional Auletes with B-R. mouthpieces, and the very fact of the cleavage of the *glossa* to the roof of the mouth is an indication of the minimum length of the vibrator and also of the material used for the mouthpiece, viz. wheat or oat straws; the stronger, stiffer mouthpieces of reed such as Theophrastus describes were obviously not yet in use.

The facility with which shades of intonation could be obtained with this mouthpiece suggests that the purity of the *ἁρμονία* may have been endangered thereby; this surely might have been the case with an Aulete unfamiliar with the Modes. But already long before Pindar's day the Harmoniai must have been firmly established in the musical consciousness of Greek Auletes. It is evident from the episode attached by Plutarch to the use of the Spondeiasmos [1] in the Spondaic Hymn, that neither the Aulete, who played in the ancient manner (or ancient Tropos in the sense of Harmonia) with the interval called *Syntonoteros Spondeiasmos* on the Tonic, nor the Aulete of Plutarch's own day, could be said to have played at random, regardless of the authentic degrees of the Perfect Immutable

[1] Plut., *de Mus.* (ed. Weil and Reinach), pp. 48–9, §§ 114 and 115.



PIPER PLAYING IN THE BACCHIC REVELS UPON DOUBLE AULOI FITTED WITH BOMBYXES ;
His head is thrown back, the pipes held aloft at an angle, implying a high *tessitura*
Bas-relief from the Farnese, Museo Nazionale, Napoli

System, and that these were still modal in Auletics has been shown in Chapter ii. What happened when an Aulete, not yet familiar enough with the Harmoniai to produce them on a modal Aulos, or to a piper trying to play the Ditonal Scale—upon which Aristoxenus theorized—upon the modal Aulos [1] may be surmised from a passage from the Philebus of Plato (p. 56A).

PLATO ON EMPIRICISM IN AULOS PLAYING

Plato had been discoursing upon the respective parts played by conjecture (i.e. empiricism) as opposed to methods based upon number and measure in the creative arts:

> Music, for instance [says Plato] [2] is full of this empiricism, harmonizing consonances (*τὸ σύμφωνον*) not by measure but by the practice of conjecture (i.e. by empirical methods) and all Aulos music tries to find the measurement of each note by conjecture, so that it is mixed up with much that is doubtful and has little that is certain.

It is evident from what Plato says that he regards this empirical method for the determination of the consonances as a falling away from musicianship and the best practice of the day; it is all of a piece with the tide of decadence, which he deplores in other passages in the Dialogues. Incidentally, it proves that the purity of the consonances was still demanded in the general practice of the age now on the wane.

The reference to the music of the Aulos makes this clear. The Aulos was the creator of the Harmoniai; the boring of the fingerholes gave the notes of the Modal Scales. But a 'new' scale was even now being introduced among musicians, viz. the non-modal one which, although it had long been in use on the Panpipes among the shepherds, had only recently been adumbrated theoretically in the Timaeus of Plato. The intervals of this scale, which was the basis of the treatise of Aristoxenus, were entirely foreign to the *ἁρμονία* and could not be played upon the Aulos by natural means; hence the feeling about for the notes, and the empirical method of obtaining the intervals. When once the piper's criteria were gone, i.e. the familiar intervals of the Harmonia produced through equidistant fingerholes, he was at sea. At this period, when the B-R. had come into more general use, the piper had to possess his scale inwardly before he could be sure of playing it absolutely in tune. He could, it is true, by using a Hypolydian Aulos, give a near approximation of the Ditonal Scale in question, aided by artificial means; his difficulty would be to remember the exact shade of intonation required. Moreover, he would not have the same luck with any other modal Aulos. In connexion with the difficulties experienced during the transitional period, which culminated with the triumph of virtuosity, we may recall the conclusion of a description of the innovation attributed by

[1] It is doubtful from many references in the sources whether an Aulos other than modal was in use in Greece in Plato's day.

[2] Philebus, p. 56A. See also my review of Théodore Reinach's *La Musique Grecque* (ed. Payot) in *Mus. Standard*, William Reeves, 1927, March 26 and April 9 and 23, in which the question of the use of the Ditonal Scale and of tempering among the Greeks is discussed at some length. The translation of this passage is by E. J.

TABLE II

TABLE OF SELECTED BEATING-REED MOUTHPIECES; STEMS OF VARIOUS LENGTHS; VIBRATING TONGUES FROM ·03 TO ·033 IN LENGTH. EXAMPLE OF REGIONAL PITCH $\frac{E\ 18}{128}$ = 156·4 V.P.S.

No. of Mp.	Length	Diameter	Tongue Length	Tongue Width	Proper Note of mp.: Norm	Results of Tests
No. 7	·172	·0025	·0305	·0015	$\frac{E\ 18}{128}$ 29/3/33	Mp. note did not agree with formula; after scraping, the correct norm was given
No. 47	·134	·003	·03	·002	$\frac{E\ 18}{128}$ 29/3/33	After scraping the tongue, *E* 18 the norm of formula came clear and strong. Rich glottis note $\frac{B\ 12}{64}$ a 4th below norm.
XXXV		·0035	·028	·001	$\frac{E\ 18}{128}$ gl. $\frac{A\ 27}{64}$	The norm. at once
H 1	·096	·003	·03	·002	$\frac{E\ 18}{128}$ gl. $\frac{B\ 12}{64}$	Both notes strong, resonant
XVI	·098	·003	·033	·0025	$\frac{E\ 18}{128}$	Glottis $\frac{B\ 12}{64}$ plays in Loret xvi
B 16	·140	·003	·03	·002	$\frac{E\ 18}{64}$ and $\frac{E\ 18}{128}$	At first played $\frac{F\ 16}{128}$; then cleared to root of tongue, it played *E* 18/128 with ictus and $\frac{E\ 18}{64}$, a soft norm.
B 9	·128	·003	·03	·002	$\frac{E\ 18}{128}$	After scraping tongue, played strong, rich full tone, norm *E* 18
T 3 Sealed cut on 11/11/33	·102	·0025	·032	·0025	$\frac{E\ 18}{256}$ norm.	Gives 4th Harmonic $\frac{E\ 18}{1024}$ very clear pure note *N.B.*—The mp. straw is cylindrical yet produces the double octave harmonic
D 2	·160	·0025	·03	·0015 very fine Tongue	$\frac{E\ 18}{128}$	The octave below norm $\frac{E\ 18}{64}$ given readily on same breath by glottis action
No. 11 made on 11/11/33	·117	·0025	·031	·001 very fine Tongue	$\frac{E\ 18}{128}$	Glottis note $\frac{B\ 12}{64}$; mp. responded at once; very good tone

N.B.—All these play $\frac{E\ 18}{128}$ in accordance with Formula viii (= 156 v.p.s.).

Athenaeus [1] to Pronomus, the Theban, (fifth century B.C.), who trained Alcibiades in playing the Aulos. 'But Pronomus, the Theban, first played (all) the Harmoniai from the (same) Auloi, but now at random *without ratio* (ἀλόγως), they meddle with Music.'

To return to the more theoretical aspect of the B-R. mouthpiece, it may be stated that for general practical purposes the influence of the length of tongue in the mouthpiece is regional as to pitch (as may be seen from Table II) (see Chap. ii, Fig. 25).

When all other dimensions are equal, a comparison of mouthpieces reveals the fact that the ratio between the lengths of *glossa* in any two such mouthpieces produces a rise or fall in pitch in the same ratio, as, for example, in B.11 and R.11 ; the tongue lengths of these B-R. mouthpieces are respectively of ·04 and ·03 in epitritic ratio 4 : 3 with a common width of ·002 ; B.11 has a diameter of ·0035 and R.11 of ·003 thus, in accordance with Formula viii.

B.11 $\cdot 080 = T \times 2$
$\cdot 007 = \Delta \times 2$
$\cdot 003 = (\Delta - T.W.) \times 2$

$\underline{\underline{\cdot 090}} \times 4 = \cdot 360$ effective length of pulse of note of B-R. mp. B.11.

$$\frac{340}{\cdot 360}\ \text{m./s.} = 8x = 944\cdot 4 \text{ and } x = 118 \text{ v.f.}$$

$$\frac{B\ 12}{64} \text{ is the normal proper note of mp. B.11.}$$

R.11 $\cdot 060 = T \times 2$
$\cdot 006 = \Delta \times 2$
$\cdot 002 = \Delta - (T.W. \times 2)$

$\underline{\underline{\cdot 068}} \times 4 = \cdot 272$, effective length of pulse of note of mp. R.11.

$$\frac{340}{\cdot 272}\ \text{m./s.} = 8x = 1250, \text{ and } x = 156\cdot 25 \text{ v.f.}$$

The normal proper note of mp. R.11 is $\frac{E\ 18}{128} = 156\cdot 25$ v.f.

Thus the formula works out correctly here : the ratio between *B* 12 and *E* 9 (18) is in effect 4 : 3.

MOMENTOUS SIGNIFICANCE OF SHORTENING THE TONGUE OF THE B-R. MOUTHPIECE

This comparison led to the discovery of an implication of the greatest importance, not only to the history of the Aulos, but also to the evolution and history of Music, viz. that a rise in pitch of the fundamental of the Aulos, when played by a B-R. mouthpiece, may be obtained by the simple expedient of shortening the vibrating tongue of the mouthpiece by a definite

[1] *Deipn.*, xiv, p. 631*e* (31).
[2] Or alternatively for mp. B 11 ; and similarly for mp. R 11.

$$\frac{340}{64(\cdot 04 + \cdot 007 - \cdot 002)} = \frac{340}{64 \times \cdot 045} = \frac{340}{\cdot 288} = 118 \text{ v.f.}$$

ratio; e.g. by a third, fourth, or half, by means of a simple movement of the lips. The collaboration of mouthpiece and resonator is so harmonious and complete that the rise in pitch operates instantly on the whole pipe, raising the fundamental to the dominant, the fourth or the octave, and likewise the whole sequence from the fingerholes, as they are uncovered. The effect is to transpose the tonality of the Aulos according to the ratio selected. The significance of this apparently simple—but hitherto quite unsuspected—property of the primitive beating-reed mouthpiece is of paramount importance in accounting for the origin and development of our own musical system; a fact which has already been discussed in Chap. ii (Fig. 24). It was seen that with a Hypolydian Aulos, of Determinant 20 having three fingerholes, which produces a sequence of intervals $\frac{10}{9} \times \frac{9}{8} \times \frac{16}{15}$ on the Tonic *C*, the effect of shortening the tongue of the mouthpiece by $\frac{1}{3}$, and starting again from the exit, produced the same series of ratios on *G*, i.e. our major scale

C D E F G A B C

The use to which the Auletes of Ancient Greece put this property of the B-R. mouthpiece, and its effect on the development of their musical system in theory and practice has already been related. The revolution thus achieved was far-reaching: at one blow modality was deprived of its characteristic feature, viz. the octave unit. The Harmonia was shorn of one of its modal tetrachords: its Ethos destroyed. What was gained thereby in compensation for the loss to modality? This change in the technique of the Aulos was responsible for a rapid access of virtuosity, by increasing the compass of the instrument in theory, and moderately in practice; for although it is possible to shorten the little tongue as described while playing, the delicacy of this part of the mouthpiece would not permit of the use of the device for frequent modulation; the tongue becomes intractable when too many demands are made upon it. The tongue of the straw mouthpiece did not respond readily to more than one change of length. It was the fragility of the wheat-straw beating-reed, no doubt, that led to the facture of the mouthpiece from the stems of river reeds, as recorded by Theophrastus.

From practical tests made with reeds from English rivers, I am inclined to think that the expedient in question could be quite satisfactorily used on such reed mouthpieces for practical music, as one would be led to expect from the evidence provided by the vase paintings.

CHANGE TO ELABORATE PLAYING ON THE AULOS

Theophrastus [1] states that up to the time of Antigenidas, a famous Aulete who flourished during the reign of Alexander the Great, the Zeuge or D-R. mouthpiece was played in a natural manner, but that 'when they changed to elaborate playing, the cutting was also changed. And they say

[1] *Hist. Plant.*, iv, 11 (3).

that they (the reeds) become useful in three years, and need but little practice' (cf. Chap. ii).

Gevaert [1] translates the last line thus: 'et les languettes se prêtent aux intonations abaissées'. If, as I believe, καταστάσματα refers to the invisible action of the lips upon the reed tongue, brought about by the visible pulling down of the Syrinx, or B-R. mouthpiece, then the effect on the Aulos was the reverse of what Gevaert imagines, i.e. the pitch would be raised because, as explained in detail earlier in the work (Chap. ii, Plate No. 6), the effect of pulling down the Syrinx together with the Aulos—the piper's head the while bent slightly over the instrument—would be to shorten the tongue of the B-R. and thus raise the pitch of the Aulos. As to the properties of the mouthpieces made from a length of river-reed, cut near the tip of the plant, Theophrastus says that they are very soft, i.e. pliable, compared with those cut from the shoot near the root.

If we subject such mouthpieces to a practical examination, it is found that their fragility is certainly considerably lessened, but at the expense of beauty of tone. They need much preparation and treatment, after the reed has been duly matured, before they can be tuned to a reliable pitch by scraping and thinning the tongue, in order to give elasticity, and to allow it to vibrate from the root or hinge. When this has been satisfactorily accomplished, as with B-R. m.p. '$\underline{|E|}$' the mouthpiece gives a powerful, if somewhat loose and coarse, note on $\frac{B\ 12}{64} = 117{\cdot}3$ v.p.s. Owing to the difficulty experienced in cutting a reed mouthpiece with a fine, narrow tongue that will bear scraping, one has to be content with less beauty of tone. The reed mouthpiece, however, when provided with an elastic tongue, will sometimes play a modal octave sequence with a little straining on the last two or three notes, providing the tongue measures not less than ·04 in length and plays the 8-foot octave. The reed mouthpiece possesses the same drawback as the straw mouthpiece: i.e. so long as the piper consciously possesses the Harmonia, and does not depend upon the fingerholes to produce the notes of the sequence in correct intonation, all may be well; but the Aulos will not teach the Aulete the Harmonia, unless it be furnished with a D-R. mouthpiece.

The shortening by the lips of the tongue of a B-R. mouthpiece of reed is easily accomplished, for the tongue responds more satisfactorily than with the straw mouthpiece. It will be the task of some scientist to explain how it is that with the length of the resonator of the Aulos unchanged, the extrusion of mouthpiece of the same length, the length from the centre of each fingerhole to the tip of the mouthpiece unaltered, the mere shortening of the little vibrating tongue by a third or a half—an affair of a few millimetres only, which does *not* affect the total length of the instrument itself—all the notes that could be obtained by any device from the pipe with the tongue vibrating at full length, are, by that slight movement of the lips, raised in pitch by exactly a perfect fifth or an octave. It is clear that the Formulae i and ii are not applicable altogether; and examples such as B.11

[1] *Probl.*, pp. 348–9, Note 4.

and R.11, cited above, in which agreement with Formula viii was reached, may be fairly equally divided between rule and exception. Thus, through the incidence of the same law of proportion that produces the Harmonic series as physical basis of sound, and the reversed series responsible for the genesis of the Harmoniai, the Aulos is endowed with the power of modulation into a different tonality or compass, and at the same time provides the germ of the idea, which out of half a Harmonia created our modern scale.

PERFORMANCE OF AULOS LORET XXIII WITH B-R. TONGUE OF MP. SHORTENED BY ONE-THIRD

For those who are not averse from the use of the formula, the working out of the result of shortening the tongue of B-R. mouthpiece xviii*c*, establishes the effect of the shift on the tongue of the mouthpiece in theory as well as in practice, and is given below.

BY FORMULA VIII

B-R mouthpiece 'xviii c'.

Wheat straw, length, ·150; Tongue length, ·035.
Tongue width, ·0025; Δ ·0035.

$$\begin{aligned} T \times 2 &= \cdot 070^{1} \\ \Delta \times 2 &= \cdot 007 \\ (\Delta - \text{T.W.}) \times 2 &= \underline{\cdot 002} \end{aligned}$$

N.B.—Since the tongue width is not reduced by the shift, this figure remains the same after the shift.

$\cdot 079 \times 4 = \cdot 316$ effective length of tongue.

$$\frac{340}{\cdot 316} = 1075 \cdot 9 \text{ v.p.s.} = 8x \text{ and } x = 134 \cdot 5 \text{ v.p.s.}$$

The v.f. of mp. xviii *c*'s proper note is in effect $\frac{C\ 21}{128} = 134\cdot 5$ v.p.s.

BY A SHIFT OF ONE-THIRD ON THE TONGUE

Effective length of mouthpiece xviii $c^{1} = \cdot 316$

$$\frac{\cdot 316 \times 2}{3} = \cdot 2106 = \cdot 211.$$

$$\frac{340}{\cdot 211} = 1611 \cdot 3 = 8x \text{ and } x = 201 \cdot 4 \text{ v.p.s., i.e. } \frac{G\ 14}{128}$$

$$\frac{C\ 21}{128} \times \frac{3}{2} = \frac{G\ 14}{128} \text{ of 201 v.p.s. (= a perfect fifth).}$$

$$\text{by ratios of length 21 to 14} = \frac{2}{3}$$

When tested alone, the mouthpiece played on $\frac{C\ 21}{128}$ and by shift of $\frac{1}{3}$, on $\frac{G\ 14}{128}$; it also played with a fine tone on $\frac{C\ 11}{128}$ and by a shift on G a 5th

[1] There is a note on record that the tongue on this mouthpiece had first been cut to ·030, the I.D. on pipe xviii, for which the mouthpiece was destined. This length of tongue, however, did not give satisfactory results in resonance, and it was, therefore, lengthened empirically to ·035 when the note produced was in correct intonation.

Test of B-R mouthpiece xviii*c* in Loret Aulos xxiii; for measurements and performance in detail, consult the Record of that Aulos.

higher. Tests October 23, 1930; November 27, 1933; and December 20, 1933.

The mouthpiece xviii*c* tested in xxiii Loret, actually plays both on C 128, and with shift of $\frac{1}{3}$ on $\frac{G\,14}{128}$. But, of course, the shift on the mouthpiece tongue, which raises the fundamental from Hole 1, as well as the notes of the sequence to the dominant, cannot be checked by theory, i.e. by Formula viii, since the altered length produced by the shift relates only to the tongue of the mouthpiece, and does not affect the length of extrusion, nor that of the Aulos + mouthpiece.

From the record of performance of Aulos Loret xxiii, it is seen that with this same B-R. mouthpiece No. xviii*c*, at an extrusion of ·071 the Aulos played in the Phrygian Harmonia, of Determinant 12, on $C = 128$ v.p.s., a 6th lower than theory demands, and that with the same mouthpiece at extrusion ·047, the Aulos played in the Dorian Spondaic, of Determinant 11, on the same fundamental $C = 128$ v.p.s.

This performance is similar to that of Aulos xxiii with D-R. mouthpiece N.32 (Chap. ii) in the Lydian and Mixolydian Harmoniai. In both cases the performance is, therefore, that of true Harmoniai—not species—since the fundamental Tonic is common to both, while the ratios of the intervals change in relation to the Tonic and to each other, although the increment of distance must, perforce, remain unaltered. And herein, owing entirely to the unique reactions of its mouthpieces, the Aulos differs from the strings in the manner in which it gives effect to modality. With both mouthpieces the cause of the change of modality, taking place within the mouth of the piper, may have no outward visible sign: in the B-R., the vibrating segment of the tongue is shortened, and in the D-R. the vibrating length, without alteration of the combined length of resonator + mouthpiece.

But, as we have seen, the converse of these conditions is also true, for the composite length of the Aulos may be varied by using a different extrusion of the mouthpiece, which alters the Harmonia without change of Tonic, although the length has been altered.

So far, Aulos xxiii has a record of four different Harmoniai, Dorian, Phrygian, Lydian and Mixolydian, obtained by means of various methods and devices. We begin to see that Plato's epithet παναρμόνιον [1] might well refer to the Aulos, even at this unsophisticated stage in its development.

PIPER PERSUADED BY PYTHAGORAS TO CHANGE THE MODALITY OF HIS AULOS FROM PHRYGIAN TO DORIAN SPONDAIC

An interesting confirmation of the double modality of an Aulos, in which the change from the one Harmonia to the other was practically uninterrupted, is given by Iamblicus [2] in his life of Pythagoras:

Among the deeds of Pythagoras, it is said, that once through the Spondaic song

[1] Plato, *Rep.*, p. 399; also D. B. Monro, *Modes*, p. 41, in which the Greek text is given with translation.

[2] *Vita Pythagorae*, tr. by Thos. Taylor, Cap. 25.

of a piper, he extinguished the rage of a Tauromenian lad, who had been feasting by night, and intended to burn the vestibule of his mistress, in consequence of seeing her coming from the house of his rival. For the lad was inflamed and excited by hearing a Phrygian song, which, however, Pythagoras most rapidly suppressed. And this was how it was done. Pythagoras, as he was astronomizing, happened to meet with the Phrygian piper at an unreasonable time of night, and persuaded him to change his *Phrygian* for a *Spondeian* song, through which the fury of the lad being immediately repressed, he returned home in an orderly manner, though a little before this, he could not be in the least restrained, nor would in short, bear any admonition, and even stupidly insulted Pythagoras when he met him.

The Spondaic song, an auletic libation hymn, is founded upon the Dorian octave Harmonia of seven notes, produced by Determinant 11; this Mode is called by Plutarch the *Spondeiakos Tropos*;[1] it is identical with the scale of the Elgin Aulos (the straight one) in the Graeco-Roman department at the British Museum. On my facsimile of this precious relic, the Spondeion can be played on $\frac{C\,11}{128}$ by six different D-R. mouthpieces, at extrusion ·110 and by B-R. 12 on $\frac{F\,16}{64} = 88$ v.p.s., and by other mouthpieces besides. By means of another D-R. mouthpiece, N.7, at an extrusion of ·132, the Elgin Aulos plays in the Phrygian Harmonia on $\frac{A\,27}{128} = 208{\cdot}6$ v.p.s. (See Chap. x, Records and Table of Elgin Aulos performance.) The piper cited by Iamblicus might thus have been using the Elgin Aulos, with the same B-R. mouthpiece (or D-R.) pulled out to ·132 for the Phrygian song and merely pushed in to ·108 or ·110 for the Dorian Spondaic.

A few more illustrations may now be given from our records of the characteristic properties of these two remarkable types of mouthpiece; they enable us to realize the nature of the problems that arise in connexion with the theory and practice of the Aulos of the Greeks. These data may also, it is hoped, help to make good some of the lacunae in the treatises on acoustics concerning the behaviour and attributes of the vibrating-reed used as mouthpiece.

Feeling the need for authoritative information concerning the practical behaviour of the modern clarinet reed, I asked Mr. Charles Draper (May 2, 1931 and again in 1937). Here are his replies to my queries:

K. S. What manipulation of the clarinet reed produces the rise to the twelfth through each hole of the Chalumeau register?

C. D. None whatever. The reed has no part in producing the twelfths on the clarinet; they are entirely the affair of the left-hand thumb key.

K. S. And what about the breath?

C. D. Just as in singing.

K. S. Then the muscles of the larynx are brought into play?—relaxed for low notes, contracted for higher?

C. D. Yes, and more than that: the muscles of the stomach and legs are all brought into play and you feel the clarinet in your shoulders!

[1] Plut., *de Mus.*, (ed. Weil and Reinach), pp. 48 sqq. and 72–3.

K. S. So you play with the whole man.

I then told Mr. Draper of the results of shortening the little tongue of the Aulos mouthpiece in primitive pipes.

C. D. Nothing of that kind is done with the clarinet mouthpiece.

K. S. Are the twelfths from the Chalumeau register Harmonics?

C. D. No! they are not what is generally understood as Harmonics, but merely notes a twelfth higher.

K. S. Therefore the mouthpiece is of standard dimensions and the holes are fixed empirically?

C. D. The blowing does the rest!

K. S. Do the lips remain on the reed tongue at the same distance for all notes?

C. D. The mouthpiece is flexible in the mouth and the lips gradually move up on the reed as pitch rises.

K. S. What would happen to the twelfths if the left-hand thumb key remained closed? Could the notes be produced as Harmonics?

C. D. No! they would simply not sound, the uncovering of the holes would merely produce the notes of the Chalumeau register.

Mr. Charles Draper's statements concerning the playing of the modern clarinet and its beating-reed mouthpiece suggest that the function of the left-hand thumb key (speaker key), and the instinctive movement of the lips on the reed, present some analogies with the shortening of the vibrating tongue of the primitive beating-reed mouthpiece, by the impact of the lips at one-third of its length, despite the disparity in the dimensions of the ancient and modern instruments.

EXAMPLES OF THE PERFORMANCE OF REED MOUTHPIECES

(1) Let us take, for instance, a little B-R. mouthpiece, H.11, cut in a wheat straw, closed at the embouchure by sealing-wax, and therefore, of cylindrical bore. The straw has a total length of only ·095 (= $3\frac{4}{5}$ inches); and a diameter of 3 mm. From its function as a closed pipe, the straw mouthpiece of less than 4 inches should speak in the 1-foot octave, i.e. above $C = 512$ v.p.s. Yet the normal proper note of H.11, when taken into the mouth and played as a separate instrument, is $\frac{F\ 17}{128}$ (= 165·6 v.p.s.) in the 4-foot octave.

This B-R. mouthpiece, H.11, should have a theoretical length of ·098: ·095 length of straw mouthpiece + ·003 diameter of straw = ·098; actual length × 4 = ·392, the effective length.

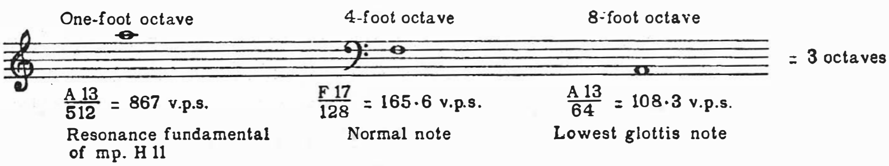
$$\frac{340}{\cdot 392} = 867\cdot 3 \text{ v.p.s.} = \frac{A\ 13}{512}$$

therefore theoretically, according to Formula ii, the fundamental resonance note of H.11, blown across the exit, with the beating-tongue securely closed should be $\frac{340}{\cdot 392} = 867\cdot 3$ v.p.s., or $\frac{A\ 13}{512}$ in the 1-foot octave. This resonance test could not be carried out in practice on this mouthpiece, for the exit is split, but judging from many other tested mouthpieces, the fundamental resonance of theory by the formula, corresponds very closely with actual practice in all specimens. Disregarding octaves, the ratio between the normal note of the mouthpiece H.11 $= F$ 17 (= 165·6 v.p.s.), and the fundamental resonance note of the mouthpiece *A*.13 (= 867·3 v.p.s.), is approximately the epitritic interval of the 4th.

$$\left[\frac{17}{13} \times \frac{3}{4} = \frac{51}{52}, \text{ i.e. a 4th flattened by } \textit{34 cents.}\right]$$

To this resonance of the 4th, operating in mouthpiece H.11, is undoubtedly due the fine, resonant and rich quality of the note of this little mouthpiece.

The ratio between the resonance fundamental note of mouthpiece H.11, and its lowest proper glottis or ictus note is:



One wonders what power is at work to convert the effective length of about 4 inches of straw into an instrument of 4-foot or even 8-foot tone, for the mouthpiece's useful resonant note, of great power and fine quality, blown with ictus or by suction from the exit, has a pitch of $\frac{B\ 12}{64} = 117{\cdot}3$ v.p.s. in the 8-foot octave.

It will be noticed on examining the record of performance of the Lady Maket Aulos with 3 holes, that B-R. mouthpiece H.11 plays the modal sequence, with a fine resonant tone, from $\frac{G\ 14}{64} = 100{\cdot}5$ v.p.s., the Tonic of the Mixolydian Harmonia. But, inserted in the Aulos, the mouthpiece has not the same wide range as when used alone.

We may here recall the practical limits of two octaves and a 5th assigned by Aristoxenus [1] to the compass of voice and instruments, a range which may, he allows, be extended to three octaves or more between the *lowest* note of an Aulete playing with normal use of his mouthpiece and the *highest* note of the Syrinx mouthpiece (*κατασπασθείσης γε τῆς σύριγγος*) (i.e. the beating-reed). The Aulete, by the shortening of the vibrating tongue of the mouthpiece with his lips, raises the pitch of the whole instrument by a 5th or an octave. Attention may also be drawn to B-R. mouthpiece No. T.3 which, when playing alone, gives a pure bell-like fourth Harmonic of $\frac{E\ 18}{256}$, the norm, in the $\frac{E\ 18}{1024}$ octave (the half-foot) without requiring a speakerhole. It is seen, therefore, that the proper note of this reed tongue lies in the 2-foot octave, whereas the other six B-R. mouthpieces, particulars of which appear in the table below, have their norm an octave lower on

[1] *Harm.*, p. 21M.

$\frac{E\,18}{128}$, with glottis notes on $\frac{B\,12}{64}$, a perfect 4th below. The reason for this difference in pitch is found in the tongue width of T.3, which is the same as the diameter of the straw, and therefore too wide to give good results, although the proper note of this mouthpiece is actually in the correct register for a stopped pipe, in relation to its resonance note, which, as we have seen, is quite exceptional.

(2) The seven examples in Table ii, selected from my collection of B-R. mouthpieces of wheat straws, measuring from ·099 to ·198, all give the same normal note $\frac{E\,18}{128}$, in the 4-foot octave = 156·2 v.p.s. (with the exception of mouthpiece T.3 which speaks an octave higher, owing to the width of its tongue). These are common features of these mouthpieces, of which innumerable specimens of every degree of pitch in the modal scale have been made, tested and recorded. The normal proper note works out with great regularity in the majority of these, according to Formula viii, and the glottis notes are usually found a 4th, 5th or octave below the norm. The records (as in Table ii) emphasize the fact that the length of the shank of the mouthpiece has no influence in the determination of pitch.

(3) Two groups, each consisting of seven D-R. mouthpieces, measuring from ·112 to ·178. The first set has a common diameter ·0035, and as proper note $\frac{C\,11}{256}$ = 256 v.p.s. or $\frac{C\,21}{256}$ = 268 v.p.s.; the second group has diameters of varying calibre. The longest of these mouthpieces at first measuring ·230 was afterwards cut down to ·161; the original normal note $\frac{C\,21}{256}$ remained unchanged, a common experience with these mouthpieces, testifying once again to the unimportance of length of straw as a determinant of pitch. When tested as simple resonators, and blown across the end, they respond very accurately to Formula i. It is when these mouthpieces are inserted into the mouth, and blown as double-reeds, that surprises in diagnosis make their appearance. In the second set, the lengths vary from ·112 to ·178, and the diameters from ·0035 to ·005, but the norm of all seven mouthpieces is $\frac{F\,17}{128}$ = 165·6 v.p.s. or $\frac{F\,17}{256}$ of 332 v.p.s. The vibrating length is the same for all seven, and there is only a very slight difference of diameter; hence the identity of their notes. All seven mouthpieces respond to Formula viii.

(4) Another instance of the apparent vagaries in regard to the incidence of length on pitch in Auloi is furnished by a record of the performance of facsimile reproductions of ancient Auloi.

For the reed-blown pipe from a sarcophagus in Egypt, numbered by Victor Loret [1] xxvii, I possess nine double-reed mouthpieces, seven of which play the pipe in the Mixolydian Harmonia—of Determinant 14—and therefore, at the same extrusion (within 3 mm.) and total length. These

[1] *Encycl. de la Musique*, ' Egypte ', Fasc. 1, p. 19 (Paris: Delagrave).

TABLE III

TABLE OF SELECTED BEATING-REED MOUTHPIECES OF VARIOUS TONGUE-LENGTHS, DIAMETERS AND TONGUE-WIDTHS, WITH THEIR PROPER NOTES, NORM AND GLOTTIS

No. of mp.	Length	Dia-meter	Tongue-length	Tongue-width	Proper Note of mp. Norm	Results of Tests. Glottis Note
XVIII C. new 25/11/33	·15c	·0035	·035	·0025	$\frac{C\ 21}{128}$ = 134 v.p.s.	Plays in Loret xviii at *Ext* = ·055 full rich notes on $\frac{B\ 12}{64}$. The Tongue was first cut to ·030, the I.D., which, however, did not produce the desired pitch; it was then lengthened to ·035 and at once the mp. gave out a correct norm $\frac{C\ 21}{128}$.
No. 29 oat	·130	·004	·034	·0025	$\frac{C\ 21}{128}$ correct norm	Gl. $\frac{G\ 14 \text{ and } D\ 20}{128}$ 3/4/33. Coarse blare at first; tongue stiff; slight clearance sent pitch down from $\frac{G\ 14}{128}$ to *D* 20; after scraping the true norm came at once = $\frac{C\ 21}{128}$ = 134·5 v.p.s.
No. 100 fine silica wheat	·211	·003	·032	·001	$\frac{D\ 20}{64}$ = 143 v.p.s. (141 v.p.s. exact)	Gl. $C = 64$ v.p.s. Very fine tongue of 1 mm. in width. Resonance fundamental = $\frac{G}{512}$.
Z	·141	·003	·035	·003	= $\frac{D\ 20}{128}$	Plays in Maket 3 on $\frac{G\ 14}{128}$; powerful note. N/B—As the diameter and Tongue-width are the same, the formula = $·035 + ·003 \times 8 = ·304$. $\frac{340}{·304} = 139·8$ v.p.s.

No. 33 17/5/33 natural knot	·101	·002	·037	·0015	$\frac{C\,21}{128}$ correct to formula	Good gl. notes on $\frac{E\,18}{128}$ and $\frac{B\,12}{64}$.
No. 31 sealed 28/6/33	·130	·003	·037	·0015	$\frac{C\,11}{128}$ correct to formula	Gl. tense $\frac{C\,11}{256}$. The difference of 1 mm. in diameter accounts for variation in pitch compared with No. 33.
No. 17* 17/5/33 Cf. with No. 15	·170 (originally ·176) 15/1/34	·0025	·040	·002	$\frac{\overset{\sharp}{B}\,12}{64}$ or $\overset{\flat}{C}$ 128 = 124 v.p.s. correct to formula	The Tongue originally cut to ·035 had worn loose, and lower on one side. I made this equal at ·040 gl. $\frac{G\,15}{64}$ beautiful rich tone.
No. 15* 15/1/34	·085	·0025	·040	·002	$\frac{\overset{\sharp}{B}\,12}{64}$ or $\overset{\flat}{C}$ = 128 correct to formula	These straws, Nos. 17 and 15, form valuable evidence concerning the factor of length in reed-blown pipes. It is the Tongue-length, not that of straw, which is effective through length. Cf. Porphyry, *Comm. on Ptol.*, the performance of these mps. in Auloi.
K Mak. 3	·152	·003	·035	·0025	$\frac{\overset{\sharp}{C}\,21}{128}$ or $\frac{\overset{\flat}{D}\,20}{128}$	At first the Tongue-length ·032 gave norm of $\frac{E\,19}{128}$ then cut to ·035 gave a norm between C 21 and D20. Inserted in Aulos Maket 3, it plays well at extrusion ·047 on $\frac{G\,14}{64}$ but gave a distorted series with mp. pulled out to ·055
W.1	·138	·0025	·039	·0015	$\frac{\overset{\flat}{C}}{128}$ = 125 v.p.s. true to formula $\frac{340}{340}$ = 125 v.p.s.	Gl. $\frac{F\,17}{64}$ magnificent tone, resonant, harmonic constituents rich.

* It will be noticed that the lengths of the shank in mouthpieces Nos. 17 and 15 are in octave relation. See further on Porphyry's *Commentary on Ptolemy*, i, 8, with my notes on debatable points; and examples of octave relation qualified in incidence.

TABLE IV

SEVEN D–R. MP.'S OF VARIOUS LENGTHS, HAVING THE SAME DIAMETER ·004 AND A COMMON NORM OF $C\ 11$ AT V.L. FROM ·079 TO ·083

No. of mp.	Length	Diameter	V.L.	Proper Note: Norm.	Proper Note: Glottis	By Formula VIII in Operation	
R.7	·204 cut to ·159	·004	·079	$\frac{C\ 11}{256}$	gl. $\frac{F\ 17}{128}$ and $\frac{F\ 17}{256}$ at V.L. ·079	·079 ·004 ·083 × 4 $\frac{340}{·332}$ = 256 v.p.s. Tested 16/11/33; Jan. 4/33	Plays in Loret xx on $\frac{G\ 14}{128}$
Cl.6	·200 cut ·161	·004	·079	$\frac{C\ 11}{256}$	gl. $\frac{G\ 15}{128}$ (strong)	*Working out of Formula the same for all seven mp.s*	Plays in Elgin Aulos on $\frac{C\ 11}{128}$ at V.L. ·075
N.7	·157	·004	·079	$\frac{C\ 11}{256}$	gl. $\frac{F\ 17}{128}$		Plays in Elgin Aulos on $\frac{A\ 27}{128}$ at V.L. ·065
N.1	·131	·004	·079	$\frac{C\ 11}{256}$			Plays in Elgin Aulos on $\frac{C\ 13}{128}$ at ·079 V.L.
N.32	·160	·004	·079 and ·065	$\frac{C\ 11}{256}$ $F\ 17$	gl. $\frac{F\ 17}{256}$	Tested 25/4/33; 28/4/33; 6/11/33	Plays in Loret xxiii on $\frac{A\ 13}{128}$ at V.L. ·065
Cl.5	·133	·004	·079 and ·060	$\frac{C\ 11}{256}$ $\frac{F\ 17}{256}$	gl. $\frac{G\ 15}{128}$ $\frac{F\ 17}{128}$	Played in Aulos on $\frac{A\ 13}{128}$ in tune Jan. 4/33, but gave norm of V.L. ·060 on $\frac{F\ 17}{128}$ also.	Plays in Loret xviii on $\frac{A\ 13}{64}$ at ·060
R.1	·150	·004	·079	$\frac{C\ 11}{256}$	gl. $\frac{F\ 17}{128}$ at V.L. ·079	Tested 8/4/33; 10/4/33; 4/1/34 (unfortunately badly cracked on being withdrawn from Aulos after the test)	Plays in Loret xvii on $\frac{C\ 11}{256}$

It will be obvious that the length of the shank of the mp. has no influence on the pitch of the proper note of the mp., which did not change in R., 7' and 'Cl.6' when the shank was cut shorter. The change in the fundamental note of Aulos + mp. is due to accommodation through resonance.

TABLE V

SEVEN D-R. MOUTHPIECES OF VARIOUS LENGTHS; DIAMETERS FROM ·0035 TO ·005; AT A V.L. OF ·060; THE MOUTHPIECES HAVE AS NORMAL NOTE $\frac{F\ 17}{128}$ OR $\frac{F\ 17}{256}$. AT V.L. ·079 THE NORM IS $\frac{C\ 11}{128}$ OR $\frac{C\ 11}{256}$

No. of Mp.	Length	Diameter	V.L.	Proper Note of Mp. Norm or Glottis	By Formula No. VIII	Performance
O.8	·178	·004	·060	$\frac{F\ 17}{256}$ gl. $\frac{A\ 13}{256}$ = 433 v.p.s.	$·060 + \frac{·004}{·064 \times 4} = \frac{340}{·256} =$ 332 v.p.s. $\frac{F\ 17}{256}$	
H.5	·141	·004	·080	$\frac{C\ 11}{256}$ gl. $\frac{D\ 20}{128}$ strong note	$·080 + \frac{·004}{·084 \times 4} = \frac{340}{·336} =$ (126) v.p.s. 252 v.p.s.	Plays in Loret xxi. on D 20 a strong glottis note which is also that of resonance fundamental of H 5.
H.1	·112	·004 exit oval ·008	·056 better at ·060	$\frac{F\ 17}{128}$ (soft) gl. $\frac{F\ 17}{256}$ powerful note alternates between the two F's	$·056 + \frac{·008}{·064 \times 4} = \frac{340}{·256} =$ 332 v.p.s. an octave above the norm	Tested 22/6/33. The norm, an octave lower than formula demands due to resonance of straw = $\frac{F\ 17}{1024}$
Cl.8	·166	·0035	·060	$\frac{F\ 17}{256}$ gl. $\frac{D\ 20}{256}$ and $\frac{A\ 13}{256}$ and gl. $\frac{A\ 13}{128}$	$·060 + \frac{·0035}{·0635 \times 4} = \frac{340}{·254} =$ 334 v.p.s.	
R.3	·119	·004	·060	$\frac{F\ 17}{256}$ gl. $\frac{A\ 13}{128}$ v. powerful norm	Formula as for 'O.8'. The rich powerful Tone of this mp. probably results from the texture of the straw and from the fact that the fundamental resonance of the mp. is in octave relation to the norm of the mp., i.e. $\frac{F\ 17}{1024}$	Plays in Loret xxiv on $\frac{A 13}{128}$ at Ext. ·062 and V.L. ·06 in the Lydian Harmonia.
N.30	·170 cut to ·161	·005	·059	$\frac{F\ 17}{128}$ norm ictus $\frac{C\ 11}{256}$ at ·075 $\frac{C\ 11}{256}$ norm gl. $\frac{F\ 17}{128}$	$·059 + \frac{·005}{·064 \times 4} = \frac{340}{·256} =$ 332 v.p.s.	
H.7	·142	·0045	·060	$\frac{F\ 17}{256}$ norm gl. $\frac{A\ 13}{128}$	$·060 + \frac{·0045}{·0645 \times 4} = \frac{340}{·258} =$ 329·4 v.p.s.	Plays in Loret xxvii on $\frac{B\ 12}{64}$ at V.L. ·065 in the Mixolydian Harmonia.

TABLE VI

AULOS LORET XXVII PLAYED BY NINE DIFFERENT D–R. MOUTHPIECES

For measurements of Aulos xxvii see Records of Performance, Chap. x

No. of mp.	No. of Finger-notes	V.L. of mp.	I. of D.	Ex-trusion of mp.	Modal Det.	Harmonia	Aulos Fundamental	Sequence of Harmonia	Date of Tests
xxviia	6	·060–5	·0245	·090	15	Unrecognized Harmonia	$A\ \frac{27}{64}$ = 104 v.p.s.	Vent $\frac{15}{15}$ $\frac{14}{15}$ $\frac{13}{15}$ $\frac{12}{15}$ $\frac{11}{15}$ $\frac{10}{15}$ *Cents* *119·4 ; 128 139 151 ; 165*	2/2/33
xxviib	—	·063	—	·065	14	Mixolydian	$B\ \frac{12}{64}$ = 117·3 v.p.s.	Vent $\frac{14}{14}$ $\frac{13}{14}$ $\frac{12}{14}$ $\frac{11}{14}$ $\frac{10}{14}$ $\frac{9}{14}$ *Cents* *128 ; 139 ; 151 ; 165 ; 182*	9/2/33
D.1	—	·045	—	·065	14	Myxolydian	$D\ \frac{20}{128}$ = 141 v.p.s.	ditto	10/2/33
R.6	—	·074	—	·115	16	Hypodorian	$B\ \frac{12}{128}$ = 234·6 v.p.s. and on $G\ \frac{14}{128}$ = 201 v.p.s.	Exit $\frac{16}{16}$ $\frac{15}{16}$ $\frac{14}{16}$ $\frac{13}{16}$ $\frac{12}{16}$ $\frac{11}{16}$ $\frac{10}{16}$ *Cents* *112 ; 119·4; 128 ; 139 ; 151 ; 165*	
Z.1	—	·060	—	·065	14	Mixolydian	C = 128 v.p.s.	$\frac{14}{14}$, &c., as above. Does not play easily, but in tune	17/6/33
H.7	—	·065	—	·068	14	Mixolydian	$B\ \frac{12}{64}$ = 117·3 v.p.s.	ditto	
R.9	—	·065	—	·068	14	Mixolydian	$A\ \frac{13}{64}$ = 108·3 v.p.s.	ditto	
0.6	—	·065	—	·068	14	Mixolydian	$G\ \frac{14}{64}$ = 101 v.p.s.	ditto	
H.6	—	·065	—	·068	14	Mixolydian	$B\ \frac{12}{6}$ = 117·3 v.p.s.	ditto	

seven mouthpieces play the Aulos in five different tonalities, viz. three on $\frac{B\,12}{64}$; one on $\frac{G\,14}{64}$; one on $\frac{A\,13}{64}$; one on $\frac{C\,11}{128}$ and one on $\frac{D\,20}{128}$.

Mps. xxvii *b*; H.7; H.6	O·6	R.9	Z.1;	D.1

Play the Aulos on

$\frac{B\,12}{64} = \frac{117{\cdot}3}{\text{v.p.s.}}$; $\frac{G\,14}{64} = \frac{100{\cdot}5}{\text{v.p.s.}}$; $\frac{A\,13}{64} = \frac{108{\cdot}3}{\text{v.p.s.}}$; $\frac{C\,11}{128}$; $\frac{D\,20}{128} = \frac{140{\cdot}8}{\text{v.p.s.}}$

The range in tonality extends over a diminished 5th (ratios 14 : 10 = *583 cents*); but in each case, although the fundamental and notes of the pipe vary in pitch, the ratio of the intervals obtained through the fingerholes remains invariable. (See Record of Loret xxvii for further details.) Such a performance on a wood-wind instrument is without parallel in modern times; it is due entirely to modality.

(5) Of the seven D-R. mouthpieces that play the Elgin Aulos at the same extrusion, viz. ·110, six give the Aulos the same tonality, on the fundamental note $\frac{C\,11}{128}$, and in the Dorian Spondaic Harmonia of Determinant 11; the 7th D-R. mouthpiece, on a longer extrusion than the others, plays in the Phrygian Mode on $\frac{A\,27}{64}$ (= 104·6 v.p.s.), a detailed record of which appears under 'Elgin Aulos'. The interval between the two fundamentals $\frac{A\,27}{64}$ and $\frac{C\,11}{128}$ is that of a major 3rd flattened by $\frac{55}{54}$ (= *355 cents, instead of 386*). This interval $\frac{27}{22}$ is better known as the *Wosta of Zālzāl*, used as a fret on the lute in the eighth century A.D. by Arab musicians.

(6) Pursuing our experiences with reed-blown pipes, the converse proposition adds to the perplexities of the uninitiated, concerning the behaviour and reactions of reed mouthpieces and their compelling influence on pitch —a fact apparently left entirely unexplained in text-books. The D-R. mouthpiece 'D.I.' plays in eight Auloi, of which the length of resonator + mouthpiece ranges from ·343 to ·624; in three of these the pitch of the pipe's fundamental note is $\frac{B\,12}{64}$ (= 117·3 v.p.s.); in one $C = 128$; in two $\frac{D\,20}{128}$ (= 140·8 v.p.s.), in one on $\frac{F\,16}{64}$ (= 88 v.p.s.) in one $\frac{E\,19}{128}$ = 148 v.p.s.

In the Auloi	Mixolyd.	Dorian	Mixol.	Phr.	Mixolydian.	
Nos.	xxviii; xx; xxiv;	Elgin;	xxvii;	xxi;	xxvi;	xx.
	$\frac{B\,12}{64}$ = 117·3 v.p.s	C = 128 v.p.s	$\frac{D\,20}{128}$ = 140·8 v.p.s	$\frac{D\,20}{128}$	$\frac{E\,19}{128}$ = 148 v.p.s	$\frac{F\,16}{64}$ = 88 v.p.s

In relating these results of tests on Auloi to our experience with other

TABLE VII

THE ELGIN AULOS, BRITISH MUSEUM ; PLAYED BY EIGHT D–R. MOUTHPIECES
For measurements of Aulos, see Records of Performance

No. of mp.	No. of Finger-holes	V.L. of mp.	I. of D.	Ex-trusion of mp.	Modal Det.	Harmonia	Aulos Fundamental	Sequence of Harmonia	Date of Tests
Elgin BB.	6	·065–070	·032 mean	·110	11	Dorian Spondaic	C = 128 v.p.s.	Vent $\frac{11}{11}$ $\frac{10}{11}$ $\frac{9}{11}$ $\frac{8}{11}$ $\frac{7}{11}$ $\frac{6}{11}$ *Cents* 165 ; 182 ; 204 ; 231 ; 267	15/12/25
D.1	—	·060	—	·110	11	Dorian	C = 128 v.p.s.	ditto	16/12/25
Cl.6	—	·075	—	·110	11	Dorian	C = 128 v.p.s.	ditto	27/9/30
Cl.7	—	·079	—	·110	11	Dorian	C = 128 v.p.s.	ditto	10/2/31
Cl.18	—	·075	—	·110	11	Dorian	C = 128 v.p.s.	ditto	24/4/33
N.1	—	·079	—	·110	11	Dorian	C = 128 v.p.s.	ditto	10/6/33
N.7	—	·065	—	·132	12	Phrygian	$A\ \frac{27}{128}$ = 209 v.p.s.	Vent $\frac{12}{12}$ $\frac{11}{12}$ $\frac{10}{12}$ $\frac{9}{12}$ $\frac{8}{12}$ $\frac{7}{12}$ *Cents* 151 ; 165 ; 182 ; 204 ; 231	

Many more undated tests have been made of all of these mps. used in Elgin Aulos. All played strictly in tune with more or less ease.

TABLE VIII

Performance of D–R. Mouthpiece D.1 in Several Auloi (8)

D.1 : length ·172 ; diameter ·003 emb. flattened to ·0055 : proper note. Norm at ·062* = $\frac{F\ 17}{128}$; at ·080* = $\frac{C\ 11}{128}$ and at ·080 also $C = 256$; at ·055* = $\frac{F\ 16}{128}$; at ·045* = $\frac{A\ 13}{128}$. The lengths marked with asterisk have v.f. true to formula.

No. of Aulos	Holes	Modal Det.	Length, Vent or Exit	Extrusion of mp.	Combined Length	I.D.	V.L.	Fundamental of Aulos	Tests	Remarks ; Sequence
Elgin	6	11	·244 vent	·110	·354 ·032 × 11 = ·352	·032	·060	$\frac{C\ 11}{128}$	14/2/33 } 17/2/33 } 6/1/34 }	Whole series in tune. Strong firm notes even from Hole 6. Mp. not so strong ; gave 5 notes only. Refused 6th Hole. (Was it K. S. at fault ?) *
Loret xxvii	6	14	·278 vent	·065	·343	·0246 × 14 = ·344	·045	$\frac{D\ 20}{128}$	14/2/33	Whole series in tune. Steady, strong, true
Loret xxvi	3	14	·322 exit	·070	·392	·028 × 14 = ·392	·056	$\frac{E\ 19}{128}$	11/2/33	14, 13, 12, 11 in tune. Strong and steady
Loret xxi	4	12	·271 vent	·080	·351	·030 × 12 = ·360	·060	$\frac{D\ 20}{128}$	14/2/33	Series 12, 11, 10, 9 easily in tune
Loret xxviii	6	14	·300 exit	·078	·378	·027 × 14 = ·378	·055	$\frac{B\ 12}{64}$	14/2/33	Whole series 14, 13, 12, 11, 10, 9, 8 easily in tune to Mese. On Hole 6 several times up and down
Loret xx	6	14	·3045 vent	·087	·3915	·0278 × 14 = ·3892	·062	$\frac{B\ 12}{64}$	14/2/33	14, 13, 12, 11, 10, 9 whole series in tune
Loret xxiv	4	14	·302	·090	·392	·028 × 14 = ·392	·060	$\frac{B\ 12}{64}$	14/2/33	Played the series 14, 13, 12, 11 in tune
Loret xvi	4	12	·309	·087	·396	·033 × 12 = ·396	·060	$\frac{B\ 12}{64}$	8/1/34	12, 11, 10, 9 fairly easy. Steady, and in tune

N.B.—Although the Vibration Length is significant in the determination of pitch in the proper note of the D–R. mouthpiece, when used alone, yet, when once the mp. has been inserted into the Aulos, and has drawn forth the fundamental, to lengthen or shorten the V.L. (or impact of the lips on the mp.), does not enable the piper to change the fundamental note of the Aulos ; as is the privilege of the B–R. mouthpiece.

The Increment of Distance multiplied by the Modal Determinant should be compared with the total length of Aulos + extrusion of mp.

wind instruments, we have to bear in mind the fact that cylindrical pipes played with reed mouthpieces, whether of D-R. or B-R. type, belong by their reactions to the category of closed pipes. For instance the Elgin Aulos has a length from the vent of ·244 + extrusion of mouthpiece ·110 $= \cdot 354$; $\cdot 354 \times 4 = 1{\cdot}416$; $\frac{340}{1{\cdot}416} = 240$ v.p.s., i.e. *B* in the 4-foot octave, a 7th above $C = 128$ v.p.s. In Aulos xx, length ·305; played by mouthpiece 'D.1.' at an extrusion of $\cdot 087 = \cdot 392 \times 4 = 1{\cdot}568$; $\frac{340}{1{\cdot}568}$ $= 216{\cdot}8$ v.p.s. $= \frac{A\ 13}{128}$. Here again the estimated theoretical length as a closed pipe gives a fundamental $\frac{A\ 13}{128}$ in the 4-foot octave, whereas the actual note is, as shown above, $\frac{B\ 12}{64}$ a 7th lower.

CONCERNING THE OCTAVE RELATION IN AULOI (Porphyry's *Comm. on Ptol.*, i, 8)

An important point concerning the octave relation in Auloi remains to be made clear and definite, for it arises frequently in the sources and has led students and authors astray. The matter at issue would seem to involve a contradiction of an undisputed acoustic law, or at least to introduce a serious reservation or exception due to the unrecognized properties of primitive forms of reed mouthpieces. The assumption that because the ratio 2 : 1 in vibration frequencies and in lengths of string produces the octave relation, this ratio must also bring about a like result in reed-blown pipes, is erroneous; this is not the case.

A comparison of the performance of B-R. mouthpieces Nos. 17 and 15 (see Table iii), having stems measuring ·170 and ·085 respectively, i.e. in the ratio 2 : 1, while other dimensions are the same for both, demonstrates the fact that the length of straw, two to one, has not brought about the octave relation in the resulting pitch of the proper notes of the mouthpieces in question. Bearing in mind the dictum of Porphyry in his commentary on Ptolemy, i. 8 [1] concerning ratios of length to pitch in the Aulos, further tests were made with these mouthpieces. Ptolemy writes (i, 8):

> Let us forbear to demonstrate our proposition by means of Auloi and Syrinxes, or by means of weights hung on to strings, because demonstrations of this kind cannot attain to the highest degree of accuracy, but rather they occasion misrepresentation upon those who try to do it. For in Auloi and Syrinxes, besides the fact that it is very difficult to discover the correction of their deviations therein, even the points between which the lengths must be measured include a certain undetermined width (Note 1); (πλάτος, K. S. the diameter of the bore) [2] and generally

[1] Ingemar Düring, *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*, (Göteborg, 1932), p. 119, line 14, and *Die Harmonielehre des Klaudios Ptolemaios* (Göteborg, 1930), p. 16, lines 32 sqq. Engl. Tr. by Professor J. F. M.

[2] πλάτος, width, can only apply to the diameter of the bore not to width of finger-holes, since there is none in the Syrinx, and Ptolemy's statement refers to both Syrinx and Aulos. (See also Note 1, by K. S.)

the majority of wind instruments have some unordered element added, apart from the blowing in of the breath.

In his commentary on this passage, Porphyry discourses as follows (*op. cit.*, p. 119, lines 14 sqq.):

Some of the Pythagoreans have set out differently things pertaining to consonances on instruments. For some, making two Auloi (2) of bronze or reeds (καλάμους) [see Notes by K. S. following] equal in thickness and equal in bore (ἰσοκοιλίους) (1) like Syrinxes, of which the one was double of the other in length (3) and blowing with the mouth into the Auloi at the same time (4) through the little tongues therein (5) (γλωσσιδίων), used to get the consonance of the octave in the ratio (6) 2 : 1; and they got the other consonances in the proper ratios, the Auloi having a ratio towards one another in accordance with length (7), i.e. sometimes of the 4 : 3; or of the 3 : 2; or of 3 : 1; or of the 4 : 1. And by means of one Aulos they used to get the same result just as well for dividing the whole Aulos (8), sometimes in half for the octave, sometimes in 4 : 3, taking the three parts towards the glossis (9) for the 4th, and in the case of the others, according to their proper ratios, and making the holes (10) in accordance with these, and likewise breathing alike into the same, they got the proper consonance. They obtained a similar result from the Hydra (11) [Hydraulis or Water Organ] from which the Auloi, superposed in a row, being unequal, produce the Harmoniai. Wherefore also Ptolemy deprecating all that had been said before, for what reason he himself has said, arrived at the division of the Kanon. For in the case of Auloi and Syrinxes, he says ' besides the difficulty of discovering the correction of their deviations (12) because the reeds are sometimes broader, sometimes narrower (13), and even their limits like (geometrical) figures depend to some extent on a certain undetermined width, and generally the majority of wind instruments have some unordered element added apart from the blowing in of the breath ' (Düring, *op. cit.*, p. 120, ll. 22–4; width = πλάτει; see also K. S., Chap. vii, p. 273).

NOTES ON DEBATABLE POINTS IN THE QUOTATION FROM PORPHYRY

(1) It has already been explained above that the diameter of the bore of the reed-blown pipe, so important for the determination of pitch in Syrinx and flute, has no pitch significance in the resonator of the reed-blown Aulos. The difficulty to which reference is made here is due to two facts unnoticed by Porphyry: (1) that Ptolemy's reference includes two instruments, Syrinx and Aulos, which differ radically in their acoustic properties; (2) that the visible length in Syrinx and Aulos between mouthpiece and exit does not correspond to the length of the column of air productive of the note heard; in the Syrinx length is increased by the width of the diameter of the bore (πλάτος) according to rule, whereas in the Aulos it is the mouthpiece which is the determinant factor of pitch, and only partly as a result of the additional length it contributes to that of the resonator. Neither Ptolemy nor Porphyry has taken the mouthpiece into account.

(2) Two Auloi are taken, equal in thickness and in bore, but the one the double of the other in length, so that according to Porphyry, who, of course, is in error, length is the sole cause of the difference in pitch, viz. of an octave, between the two Auloi.

(3) The significance of the blowing into the Auloi with the mouth through the little tongues, *at the same time*, obviously implies the fact that both volume and compression of breath were the same for both Auloi,

for it is, of course, only possible to feed the breath through two mouthpieces, taken into the mouth at once, by means of a homogeneous stream. This statement provides a third equal factor, again leaving length only as differentia. It is expressly stated that the Auloi are blown at the same time, and this throws an interesting light upon the meaning attached by Porphyry to the Symphonia, as a blend of two notes sounded simultaneously, i.e. not only one after the other as intervals, but in harmony, in the modern sense.

(4), (5) and (6). The little tongues (γλωσσιδίων) are the elusive factors, and the silence of Porphyry on this score—suggesting a lack of practical acquaintance with this all-important part of the Aulos—enables us to explain the result chronicled, without necessarily putting either Pythagoreans or Porphyry in the wrong as to the occurrence of the octave relation: for without some reservation or qualifying statement, we now know from what has gone before in this section, that the assumption and explanation of playing an octaves on Auloi in accordance with the ratio of length is altogether erroneous. But this is what may have happened, more or less behind the scenes, so far as an observer is concerned. If the piper—in the secret, or even fortuitously, were to draw down, by an imperceptible movement (probably by less than an inch), the shorter of the two Auloi (having tongues of the same length), so that the impact of the lips on the little tongue now occurred half-way up the tongue instead of at the base or hinge, we are already aware that the effect would be to send the fundamental of the Aulos up an octave, without any apparent reason. The piper —with a twinkle in his eye, and a metaphorical tongue of his own in his cheek (without prejudice to the tongue of the beating-reed), proud, no doubt, of the secret devices of his craft, as were the medieval pipers and sackbut players who rigidly protected those of their guilds—our piper would enjoy the errors of judgement of the uninitiated observers. Alternatively, to use one B-R. mouthpiece in the longer Aulos with a tongue to say ·046 in length and the shorter Aulos with a mouthpiece having a tongue of ·023 in length—width of tongue and diameter of reed or straw being the same in both mouthpieces—the effect would be that described by the Pythagoreans or Porphyry, but with the one proviso, viz. that both mouthpieces were of the same strength and power of utterance, a property mainly dependent upon the texture of the straw or reed.

(7) To prove that this statement made by Ptolemy [1] is entirely erroneous is one of the main theses of this section.

(8) and (9). After the erroneous claim that the pair of Auloi have a ratio towards one another in accordance with length when played through the little tongues, Porphyry considers the Aulos *per se*, i.e. the resonator alone, and states that the divisions of the length producing the ratios of the Symphoniai are carried out in the boring of holes, and he gives as an example the 4 : 3 of the diatessaron, the three parts towards the *glossa*. (9) If the Aulos actually sounded the interval of the 4th from the exit to a hole opened at one 4th of the length of the resonator, it would certainly

[1] Cf. *Harm.*, ii, 12 (Wallis, Oxford, 1682), p. 157, lines 5 sqq.

be because the three-quarters of the length lay between that hole and the *glossa*.

(10) Porphyry, however, has got into very deep waters here, for to divide the Aulos resonator alone into four parts, without taking into account the mouthpiece, would falsify the ratios. On the other hand, to place the fingerholes at equal distances, of which the length of Aulos resonator plus extrusion of mouthpiece is a multiple, will produce a series of intervals in superparticular ratios: what the ratios of these intervals are depends entirely upon the Determinant number of increments of distance. The reader may here be reminded of the performance quoted above from our records of Loret xxiii, played with D-R. mouthpiece N.32, in which, after starting with the Dorian Harmonia requiring eleven increments of distance, a lengthened extrusion of the mouthpiece, adding one more increment of distance, was found to produce the Phrygian Harmonia, of Determinant 12, *through the self-same fingerholes.* Two different series of intervals on the same fundamental were thus given out through the fingerholes, e.g. from exit to Hole 1, as a Dorian Aulos, the interval of ratio 11 : 10; and as Phrygian Aulos the interval of ratio 12 : 11. Such a feat would be impossible were Porphyry's contention correct (see Records, Chap. x, of Cairo Auloi 'G' and 'R').

(11) An interesting reference is then made by Porphyry to the Hydraulos or Water Organ, in which the fixed row of Auloi, being unequal in length, get their Harmoniai. This statement might appear to contain a contradiction of the points raised in my Note 10, but it is not so; for the conditions are now radically changed. In the Hydraulis of Porphyry's day, the air compressed by means of water was fed to all the pipes at the *same* degree of compression, whereas in the Aulos played by the action of the human breath and larynx, the compression of air is adapted and regulated for each proportional length put in operation by the opening of the fingerholes, as already explained at length. Moreover, each Aulos in the Hydraulis had the mouthpiece suited to give one note only; whereas the piper with his Aulos had one mouthpiece and reed tongue only which was enabled to accommodate itself to the different length proper to each fingerhole, through his power of varying the compression of his breath stream.

(12) This reference by Ptolemy to the need for correction of the deviations in the intonation of the Aulos, due to difference in the diameter of the bore of the reeds, seems to imply that Ptolemy had made a study of the properties of pipes and flutes, and had discovered the influence of diameter on pitch, but that his research and experiments had not been extended to the properties of the mouthpiece: he had not discovered its dominance over length.

We may now examine the behaviour of B-R. mouthpieces Nos. 17 and 15 (see for dimensions, proper notes, &c., Table iii), when tested in connexion with the question of the proportional influence of length in the determination of pitch in Auloi as propounded by Ptolemy and Porphyry.

The Table shows that the length of the two mouthpieces in the ratio of 2 (No. 17) to 1 (No. 15) did *not* produce the octave relation in the proper notes of the mouthpieces which, on the contrary, were identical and in agreement with formula. The result of this test—one of many similar ones carried out under the same conditions—is due to the fact that the factors of tongue-length and width, and of the diameter of the straw which are the determinants of pitch in the mouthpiece (and in a sense also of the whole Aulos) are the same in both specimens. This result thus negatives the conclusions of the Pythagoreans of Ptolemy and of Porphyry in so far as the mouthpiece used alone is concerned.

The two mouthpieces were in turn inserted into Loret xxxv (Brit. Mus.). 'No. 15', a tiny mouthpiece, at an extrusion of ·050, when it played from exit a fundamental glottis note $\frac{G15}{64} = 93{\cdot}8$ v.p.s. The pipe resonator (an open pipe) measures $\cdot 222 + (\cdot 050 + \Delta\, \cdot 003) = \cdot 275$; $\cdot 275 \times 4$ (for a closed pipe) $= 1{\cdot}100$; therefore $\frac{340}{1{\cdot}100} = 309$ v.p.s. $= \overset{\flat}{E}\,18$ (the exact v.f. of which is 313 v.p.s.). Thus No. 15 in this Aulos plays an octave and a 6th lower than the pitch computed by the formula for the resonator + mouthpiece.

Since the mouthpiece is the determining factor, 'No. 17', the longer mouthpiece, was inserted into the same Aulos xxxv at an extrusion of ·100 (twice that of No. 15); thus $\cdot 222 + \cdot 100 + \Delta\, \cdot 003 = \cdot 325$; $\cdot 325 \times 4 = 1{\cdot}300$; therefore $\frac{340}{1{\cdot}300} = 261{\cdot}5$ v.p.s. $= \frac{\overset{\sharp}{C}\,11}{256}$. The actual fundamental from exit, given by test from Aulos + mouthpiece—a glottis note—somewhat richer in Harmonics and more powerful than that of No. 15, was again $\frac{G\,15}{64}$. This note is thus an octave and a sharpened 4th lower than the theoretical one computed by formula.

A second test was then made with Aulos Loret xvi, playing from Hole 1. By formula: mouthpiece, No. 15 at extrusion ·062; length of resonator from centre of Hole 1 $= \cdot 308 + \Delta\, \cdot 005 + \cdot 062 = \cdot 375$. $\cdot 375 \times 4 = 1{\cdot}500$; thus $\frac{340}{1{\cdot}500} = 226{\cdot}6$ v.p.s. $= \frac{\overset{\flat}{B}\,12}{128}$ (exact v.f. of B 12 $= 234{\cdot}6$ v.p.s.). The fundamental glottis note produced from Hole 1 by test was $C = 128$ v.p.s., a note nearly an octave below that computed by formula, and a 4th above the proper note of the mouthpiece used alone.

Mouthpiece No. 17, inserted likewise into Aulos xvi, at extrusion ·124 (twice that of No. 15), played from Hole 1 a fundamental glottis note, still of the same pitch as before, viz. $\frac{G15}{64}$ (a 4th lower than with mouthpiece 'No. 15'), thus proving once again the dominant power of the mouthpiece in the determination of pitch, independently of the length of the whole Aulos upon which the mouthpiece frequently but not invariably —owing to accommodation with resonance—imposes its own proper glottis note.

By formula: $\cdot 308 + \cdot 005 + \cdot 124 = \cdot 437$; $\cdot 437 \times 4 = 1\cdot 748$, and $\frac{340}{1\cdot 748} = 194\cdot 5$ v.p.s., i.e. $\frac{G15}{128}$ (exact v.f. = 188 v.p.s.); therefore, exactly an octave above the actual fundamental note of Aulos xvi + mouthpiece No. 17. Although many similar tests might be quoted, as examples of the erroneous nature of the assumption: that the ratio 2 : 1 in length invariably produces an octave pitch relation, these tests will probably suffice to show the law in operation in reed-blown pipes when furnished with a mouthpiece of one of the two primitive types. More especially will these tests be found convincing, since the results of other tests of different kinds already used in illustration, and the records of performance of individual Auloi which follow, all point to the same conclusion, viz. that the mouthpiece, be it B-R. or D-R., is the dominating partner, to which is allotted the casting vote in the modality and tonality of the Aulos.

THE ETHOS OF THE HARMONIA

Much attention has been directed in connexion with the Aulos to the function of the glottis muscles in regulating the degree of compression of the breath necessary for a rise or fall in pitch of the notes obtained through both types of mouthpiece. The interrelation of this function of the muscles of the glottis with modality and the light it throws upon the vexed question of the Ethos attributed by Plato,[1] Aristotle and other writers to the Harmoniai still remains to be briefly considered. No one who is familiar with the modal sequences of the Harmoniai of Ancient Greece remains in doubt as to the potency of this Ethos. To play a simple melodic phrase on the same degrees of each Harmonia in succession furnishes a convincing demonstration of the reality of the characteristic Ethos of the Modes. I am now going to suggest that the general classification of Ethos as μαλακός (soft, relaxed) and σύντονος (tense) is probably intimately connected with the physical process of sound production on the Aulos and in singing. There is, however, a second modal feature which is likewise contributory to the characterization of Ethos; this is the *tessitura* of the modal scale, due to the position of Mese on its own characteristic degree of the Harmonia, which bears with it certain implications, mathematical (through the ratios of the genesis), modal, musical and psychological. The latter is brought about through the essential Ethos of Mese as a member of the Harmonic Series in its function as ἀρχή (see Chap. ii). By virtue of the genesis of the modal material, Arche bestows its own essential quality upon each member of the modal series. It has already been pointed out how this essential quality of Arche—which has descended in due course upon its lower octave, Mese—stands out in a varying degree of contrast (according to the Harmonia) to the modal Tonic common to all the Harmoniai. The Dorian Mese, as sharp Harmonic 4th $\left(\text{ratio } \frac{11}{8}\right)$ and the Mixolydian flat Harmonic 7th $\left(\text{ratio } \frac{7}{4}\right)$ being the most striking.

[1] *Rep.*, 398–9.

Let us now turn to one or two quotations germane to the subject: first come Aristotle's definition and characterization of Ethos:[1]

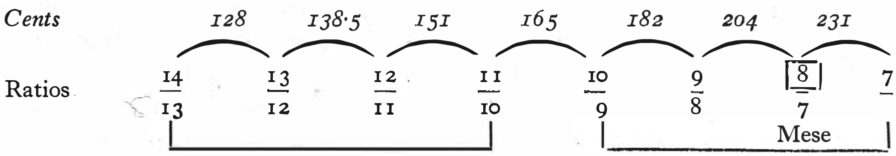
> To begin with there is such a distinction in the nature of the Harmoniai that each of them produces a different disposition in the listener. By some of them, as for example the Mixolydian, we are disposed to grief and depression; by others, as for example the low-pitched ones, we are disposed to tenderness of sentiment.

And then: 'Thus for those whose powers have failed through years, it is not easy to sing the syntonic Harmoniai, and their time of life naturally suggests the use of the low (ἀνειμένας).' Plato's[2] treatment of Ethos is well known:

> What then are the Threnodic Harmoniai? 'Mixolydian,' said he, 'and Syntonic Lydian, and some others of the same character'—'Which of the Harmoniai then are soft and convivial?'—'The Ionian,' he replied, 'and Lydian, and such as are called slack' (χαλαραί).

We notice at once that on the one hand *tension* and *relaxation* point directly to the action of the glottis in producing a rise or fall of pitch, e.g. the Mixolydian Harmonia in its first ascending tetrachord induces at once a plaintive, mournful feeling in accord with the lack of effort required in

MIXOLYDIAN HARMONIA



playing or singing the notes, while the second tetrachord, with its upward straining to produce ever wider intervals up to the high Mese perched on the 7th degree, when the singer or Aulete only reaches the octave after the further tension of a septimal tone $\left(\frac{8}{7}\right)$, satisfies the claims of both quotations. In using the Mixolydian Harmonia, one may sympathize with those whose powers have failed, and who have to keep the voice at a high *tessitura.*

[3]

In the descending modal scale or in a melodic use of the Mixolydian modal sequence, such as the above, the grief deepens into an atmosphere of gloom and depression which becomes well-nigh intolerable as the melos sinks by slow steps from the Mese through the second tetrachord (of the Mixolydian) back to the Tonic on Hypate Hypaton. Such a melodic

[1] *Politics,* v (viii), 5, p. 1340, *a,* 38. [2] *Rep.*, iii, 398E.

[3] Superscript accidentals are given, but they may be misleading; the notes must be heard tuned true to ratio. The Mixolydian is taken here as a species of the Dorian Harmonia on $C = 128$.

phrase enables us to realize how appropriate was the abstention from the use of the Hypaton tetrachord in the Dorian melodies (ἐν τοῖς Δωρίοις) in order to preserve the Ethos unspoiled, as recorded by Plutarch.[1] The Syntonic Lydian included by Plato will need a little consideration. To what form of the Lydian Harmonia does he refer? We may here recall the change brought about in the Tonic of the Lydian Harmonia, used as species in the P.I.S., through the extension of the Dorian Tonos from Hypate Meson down to Hypate Hypaton in the Mixolydian Species. That tetrachord bore the ratios 28, 27, 24, 22 or 21, thus obliterating the Lydian Tonic of the Lydian Harmonia, of ratio 26, which in the P.I.S. henceforth appeared only as Oxypyknon Chromatic. The formula of this Mixolydian first tetrachord is ascribed, by Ptolemy, to Archytas, but with the two tones, $\frac{9}{8} \times \frac{8}{7}$ reversed thus: $\frac{8}{7} \times \frac{9}{8}$. If the first tetrachord of the Lydian Species in the P.I.S. is compared with that of the original Lydian Harmonia of Determinant 13 or 26, it will at once be apparent that, with both forms at the same high *tessitura*, due to the Mese on the 6th degree of the scale, it is to the Lydian Species of the P.I.S. that the epithet Syntonos belongs.

From the two tetrachords given below, with the values of intervals in ratios and *cents*, it must be evident that a greater tension of the glottis action is required to produce the species in the P.I.S. than the first tetrachord in the Harmonia. With the latter sequence, the Aulete passes

COMPARISON OF THE FIRST TETRACHORD OF THE LYDIAN SPECIES IN THE P.I.S. WITH THAT OF THE ORIGINAL LYDIAN HARMONIA

Lydian Species Kata Dynamin	Degrees of the P.I.S.	PH. Hyp.		Lich. H.		Hyp. Mes.		PH. Mes.	Tetrachord
	Ratios	27		24		21		20	$\frac{27}{20}$
	Cents		*204*		*231*		*84·4*		= *520 cents*
Lydian Harm. Kata Thesin		Hyp. Mes.		PH. Mes.		Lich.		Mese	Tetrachord
	Ratios	26		24		22		20	$\frac{13}{10}$
	Cents		*138·5*		*150·5*		*165*		= *455 cents*

imperceptibly through the first two intervals and with a very slight effort only to the third interval. In the species, the tetrachord is seen to be greater than a perfect 4th by a comma, whereas in the Harmonia the tetrachord $\frac{13}{10}$ = *455 cents*, that is, a 4th diminished by *43 cents.*

In the light, therefore, of the judgement of Plato and of Aristotle, it seems that it is the Lydian Species of the P.I.S. that is meant by Syntono-Lydisti.

[1] *de Mus.*, Cap. 19 (ed. Weil and Rein., § 183).

CHAPTER IV

THE HARMONIA AS MODAL BASIS OF THE PERFECT IMMUTABLE SYSTEM OF THE TONOI AND OF THEIR NOTATION

Introductory—Evolution of the Greater Complete System from the Species of the Seven Harmoniai. The Four Periods in the Development of the Greek Musical System. The Nomenclature of the Degrees of the Scales: The Onomasiai Kata Thesin and Kata Dynamin. Modal Implications of the Historical Development of the Kithara. The Practical Basis of the Species. The Order in which the Species occur is the Reverse of that of the Harmoniai and of the Tonoi. Omega, the Common Tonic of the Harmoniai. Birth of the Tonos. Modal Significance of the Tetrachord Synemmenon. The Modal Basis of the P.I.S. confirmed by Ptolemy's Formulae. Modal Origin of Ptolemy's Syntonic Chromatic. The Seven Harmoniai restored by Ptolemy through the Mechanism of the Tonoi. Four Stages in the Development of the Tonos marked by a change of Starting-note. Stage I: Hypate Meson as Starting-note and Modal Pivot. Stage II: Hypate Hypaton as Starting-note now becomes the Modal Pivot. Stage III: The Modal Pivot changes to Proslambanomenos as Starting-note. Stage IV: Fundamental Modal Change in the P.I.S. from Dorian to Phrygian. The Tonoi as Curtailed Modes.

INTRODUCTORY: EVOLUTION OF THE GREATER COMPLETE SYSTEM FROM THE SPECIES OF THE SEVEN HARMONIAI

THE main purpose to be achieved in this chapter is to present the Harmonia, the Species and the Tonos as different forms and uses of the Mode, as understood in this work, the identity of each being unmistakably manifested through its series of ratio numbers. In proof of this statement the author relies upon the agreement of the *a priori* demonstration of the basis and principles of the Modal Genesis given in Chapter i, with evidence from the Theorists presented *passim* and in greater detail in Chapter v.

From the Harmonia as a modal octave, the next step in the development of the Modal System was the formation of the Greater Complete System (G.C.S.) by the gradual grouping of Modal Species round the nucleus formed by the Dorian Harmonia. This grouping was brought about as a natural consequence of the arithmetical progression of the ratios, which, as seen in the first chapter, result from the division of the string or pipe through equal measure by the Determinant number of the Mode.

Guided by the special appeal of one or other of the Modes—bearing tribal distinctions (such as Dorian, Phrygian, &c.) the origin of which remains a matter of conjecture—the Greater Complete System evolved out of the seven Harmoniai used as species, not as a theorists' scale, but in response to the modal urge felt by the practical musician.

The system of the Tonoi will be shown to consist of an ingenious use made of the Harmonia for the primary purpose of presenting the Dorian Harmonia in its extended form (Dis-diapason), transposed as species into each of the other six Modes in turn, used as keys. In order to carry out this operation systematically, the unused ratios of each Mode not required for the Dorian or standard scale in the three genera have been omitted by the Greeks, so that the Tonos may aptly be described as a curtailed Mode. Since the seven Modes are taken within a common octave, and are generated from the common fundamental, interpreted as *F*, the scale formed from their Mesai or Proslambanomenoi is that of the Harmonic Series on *F* from 8 to 16, 15 being considered as an alternative to 14.[1]

The scheme of the *Tonoi*[2] or *Tropoi* presents in addition each Harmonia as an octave species partaking of the Ethos and tonality of each of the other Modes through the genesis of the modal material from the Arche (ἀρχή) or keynote of the Mode. Each Mode is, therefore, represented in the scheme as a true Harmonia from *F* to *F* in its homonym Tonos, and in addition as a modal species in seven different tonalities or keys in the seven original Tonoi.

The Greater Complete System (= G.C.S., σύστημα τέλειον μεῖζον) and the inception of the idea of the Perfect Immutable System (= P.I.S., σύστημα τέλειον ἀμετάβολον) are manifestly related to the development of the Modes as octave species on the Kithara, the number of whose strings was gradually increased from 8 to 15.[3] Moreover, the Perfect Immutable System, given above, as revealed by the notation in the tables of Alypius, differs in certain particulars from the original modal scheme. The significance of the modifications brought about by time is of some importance, and will be dealt with briefly in this chapter and more fully hereafter.

Any attempt to throw light upon the systems of Greek Music known as the Greater Complete, the Lesser Complete and the Perfect Immutable Systems, must deal with the subject from at least four points of view, which, however, it would not be possible to consider separately in succession without the disadvantage of frequent repetition. These four aspects of the systems are the following:

(*a*) Historical; (*b*) structural; (*c*) theoretical; (*d*) practical.

The systems are thereby exhibited in the light of:

(*a*) the Modes; (*b*) the Species; (*c*) the Tonoi; (*d*) the Genera; (*e*) in relation to 'the twenty-eight sounds' of the Greek Scale.[4]

[1] An explanation of the alternative use of 15 and 14 is given further on.

[2] The Tonoi are termed Tropoi in the Tables of Alypius.

[3] The practice known as magadizing (from μαγάς = bridge of the Kithara and μάγαδις), i.e. of obtaining the octave overtone of the string by a light touch on the half-length at the Node, may have preceded the increase of strings to 15. This practice does not prejudice the quality of tone, as is the case with the nail technique (see further on). Magadizing may be applied to all the strings of the Kithara with ease and success. (See Ps-Aristotle, Probl. xix, 39*b*, ed. Gev., pp. 20–1.)

[4] *Excerpta ex Nicom.*, ii, pp. 36, 38–40M., *Eucl.*, *Int. Harm.*, pp. 5–6M., *Arist. Quint.*, pp. 9, 10M.

THE FOUR PERIODS IN THE DEVELOPMENT OF THE GREEK MUSICAL SYSTEM

We shall in addition classify the four periods of musical development characterized by the reign of the above-mentioned systems from the standpoint of Modality as follows :

(i) The earliest Modal Period of the G.C.S. (pre-Aristoxenian) during which Hypate Meson, as the first note of the Harmonia, was regarded as the true beginning of the scale, and Mese was on the *fourth* degree. This was the period of the pure Dorian Harmonia.

(ii) The second period, during which Hypate Hypaton was regarded as the beginning of the scale, and Mese was on the *seventh* degree. This was the period of the Mixolydian or Hyperdorian Species.[1]

(iii) The third Period or Ptolemaic, during which Proslambanomenos was regarded as the beginning of the scale, and on it, as on Mese, was based a Tone of Disjunction. Mese was now on the *eighth* degree, a change which marks the passing of the true Modal System of the Harmonia. This period was characterized by a bastard species falsely denominated Hypodorian by the Graeco-Roman theorists.

(iv) The fourth period developed in the Christian East through the Greek Church during the early centuries of our era. Proslambanomenos is still the beginning of the scale ; but instead of standing outside the tetrachords, as stated by Aristides Quintilianus (M., p. 10), Proslambanomenos is now seen to form part of the series of tetrachords itself ; for the first of the conjunct tetrachords—formerly beginning on Hypate Hypaton—is now based upon Proslambanomenos. A momentous change has come over the P.I.S. : its whole scheme has been changed from Dorian to Phrygian.[2] The sources of our knowledge of the post-Aristoxenian P.I.S. are unimpeachable ; more especially is this true of the fragment of the work of Alypius, which originally contained complete tables of this system in the three Genera and in all the fifteen Tonoi (*a*) in Vocal and Instrumental Symbols of Notation, the basis of which is pitch value determined by modal ratios ; (*b*) in the Thetic Nomenclature (*ὀνομασία κατὰ θέσιν*) founded upon the degrees of the Dorian Modal Scale of two octaves upon the Kithara, which is precise as to functions, but entirely indeterminate as to pitch, relative or absolute. It is precisely this ambiguity as to pitch

[1] See Lamprocles, who foreshadowed the change ; and Ptolemy's transposition scales (Plut., *de Mus.*, Cap. 16E. (Weil and Rein.), p. 64, § 156) ; Ptol., *op. cit.* (Wallis, 1682), pp. 177 sqq.

[2] See 'The Origin of the Ecclesiastical Modes', Appendix ii, and also 'The Four Stages'. It is a highly significant fact that the system which forms the subject matter of this chapter consists of a synthesis of all the chief elements of the Music of Ancient Greece, viz. the Harmonia (*ἁρμονία*) and its genesis or *καταπύκνωσις* ; the Synaphe (*συναφή*) or conjunction ; the Tonoi or Keys ; the three Genera ; the species (*εἴδη*, *σχήματα*, or Modes), in practical music ; the Dynamic Mese ; the system of Notation ; and finally the Aristoxenian System which was superimposed upon the framework devised for the Ancient Modes. Since the elements themselves, moreover, constitute stages in development it would seem that a careful investigation of the evolution of the P.I.S. would almost resolve itself into a study of the history of Greek Music.

values in the notation by degrees which made it possible to read into the system devised for the Modes and Tonoi (τόνοι), the non-modal Aristoxenian system; for a knowledge of the Modal System is required before the scheme of Notation can be correctly interpreted.[1] As the Aristoxenian is the only system of which the Greek theorists give any satisfactory explanation, and as all definite traces of the ancient Modal System had vanished, modern writers on Greek Music have never suspected the substitution.

The Perfect Immutable System of the ancient Greeks represents the most advanced stage in the development of the Modal System; its subtle, compact, but somewhat complicated scheme betrays the hand of the theorist. There is no part of the Greek musical system which has been described with such minuteness; no loophole for errors seems to have been left: the names of the tetrachords, of the degrees of the scale, the magnitude of the intervals from degree to degree have all been carefully recorded,[2] and in the *Sectio Canonis* of Euclid the method of dividing the string in order to obtain each of the notes, is explained in Theorems xix and xx, illustrated by diagrams. In all these descriptions there is no hint given of the real nature of the Modal System.

The double-octave system was first known as the Greater Complete, and later when the Lesser Complete or Conjunct System (σύστημα τέλειον ἔλαττον) was amalgamated with it—whereby the Synemmenon was added to the G.C.S. as an alternative tetrachord to the Diezeugmenon—the double-octave system in this final form received from the theorists the name of Perfect Immutable. This was a recognition of the fact that it was to be no longer merely a standard Kithara scale, primarily modal, for use in a single tonality and modality like the G.C.S., but to form part of a system of seven or more Tonoi, into each of which the Dorian Harmonia was transposed. Whereas the G.C.S. represents the natural development of the Harmonia upon the Kithara, obtained as a consequence of the grouping of the six other Modal Species of the octave above and below the Dorian Harmonia, in obedience to a merely modal necessity, the P.I.S. was something more than this: it was a system of all the Modes, present as Harmoniai, as Species and as Tonoi or Keys, as we shall proceed to demonstrate. At the inception of the scheme each Tonos had the structure of the Dorian Harmonia extended to two octaves.

THE TWO NOMENCLATURES OF THE DEGREES OF THE SCALE: THE ONOMASIAI KATA THESIN AND KATA DYNAMIN

The nomenclature of the degrees of the scale by position, the *Onomasia Kata Thesin*, reveals its own origin, however, when the true nature of the Harmoniai is known, and bears witness to the evolution of the Greek Musical System.

[1] Cf. Polemics of Aristoxenus on the claim of the Harmonists *re* Notation. (See App. No. 1 and Chap. v.)

[2] *Eucl.*, *Int. Harm.*, pp. 3–6M. and pp. 14–16M. through species of 4ths and 5ths; Ptol., *Harm.*, *passim*; Nicom., *Harm.*, pp. 21–3M.; Gaud., *Harm. Int.*, pp. 15–18M. by ratios; pp. 18–20M. through species of 4ths and 5ths; Bacchius, *Intr.*, pp. 18, 19M. through species. Eucl., *Sect. Can.*, pp. 37–40M.

It was shown in Chapter i that the position of Mese on a certain degree of the scale, which must always bear an octave ratio to the Arche (ἀρχή), viz. 8, 16, or 32, is an inherent and inevitable index of the Harmonia or Mode, revealing its identity beyond all possible doubt. The Mese on the 4th degree of the scale, therefore, betrays the Dorian origin of the nomenclature, since the division by 11 or 22 is the only one which places an octave of Arche on the 4th degree. Moreover, the fact that this nomenclature—identical in the Dorian Harmonia alone with that of the *Onomasia Kata Dynamin* (ὀνομασία κατὰ δύναμιν)—was in general use, and was in fact the only one handed down to posterity until Ptolemy's day, certainly points to the domination of the Dorian Harmonia,[1] which at some time, yet to be determined, had come to be regarded as the standard scale; and therefore, no longer required any specific tribal appellation.

FIG. 30.—Birth of the Seven Modal Species of the G.C.S. from the Kithara of 15 strings (without retuning).

The Dorian Harmonia of M.D.22 forms the Nucleus

	HYPATON				MESON				DIEZEUGMENON				HYPER-BOLAION		
The Strings of the Kithara	Hypate	Parhypate	Lichanos	Hypate	Parhypate	Lichanos	MESE	Trite Syn.	Paramese	Trite	Paranete	Nete	Trite	Paranete	Nete
Nos.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hypodorian Species							$\boxed{16}$	15		13	12	11	10	9	8
Hypophrygian						**18** T	$\boxed{16}$	15		13	12	11	10	9	
Hypolydian					**20** T	18	$\boxed{16}$	15	14	13	12	11	10		
Dorian				22 T	20	18	$\boxed{16}$		14	13	12	11			
Phrygian			**24** T	22	20	18	$\boxed{16}$		14	13	12				
Lydian		**26** T	24	22	20	18	$\boxed{16}$	15	(14)	13					
Mixolydian	**28** T	26	24	22	20	18	$\boxed{16}$		14						
	C	*D*♭	*E*♭	*F*	*G*♭	*A*♭	*B*♭	*B*♮	*C*	*D*♭	*E*♭	*F*	*G*♭	*A*♭	*B*♭

N.B.—T = Tonic; the Mesai are enclosed in squares. The Tonics bear the Determinant number of the Harmonia. The Hypolydian has two forms: (A) passing from Mese through Trite Synemmenon with ratio 15 on the perfect 4th, (B) passing from Mese through Paramese with ratio 14 on the Tritone. The Nomenclature of the degrees from Hypate Meson to Nete Diezeugmenon of the Dorian Harmonia is the origin of the Onomasia Kata Thesin.

This view of the modal origin of the two nomenclatures of Ptolemy negatives once for all the possibility of the Thetic nomenclature having

[1] See D. B. Monro, *op. cit.*, p. 42.

preceded the Dynamic in time: the *Onomasia Kata Thesin* could have had no existence independent of the *Dynamis* of the Dorian [1] Mode.

Any attempt to reconstruct the G.C.S. from the evolutionary point of view reveals the two great music-making principles at work: the *Modal* and the *Tonal*. The Modal System evolving upon the Aulos (see Chapters i and ii) was in essence an octave system; for although many early reed-blown pipes were incontestably bored with three holes, the four notes which they are capable of producing do not necessarily constitute a tetrachordal unit,[2] but only a range restricted to four notes owing to technical considerations. If it be accepted that the Dorian Harmonia or disjunct octave, in essence octachordal, was the nucleus of the G.C.S. and that the Kithara nomenclature was both Dynamic and Thetic, then there still remains the addition at the acute end of this disjunct octave, of the conjunct tetrachord Hyperbolaion, and at the grave end of the tetrachord Hypaton to be accounted for. Can such an addition at the grave end be made without interference with the position of the Dynamic Mese (ἀρχή) which seems to have been thereby shifted from the 4th to the 7th degree, involving a change of species from Dorian to Mixolydian? The answer is, of course, in the affirmative, conditionally on Hypate Meson retaining the status of first note or beginning of the scale.[3]

MODAL IMPLICATIONS OF THE HISTORICAL DEVELOPMENT OF THE KITHARA

Certain indications of the most probable steps in the transition from Harmonia to G.C.S. are to be found in the historical development of the Kithara. The statements which attribute every additional string to some definite musician, while not constituting authentic evidence, yet emphasize a musical necessity of some kind. Let us consider what this musical necessity may have been with a Kithara of eight strings [4] tuned to the Dorian standard Harmonia as starting-point. A ninth string was added, some say by Phrynis of Mitylene, a victor in the Panathenaia in 456 B.C., a 10th string by Histaeus of Colophon, an 11th by Timotheus of Miletus, a 12th by Melanippedes, a dithyrambist and musician, who lived at the court of Perdiccas, King of Macedonia (454–413 B.C.).

It is known that Ion of Chios, who won a dramatic victory at Athens in 452 B.C., used a ten-stringed Kithara on which he played in three Har-

[1] For an example of the two nomenclatures in practical use, see 'The Harmonic Canon of Florence', Chap. v.

[2] For an example of the tetrachordal unit produced by the Aulos, together with an explanation of the *modus operandi* whereby our modern major scale came to birth, see Chaps. ii and iii.

[3] See Ptol., *Harm.*, ii, xi, where Hypate Meson is regarded as the common Tonic or initial note of all the species *Kata Thesin* while the Mese, *Kata Dynamin*, of each species is given by Ptolemy the position of the real keynote, a specified number of degrees above the Tonic or Hypate Meson.

[4] Plut., *de Mus.*, Cap. 30–1 and p. 1142. If the edition by Weil and Rein. is used, the critical apparatus should be very carefully examined with the text, as debatable alterations and deductions have been made, with which I am not in agreement. See also Nicom., *Intr. Harm.*, p. 35M. and Boethius, *de Mus.*, 1, 20.

moniai (and, therefore, through the species). These data indicate the fifth century B.C. as the period of rapid modal development through the species upon the Kithara. When the tribal Modes made their appearance in Greece, it is probable that from the first, different instruments were in use for the principal Modes; that, in fact, there were Dorian, Phrygian and Lydian Kitharai, just as at first there was a separate Aulos for each Mode.[1] The 9th string might have been added either above Nete (for Trite Hyperbolaion) or below Hypate for Hyperhypate,[2] if the necessity was a purely melodic one, affecting the *tessitura* only. If we suppose the 9th string (*E*♭) to have been added below Hypate Meson, a note afterwards known as Lichanos Hypaton or Hyperhypate, the addition may have been made in response (1) to a melodic, or (2) to a modal necessity. In the first case, Hypate Meson would remain the initial note or Tonic of the scale and Mese would retain her dynamic power, due to the division by Determinant 22. In the second instance, it is plain from the ratios that there will now be a possibility of obtaining on the Kithara, *without retuning*, a Phrygian Harmonia beginning on the Hyperhypate, *E*♭ of ratio 24, and with the sequence of ratios that belongs to the diatonic scale of the division by 24. But Hypate will no longer be a fundamental or Tonic common to the two Harmoniai: the Phrygian has forfeited its status as a Mode on *F* to become a species of the Dorian with *E*♭ for its Tonic. The necessity here was to play in the Phrygian Mode on a Dorian Kithara. By the simple expedient of adding a string to the Kithara tuned to one special Mode, therefore, the difference between a Mode and a species is made apparent. In order to obtain a Phrygian Harmonia on *F* on the Dorian Kithara of eight strings, every one of the strings would need retuning except Hypate, which remains the common Tonic. It sounds plausible enough to retune the Kithara for a change of Mode, but in practice it is a lengthy process, for the strings do not take kindly to changes in tuning. The dynamic Mese of the Phrygian Harmonia on the 5th degree would fall to the 5th string, the Paramese, of the eight-stringed Kithara, so that the Dorian and Phrygian Mesai would differ in pitch. A nine-stringed Kithara is therefore postulated: in terms of the Dynamic nomenclature, it is found that the Dynamic Mese of the Phrygian species on the 5th degree, is identical in pitch with the Dynamic Mese on the 4th degree of the Dorian Kithara. On a nine-stringed Kithara the 4th string has thus become the Thetic Mese of the Phrygian species. It seems clear that the *Onomasia Kata Thesin* is a device engendered by an attempt to use the modal nomenclature designed specially for one Mode for a different one. The use of the species in practice had many advantages: in particular, the Mese string remains the same for all.

THE PRACTICAL BASIS OF THE SPECIES

The principle of the modal division into equal parts of string or column of air is so logical in itself, once the underlying harmonic law is realized, that no effort is required to grasp the practical basis of the species. Given

[1] Athen., xiv, p. 631E.; Paus., ix, Cap. 15. [2] Boethius, *loc. cit.*

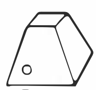
an *F* string divided into 22 parts, another string, longer than the first by two of these equal parts, will produce a sequence of ratios identical with that of a division by Determinant 24, but starting on an *E* instead of on an *F*. There will, therefore, be a change of Modal Species accompanied by a subtle change of tonality; for although the 22 division on an *F* string and the 24 division on an *E* string both have as Arche *B*, producing according to modern ideas the key of *B* minor in both species, the relation between the *Mesai* and the Tonics is now entirely changed, being respectively for 22 a sharp Harmonic 4th, and for 24 a Perfect 5th. It will be recognized that tonality, as understood by the ancient Greeks and by musicians of the twentieth century A.D. is a very different power: in antiquity the power or function of the Mese was of the very essence of the Modes, causative and qualitative; the Mese was responsible for Ethos (ἦθος) and *tessitura*, both of which are conditioned directly by the position of Mese in the Harmonic Series of the fundamental, or common string-note of the Modal System, as a result of the division of the string by the Modal Determinant. In Fig. 30 the hypothetical development of the species upon the Kithara is shown; it will be noticed that the hypothesis agrees with accepted data. The distribution of the additional strings up to fifteen, some above and some below the original eight,[1] is both rational, and the only course which could have any practical success. The tuning of fifteen strings of equal length to a diatonic scale of a range of two octaves throws all the onus of differentiation upon diameter and tension, both of which factors need to be balanced to a nicety with the common factor of length if the quality of tone is not to suffer thereby. It may be seen from a reference to Ptolemy (i, 11) that the Ancients were well aware of this problem without, however, possessing a formula for its solution which was reached empirically.

This arrangement gives a result in accordance with the established and authentic data of the accomplished fact. Attention must also be drawn to the fact that in order to be able to use species of the group of Modes bearing the prefix *Hypo*, it will be necessary to adopt one of the three following expedients:

(*a*) to add a string for Trite Synemmenon (ratio 15);

(*b*) to retune Paramese as Trite Synemmenon for those Species which require 15 instead of 14;

(*c*) to use some such device as the block[2] described and illustrated for modifying the intonation temporarily without retuning, in which case the string is normally tuned for 15 and shortened by the insertion of the little block for 14.



Block

[1] Adrastus, a Peripatetic Philosopher of the second century A.D., quoted by Theo of Smyrna (ed. J. Dupuis, p. 86) confirms this distribution of the additional strings, above and below the original eight as still practised in his day.

[2] This device, in the shape of a truncated wooden pyramid, is used in order to raise temporarily the tension of a string on the Kithara. The blocks vary in height

It is evident that in the Modal Species the three musically important features are:

(1) The graded relationship between the common Mese and each individual Tonic of the species, which bears a numerical ratio identical with that of the Modal Homonym: that is to say, that all the species have a Dynamic Mese of the same absolute pitch, but placed upon a different degree in each scale and consequently bearing a different relationship to each individual Tonic or Thetic Hypate.

(2) That the Tonic, instead of being the same for the seven sequences, as in the Modal Series, is the element that changes in each species; and therefore, it is from the Tonic that the species takes its name; whereas in the Mode it is the Mese which is the characteristic element of differentiation from which the Modes are named. This is made evident by the ratios.

(3) That in the modal octave all the degrees except the first and last have a sevenfold differentiation due to the fact that although all begin on the same note, the Tonic has each time a different value as characteristic modal denominator in the aliquot division which determines the ratios of the whole sequence.

At this juncture the reader may be reminded that Aristides Quintilianus refers to this very point as follows: 'Accordingly, it is clear that if one takes the same sign as starting note, and calls it at different times by the differing value of the note, the nature of the *Harmoniai* is made manifest from the succession of consecutive sounds.' [1]

The latter part of the same quotation applies equally to the Tonic of the species, with the difference already mentioned above that the Tonic varies in pitch as well as in ratio number for each species.

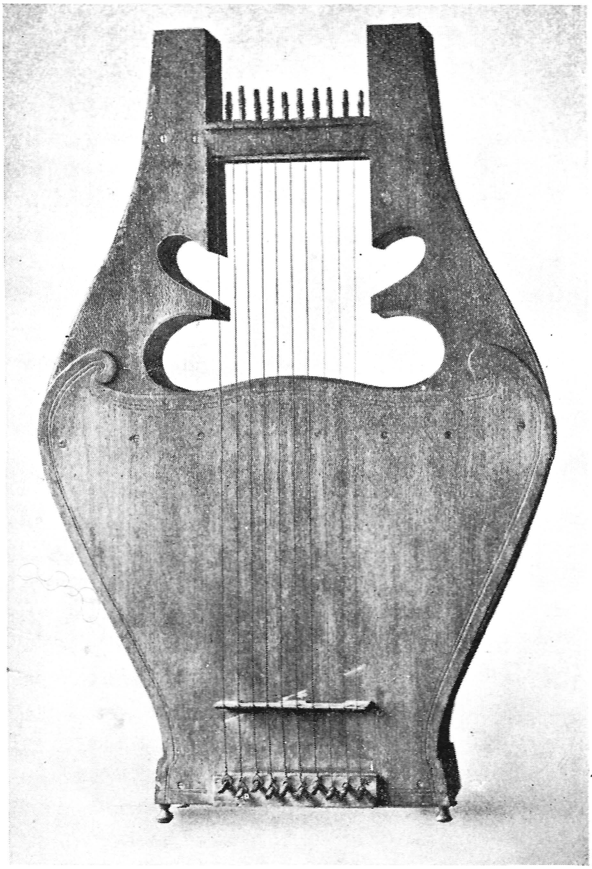
One might say, therefore, that in the system by species, there are seven Tonics and one Mese, but that all the other degrees are of the same absolute pitch as the notes bearing the same ratio number in any other species of the same Modal Scale of which it is a species; whereas in the system of the Modes, there is one common Tonic and seven varieties of each degree, differentiated by their ratios and, therefore, by their exact pitch; and the seven Mesai fall upon different degrees of the scale.

A glance at the table of the species (Fig. 30) makes it clear that by keeping the Dorian Harmonia in the centre, the Phrygian and Lydian and Mixolydian lie below it, and the Hypolydian, Hypophrygian and Hypodorian above it, in the exact order given by Ps.-Euclid [2] from Hypate Hypaton to Nete Hyperbolaion.

according to the ratio of the note to be sharpened, the number of which may be printed on the front of the block, for these little stops cannot be used indiscriminately under the strings. A small hole is bored with a red-hot knitting-needle through the base of the pyramid diagonally, and out at one of the sides, to hold a string by which it may be suspended from the tuning peg. Each block is filed down to the height required to produce the note, when the stop is slipped under the string. The blocks are mainly required to change ratio 15 to 14; ratios 27 to 26, and 22 to 21, in order to play the species of the later P.I.S. in the Tonoi.

[1] *de Mus.*, p. 18M., line 7. [2] *Intr. Harm.*, pp. 15, 16M.

PLATE 15



THE KITHARA, CIR. 500 B.C.
Facsimile made by H. Kent in 1915 from a vase painting (red on black)
British Museum

THE ORDER IN WHICH THE SPECIES OCCUR IS THE REVERSE OF THAT OF THE HARMONIAI AND OF THE TONOI

It is obvious, therefore, that within the P.I.S., this order of the species based upon different Tonics must be the reverse of that of the Harmoniai based upon the different Mesai or Archai. In fact, the Archai being chosen from the Harmonic Series, follow the arithmetical progression from grave to acute in their ratios, while the Tonics of the sequence of species, bearing the ratios of lengths of string or columns of air, progress in arithmetic succession from the opposite direction according to the natural law, viz. grave to acute (see Fig. 31). To resume briefly: the G.C.S. must be regarded primarily as the vehicle of the Octave Species of the Dorian Harmonia, with which, as nucleus, is formed in the aggregate a scale of two octaves, acknowledged at the outset as Dorian in modality.

The inevitable adoption of the species upon the Kithara as the result of increasing the number of strings beyond eight, might have proved a blow to the dominance of the Dorian Harmonia but for the equally inevitable discovery of the twin birth of the Tonos with the species, which may have come about as follows:

At the Pythian Games which attracted musicians from all parts of Greece and of the known world, three Kitharistai from Athens, Lydia and Phrygia are comparing notes about the tunings of their instruments, all of which have ten strings, thus allowing each Kitharist to modulate into two Modal Species. The Athenian's Kithara (or *κίθαρις*) tuned to the Dorian Harmonia admitted of modulations into Phrygian and Lydian species; the Lydian Kithara into Dorian and Phrygian, the Phrygian Kithara into Lydian and Dorian. The three Kitharai were tuned according to custom to begin the tribal Harmonia, let us say, on *F*. The Phrygian finds that his own Harmonia can be played upon the Dorian and Lydian Kitharai, but at a pitch different from his own, lower upon the Dorian and beginning on *E*♭ flattened, higher upon the Lydian (in *G*). The Athenian can play his Harmonia at a higher pitch (on *G*) upon the Phrygian instrument and still higher upon the Lydian (on *A*). The Lydian recognizes his Harmonia on both of the other Kitharai but at pitches lower than his own, on the Dorian on *D*, on the Phrygian on *E*.

Moreover, although the *F* string has the same pitch on the three Kitharai, the next note lower (*E*) and higher (*G*) will bear a different intonation on each Kithara.

OMEGA THE COMMON TONIC OF THE HARMONIAI

A slight digression may be made here in order to discuss the significance of the point that the three Kitharas were tuned, according to custom, to begin the Harmonia on Tonic *F*. Why on *F*? And how do we know that this was a common practice among the Greeks?

It may be definitely asserted, on the authority of Alypius, that the seven original Harmoniai used as Modes had a common Tonic, indicated for each in the Homonym Tonos, by the Symbol of Notation Ω (omega)

FIG. 31.—Concerning the Confusion between the Harmoniai, their Species and the Tonoi, which arose from the fact that the Order of the Species in the P.I.S. was the Reverse of that of the Modes in the Tonoi

The Order of the Species in the Dorian Harmonia of the P.I.S. ; named from their Tonics each of which bears the number of the M.D. of its Homonym Harmonia

	Proslamb.	Hypate (Grave-Hypaton)	Parhypate (Grave-Hypaton)	Lichanos (Grave-Hypaton)	Hypate (Meson)	Parhypate (Meson)	Lichanos (Meson)	MESE	(Tr. Syn. Paramese &c.)	
The M.D. of the Tonics of the Species	32	28	26	24	22	20	18	16	(15)	(14)
	Octave of Hypodorian Species which begins on MESE	Tonic	Tonic	Tonic	Tonic	Tonic	Tonic	Tonic		
		Mixolydian	Lydian	Phrygian	Dorian	Hypolydian	Hypophrygian	Hypodorian	(Mixolydian &c.)	
	16	14	13	12	11	10	9	8		

The Mesai Kata Dynamin of the Harmoniai in the Tonoi

The Order of the Harmoniai, named from the Proslambanomenoi of the Tonoi which are derived through their Mesai from the Harmonic Series

8 16 32	9 18	10 20	11 22	12 24	13 26	14 28	16 32

The Order of the Mesai (Archai) in the Harmonic Series

Prosl. and Mese	"	"	"	"	"	"	"
	"	"	"	"	"	"	"
Tonos	Tonos	Tonos	Tonos	Tonos	Tonos	Tonos	Tonos
Hypodorian	Hypophrygian	Hypolydian	Dorian	Phrygian	Lydian	Mixolydian	Hypodorian Hypophrygian
8	9	10	11	12	13	14	

N.B.—The confusion between Species, Harmoniai and Tonoi reigned throughout the Middle Ages, owing to a misunderstanding of the Treatise *De Musica* by Boethius: the confusion, further emphasized by Glareanus, persists to the present day in the equivalent Greek appellations given to the Ecclesiastical Modes (q.v. Appendix Tab.), e.g. the Protus or *D* Mode being misnamed Dorian, the *E* Mode Phrygian, following the order of the Tonoi while applying the modal names to the species as they occur on the white notes of the keyboard, thus reversing the order.

which appears in each Tonos on the degree of the P.I.S. proper to it as species. The Lydian group in the Tables of Alypius is the exception. The distinctive symbol of the Lydian and Hypolydian Harmoniai according to Alypius is R, two dieses lower than the common Tonic, Ω, a fact which is in agreement with our discovery that the M.D. of the Lydian Harmonia had at some time been lowered in the P.I.S. from 26 to 27 on Parhypate Hypaton, as signalized by the ratios of the tetrachords ascribed to Archytas. The Hypolydian shared the same fate in the Alypian tables, although no change in the M.D. of the Hypolydian Harmonia has been observed in theory or practice. The purpose may have been the preservation of the relation of a 4th between the two Tonoi. (See Appendix No. i, on Notation.)

Competent modern authorities, such as K. von Jan, have adopted *F* as the pitch corresponding with the Ω of Greek vocal notation : the average male vocal compass of the Ancient Greeks may well have ranged through two octaves or more from *F*. Dr. Hugo Riemann is the only authority of note who rejects this interpretation in favour of *E*.

It may be recalled in this connexion that the standard pipe known as Lu of Hoang Chung, on which the canons of Ancient Chinese [1] music were based had as fundamental note an *F* of 352 v.p.s., if we may judge from the dimensions of the pipe transmitted by Ancient Chinese authors. An *F* of that vibration frequency occurs as 11th Harmonic on a fundamental *C* of 32 v.p.s. It may be added that it has been my frequent experience during research in the music of primitive folk to find their musical instruments tuned to an *F* of 352 v.p.s. and to *C* of 256 v.p.s., also to find notes of those frequencies prominent in their songs.

This hypothetical episode is offered as a plausible suggestion of the expedient which led to the discovery of the Tonos as transposition scale, whereby the point arises : was the musical urge at the back of this development of the Kithara modal or tonal or both ? The urge was a desire of the musicians for greater modal facilities, which incidentally brought in their train the idea of transposition, afterwards utilized by the theorists in the P.I.S. ; but it was probably not until the Kitharists came together, each Kithara tuned to begin its own tribal Harmonia on *F*, and began to compare notes, that the twin development of the species, viz. the Tonos, was discovered. The strings of the Kithara might be increased to fifteen for modal purposes, and yet the existence of the Tonos, or of actual transposition, might never be realized except for the sense of absolute pitch, and for the exact musical memory which is implied in the faculty. But let there be a gathering together of seven Kitharai, each tuned to a double octave of a different Mode from the same fundamental note, and it will be a marvel if the Athenian Kitharist does not discover that the Dorian Harmonia can now be played at seven different points of pitch and that, moreover, all the other Harmoniai benefit by differentiation to the same

[1] For an account of the pipe see *The Shoo-King or the Historical Classic, being the most ancient authentic Record of the Annals of the Chinese Empire*, trans. by W. H. Medhurst, Senior, p. 20.

extent. Many other interesting facts are brought into relief by this experience: for instance, that any Harmonia when taken in a Tonos, has as keynote the Mese of that Tonos and its individual Tonic may be distinguished by its Determinant number: 24 for Phrygian, 32 for Hypodorian and so on. Each Tonos or transposed scale is a species not of the one same scale or Harmonia, but of a different one each time.[1]

BIRTH OF THE TONOS

As soon, therefore, as the Kitharists became aware that the Modal Species obtained on the three Kitharai were identical as to sequence of intervals, but different in pitch on each Kithara, the *Tonos* was born as the twin sister of *Species*. There would not, nevertheless, in spite of this tonal richness, be any scheme analogous to that of Alypius, unless the same Modal Species were to be taken in turn upon each of the tribal Kitharai; for instance, the Dorian species taken upon the Phrygian, the Aeolian (Hypodorian), the Ionian (Hypophrygian), the Mixolydian and the two Lydian Kitharai, and all tuned to *F*. The result of this musical feat would be the Dorian Harmonia played in seven Keys or Tonoi, which have every right to be known by their tribal names, since they were obtained from the fundamental *F* as Modes. And this transposition of the Harmonia was a practical possibility likewise for each of the other Modal Species.

The issue here is far from simple; it involves all that is implied by absolute and relative pitch, as well as an understanding of Harmonia, Tonos and Species and a feeling for Modal Ethos. Thus a Tonos effective on the octave Harmonia alone must have preceded the double octave Tonos of the P.I.S.

We now find that we have traced three orders of scales or systems (συστήματα):

(*a*) The Harmonia or modal scale of one octave.

(*b*) The Species, which as considered here, have each a twofold existence: as Species and as Harmonia of the same name.

(*c*) The Tonos which has a threefold existence: as Tonos, as Species and as Harmonia. As the result of the difference between (*a*) and (*b*), the Theorist, turning his attention to these three orders exhibited on a fifteen-stringed Kithara, sees yet a 4th and a 5th order emerge, viz. (*d*) the Greater Complete System.

(*e*) The P.I.S. or grouping of the Tonoi into a Modal System carrying with it important implications.

We are thus brought face to face with the G.C.S. as an accomplished fact before ever the theorist had become aware of the metabasis, or had had time to analyse the System into a succession of four tetrachords, conjunct in two pairs, divided by the Tone of Disjunction (τόνος διαζευκτικός between Mese and Paramese and considered as the original disjunct octave from

[1] The tables of Alypius as interpreted by Macran and K. von Jan give a false view of the Tonoi, as systems identical in structure and differing only in pitch. See Appendix, 'The Modal Interpretation of Notation'.

Hypate Meson to Nete Diezeugmenon, enlarged by the addition of a tetrachord at each end into a double octave with one note missing. As soon, in fact, as the Kithara had been given fifteen strings for the purpose of playing the seven Harmoniai as species, the G.C.S. was there, unsuspected by the theorists.

It is not exactly known what space of time elapsed between the selective use of the fifteen strings for the purpose of Metabole from one species to another and the use of all fifteen strings in succession from beginning to end as one scale, to which the system undoubtedly owes its name. The G.C.S. remained for a long time an octachordal system in spite of its compass, just as it would at the present day in a song having a compass of two octaves. A scale of this kind is obviously implied by Plato [1] in the well-known passage from the *Republic*, in which Socrates affirms that the music of the ideal state will not need a multitude of strings or every Harmonia (*παναρμόνιον*), i.e. on which the G.C.S. can be played. After specifying the instruments for which there will be no use in the State, he says with regard to the Aulos : ‘ Has not the Aulos (reed-pipe) a great number of notes and are not instruments embracing all Harmoniai (*τὰ παναρμόνια*) simply imitations of the Aulos ? ’

This passage seems to establish as fact the use before Plato's day of a scale comprising all the Modes, and of the existence of instruments upon which such a scale could be played. The hypothetical development of the G.C.S. sketched above accords fully with the practice described by Plato, and the fact that he does not make use of the theorists' more technical G.C.S. does not detract from the value of the historical evidence afforded by the passage in question.

It is obvious from the meaning of the word that *Proslambanomenos*, ‘ the added note ’, was not incorporated into the system until very late—and not into the G.C.S. at all. The need for Proslambanomenos which, as Aristides Quintilianus [2] explains, was added from without (*ἔξωθεν*) and bore no relation to any of the tetrachords, being merely intended to form the octave consonance to Mese, did not arise until the second great period of the development of the modal system had dawned, when Hypate Hypaton was regarded as the beginning of the scale.

The name *Hyperhypate* [3] used by one or two of the theorists for Lichanos Hypaton, on the other hand, was evidently a relic of the days when the Kithara had received its ninth string added below Hypate Meson. It was only much later, after the whole series was ‘ complete ’ or perfectly developed

[1] Plato, *Rep.*, p. 399 *c* and *d*. ‘ *Οὐκ ἄρα, ἦν δ' ἐγώ, πολυχορδίας γε οὐδὲ παναρμονίου ἡμῖν δεήσει ἐν ταῖς ᾠδαῖς τε καὶ μέλεσιν. Οὔ μοι, ἔφη, φαίνεται. Τριγώνων ἄρα καὶ πηκτίδων καὶ πάντων ὀργάνων, ὅσα πολύχορδα καὶ πολυαρμόνια, δημιουργοὺς οὐ θρέψομεν. Οὐ φαινόμεθα. Τί δέ ; αὐλοποιοὺς ἢ αυλητὰς παραδέξῃ τὴν πόλιν ; ἢ οὐ τοῦτο πολυχορδότατον, καὶ αὐτὰ τὰ παναρμόνια αὐλοῦ τυγχάνει ὄντα μίμημα ; Δῆλα δή ἦ δ' ὅς. Λύρα δή σοι, ἦν δ' ἐγώ, καὶ κιθάρα λείπεται κατὰ πόλιν χρήσιμα. καὶ αὖ κατ' ἀγροὺς τοῖς νομεῦσι σύριγξ ἄν τις εἴη.*’

[2] *de Mus.*, p. 10 M., line 7 sqq.

[3] Arist. Quint., *op. cit.*, p. 10 ; *Theo of Smyrna*, ed. Dupuis (Paris, 1892), pp. 145, 147, 151.

to fifteen strings, that the theorist set to work to co-ordinate and explain the sequence as a system, and that a nomenclature for the additional tetrachords was found by the simple expedient of repeating the nomenclature of the original tetrachords with the addition of a qualifying term applied to each tetrachord, such as Hypaton, Meson, &c. The name Hyperhypate then ceasing to have any further significance, fell into disuse.

THE MODAL SIGNIFICANCE OF THE TETRACHORD SYNEMMENON

The Lesser Complete or Conjunct System, known as the *σύστημα τέλειον κατὰ συναφήν* or *ἔλασσον*, is described by Ps-Euclid [1] as extending from Proslambanomenos to Nete Synemmenon, and as consisting of the three conjunct tetrachords Hypaton, Meson, and Synemmenon. Elsewhere Euclid states (p. 17, line 11) that the two conjunct tetrachords must be of the same species (*ὅμοια κατὰ σχῆμα*), and that there are three such conjunctions, Hypaton + Meson; Meson + Synemmenon; Diezeugmenon + Hyperbolaion: a statement which definitely rules out modal sequences, whose octaves never consist of two tetrachords of the same species. Other Graeco-Roman theorists give similar definitions, and the elaborate distinctions made by Bacchius between Synaphe, Episynaphe and Hyposynaphe do not take us much further. The Lesser Complete System is usually regarded as affording facilities for modulating, but Greek scholars have concerned themselves but slightly with its significance, and still less with its origin. One might, indeed, almost doubt whether the Lesser Complete System ever existed elsewhere than retrospectively in the imagination of theorists who have misunderstood the real nature of the pre-Aristoxenian musical system.

Why should the conjunction of the Synemmenon tetrachord bring about a change of key and of species, whereas the conjunctions between the Hypaton and Meson and the Diezeugmenon and Hyperbolaion tetrachords do not bring about a Metabole? The somewhat obvious answer is because Trite Synemmenon introduces a new note foreign to the Dorian Harmonia beginning on Hypate Meson, whereas the Hypaton is but a repetition of the Diezeugmenon an octave lower, and therefore contributes no new element. This is seen clearly through the modal ratios. A comparison of the two Modes, Dorian and Hypodorian, shows clearly the origin of the tetrachord Synemmenon: the tetrachord 22, 20, 18, 16, or (11, 10, 9, 8) is common to both Modes, the turning-point or pivot being (16). When this ratio number is followed by 15, the tetrachord is Hypodorian; when the sequence runs 16, 14, 13, 12, 11, the Mode is Dorian. The Metabole is thus seen to be due solely to the substitution of ratio 15 for 14; 15 being a ratio which does not enter into the composition of the Dorian Harmonia. If this be granted, then one may assert that conjunction only involves a change of Mode, Species or Key when a new note or ratio breaks through the melodic succession or *Emmeleia*, as does the 15 of the Trite Synemmenon. Synemmenon is, therefore, a modal conjunction, whereas Hypaton and Hyperbolaion considered as conjunctions are pure theoretical conventions,

[1] Ps. Eucl., *Intr. Harm.*, p. 17 M., ll. 28 sqq.

arising merely from the necessity of passing from a higher to a lower octave of the Dorian Harmonia, or the converse.

A very important question has been hereby introduced which throws some light upon the function of Trite Synemmenon. It is clear that the theorists through ignorance or perversion of the Modal System have failed to see that the Hypodorian and Hypophrygian species can only be obtained by using the tetrachord Synemmenon instead of Diezeugmenon or Hypaton. The mere inversion of the two tetrachords of the Dorian Harmonia, for instance, with the addition of a tone of disjunction at the beginning of the scale, does not in any sense constitute a new Mode; it is certainly not the Hypodorian Modal species that lies between Proslambanomenos and Mese,[1] for the ratios, as seen in the figure, are not the same (see Fig. 19, Chap. i). The Hypodorian Harmonia does indeed begin on ratio 16 (as shown above) since that is the number of its division, but the 16 must fall upon Hypate Meson, the first note of the Harmonia. If Hypate Hypaton were taken as the beginning of the scale—a practice adopted as we know—then the Determinant ratio for the string-note as Hypate Hypaton would be 22 and Proslambanomenos in the Hypodorian Mode would bear the ratio 24 and not 32. This structure completely carried out, with some of the ratios correctly indicated, actually survives in a unique thirteenth-century Greek manuscript preserved in Florence [2] which is quoted, and discussed with translation further on. It is impossible in the face of this manuscript modal division of the Canon or Monochord in a thirteenth-century transcript—which must have been one of many extending through several centuries—not to accept the principle of the Modal Division as a proven fact in the development of Music in Ancient Greece.

The Hypodorian Harmonia was evidently considered to be of the highest importance at the time of the inception of the scheme of Notation, for the alphabetical sequence constituting the vocal Notation began its descent from Alpha through the Pykna in the Synemmenon tetrachord of the Hyperphrygian Tonos—located an octave above the Hypodorian Tonos, and possessing all its structural characteristics but none of those belonging to the Phrygian group. The sequence of symbols concludes with omega (Ω) as Mese of the Hypodorian Tonos. The scheme of Notation thus began and ended with the Hypodorian (see Appendix No. i). The scheme of Notation, therefore, has for its nucleus Mese, thus emphasizing the dynamic properties of the keynote, and indicating the inherence of Synemmenon—or

[1] As Macran (and others) would have it, following the Graeco-Roman theorists, *The Harm. of Aristox.*, Intro., pp. 22, 23 and 14, 15. See also on Macran's theory of the Modes in J. D. Denniston, ‘Some Recent Theories of the Greek Modes’, *Class. Quart.*, Vol. vii, April, 1913, pp. 87 sqq. Macran does not appear to have noticed that his theory of the Dorian Harmony, as noted on Tables 5 and 10 and pp. 13, 14 and 23, precludes its use for more than one octave without a change of Key; it could not be played, for instance, on the white notes of the keyboard.

[2] The scheme of this Canon is given in Chap. v. The Codex Plut. 56, fol. 10, is preserved in the Laur. Medic. Library in Florence and has been published with Latin and German translations, in which the text has been subjected to many destructive emendations under an entirely erroneous interpretation.

at all events of the ratio 15 falling to Trite Synemmenon—in the P.I.S., the purpose being to provide the characteristic note of the three *Hypo* Modes in the Diatonic Genus. This is one of the important facts which have been elicited through the analysis of the scheme of Notation. It seems highly probable that Trite Synemmenon was already included in the G.C.S., there could otherwise have been no *Hypo* Species. Moreover, since the G.C.S. was a system of 15 notes, and Proslambanomenos was a late addition, the number was obviously made up originally by the inclusion of ratio 15 as well as 14. Any other conclusion must inevitably dissociate the G.C.S. from the scheme of Notation, since it was precisely the Pyknum based upon Mese, i.e. 16, 15, 14, or 32, 31, 30, taken in succession through the seven original Tonoi which formed the nucleus of the scheme.

If the ratios of the Lesser Complete System be compared with those of the Greater, it will be seen that the sole difference between the two was the one note next to Mese, viz. Trite or Paramese, respectively, 15 in the Lesser, 14 in the Greater Complete System. We have it on the authority of Nicomachus [1] and other of the Graeco-Roman Theorists, that among the ancients, Trite and Paramese were interchangeable terms for the note next above Mese, and this is suggested also in the Pseudo-Aristotelian Problems. [2]

A query which will naturally arise out of the foregoing is that since the two tetrachords Synemmenon and Diezeugmenon differ by one note only; viz. Trite, represented by ratio 15 for the former, and Paramese by 14 for the latter, why was it considered necessary to add the whole tetrachord in the P.I.S., which, according to the theorists, consisted in the amalgamation of the Greater and Lesser Complete Systems? The answer is that the inclusion of the whole of Synemmenon was necessary for the rendering of the Hypodorian, Hypophrygian and Hypolydian Species, [3] which pass through the Synemmenon Tetrachord, and omitting the Diezeugmenon, finish in the Hyperbolaion. It will be noted that Diezeugmenon, as alternative to Synemmenon, is omitted in those species.

THE MODAL BASIS OF THE P.I.S. CONFIRMED BY PTOLEMY'S FORMULAE

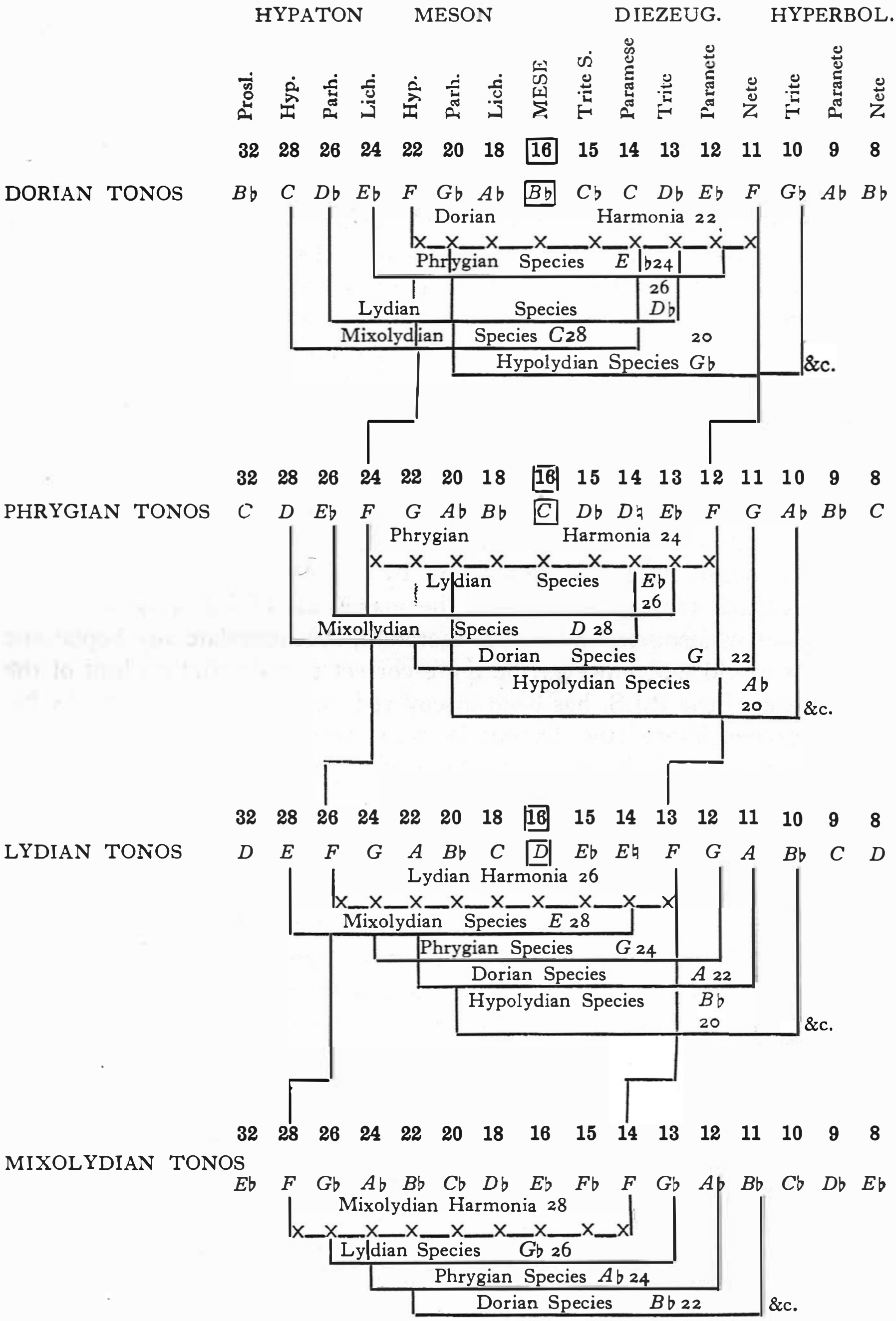
The P.I.S. marks the era of the complete cycle of the Modal Tonoi, or transposition scales, sometimes called the pure key period, which is suggested by the term *Immutable* or unchanging, signifying the one type scale to be repeated without alteration at various points of pitch, determined by the position of the Dorian extended species in each of the other Harmoniai: or, it might equally well be said, by the position of Proslambanomenos as octave of Mese. The Harmoniai are used in Katapyknosis for this purpose (i.e. according to modal genesis), but only in so far as the ratios of the

[1] *Harm.*, p. 17, l. 24 sqq.; M., p. 9, l. 31 sqq.; Boethius, *de Mus.*, Lib. i, Cap. 20.

[2] *Probl.*, xix, *47* (ed. Gev.), pp. 30–1; *7* and *32*, Gev., pp. 32–3.

[3] The reader will bear in mind that the Hypolydian Mode and species are known in two forms, and that this statement applies to the form upon which our major scale is based, i.e. having ratios 20, 18, 16, 15.

FIG. 32—Examples of the Tonos with its Homonym Harmonia and its Species.



Dorian type scale—the same for every Tonos—are required for the three genera. The Mese in each Tonos is the Arche of the Harmonia of the same name, produced by its characteristic division of the *F* string ; as a Tonos, therefore, it had the tonality due to the incidence of the Arche upon the *F* string.

The first fact that emerges from a careful analysis of the scheme of Notation is the evidence of great alterations which have been carried out since the original plan was drafted, and which are sufficient to belie the qualification of *immutable* : for it is no longer a single type scale that is here presented, but several which may readily be identified by means of the working basis of the system of Notation. The subtlety, combined with rational simplicity, displayed in the inception of this scheme is truly amazing and leaves one incredulous that it should have emanated from a single human intelligence of that age. Who was the author of the scheme ? Was it Pythagoras, as is implied by Aristides Quintilianus ? [1]

The second query which will certainly arise is : How is it that no hint of this extraordinary modal basis of the Perfect Immutable System has been discovered in the classical sources, or in the works of the theorists ? This may perhaps be because all traces which might have furnished a clue have been obliterated through the substitution of the Aristoxenian scale system, favoured by the nomenclature by degrees which was in general use, and which took no account of the magnitude of intervals, or of any pitch values whatsoever, and could, therefore, accommodate any heptatonic scale. It would not, however, be quite correct to state that no hint of the modal basis of the P.I.S. has been discovered, for Claudius Ptolemy by his double nomenclature (the Onomasia Kata Dynamin and the Onomasia Kata Thesin), which brings the section of a Tonos corresponding to the Harmonia of the same name within the limits of an octave common to all the Modes, has in a measure restored the original basic idea of the ancient Modal System.

It is quite evident from the position of Mese on one or other of the degrees of the scale in the Onomasia Kata Dynamin, that the Modes were known to Ptolemy in practice and to a limited extent in theory. What he has done is to establish the Harmoniai as a system of seven related scales, beginning and ending on the same note interpreted as *F*, but with Mese occurring in each Mode on a different degree of the octave. By realizing that the Harmonia was actually located in some way within the Tonos, another valuable hint was given, but unless the natural law underlying the modal basis was understood, students of Greek Music could only realize Ptolemy's meaning vaguely from the standpoint of the degrees and their functions. Other important points in theory and practice described by Ptolemy are discussed further on in their proper sequence.

Among the formulae recorded by Ptolemy are to be found several which constitute a complete vindication of the Modal System discovered by the present author. Some of these formulae (given below) are attributed

[1] *de Mus.*, M., p. 28 : '*Πυθαγόρου τῶν στοιχείων ὅλων, ἐκθέσεις τῶν τε τρόπων κατὰ τὰ τρία γένη.*'

FIG. 33.—The Modal Ratios of the Perfect Immutable System identified with Ptolemy's Formulae for the Chroai

DIATONIC GENUS

		HYPATON			MESON			SYNEM.			
	Prosl.	Hypate	Parhyp	Lichanos	Hypate	Parhyp	Lichanos	MESE	Trite	Paranete	Nete
Ratios of the Modal Tonos	32	28	27 / 26	24	22 / 21	20	18	16	15	26 / 27	24

Ratios of the Modal Tonos

Modal Formulae (flat 4th)

28 : 27 24 21 20 18 16 15 (30) 27 24

9 : 8 8 : 7 10 : 9 9 : 8 $\frac{16}{15}$ 10 : 9 9 : 8

Identical with Modal Formula

Diatonic of Archytas* or **Toniaion of Ptolemy**

$$\frac{28}{27} \times \frac{8}{7} \times \frac{9}{8} = 4 : 3$$

Diatonic of Didymus

$$\frac{16}{15} \times \frac{10}{9} \times \frac{9}{8} = \frac{4}{3}$$

Ptolemy's Diatonon† Malakon

$$\frac{21}{20} \times \frac{10}{9} \times \frac{8}{7} = \frac{4}{3}$$

Corresponding with Modal ratios : 21 20 18 16

Ptolemy's Homalon

$$\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8} = \frac{3}{2}$$

Identical with Modal Formula from Lich. Hypaton to Mese.

CHROMATIC—ENHARMONIC GENUS

HYPATON				MESON			SYNEMMENON				
Prosl.	Hyp.	Parh.	Lich.	Hyp.	Parh.	Lich.	MESE	Trite	Paran.	Nete	Paramese
32	28	27	26 / 24	22	21	18	16 / 32	31	30	24	28 &c.

Modal Formulae

24 22 21 18

12 : 11 22:21 7 : 6

Ptolemy's Syntonic Chromatic according to all the MSS.

$$\frac{12}{11} \times \frac{22}{21} \times \frac{7}{6} = \frac{4}{3}$$

Identical with Modal Formula

The Enharmonic of Didymus

$$\frac{32}{31} \times \frac{31}{30} \times \frac{5}{4} = \frac{4}{3}$$

Identical with Modal Formula

Ptolemy's Enharmonic of Eratosthenes $\frac{40}{39} \times \frac{39}{38} \times \frac{19}{15} = \frac{4}{3}$ **is identical with the Enharmonic of the Hypolydian Harmonia from Parhypate Meson to Trite Synemmenon.**

N.B.—The Modal ratios of the Tonos worked out on an *a priori* basis are confirmed by the aggregation of four of the formulae recorded by Ptolemy for the Diatonic Genus, and by three more for the Chromatic-Enharmonic Genus.

* **Transposition of ratios $\frac{8}{7}$ and $\frac{9}{8}$.**

† **Ptolemy, in order to obtain a perfect 4th, has increased the third ratio from $\frac{9}{8}$ to $\frac{8}{7}$.**

to musicians and mathematicians of the Alexandrian School. Additional evidence, gleaned by research among data of the early Greek Church and of the Eastern Arabs of the eighth to the tenth century A.D., affords further proof of the general use of the Modes among the Greeks of Hellenistic Asia, the Eastern Arabs and the Persians. From the figure it will be seen that certain of these formulae for the Chroai or Shades of the Genera recorded by Ptolemy are actually sections of the modal P.I.S. propounded in the present work, every tetrachord of which is shown to be in agreement with the testimony of Ptolemy.[1] The ratios of the tetrachords from the two sources are collated in the table and it will be found that the modal ratios for the tetrachord Hypaton correspond with the formula for the Diatonic of Archytas and the Tonic Diatonic of Ptolemy : the ratios of the tetrachord Meson with those of the Diatonon Malakon ; the Synemmenon tetrachord in certain Tonoi is identical with the formulae for the Diatonic of Didymus ; the whole subject is treated in some detail in Chapter V. Of special interest is the Homalon Diatonon of Ptolemy

$$\left(\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8} = \frac{3}{2}\right)$$

which is identical with the first five degrees of the modal Phrygian species, between Lichanos Hypaton and Mese, in the Dorian and Hypodorian Tonoi ; the Syntonic Chromatic formula of Ptolemy is derived from the same Phrygian Harmonia.[2]

[1] *Harm.*, ii, Cap. 14, pp. 170–72 (ed. Wallis, 1682), and ii, Cap. 15, p. 186.

[2] The Syntonic Chromatic is defined in the Codices of Ptolemy's Harmonics in two tetrachordal formulae which, but for their significant implications, differ merely in the relative position of the first two ratios in the formula. The formula as propounded by Ptolemy, shall be designated A.

THE FORMULA A $\frac{22}{21} \times \frac{12}{11} \times \frac{7}{6} = \frac{4}{3}$

A (1). Is described by Ptolemy in the text : Wallis, 1682, 4*to*, pp. 72–3, i, xv, p. 171, &c. ; Düring, *op. cit.*, text, 1930, i, xiv, p. 35.

A (2). It appears again in a table entitled ' The Shades of the Genera ', according to the five Musicians, Archytas, Aristoxenus, Eratosthenes, Didymus, Ptolemy (who has not discovered its kinship with his Homalon Diatonic). Wallis, pp. 170–2 ; Düring, text, pp. 70–3 ; Düring, translation, pp. 86–7.

A (3). In tables of the mixed tetrachords of the P.I.S. for the seven Harmoniai, *apo Netes* and *apo Meses.* Wallis, *op. cit.*, pp. 177–83 ; Düring, text, pp. 76–9.

FORMULA B $\frac{12}{11} \times \frac{22}{21} \times \frac{7}{6} = \frac{4}{3}$

appears in the table of the Chroai (see A (2) above), found at the end of Book ii, just above the Title of Book iii, in a very large number of the Codices.

This table differs from A (2) inasmuch as it contains only Ptolemy's own contribution to the Chroai, of which it seems to be a correction.

The formula A has been stated according to the practice followed by Ptolemy when giving ratios in sequence, as the superparticular ratios proper to the arithmetical progression of the Harmonic Series, while reckoning the values as vulgar fractions or as lengths of string. This is an obvious contradiction which suggests

MODAL ORIGIN OF PTOLEMY'S SYNTONIC CHROMATIC

Thus the testimony of the medieval theorist provides one more link in the chain of evidence of the survival of the Phrygian Harmonia of M.D. 12 or 24 through the ages, starting from the prototype on the monochord of Pythagoras (*ap.* Gaud., M., p. 14) through the writings of Thrasyllus (*ap.* Theo of Smyrna); the Homalon diatonic of Ptolemy correctly stated by him and discussed (in I, xvi; Wallis, *op. cit.*, pp. 80–8); the treatises of Al-Fārābī (ninth–tenth century), and of Safi-ed-Din, fourteenth century; the Canon of Florence (see Chaps. v, ix); the monochord of Praetorius with the division into 48 equal segments, viz. Phrygian Harmonia, Enharmonic Genus. (Other references will be found in Chap. vii, and in Chap. ix). Had Düring's edition been the only one available, this valuable testimony of the existence and survival of the Harmonia would have been lost.

THE SEVEN HARMONIAI RESTORED BY PTOLEMY THROUGH THE MECHANISM OF THE TONOI.

The significance of these correspondences furnished by the recorded teaching of eminent scholars of the Alexandrian school, second century A.D., is the more important when it is realized that the tetrachord in question is

that the ratios in the formula have been reversed, as for instance in the Syntonic Diatonic.

Ptolemy's formula 16/15 × 9/8 × 10/9, i.e. 15, 16, 18, 20 in the Harmonic Series.

Modal formula 16/15 × 10/9 × 9/8, i.e. 32, 30, 27, 24.

The formula in B is in fact that of the first tetrachord of the Phrygian Harmonia in the Chromatic genus, which may be stated (according to my practice) thus:

24/24 22/24 21/24 18/24 = 4/3

or simply with constant denominator

24/24 22 21 18 = 4/3

Wallis states that he has given the table because it appears in all the codices at the end of Lib. ii. (*op. cit.*, pp. 186–7), but as he considers it corrupt, he has introduced another table, in which the values have been calculated by him in accordance with Ptolemy's exposition in the text (*op. cit.*, pp. 87–8).

Düring omits the table altogether, dismissing it as corrupt. Obviously, Formula A is the one Ptolemy had in mind. It is, however, equally obvious that at some time during the early Middle Ages there existed a scribe or theorist who was so intimately acquainted with the Phrygian Harmonia that he was able to detect the errors in Ptolemy's statement of the scale, and that he restored the order of the modal ratios, in accordance with the Chromatic scale of the Phrygian Harmonia. This knowledge may even not have been restricted to one theorist alone, since the table in question is contained in a very considerable number of codices belonging to different classes and centuries, amongst which are the ten consulted by Wallis and the following of which rotographs were obtained by me.

VAT., G. 191; xiii–xiv, S., fol. 347*v*. Düring, No. 64, *W M.* Klasse.
VAT., G. 192; xiii–xiv, S., fol. 209*r*. Düring, No. 65, *V M.* Klasse.
VAT., G 187; xiv, S., fol. 50*r*. Düring, No. 61, *F.* Klasse.
VAT., G. 1045; xvi, S., fol. 85*r*.
VAT., G. 2365; xvi, S., fol. 111*r*. In 1924 this codex was a new acquisition.
FLOR. *PLUT.*, G. 58, No. 29; xv, S., fol. 258*v*. Düring, No. 11, *M.* Klasse.

the original form of the Authentus Protus of the early Greek Church and that it is still in use in the Syrian Church at the present day.[1]

Ptolemy has actually restored the seven ancient Modes, through the mechanism of the Tonoi, as a modal system in the aggregate, and as regards the dynamic position of Mese characteristic of each. Each Tonos, moreover, is made to bear the burden of the Dorian Species, while concealing its own identity as a Mode. This is how Ptolemy explains the matter :

> Nor should we find that modulation of Tonos (Key) was introduced for the sake of the higher or lower voices ; for this difference can be met by the raising or lowering of the whole instrument, as the melody (*μέλος*) remains unaffected whether it is performed consistently throughout by artists with high, or by artists with low voices. The object of modulation is rather that the one unbroken melody (*τὸ αὐτὸ μέλος*) sung by the one voice may produce a change of feeling by having its tonic (lit. having its beginning) (not Tonic in the modern sense, but *ἀρχόμενον* = Arche or Mese) now in the higher, now in the lower regions of that one voice.[2]

Macran, in discussing Ptolemy's innovation, however, states that ' we are not justified in tracing any new sense of the possibility of different modalities in this innovation. For Ptolemy himself asserts that the object of passing from one Mode to another is merely to bring the melody within a new compass of notes '.[3]

This is surely a strange conclusion to draw from the passage, for Ptolemy's nomenclature reveals the exact position within the modal octave *F* to *F* of the Mese or Arche, which varies in each Mode, thus changing the sequence of intervals within the octave and the species, but not the compass of notes.

Macran's Table 23 shows this clearly. He has missed the significance of his own exposition in this diagram. The melody is only brought within a new compass of notes by changing the Tonos and when the extension of the Modes (by means of the P.I.S.) illustrated by Macran in Table 21 is retained. As soon as the Modal System has been restored by the introduction of the system of double nomenclature, the compass of notes used in the melody is, according to Ptolemy's statement, on the contrary restricted in each Tonos to the same octave *F* to *F*. What, then, is the metabolism to which Ptolemy alludes in ' the one unbroken melody sung by the one voice . . . and having its beginning now in the higher, now in the lower regions of that voice ' ?

It is a purely modal metabolism conditioned, not by an alteration of compass of notes, nor by the *tessitura* resulting merely from the accident of a movable Mese or Tonic, situated upon a higher or lower degree of the scale, but upon the modal configuration of the magnitude and sequence of the intervals themselves, resulting from the nature of the Dynamic Mese or Arche, with its power of originating its own structural scheme.

Thus, owing to his failure to recognize the implication of the ingenious

[1] Dr. Joh. Tzetzes, *Über die Altgriechische Musik in der griech. Kirche* (Munich 1874), p. 77, also pp. 30–1 ; 43, 44, 82, 83.

[2] Ptol., *Harm.*, ii, 7 (trans. by H. S. Macran), ' Aristoxenus ', Intro., pp. 39–40, note.

[3] *Op. cit.*, p. 65 ; see also pp. 63–5, Table 23.

theory of a movable Mese, with the introduction of which he has been credited,[1] Macran's Mese was a pure convention, arbitrarily selected, not a dynamic Arche which conditioned all the other degrees.

FOUR STAGES IN THE DEVELOPMENT OF THE TONOS MARKED BY A CHANGE OF STARTING-NOTE

The main points in the subject matter of this chapter which are affected by Ptolemy's exposition relate to the significance of the four possible degrees of the Tonos considered as the beginning of the scale, as already briefly stated above.

(i) Hypate Meson, (ii) Hypate Hypaton, (iii) Proslambanomenos, (iv) the modal octave *F* to *F* within each Tonos, which falls on a different degree in each Tonos. All of these contingencies were evidently familiar to Ptolemy : (i), (ii), (iii), affect the species of the type scale ; (iv) the modality of the system as a whole. The scale beginning on Hypate Meson (i), Ptolemy calls *Apo Netes* (ἀπὸ νήτης), and (iii) beginning on Proslambanomenos *Apo Meses* (ἀπὸ μέσης). It must not be forgotten that these four stages in the development of the P.I.S., implied or chronicled by the Theorists, were realities in practical music, and emphasize the essentially modal basis of music in antiquity.

We may here recall, for instance, the subtle distinction in essence between Modes and species inferred from the seemingly irreconcilable order of their sequence in diametrically opposed directions, which we perceive to be due primarily to the outcome of the two aspects of equal measure applied to cause and effect in the realm of sound and music. The Mode has an independent genesis and one implicit Tonality ; the species has the genesis of its homonym Mode and the same sequence of interrelated proportional ratios ; its Mese is on the same degree of the scale as in the Mode itself ; but all the species derived from any modal octave scale have a common keynote of the same pitch, which greatly facilitates modulation, both modal and tonal.

The Modes could only be used as such singly (as laid down for the Nome) or with modulation in vocal music only, owing to the technical exigencies of musical instruments ; it was owing to this technical necessity that the species were discovered, inherent in the Modal Scale to which the Kithara was tuned.

It may be said, therefore, that whereas the seven Harmoniai were united through their common fundamental or Tonic (*F*)—the monochord string in practice, or the fundamental note of the Aulos within the limits of three Modes on each Aulos—the modal species were united through a common keynote ; they are differentiated and recognized through their individual Tonics and the degree of the Modal Scale on which the Tonic falls. The order of the species must necessarily vary with the scale of which they form species : the order of the Modes never changes. Each Mode has

[1] J. D. Denniston, ' Some Recent Theories of the Greek Modes ', *Class. Quart.*, April, 1913, pp. 87–8.

one implicit key; each species has an implicit key in each Mode, therefore seven keys in the seven different Modes.

In modern practice the number of keys available might be enormously increased by tuning the monochord string to various notes—in the Aulos by the use of different mouthpieces (see Chap. x, Elgin Aulos Records). The ratios form an immediate and infallible clue for the identification of Mode and species.

The significance of these stages in the evolution of the P.I.S. may now be considered separately.

Stage i.—Hypate Meson as Starting-Note and Modal Pivot

(i) The retention of Hypate Meson for many centuries as the real beginning of the scale and the preservation of the purity of the Mode long after the Kithara had strings enough for two octaves, is more especially significant of a great sensibility to Modal Ethos. It explains a fact that has puzzled students of Greek Music, viz. why there are so few allusions to the species in the classics. It was evidently taken for granted that the addition of strings or degrees, above or below the modal octave, was for the purpose of modulating into the species. The scheme of modal ratios, into which the Modes may be analysed, whether followed consciously with the mind, or through their equivalents in sound by the ear, lends itself with the greatest ease to the modulation through cycles of modal species, with Mese, as central pivot to guide the ear or the mind, through its relation to the Tonic: the Harmonic 4th 11 : 8, for instance, denoting the Dorian, the perfect 5th 12 : 8 the Phrygian; the diminished Major 6th 13 : 8 the Lydian, and so on. It is this unique structure of the Greek system of scales known as the P.I.S. which preserved its elasticity. In each Tonos is combined the possibility of modulating by changing the Tonality and the *tessitura* through the use of the species without retuning the instrument, merely by inherent dynamic resources, including the power of bringing to life the Mode latent within the limits of the octave *F* to *F* (Ω to Γ), which only coincides with Hypate Meson in the Dorian Mode.

Stage ii.—Hypate Hypaton as Starting-Note now becomes the Modal Pivot

Hypate Hypaton as starting-note now becomes the modal pivot.

(ii) In Ptolemy's description of his transposition of the seven Tonoi a 4th lower may be recognized a conscious or unconscious allusion to the change which takes place in Tonality and species when the Tonos is considered as beginning on Hypate Hypaton, or as the process would have been described in terms of the Modal System, when the string-note (or τάσις) was changed from the *F* of Hypate Meson to the *C* a 4th below of Hypate Hypaton. A curious anomaly, very puzzling to students, results from this radical and disturbing alteration involving a change of species, a change of pitch and a change from the disjunct to the conjunct scale, or of the position of the interval of disjunction. The effect of Ptolemy's transposition may thus be resumed as a modulation to the sub-dominant Hypate Hypaton involving (1) a change of species in the type scale of the P.I.S. from Dorian to Mixolydian; (2) a change of Tonic from the *F* of Hypate Meson to *C* of Hypate Hypaton in the Dorian Tonos; (3) a change of Tonality—although Mese still remains the same in pitch—through the change in the dynamic relation between Mese and Tonic: i.e. from 11 : 8 to 28 : 16. Hitherto 22 has been the Modal Determinant of the string for the Dorian Harmonia, beginning on *F*; and whether the descent through *E*♭, *D*♭, *C*, be regarded as a mere tetrachordal expansion downwards, or through the Tonics of the Phrygian Species 24, of the Lydian Species 26 or 27, or of the Mixolydian 28, the time comes at last when either theorist or practical musician recognizing by ear the sequence from Hypate Hypaton to Paramese as Mixolydian, or knowing 28 as the number of the division of the string for the Mixolydian Harmonia—that it is, in fact, the initial number upon which, as Hypate Hypaton, the Mixolydian Species begins—conceives the idea of making that note the starting-point, thus indicating a predilection for the Mixolydian over the Dorian. The Mese *B*♭ of the Dorian Species is the same for the Mixolydian Species, but the alteration in the beginning of the scale shifts the position of the Mese from the 4th degree in the Dorian to the 7th in the Mixolydian. Without retuning a single string, the Dorian Harmonia, therefore, has suddenly acquired a double significance as type scale of dual species, according to whether the string note *F* is retained as Hypate Meson with Modal Determinant 22 as Modal Pivot, or whether a new string-note or *Tasis*, a 4th lower, as Hypate Hypaton with a division by 28 shall be adopted. The event may be chronicled as a transposition of the Tonos or of the whole P.I.S. (since the Tonoi are transpositions of the extended Dorian Harmonia) a 4th down, or as a change from the Dorian to the Mixolydian Species, or even in the Aristoxenian sense as a modulation to the Dominant (Paramese). In the Modal System, the matter is considerably more subtle; where all tetrachords are composed of semitone, tone, tone, of the exact aggregate value of a perfect 4th, it is a difficult matter to find any distinction between the Dorian Species of the G.C.S. or of the P.I.S. on *F* continued downwards to *C*, and the tetrachords of the Mixolydian Species on *C* rising to *F*, except in the rhythmical quality of the two scales in melody, the one with *F* as Tonic, and the other with *C*. With the Modal System it is different.

Since the whole system of the Modal Tonoi has been built upon the one note *F* as fundamental or string-note and, therefore, invariable in pitch, it stands to reason that the whole of the difference due to the modal ratios must have been borne by the *C* as Hypate Hypaton, so that there were for the seven original Tonoi seven different *C*'s. When, however, it is proposed to interchange the functions of these two notes, and to transfer to *C* as Hypate Hypaton the fundamental stability of the whole system that formerly resided in *F*, the sevenfold differentiation will now become the apanage of *F*, and there must henceforth be seven *F*'s. Until this fact is realized the radical nature of the change of pivot cannot be gauged. Which of the former seven *C*'s will be the chosen one for the new modal basis? Will it be the very high Dorian *C* of 138·029 v.p.s., or the very low Mixolydian *C* of 123·2 v.p.s., or the Hypodorian *C* of a frequency of 128?

Whether it be possible to fix, through the system of Notation which holds the clue to the Modes, the original pivot or fundamental basis at the time of the inception of the scheme as *F* or *C*; or whether the modifications to which the scheme was subjected in the course of time had affected this pivot likewise, must be left for final settlement during the discussion on Notation (Appendix No. i), which demonstrates clearly that fine distinctions of pitch do not form the determining factor. This much is certain, however: that the common fundamental *F* has for its symbol Ω or its octave Γ (omega or gamma)—suggestive in itself of the finality of the modal octave—which actually does appear in each Tonos, but each time with a different ratio, so that *F* has as successive denominators 14, 13, 12, 11, 20, 18, 16, according to the Determinant of the Modal Homonym of the Tonos in question, with the exception of the Lydian and Hypolydian Tonoi in which the symbol Ω or Γ does not occur. It follows consequently that the *C* a 4th below *F* forms with it a different ratio in each Tonos, and is represented by one of the three symbols M, N or Ξ.

It is evident that the shifting of the pivotal note was revolutionary enough to be recorded. The failure of the Lydian group to conform to the general rule, as regards the modal pivot, requires some explanation. The alteration of the 2nd degree of the Diatonic Hypaton tetrachord from 28 : 26 to 28 : 27, lowering the Tonic of the Lydian species by a diesis from 26 to 27, accounts for the lower symbol for *F* used in the Lydian Tonos, viz. Ꝑ instead of Ω (actually two dieses lower, for the same change was made in the Lydian Determinant also, from 26 to 27), except in the Diatonic Synemmenon Tetrachord, where *Gamma* appears as ratio 51 and Chromatic Paranete Diezeugmenon as 26 is represented by *Delta*, a diesis lower. The case of the Hypolydian is analogous owing to the ratio of Hypate Meson having been modified from 22 to 21, so that Parhypate as 20 has been lowered from its original pitch, because the ratio from Hypate to Parhypate Meson is now 21/20 instead of 22/20.

The basis of the Modal System in Ancient Greece is thus, by weight of internal evidence, shown to have been Ω interpreted as *F*.

To resume: the change of Tonic to Hypate Hypaton, while maintaining the *F* as pivot, does not in consequence bring about a change of species so much as a change of structure—the disjunct formation being abandoned definitely in favour of the conjunct—so that the interval of disjunction between the tetrachords, characteristic of the Harmonia, is now regarded as an interval of the same kind occurring between the 7th and 8th degrees, thus forfeiting its significance as Tone of Disjunction.

The significance of the structural change brought about in the Tonos by conjunction will thus be realized from Fig. 46, Chap. v; the Mixolydian tetrachord is now seen yoked to the Dorian by conjunction on the 22 of Hypate Meson as recorded by Plutarch, whereas in the Mixolydian Harmonia the complementary tetrachord is Hypolydian.

In the Mixolydian Mode, as in all the Harmoniai, the interval of disjunction occurs between the 4th and 5th degrees, separating the Hypolydian half of the Mode from the unnamed first tetrachord to which it has been yoked. The ratio of the interval of disjunction is thus 11 : 10 in the Mixolydian Harmonia and 16 : 14 in the type-scale. If in Ancient Greece the tetrachordal structure of the scale was realized in practice as a musical fact, the change of rhythm in the well-known Mixolydian Harmonia, when used as a Tonos, was quite sufficient to account for its being chronicled as a discovery by Lamprokles [1] (fl. *c.* 500 B.C.), which, it will be recalled, was concerned with this very point, and which, therefore, seems to indicate a much earlier origin of the occasional practice of beginning on Hypate Hypaton than the age of Ptolemy. If regarded as a matter of modal predilection, the importance of the record of its theoretical usage is greatly diminished; it was rather the concern of the musician than of the theorist. The point at issue in the age of Lamprokles was the confusion that existed in the minds of musicians between the Harmonia and the species in the Tonos. Plutarch has placed on record the fact that the Greeks did not use the tetrachord Hypaton in the Dorian Tonos on account of the change it produced in the Ethos.[2] The full significance of this change of Ethos can only be gauged when the Modal Tonos is heard. It was, in fact, not only a change in Ethos—which in itself is very striking—but a change of species that is effected by beginning or ending a scale on Hypate Hypaton, which was, according to Plutarch's statement, the actual form of the Tonos in his day, between Hypate Hypaton and Paramese. This new form or species of the Tonos, entailing a division of the string by Determinant 28 instead of 22, recalls several passages in Aristoxenus and other writers, in which the number 28 figures in connexion with the Greek Scale of pre-Aristoxenian days; these are examined in the next chapter.

Stage iii.—The Modal Pivot changes to Proslambanomenos as Starting-Note

(iii) The era of Proslambanomenos ushering in the bastard Hypodorian species of the theorists has but little in common with the Modal System. When used as the first and lowest of the species by the Harmonists, however, we may safely conjecture that they, in their musical practice, lowered Hypate Hypaton from ratio 28 to 30, as lower octave of Trite Synemmenon, and proceeded to sing the Hypodorian in all its purity.

To recapitulate the principal points of this chapter, it may not unreasonably be claimed that many of the mists which had gathered from the remote past around the music of Ancient Greece, forming an almost impenetrable barrier between the earlier and the post-Aristoxenian periods, have been dispelled by the discovery of the Modal System.

That a system of such indisputable importance, and capable of that logical development in many directions which a mathematical basis alone ensures, should have remained concealed, unsuspected even, for centuries will at first sight, perhaps, appear incomprehensible and highly debatable. It is hoped, however, that the satisfactory explanation provided by the Modal System for many knotty points and apparent contradictions, which have hitherto confronted the student, will induce many to go carefully and

[1] Plut., *de Mus.* (ed. Weil and Reinach), 16E; and also D, pp. 62–7.
[2] Plut., *op. cit.*, Cap. 19, p. 79 and note 185.

impartially into the matter. This discovery should arouse no spirit of controversy, since this is the first time that an explanation of the Mode has carried with it *a new musical fact* unknown before, instead of being a recasting of the old arguments. This, in fact, is ' the hitherto overlooked factor ' which is to solve the problem of Ancient Music about which Macran expresses the keenest scepticism in his Introduction (pp. 81–2).

It has been demonstrated that the origin of the G.C.S. may be traced to the use of the Modal Species within the Dorian Harmonia, upon a Dorian Kithara; that the G.C.S. at its inception, in fact, was not realized as one scale of a range of two octaves consisting of an extension of the Dorian Harmonia, but as a cycle of seven octave scales linked together in species.[1] The discovery that the cycle in itself formed a scale was probably not made until the full cycle had been played upon a fifteen-stringed Kithara.

In practice the G.C.S. may, therefore, have continued for centuries to present a series of Modal Species of the Harmonia in the Tonality of one single Mode, the Dorian. Historically and practically, the G.C.S. thus represents the period of the dominance of the Dorian Mode which supplied the urge behind the development of the system; and it only became an evident theoretical fact on completion.

But although the fact that the same practical possibilities were offered by the P.I.S., and the completed scheme, as a system of transposition scales, emphasizes the dominance of the Dorian Mode still more strongly, yet the hidden basis of the system is seen to depend upon *the co-operation on equal terms of all the seven Modes.*

As a theoretical scheme of transposition scales, the seven Tonoi are shown to have been in origin the seven Modes in Enharmonic Genesis, curtailed in order to present exclusively the Dorian Harmonia, extended to two octaves, as species in each of the other Modes; all the superfluous notes characteristic of the other Modes themselves being omitted from the scheme. In this guise the group of Tonoi appears to modern eyes as an instrument of Tonality—a mere system of keys. We have seen that in reality it is far more than this. When the modal origin of the P.I.S. expressed in ratios is revealed, the ingenuity of the scheme, favoured by a compelling arithmetical progression, is striking. How did the scheme operate in practice among the Ancient Greeks? The inadequacy of the fifteen-stringed Kithara for rendering the scheme of the seven Tonoi is obvious. Ptolemy used an octachord Kanon. The single string of the Monochord or Kanon suffices, however, to produce the whole series of Tonoi, for tuning or experimental purposes, but the Monochord does not lend itself to an artistic performance.

Stage iv.—Fundamental Modal Change in the P.I.S. from Dorian to Phrygian

Upon the string tuned to *F*, the sequence of Modal Determinants as successive denominators, dividing the string at will to produce any of the

[1] ' The seven octave-scales which they called Harmoniai ', *Aristox.*, Macr., p. 127 (36); see also *Polemics of Aristox.* Chap. v, *infra.*

Modes, takes care of Tonality and produces the seven keys. The recurrent series of numerators expressing the degrees of the double octave scale—common to the seven Tonoi—imposes an apparently uniform modality. The modal monochord, with its soundboard divided in turn by the seven Modal Determinants, shows how the seven Tonoi may be visualized at a glance and tested one by one by means of the movable bridge.

It may be explained at this point that the hypothetical shortening of the strings of the Kithara to produce semitones and other intervals by means of a kind of nail technique suggested by some writers on Greek Music has no possible basis in practice. The device is only successful with strings vibrated by means of the bow; on a plucked string the modification in pitch is obtained at a complete sacrifice of beauty of tone, a result which in itself would have sufficed to doom the device among the Greeks. Apart from aesthetic considerations, there are others which militate strongly against the practice, such as the extreme sensibility of a string to tension. In the upper part of the Kithara, where the strings lie open, an elastic string pressed back with increasing tension will easily rise in pitch as much as a 5th; it is, indeed, difficult to keep the nail sufficiently steady against the stretched string to ensure repetition of any note at exactly the same pitch.

The species imply the Tonos. To the sensitive Greek ear accustomed to discriminate among the *Chroai* and the Modes, the difference in pitch between the Mode and the species of the same name must have been at once apparent and have led in time to a deliberate use of species as keys of a Harmonia.

THE TONOI AS CURTAILED MODES

The P.I.S., sometimes termed the Pure Key System, or scheme of Transposition Scales, is founded upon the Modes themselves taken in their Modal Genesis in order to provide for the three Genera. *The Tonoi* or Keys result from the use of the Dorian Species, extended as in the G.C.S., and taken in each of the Modes in turn, hence the term ἀμετάβολον, immutable or invariable, applied to the Dorian Species or form of the Standard Scale.

In other words, the G.C.S. was at first a cycle of all the species obtainable from one Harmonia—the Dorian—whereas the P.I.S. was a cycle of all the keys into which the Dorian Harmonia could be transposed by taking it as a species of each of the Harmoniai in turn. This is supported by an analysis of the scheme of Notation, specially designed for the Modal System of the P.I.S.

The ratios of the extended Dorian Harmonia from 28 to 8, or from 32 to 8 inclusive of Proslambanomenos, were thus taken from the Genesis of each Mode, the superfluous ratios unused in any of the three Genera of the Dorian Harmonia being simply omitted from the Tonos. These unused ratios did not, however, drop out altogether from the scheme of Notation, for they are preserved in latency, through the inevitable alphabetical sequence, by the mechanism of the complete scheme. It has been found that in order to obtain the ratios of the extended Dorian Harmonia in the three Genera within the other Modes, it is necessary to double the number of the Determinant for each Mode: 28 thus becomes 56; 24, 48, and so on. See Fig. 34.

Finally the four stages in the evolution of the P.I.S. characterized from the theoretical point of view by the degree upon which the stress of the beginning was laid, and from the musical point of view by the species or modality of the whole scale, have, on examination, revealed the following implications.

The change involved by the immanence of Stage ii is far-reaching in its effect on the structural form, on the modality and on the rhythmical life of the scale or Melos. If carried to a logical conclusion the change was revolutionary, affecting the whole operation of the Modal System, by shifting the modal pivot from Hypate Meson to Hypate Hypaton (from *F* down to *C*), with all that this implies in a modal sense. Whether this radical change in the modal pivot was actually effected, and whether it can be traced through the System of Notation or through any further development of the Modal System, is a question that must be left for subsequent discussion.

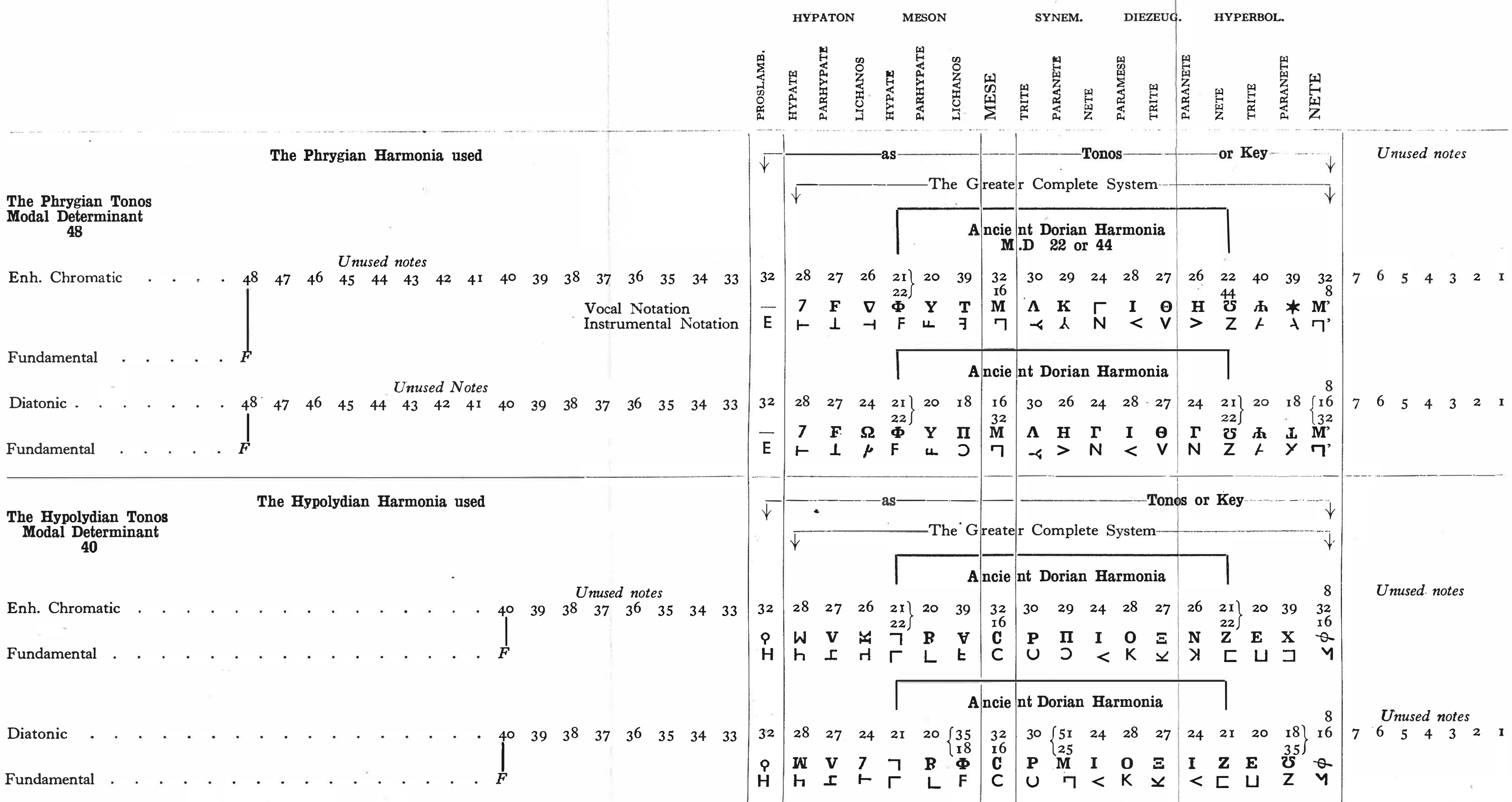
But another important implication arises apart from the P.I.S. There are two possible contingencies signified by this second stage in the development of the P.I.S.

In the first, the beginning on Hypate Hypaton is purely nominal; the modal pivot remains fixed on *F* (Hypate Meson, Kata Thesin) and constitutes a standard note of the same exact pitch in all the Tonoi, and an initial or starting-note common to all the Harmoniai within the Tonoi; but in each Tonos this modal pivot, as common point of pitch, bears a different significance due to its characteristic modal ratio and affords special facilities for modulation into other Harmoniai. With the pivot on *F*, the characteristic modal tetrachord responsible for the Modal Ethos is the *upper* one. The Harmonia with its essentially disjunct formation extending to Nete Diezeugmenon remains unimpaired. The added conjunct Hypaton tetrachord is fortuitous in origin and of secondary importance. With the modal pivot removed to Hypate Hypaton (i.e. the fundamental note of the whole series of Tonoi changed from *F* to *C*), on the other hand, the change is radical. If the rhythm and structural features are retained intact, as they probably would be in musical practice, then a new order of Modes is inaugurated, consisting of two conjunct tetrachords on the Tonic with an interval of disjunction between the 7th and 8th degrees, and that constitutes for the Mode an entirely new rhythm.[1]

More fundamental, however, is the modal change effected. It will be remembered that each Mode consists of two tetrachords yoked together, paradoxically through disjunction; the first on the Tonic characteristic of the Mode itself, and the upper tetrachord of a related Mode which enters on the 5th degree. In the Conjunct Modes the 2nd Mode entering on

[1] See Appendix No. iii. Examples of use of the Modes in Modern Composition by Elsie Hamilton.

FIG. 34.—Examples of the Tonos as Curtailed Mode exhibited in Three Stages of Development from the Ancient Dorian Harmonia



N.B.—See Appendix 'Notation' for an example of the principle on which the symbols of Greek Notation are allotted in conformity with the succession of ratios in each Harmonia: for each letter of the Alphabet—a ratio—both in sequential order. Due regard has been paid to the requirements of the Enharmonic, Chromatic and Diatonic Genera.

the 4th degree is necessarily a different one from that of the Harmonia of the same name, so that the association of the two Modes takes on another Ethos altogether.

The interesting question that emerges from this new order is, of course, this : Are there any traces extant in theory or practice of these conjunct Modes used independently ? (i.e. κατὰ δύναμιν). To this an affirmative answer must be returned when the origins of the Ecclesiastical Modes are investigated (see Appendix No. ii and *The Canon of Florence*, Chap. v), since we have now arrived at a point at which the theoretical form of the Dorian Mode, as the scale of the P.I.S., is forsaken for the wider sphere of all the Modes used independently, though still expressed through the nomenclature of the P.I.S., as no other was in use. Musically and theoretically, the issue is of very considerable importance to the evolution of Music in East and West.

CHAPTER V

EVIDENCE IN SUPPORT OF THE MODAL SYSTEM

The Harmonia's Modal Principle of Equal Measure confirmed by Aristotle. The Evidence of the Canon of Florence. The Common Modal Starting-note according to Aristides Quintilianus. The Mese as Arche. The 12 Polemics of Aristoxenus against the Harmonists. Polemics 1 and 2: Concerning the Harmonia. Polemic 3: The Close-packed Scales of the Harmonists. Polemic 4: Concerning the Tonoi. Polemic 5: Notation as the Goal of Harmonic. Polemic 6: The Theory of the Aulos and of the Pipe-scales. Polemic 7: The Aulos as the Foundation of the Order of Harmony. Polemic 8: Eratocles and the Harmonists in general treat only of the Octave. Polemic 9: Concerning Systems. Polemic 10: Eratocles determines the Species by the Recurrence of the Intervals. Polemic 11: The Harmonists assert that Points of Pitch consist of Ratios and Rates of Vibration. Polemic 12: On the Twenty-eight Consecutive Dieses. The Characteristic Ratio 11/10 of the Dorian Harmonia, ascending from the Tonic, Hypate to Parhypate Meson. Eleven, the only Determinant Number that could place Mese upon the Fourth Degree of the Scale. The Ratio 11/10 as first Diesis on the Tonic confirmed by Aristides Quintilianus (p. 123 M.). Further support for the use of Ratio 11/10 in the Tonos from Ptolemy. Definition of the Diesis by Aristides Quintilianus (p. 123 M). The Ratio 11/10 as Spondeiasmos and Eklysis. The Ekbole, interval of Five Dieses. The 28 Dieses of the Harmonists according to Aristoxenos and Aristides Quintilianus. '28' as the Ratio Number of the Mixolydian Tonic, and as Hypate Hypaton in the Tonos. Brief Recapitulation.

THE HARMONIA'S MODAL PRINCIPLE OF EQUAL MEASURE CONFIRMED BY ARISTOTLE

IN the absence of actual descriptions of the Modal System in any classical sources extant, and in the works of the Graeco-Roman theorists, certain significant passages have been brought together in this chapter to form a body of evidence in agreement with the main features of the Modal System as explained in the foregoing chapters. These passages, many of which are discussed more fully elsewhere in different contexts, have been selected for the present purpose not only on account of the support they give to the recovered Modal System, but also as examples of the peculiar value of the vantage-point afforded by a system having a mathematical basis, which proceeds from a precise and inevitable order, such as that provided by an arithmetical progression.

A matter of primary importance in a consideration of the Modal System is obviously the principle itself, according to which the Harmonia is constituted and operated in practice. This basic principle is that of the arithmetical progression known in the domain of music as the Harmonic Series, constituting the physical basis of sound. This progression when applied to length—of string or air column—through *equal measure* or aliquot division, is responsible for the creation of the Harmonia and indeed of the whole Modal System. The Harmonia may be said to be due absolutely to the principle of equal division without which its existence could not be demonstrated.

The Modal System, therefore, must stand or fall by *equal measure.* A record of this principle as the basis of the Harmonia has been preserved for posterity in germ form by Aristotle, in a terse fragment, in which Pythagorean and Platonic influences may be detected.[1]

The passage in question, quoted by Plutarch [2] (Arist., Fr. 43, Bekker) and pronounced obscure by certain authorities on Greek music, demands careful consideration, in view of its weighty implications, and of its extraordinary significance. It runs as follows:

[§ 226] *ὅτι δὲ σεμνὴ ἡ ἁρμονία καὶ θεῖόν τι καὶ μέγα Ἀριστοτέλης ὁ Πλάτωνος ταυτὶ λέγει· " ἡ δὲ ἁρμονία ἐστὶν οὐρανία τὴν φύσιν ἔχουσα θείαν καὶ καλὴν καὶ δαιμονίαν.*
[§ 228] *τετραμερὴς δὲ τῆι δυνάμει πεφυκυῖα δύο μεσότητας ἔχει ἀριθμητικήν τε καὶ ἁρμονικήν.*
[§ 229] *φαίνεταί τε τὰ μέρη αὐτῆς καὶ τὰ μεγέθη καὶ αἱ ὑπεροχαὶ κατ᾽ ἀριθμὸν καὶ ἰσομετρίαν* <*ἡρμόσθαι* add. W. and R. >, *ἐν γὰρ δυσὶ τετραχόρδοις ῥυθμίζεται τὰ μέλη* (*τὰ μέρη* Westphal).

The fact that Plutarch has introduced this quotation from Aristotle on the *Harmonia*, immediately after the chapter devoted to Plato's exposition of the theory of the Creation of the Soul in the *Timaeus* likewise as a *Harmonia*, throws into high relief the difference in the conception of Harmonia, as used in this context, by the two philosophers. While Plato considers Harmonia as part of primordial substance, Aristotle treats Harmonia as a definite, heaven-sent principle, ready for use in the production of a musical scale. Whereas the scale deduced by commentators from the *Timaeus* was obtained through the operation of the geometrical mean and progression, by powers of two and three, the Harmonia, actually described by Aristotle, illustrates the other two means—the arithmetic and the harmonic—Aristotle omits the geometric, for in practice the geometric mean and the arithmetic mean are incompatibles.

The scale derived from Plato's *Timaeus* is the ditonal, due to a cycle of seven Perfect 5ths, as a result of the *τά τε διπλάσια καὶ τριπλάσια διαστήματα.* Arithmetic progression has no part in this theory, and Aristotle's description of the Harmonia owes nothing to Plato's.

The scale of Aristotle, on the contrary, depends for its sequence on arithmetic progression, brought into operation and differentiated in accordance with a number and equal measure. Plutarch was at sea here and

[1] The Pythagorean theory of the soul as a Harmonia has been treated by Plato in *Phaedo*, 86 B 7, and in *Timaeus*, *passim.*

[2] *de Mus*. (ed. Weil and Reinach, p. 92, Cap. 23, §§ 226–30, *ἰσομετρίαν*; Reinach, § 229, substitutes *γεωμετρίαν*, his own reading, for *ἰσομετρίαν*, although this word is attested in all the manuscripts of Plutarch consulted by Weil and Reinach. Reinach holds that the word *ἰσομετρίαν* has no sense and that *γεωμετρίαν* prepares the way for § 237; but the Modal System supplies the sense and provides an explanation for *ἰσομετρίαν*. The significant part of the quotation may be translated thus: '[The Harmonia] its parts, magnitudes and excesses appear according to number and equal measure; for melodics are rhythmized in two tetrachords.'

does not attempt any explanation or illustration of this principle of equal measure. Outside the domain of the Harmonists, the practical significance of equal measure, or aliquot division by a Determinant number on string or pipe, was obviously still unknown. It is strange, however, that a shrewd and erudite scholar like Reinach found it necessary in this quotation to substitute for *ἰσομετρίαν*, *γεωμετρίαν*, a term entirely inapplicable to a scale due to the arithmetic mean or progression.

The ditonal scale resulting from a geometrical progression by 3 produces a sequence of equal intervals of one tone (of ratio 9/8) ; such a scale cannot be produced by any aliquot division by a number. The Harmonia of Aristotle, on the contrary, consists of unequal intervals—the result of a number taken conjointly with equal measure.

Aristotle's definition of the Harmonia must now be subjected to further examination :

(*a*) The four members or parts (*τετραμερής*) as to power or value (*τῇ δυνάμει*) comprised in the Harmonia are probably a reference to the initials and finals of the two tetrachords, the *φθόγγοι ἑστῶτες* or fixed notes in the P.I.S. (see further under *c* below).

(*b*) The two means are expressly stated by Aristotle to be the Arithmetic and the Harmonic—the geometric being significantly omitted because, as already mentioned, the geometric mean cannot operate with the arithmetic, nor with equal measure ; an omission, therefore, that proves that Aristotle knew what he was talking about. The Harmonic Mean implies the relation of the four members to the whole and to each other : the familiar illustration is for (A), when vibration frequencies are required,

(A) $$6 : 8 : 9 : 12, \text{ where } \frac{8}{6} = \frac{12}{9} \text{ and } \frac{9}{6} = \frac{12}{8}$$

but for equal measure or lengths of string the formula is :

(B) $$12 : 9 : 8 : 6, \text{ where } \frac{12}{9} = \frac{8}{6} \text{ and } \frac{12}{8} = \frac{9}{6}$$

The arithmetic mean implies the arithmetical progression of the Harmonic Series from 1 to infinity which (as is known from preceding chapters) may operate in two opposite directions from a grave or an acute fundamental, thus forming scales which belong to two widely differing systems. We are not left in doubt as to which of these is the system of the Harmonia according to Aristotle. The crux is, of course, the mention of the number coupled with equal measure in the text, implying a *causative* or *structural* process productive of the parts, magnitudes and excesses of the Harmonia. Equal measure likewise implies the descending arithmetical progression from high to low, and refers to the string of the monochord—for the question of length applied to pipes was still obscure even in Ptolemy's day. But if it be recalled that the division of the canon by M.D. 12 was said by Gaudentius (M., p. 14) to have been used by Pythagoras, the whole passage becomes clear.

(*c*) The final statement in the quotation ' for melodies are rhythmized in two tetrachords ', is puzzling ; it certainly does not seem to fit in with the context; it may be in some way corrupt, e.g. possibly due to an early gloss. Nevertheless, the fact must be conceded that the two tetrachords in which melodies are said to be rhythmized or ordered do actually constitute a special attribute of the modal Harmonia in which the operation of the Modal Determinant of the aliquot division brings to birth two individual tetrachords: the first on the Tonic is proper to the Mode created by the Determinant; the second, continuing the arithmetical progression, starts on the 5th degree of the sequence and is recognized as the first tetrachord of a related Mode. The Harmonia therefore, extending over eight notes, contains two tetrachords of different structure—not arbitrarily yoked together—but related through the operation of the principle underlying the aliquot division by a Modal Determinant. Thus, the claim made in this work, that the Modal System of Ancient Greek Music was based upon the principle of equal measure by a specific Determinant number, is not only in accord with these lines of Aristotle but also provides an explanation of a difficult term which editors of the passage have sought to emend.

FIG. 35.—The Dorian Octochordal Harmonia Expanded from M.D. 11 to 22

M.D. as constant denominator	Dorian				Mixolydian			
	11/11	10	9	8	7		6	$5\frac{1}{2}$
	22/22	20	18	16	14	13	12	11
	Hyp.	Parh.	Lich.	Mese	PM.	Tr.	PN.	Nete
	Meson				Diezeugmenon			

The four members

22 16 14 11

The basic law out of which grew the Modes seems on the surface extraordinarily simple; not so its implications: they are infinite; it is in these that the solution of the many difficulties that surround the subject of Greek music are to be found. The basic principle of equal measure does in effect produce seven Harmoniai each having Mese on the characteristic degree of the scale; it is, however, still necessary to prove that what is inevitably produced by the operation of the principle itself was ever actually developed by the Greeks into a Modal System; into all that the P.I.S. and the Tonoi owed to that law. No detailed evidence of that nature is obtainable from Aristotle. It exists, nevertheless, in the compilation of Ptolemy and in the opening lines of a remarkable document, removed by centuries from the evidence of Aristotle, to which attention must now be called.

THE EVIDENCE OF THE CANON OF FLORENCE

In the late thirteenth-century Florence codex (Bibl. Laurent., Plut. lvi), amongst excerpts from the *Rhetorica* of Menander of Laodicea,[1] there are interpolated three short musical canons. The first, with which we are concerned, occupies fol. 10*r*, lines 6–27, and its first four lines run as follows :

> *καταγράφεται τοίνυν ὁ κανὼν τὴν ὅλην τάξιν ἔχων· τοῦ προσλαμβανομένου | ὡσανεὶ ἦχον ἀπὸ τοῦ παντὸς γωνιῶν κατ' εὐθείαν τεταγμένου· διαιρεῖται | δὲ εἰς χωρία κη̅ : ἴσα τοῖς διαστήμασι· ἡ οὖν ὑπάτη ὑπατῶν ἐστίν χωρία κα̅ | ῑ καὶ ἐπὶ τοῦ ἑνὸς χωρίου· ἡ δὲ παρυπάτη ὑπάτων χω' κδ̅· ἡ δὲ χρωματικὴ | ὑπάτων· ιθ̅.*

As custodian of the Modal System, the Harmonic Canon of Florence No. 1 eclipses all other such documentary evidence. Its value is unique, for it stands as proof that the intimate connexion between the modal division by equal measure by means of a Determinant number, and the P.I.S. of the Classical Greek Period was recognized in the theory and practice of Greek music. Further, I suggest *re* ἦχος restoring the *spiritus lenis* ' as it were a Mode ' of the Greek Church since the manuscript also implies some knowledge of the later development of that system in Hellenistic Asia, which culminated in the *Octoechos* of the early Greek Church (see Appendix ii, ' Eccles. Modes ').

The salient points to which the Canon bears witness are the following :[2]

(1) It provides a practical example of the modal principle of equal measure by a Modal Determinant, applied specifically to the degrees of the P.I.S. in the Diatonic and Chromatic genera.

(2) The modal sequence extends from Proslambanomenos to Nete Hyperbolaion.

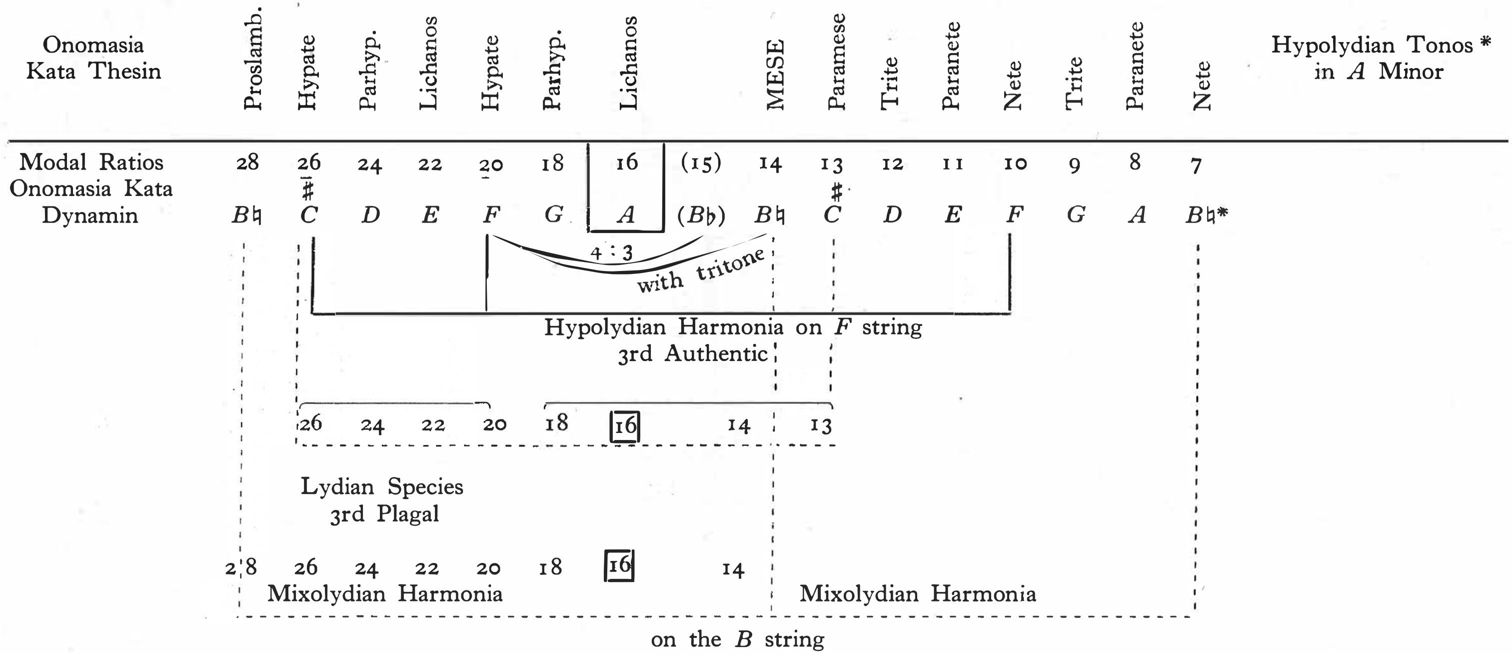
(3) The full description (in 22 lines), launches two separate, inherently modal propositions (*a*) by M.D. 28 (lines 1 to 3) and (*b*) by M.D. 24 (implied in line 3 sqq. and 15), both starting from Proslambanomenos.

(4) The text of the manuscript appears to emanate from three different authors or scribes (whom I designate as A, B and C). The original document (possibly derived from A's exemplar) is suggested by the first three lines, in which A describes in clear terms an aliquot division by 28 from Proslambanomenos—if the addition by A. Stamm of ‹ἀπὸ› in front of *τοῦ προσλαμβανομένου* be accepted. B implies a division by M.D. 24 from Proslambanomenos (in line 3 and confirms this in line 15). B, however, treats the 24 segments *χωρία* as mere units of measurement, as we should inches. Further, he assumes the Ditonal scale which necessitates the occasional fractionizing of the segments ; in this, he follows Ptolemy in principle rather than the Modal System. The chief value of B's contribu-

[1] See V. Rose, *Anecdot. Graec.*, i, 27.

[2] For the full text and discussion see A. Stamm, *Tres Canones Harmonici* (Berlin, 1881) (reviewed by F. Vogt in *Philol. Rundschau*, ii, No. 36, and by K. von Jan in *Philol. Wochenschrift*, 1882, No. 46). There is a French translation by C. E. Ruelle published with *L'Introduction harmonique de Cléonide* in his *Coll. des Auteurs Grecs relatifs à la Musique* (Paris, 1884).

FIG. 36.—The Modal Implications of the Canon of Florence according to Author 'A', lines 1–3



* The Tonos and equivalents in modern notation are hypothetical.

tion is that he establishes the fact that in his day it was customary to find the P.I.S. combined with the division of a canon into equal segments, each of which corresponded to a degree in the Diatonic or Chromatic scale.

(5) B was also aware of the procedure in using such a canon: that the numbering of the segments must proceed from the grave end, i.e. from Proslambanomenos (see Ptol., ii, 16, ed. Wallis, 1682, pp. 209–10). B knew, moreover, that the note of the numbered segment was produced by the aggregate length of that number of segments; that, therefore, fractionizing a segment extended that length into the next lower segment by the amount of the fraction, e.g. (lines 3 to 4) for 21 segments and one-tenth. B writes, ‘*χωρία $\overline{\overline{κα}}$ ῑ καὶ ἐπὶ τοῦ ἑνὸς χωρίου.*’

(6) Taken in conjunction, therefore, with the division by 28 into equal segments, the designation of each of these, numbered from 24 onwards as a degree in the P.I.S., constitutes a virtual use in practice of the two *ὀνομασίαι κατὰ θέσιν* and *κατὰ δύναμιν* of Ptolemy (op. cit., ii, 11). The numbered segments on the rule of the canon correspond to the Thesis, while the Dynamis is furnished by the implied ratios—segment by segment—of the arithmetical succession according to the M.D. This appears to be the only example extant of the practical use of the two nomenclatures, according to position and to value, signalized by Ptolemy.

(7) The Canon is introduced (at the top of fol. 10*r* of the Codex), among excerpts from Menander, as a birthday offering, and purports to present something new, for which a great future is predicted (lines 1 to 5 of fol. 10*r*). The modal implications of the text are of great interest (see Fig. 36). The division by M.D. 24 denotes the Phrygian genesis, but by basing the Thetic nomenclature upon Proslambanomenos as 24, with Mese on segment 12, we get the Dynamis from Hypate Meson on the 16th segment, which belongs by its M.D. (16) to the Hypodorian Harmonia, with its Dorian Plagal on Hypate Hypaton as 22nd segment. (B, intent on the ditonal scale, estimates it as 21 and one-tenth.)

C's contribution is of little interest to our subject: it consists merely of a ***lusus arithmeticus*** constructed from the numbered segments of the Canon.

* * *

THE COMMON MODAL STARTING-NOTE ACCORDING TO ARISTIDES QUINTILIANUS

Passing on from the underlying principle of the Harmonia to the Harmoniai themselves considered as a system, it will be conceded that its most striking feature, observed through the medium of Ptolemy and therefore already grasped by all students of Greek music, is undoubtedly the common Tonic assigned to the seven original tribal Modes, and interpreted as corresponding to our *F*. A common factor naturally suggests grouping for the purpose of comparison in order to determine points of differentiation. In a scale of tones and semitones only, however, no essential modal differentiation, such as the passage from Aristides Quintilianus quoted below would lead us to expect, is possible.

Discussing the Harmoniai of the Ancients, each of the compass of an octave, Aristides names them as they may be observed *κατ' εἶδος* on the degrees of the P.I.S. from Hypate Hypaton, and then he continues thus : 'Accordingly it is clear that if one takes the same sign first [i.e. the same starting-note] and calls it at different times by the different value of the note (*δύναμις φθόγγου*), the nature of the Harmoniai is made manifest from the succession of consecutive sounds.[1]

To anyone acquainted with the *modus operandi* for providing a differentiation of the seven Modes within the limits of the same octave, there can be no possible doubt as to the meaning of this significant passage descriptive of the correlated series of Harmoniai among the Ancients. Aristides, however, like most theorists, has not grasped the distinction between Modes and species, since it is the latter he has selected to illustrate the modal principle of differentiation.

Thus, while there is for all the Harmoniai one same initial note or Tonic, say F, this common pitch note is invested for each Harmonia with a different value. Moreover, the value—a numerical ratio, consisting of differentiated unity, having the same numerator and denominator, $\frac{22}{22}$ for the Dorian, $\frac{24}{24}$ for the Phrygian, &c.—sets going a series (*ἀκολουθία*) of sounds or ratios which determines the nature of the Harmonia. No formative values of this nature in either Mese or Tonic are to be found in the interpretation by the theorists in tones and semitones only. Aristides has thus recorded in embryo an exact definition of the basic differentiation of the group of seven Harmoniai in the Modal System, of which a graphic representation is given below. The ratio number and v.f., and the symbol of vocal Notation according to Alypius accompany each note (see Appendix i on 'Notation').

On examination of the sequence of ratios starting from the Tonic—common to all Harmoniai in respect of pitch, but invested each time with a value proper to each Harmonia—another passage from Aristides will recur to our minds. The intervals signified by these ratios are obviously those intervals other than tones and semitones 'which were used by the most ancient for their Harmoniai' (p. 21M.).

On the subject of the intervals constituting the Modal Harmoniai and the P.I.S., so far removed from the ordinary tones and semitones hitherto accepted, a few passages from classical sources may be recalled here.

In the course of Plato's dialogue between Socrates and Protarchus in the *Philebus*[2] occur the following lines :

Soc. But when you have learned what sounds are high and what low, and the number and nature of the intervals and their boundaries and proportions, and

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[1] *de Mus.*, M., p. 18, line 7 sqq. '*ἐκ δὴ τούτου φανερόν, ὡς δὲ ταὐτὸν ὑποθέμενοις σημεῖον πρῶτον, ἄλλοτε ἄλλῃ δυνάμει φθόγγου κατονομαζόμενον, ἐκ τῆς τῶν ἐφεξῆς φθόγγων ἀκολουθίας τὴν τῆς ἁρμονίας ποιότητα φανερὰν γενέσθαι συμβαίνει.*'

[2] *Philebus*, p. 17 ; Jowett's *Dialogues*, 1892, Vol. iv, p. 582.

FIG. 37.—The Seven Harmoniai on a Common Fundamental *F* within the System of

MIXOLYDIAN F=	Hyp. Hypaton **Ω** 28/28 176 v.f.	Parh. Hyp. **Ψ** 27 182·4	Lich. Chr. Hyp. **X** 26 189·5	Lich. Diat. **T** 24 205·3	Hyp. Meson. **Π** 21 234·6	Parh. Mes. **O** 20 246·4
LYDIAN F =	Parh. Hyp. **Ρ** 27/27 169·5 v.f.	Lich. Chr. H. **Ψ** 26 176	Lich. Diat. H. **Φ** 24 190·6	Hyp. Mes. **C** 21 217·9	Parh. Mes. **P** 20 228·8	Lich. Chr. M. **Π** 19 240·8
PHRYGIAN F =	Lich. D. Hyp. **Ω** 24/24 176 v.f.	Hyp. Mes. **Φ** 21 202	Parh. Mes. **Y** 20 211·2	Lich. Chr. **T** 19 222·2	Lich. Diat. **Π** 18 234·6	MESE **M** 16 264
DORIAN F =	Hyp. Mes. **Ω** 22/22 176 v.f.	Parh. Mes. **Ψ** 20 193·6	Lich. Chr. **X** 19 203·8	Lich. Diat. **T** 18 215·1	MESE **Π** 16 242	Trite Syn. **O** 15 258
HYPOLYDIAN F =	Parh. Mes. **P** 20/20 169·5 v.f.	Lich. Chr. **Ψ** 19 178·4	Lich. Diat. **Φ** 35 (18) 193·7	MESE **C** 16 212	Trite Syn. **P** 15 226	Paramese **O** 14 (28) 242·1
HYPOPHRYGIAN F =	Lich. Diat. M. **Ω** 36/36 176	MESE **Φ** 32 198	Trite Syn. **Y** 30 211·2	Paranete S. Chr. **T** 29 218·4	PN. Diat. S. **Π** 26 243·6	Nete. S. **M** 24 264
HYPODORIAN F =	MESE **Ω** 32/32 176 v.f.	Trite Syn. **Ψ** 30 187·7	PN. Chr. S. **X** 29 194·2	PN. Diat. S. **T** 26 216·6	Nete Syn. **Π** 24 324·6	Nete Diez. **M** 22 256

the Tonoi with their Modal Ratios by K. S. and Symbols of Notation from Alypius

Lich. Chr. **N** 19 259·4	Lich. Diat. **K** 18 273·6	MESE **M** 16 308	Trite Syn. **Z** 15 328·4	Paramese **Γ** 14 352 v.f.	**MIXOLYDIAN**
Lich. Diat. **M** 18 254·2	MESE **I** 16 286	Trite Syn. **Θ** 15 305·1	Paramese **Z** 14 (28) 326·8	Trite Diez. **E** 27 339	
Trite Syn. **Λ** 15 281·6	Paramese **I** 14 (28) 301·6	Trite Diez. **Θ** 27 312·8	Paranete Chr. **H** 26 324·8	PN. Diat. **Γ** 24 352	
Paramese **M** 14 (28) 277	Trite Diez. **Λ** 27 287	Paranete Ch. **K** 26 297·8	PN. Diat. **H** 24 322·6	Nete. Diez. **Γ** 22 352	
Trite Diez. **Ξ** 27 251	Paranete Ch. **N** 26 260·7	PN. Diat. **I** 24 282·5	Nete Diez. **Z** 21 322·8	Trite Hr.bol. **E** 20 339	
Nete Diez. **I** 21 301·6	Trite Hr.bol. **Θ** 20 (40) 316·8	PN. Chr. H. **H** 39 325	PN. Diat. H. **Γ** 36 352		
Trite Hr.bol. **Λ** 21 268·2	PN. Chr. H. **K** 39 288·8	PN. Diat. H. **H** 18 312·4	Nete. Hyp. **Γ** 16 352		

the scales (συστήματα) which arise out of them, which our ancestors discovered and handed down to us who follow, under the name of Harmoniai, and the affections corresponding to them in the human body . . .' &c.

It is evident that this allusion to the differentiation of the Harmoniai through the magnitude and order of their intervals could hardly have been called for by systems reducible to a single tetrachordal unit of S.T.T. structure.

Aristotle, in the *Politics*,[1] discussing citizenship and community, uses the following analogy: 'Just as we say a Harmonia of the same notes (τῶν αὐτῶν φθογγῶν) is different when it is Dorian and when it is Phrygian.' The Harmonia of the same notes means, of course, the notes from Hypate Meson to Nete Diezeugmenon κατὰ θέσιν or the strings of those names on the Kithara, but tuned κατὰ δύναμιν—in one instance to the Dorian Harmonia, in the other to the Phrygian, and differentiated by their intervals. Evidence may be seen in this of the crying need of the theorist for the double nomenclature while the Modal System was in general use, quite independently of whether the actual names made known by Ptolemy were in use or not; they were the affair of the Theorist, while for the Musician, there was the tuning. Then we have a statement by Cicero:[2] 'But we can recognize the Harmonia from the intervals of sounds, the different grouping of which makes more Harmoniai.'

Even Aristoxenus lends assistance at this juncture: in his propositions, near the end of his treatise, where he discusses divisions of the 5th according to *quanta* (μεγέθη):[3]

Again he says, if two [quanta] become equal, and two remain unequal, which will result from the lowering of the Parhypate, there will be three quanta constituting the Diatonic scale (γένος) namely, an interval less than a semitone, a tone, and an interval greater than a tone. Again, if all parts of the Fifth become unequal, there will be *four quanta* comprised in the genus in question. It is clear then that the Diatonic genus is composed of two, or of three or of four simple quanta.

This last sentence shows that the Modal System with its octave unit was not familiar to Aristoxenus as a matter of theory. It is here expressly stated that Aristoxenus recognizes a division of the 4th consisting of three unequal quanta, viz. a small semitone, a tone and a greater tone. That, of course, is one of the essentials of modality, namely a sequence beginning in the lower part of the scale with a diesis, and passing on towards the acute with intervals gradually increasing in magnitude, as in the Dorian and Phrygian Harmoniai for instance. But when Aristoxenus states that in the 5th all the four quanta may become unequal he does not explain how this may be. He has repeatedly valued the tone as the difference between the 4th and the 5th—the only tone his treatise takes into account

[1] *Pol.*, iii, 3, 1276B.
[2] *Tusc. Disp.*, i, 18: 'Harmoniam autem ex intervallis sonorum nosse possumus, quorum varia compositio etiam harmonias efficit plures.'
[3] Macran, *op. cit.*, 73 and 74, pp. 221–2; Greek text, p. 163.

—he has already allocated to the 5th a tone followed by an interval greater than a tone, so that if there are to be four unequal quanta, the 3rd tone cannot be identified with the tone as understood by Aristoxenus in his treatise, and yet he does not define it: he is clearly out of his depth here, or he would not have failed to explain the fact that he names two intervals both called tones without qualification, and yet reckons them as unequal quanta. The modal Dorian pentachord of ratios, $\frac{11}{10} \times \frac{10}{9} \times \frac{9}{8} \times \frac{8}{7}$ fulfils the conditions, providing we assume that neither 4th nor 5th was what is known as Perfect. The interval greater than a tone in the proposition is the Septimal, functioning here as Tone of Disjunction. The Phrygian Harmonia does not fulfil in its Tonic pentachord the conditions of the proposition, since it includes no interval larger than a Tone, $\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8}$. The important fact remains that the four quanta of Aristoxenus prove the existence in his day of scales consisting of intervals other than tones and semitones, as, indeed, do many other passages in his treatise. Had Aristoxenus continued his analysis of the Diatonic with four quanta with real knowledge of the subject, he would have found himself obliged to admit an increase in his proposition to seven unequal quanta in the octave scale.

It will later be suggested that the Shades of the Genera of Aristoxenus, according to his own evaluation by 12ths of tones, were in reality modal tetrachords.

Plutarch [1] likewise contributes evidence of this nature at the end of a long and interesting passage on the respective values of ancient and modern music. He begins by recalling the fact that the venerable Pythagoras rejected the judgement of Music through sensation . . . wherefore he did not judge by the hearing but by the proportional Harmonia (τῇ δ' ἀναλογικῇ ἁρμονίᾳ), which is clearly a reference to the modal Harmonia. Then follows a discussion on Enharmonic and other so-called odd (περισσός) and even (ἄρτιος) intervals in a vein reminiscent of Aristoxenus. Those terms *Artios* and *Perissos* among Harmonists would certainly have borne a more rational meaning connected with the modal progression, and not with the dividing up of intervals into an even or odd number of dieses. The significance of the passage lies in the admission that musicians were in the habit of slackening not only the *Tritai* and *Paranetai*, the *Lichanoi* and *Parhypatai*, but even the fixed notes (ἑστῶτες) also of the tetrachords, and indeed esteeming as best of all the use of systems containing a majority of such irrational intervals, by which is clearly meant, not intervals for which no ratio can be found, but rather those which cannot easily be determined by ear, since Plutarch is evidently using an unacknowledged quotation from Aristoxenus. Moreover, the octave scale which forms the subject of discussion here consists of tetrachords which are not Perfect 4ths; and this alone excludes any possible identification with the *Chroai*

[1] *Op. cit.*, (Weil and Reinach, pp. 148–56, Cap. 27 and 28, p. 1144F., pars. 389 sqq.).

of Aristoxenus or Ptolemy. It seems justifiable to conclude that they were Modal Species; of these only the Hypodorian

$$\underbrace{16 \quad 15 \quad 13 \quad 12}_{4:3} \quad \underbrace{11 \quad 10 \quad 9 \quad 8};$$

the Phrygian,

$$\underbrace{12 \quad 11 \quad 10 \quad 9}_{4:3} \quad 8 \quad 7 \quad 6;$$

and the Hypolydian with Synemmenon,

$$\underbrace{20 \quad 18 \quad 16 \quad 15}_{4:3} \quad 13 \quad 12 \quad 11 \quad 10;$$

have Perfect 4ths on the Tonic. After a perusal of the Polemics of Aristoxenus and of the analysis of the *Chroai* as modal tetrachords (given further on), the evidence afforded by this passage will probably be felt to be still more conclusive in favour of the Modal System as actual fundamental basis of the Musical System of Ancient Greece, one phase alone of which has been described by the Theorists, viz. the Aristoxenian.

THE MESE AS ARCHE

There is no single element in the Musical System of the Greeks, the importance of which is so insistently emphasized as is that of the MESE or keynote, further qualified by Aristotle and others as *ἀρχή* = beginning, generator, cause, essential element, and also as *ἡγεμών*, leader, ruling element. Certain important passages dealing with Music in the Classics, and in the writings of the Theorists, still provoke discussion on account of difficulties concerned with the nature of Mese, which require elucidation. To the query about the identity of Mese with the Tonic of modern Music, an unqualified negative may be returned. The Mese (*μέση*) or Arche (*ἀρχή*) of the Modal System was primarily causative, conditioning, by the inherent power of the note (*δύναμις φθόγγου*), the values or ratios of all other notes constituting the modal sequence; the position of Mese on its own characteristic degree of the modal octave was, as already seen, not fortuitous but due to its essential nature and order in the Harmonic Series, which is inevitably brought into operation by the principle of equal measure or division.

The essential nature of Mese was responsible for the *Ethos* of the Harmoniai, and its position in the Harmonia for the *tessitura* of the scale, which has frequently been mistaken for an indication of pitch or key.[1]

Evidence supporting these essential characteristics claimed for the Modal Mese exists in a passage in the *Εἰσαγωγή* of Ps-Euclid,[2] which may be thus interpreted: 'The Mese is the power [or value] of the note',

[1] See H. S. Macran, *Aristoxenus*, p. 73.

[2] *Intro. Harm.*, M., p. 18, line 26 sq.; p. 19, line 1: '*ἔστι δὲ μέση φθόγγου δύναμις*' (p. 18, l. 26) and further, '*ἀπὸ δὲ τῆς μέσης καὶ τῶν λοιπῶν φθόγγων αἱ δυνάμεις γνωρίζονται, τὸ γὰρ πῶς ἔχει ἕκαστος αὐτῶν πρὸς τὴν μέσην φανερῶς γίνεται*' (p. 19).

and further, 'By means of the Mese the values of the rest of the notes become known. For in what manner each of these is related to Mese is apparent.'

The Dynamis, i.e. Modal Determinant, ascribed by Euclid to Mese, is the inherent proportional value of the keynote in the Modal System in relation to its Tonic, and the problems of Ps-Aristotle [1] afford further evidence of this Dynamis. These oft-quoted problems are numbered 20, 25, 36, and 44; all give versions of the same fact, observed but not understood by the writer:

> Why is it [he asks] that if, when the strings of the Kithara have once been tuned, the pitch of the Mese is subsequently altered, all the other notes sound out of tune when the Kithara is played, not only is this the case when the Mese itself is played; but for the whole melos, whereas, when the pitch of the Lichanos or of any one of the other notes is altered, either accidentally or purposely, that note alone is observed to be out of tune?

On reading these problems over carefully, it will at once be seen that we have to do here with an entirely new conception of a keynote, possessed of powers foreign to the Tonic in the modern sense, but corresponding in every respect with those of the modal keynote. In Problem 44 (pp. 34–5), it is expressly stated that the discussion round the Mese relates to the Harmonia of the Ancients, having seven strings only between Hypate and Nete: 'ἢ ὅτι ἑπτάχορδοι ἦσαν αἱ ἁρμονίαι τὸ παλαιόν,' and therefore, it may be concluded, to the scale of Terpander, having ratios:

Hyp. Mes.			Mese	PM.	PN.	Nete
11	10	9	$\boxed{8}$	7	6	11 (5½)

(The Primitive Dorian Mode)

The two Elgin pipes in the Graeco-Roman Department at the British Museum (*c.* 500 B.C.) have been bored to give the ratios of this scale from the first hole, as explained further on. The answer given in Problem 20 burks the real issue, and is very lame indeed. The attempt to explain the causative power of Mese implied in the statement, by the use made of it in composition and in playing, seems futile. In Problem 36 (pp. 36–7) a satisfactory attempt has been made to explain how the mistuning of Mese comes to create the impression that the strings are out of tune:

> Is it because, in the case of the whole of the strings, to be in tune consists in the notes of the scale being in a definite relation to Mese which determines the [tension] [τάσις, *emendavit Th. R.*, τάξις *codices*] or order of each one of them; so that the destruction (or loss) of the Mese means loss of the responsible basis (or cause) and of the unbroken series.
>
> For the Mese alone of those [notes] between the extremes is the *Arche* or first principle [Probl. 44, p. 34].

The Arche or Mese of these Problems is the dynamic or causative keynote of the Greek Modal System, occupying in each Mode an individual degree in the Harmonia. It was responsible for the magnitude and order

[1] *Les Problèmes Musicaux d'Aristote*, par F. A. Gevaert et J. C. Vollgraff, *Texte grec avec Traduction française, commentaire*, &c., Gand, 1903, pp. 34–9; cf. Monro, *op. cit.*, pp. 42–7; Macran, *op. cit.*, pp. 70 sqq.

of the intervals in every Modal Scale, and for the exact intonation of the different notes in relation to itself and to each other, by virtue of the ratio inherent in the equal division by the Modal Determinant. Such a Mese is found in the Modal System alone; it bore in each Harmonia the ratio of some octave of the Arche, therefore, 8, 16 or 32, and was invariably followed in descending motion by the same ratios in every Harmonia in the Diatonic genus, the modal characterization consisting in the point at which the Tonic interrupted the descending sequence, and the modal interval it consequently formed with Mese. This common sequence downwards from Mese was, therefore, independent of the degrees of the scale, since the position of Mese was inevitably fixed by the Modal Determinant of each Harmonia. The sequence runs as follows (see page 185).

It will be realized that in a musical system in which attention was primarily focused on Mese or Arche, the mistuning or altering of this note after the other notes of the sequence had been tuned from it, would at once be perceived by the musician, for every interval expected by the ear would be falsified. Such an absolute dependence on Mese for the melodic elements of the scale may perhaps be best realized through the modal significance of each interval in the descending sequence from Mese: the tone led the way down through the Hypophrygian; the major 3rd through the Hypolydian; the Harmonic 4th through the Dorian; the 5th through the Phrygian, &c. The Harmonia itself was in fact a realization of the unity of the seven Harmoniai in one.

Mese was thus at the same time the Arche, and the Hegemon or leader into the spheres of the Modal Species, and Mese was also the bond of union (σύνδεσμος), the central pivot or connecting-link binding together the Modal Species through a central identity of pitch—though not of compass—into a rational system.

A passing reference may be made here to Gevaert's[1] commentary on these Aristotelian Problems, which occurs after quoting an interesting statement by Dio Chrysostom of Prusa (A.D. 40–115) on the tuning of the Greek lyre:

> We must, as in the lyre, after establishing the note Mese, then tune the rest [of the strings] to it, for if we do not, they never produce any Harmonia; so in life, comprehending the best and displaying this as a limit, we must do everything else with reference to it, for if not our life is likewise untuned and inharmonious (ἐκμελής).

Gevaert accepts these lines as a confirmation of his own interpretation of the Mese question in the Ps-Aristotelian Problems. According to his conception, Mese figures as 'the daily generator of the musical scale on the lyre and Kithara of the Greeks' which he imagines to have been carried out by tuning in 4ths and 5ths up and down, from the Mese *A* within the pre-determined octave *E* to *E*.

Gevaert claims that given a musician gifted with a correct ear, this

[1] Ps-Arist., *Probl.*, 20, 25, 36, 44, Gevaert's ed. The Mese from which all other notes are tuned.

Ps-Euclid, M., pp. 18, bottom, and 19, top, the φθόγγον δύναμις. Dio Chrys. of Prusa, Or. 68, *de Gloria*, Teubn. Texts, 1919, Vol. ii, Orations, p. 219.

FIG. 38.—The Descending Modal Sequence from Mese

Descending Ratios from Mese or Arche (Modal Sequence)			
8 or 16	Mese Hypodorian		
9 ,, 18	16 / 18	Tone 9 : 8	**Hypophrygian**
10 ,, 20	16 / 20	Major 3rd 5 : 4	**Hypolydian**
11 ,, 22	16 / 22	Harmonic 4th 11 : 8	**Dorian**
12 ,, 24	16 / 24	Perfect 5th 3 : 2	**Phrygian**
13 ,, 26	16 / 26	Harmonic 6th (flattened major) 13 : 8	**Lydian**
14 ,, 28	16 / 28	Harmonic 7th (flattened minor) 14 : 8	**Mixolydian**
15 ,, 30	Major 7th, distinctive of ' Hypo ' modes		

method of tuning will unfailingly produce a scale, the members of which are related by consonances of 5ths and 4ths, and by diaphonies of 3rds in accordance with the Pythagorean ratios. Whether this complacent method of dealing with cycles of 5ths leads to a correct estimate will become apparent: it exhibits a Mese which bears no relation to the one described in the Problems; it could not be said to determine the pitch of the other notes, for any one of these, taken as Mese, could equally well be made to produce the foregone result (viz. the white notes on the keyboard from *E* to *E*) by allowing an arbitrary interruption of the progression of 5ths as illustrated by Gevaert. Therefore, to change the intonation of the Mese in such a scale obviously could not have the result described in the Problems. To take *A* as causative Mese within the octave *E* to *E* and to practise a systematic tuning, by 5ths and 4ths, as opposed to Gevaert's arbitrary method, produces the scale of *E* major, but having the intonation of the ditonal scale instead of the one given by Gevaert. Elsewhere in his commentary, Gevaert, by his reference to Mese conferring upon the other notes an exact degree of intonation, marks the difference between his idea of the function of Mese in fixing the *key* of the scale, whereas in the Harmonia the Mese created the *Mode* first, and as inevitable implication, a certain key. The point raised by these Problems and satisfactorily solved by the modal keynote is, after all, not so much the effect of mistuning this or that note, as the bringing out of the fact that the scale in question has a rational basis dependent on the Mese, and that although the compiler is not equal to giving an explanation of that rational basis, he recognizes the existence of such a law underlying the structure of the scale in general use upon the Kithara.

Another puzzle is presented for discussion in Problem 33 (pp. 30–1). 'Why is a progression [of sounds] from high to low better adjusted than from grave to acute? Is it that in this order we begin with (Arche) the beginning, since the Mese and ruling element (Hegemon) is the highest of the tetrachord (τετραχόρδου)[1] but with the reverse order we begin with the end (τελευτή). Monro,[2] who discusses these Problems in his chapter on Tonality, is frankly perplexed and asks in this context: 'In what sense, then, was the Mese a beginning (ἀρχή) and the Hypate an end (τελευτή)?' If this question be referred to the Modal System it presents no difficulty whatever. It is clear that the reference here is to the generation of the modal material from the *Arche*, and not to the laws of composition. The proportional progression downwards from *Arche* is the natural outcome of the equal division of the string by a Modal Determinant, and is indicated in the text by the use of the opening words *Διὰ τί εὐαρμοστότερον.*

It is precisely the allusion to something besides the mere succession of sounds, something basic having a recognized beginning and end; the reverse in some undefined, subtle way of those of a melody, which gives the Problem a special value as a confirmation of the existence and use of the Modal System, which is characterized by these very features.

[1] παραχόρδου codices; τετραχόρδου correxit Bekker. The reading of the manuscripts is probably correct.

[2] *Op. cit.*, pp. 44–6.

The difficulty is created here by the confusion in the mind of the writer between two different propositions: (1) The generation of a Modal Scale from its root or ἀρχή, i.e. the question of the internal structure of the sequence, according to the ratio number, and order of the notes produced by Modal Genesis through the Determinant number used in the equal division: the Mese in this sense is dynamic. (2) The function of that ἀρχή, as an element of melody in the Modal Scale, when the answer to the Problem very properly points out that the beginning is made from the end (τελευτή), Hypate Meson, the Tonic or string-note. But the string-note was the first cause, which gave Mese the pitch value of its Determinant number; so the Mese under the guise of Arche becomes generator in turn of a series, which finds its natural and inevitable limit, τελευτή, in the string-note, as differentiated unity, consisting of equal numerator and denominator; the whole string is, in fact, the impassable barrier to further generation, and therefore bearing the *highest* number in the genesis $(\frac{22}{22}$ for the Dorian): hence the apparent contradiction in the nomenclature of Hypate = highest supreme, as the lowest note of the Harmonia considered as modal material. But Hypate was also the legitimate beginning of the ascending Harmonia as Tonos.

It may be recalled in this connexion that Plutarch[1] has a word on the same subject: 'For the highest and first was by the Ancients termed Hypaton'.

Problem 33 is a good example of the method of exposition adopted in this compilation. The opening statement in each probably represents an observed fact, or a dictum taken from the teachings of Aristotle, or of such portions of those of the Harmonists as were allowed to penetrate into the outside world from the inner sanctum of the schools of those custodians of the Modal System. For these questions the compiler seeks to provide elucidation, given usually in tentative fashion, as though conscious of treading on debatable ground. The fact that the same opening statement occurs in more than one problem, while different solutions are suggested (as, for instance, in Nos. 36 and 20) seems to indicate that the point was considered of some importance, but that no authoritative explanation was available.

To resume: the Mese of Problems 36, 33, 20, 44 and 25 is presented under five different aspects, which sum up the principal functions of the Dynamic Mese thus:

(1) *Mese:* The central note of the standard Dorian Harmonia, falling on the 4th degree as centre of the Tonos, and the Modal or Dynamic keynote.

(2) *Arche:* as the beginning, root or generator of the modal material, from which all the other members are generated according to exact ratios inherent in the arithmetical progression implied by the equal division through the Modal Determinant.

[1] *Plat. Quaest.*, ix, 1; Didot, ii, 1233–4: 'τὸ γὰρ ἄνω καὶ πρῶτον ὕπατον οἱ παλαιοὶ προσηγόρευον', quoted by Weil and Reinach, *de Mus.*, p. lviii.

(3) *Hegemon :* the ruling note—not in the modern sense—but as the principal element in the Modal Genesis ; the leader in the descending progression.

(4) *Synechon :* the keynote as cohesive element, keeping the line unbroken in a continuous sequence, by virtue of the arithmetical progression which it engenders as *Arche*.

(5) *Syndesmos :* that which binds together the Mese as connecting-link or conjunctive element in the Modal Scale, exercising its influence in both directions, acute and grave (Pr. 20, pp. 38–9). Syndesmos also implies the connecting-link of a common Mese of the same pitch for the seven correlated Modal Species.

There is one more passage of interest in this connexion from Plutarch : [1]

> To give the nomenclature by virtue of their positions, the first ones and the middle ones and the last ones, would be absurd seeing that the Hypate itself in the lyre is the topmost and first, but in the *Auloi*, the lowest and last, and moreover, it is the Mese, in whatever place one would put it in the lyre, if it be tuned in the same way, that sounds higher than the Hypate and lower than the Nete.[2]

This appears to contain an allusion to the nomenclature by position (the *ὀνομασία κατὰ θέσιν* of Ptolemy) and also to the modal feature of a Mese placed on a different degree in each Harmonia. The fact that Ptolemy was the first to give a name to the two nomenclatures does not, of course, indicate that a new necessity had only then arisen. On the contrary, the fixed names of the strings of the Kithara, whatever their number, constituted from the beginning the *Onomasia Kata Thesin*, and the modal tuning given to the strings—which was the affair of the musician—was the *Onomasia Kata Dynamin*, solely dependent on the Mese or Arche, from which all other notes are tuned. If the Kithara were tuned to the Phrygian Harmonia, the Mese would fall on the 5th degree above Hypate Meson, i.e. on Paramese ; if tuned to the Mixolydian on the 7th degree, Paranete Diezeugmenon. Modality provided the necessity for the theorists' double nomenclature.

From the Mese, we pass on to evidence concerning Harmonia, Tonos, Tropos and Species (*εἴδη*), which sometimes seems to imply identity, at others distinctions.

It has been claimed in Chapter iv that Harmonia, Tropos, Tonos and Species are all four in essence Modes differentiated as to function. The Harmonia was the modal octave, which may be obtained on the half string, after equal division by a constant Determinant has been carried out. The Harmonia was the central unit or nucleus, which developed into the P.I.S. through the grouping of the Species on the Kithara, as string by string was added,[3] until a complete octave of each of the original seven Modes

[1] *Plat. Quaest.*, ix, 2 (p. 1234, Didot, ii).

[2] '*Ἢ τὸ μὲν τοῖς τόποις ἀπονέμειν τὰ πρῶτα καὶ τὰ μέσα καὶ τὰ τελευταῖα γελοῖόν ἐστιν, αὐτὴν τὴν ὑπάτην ὁρῶντας ἐν μὲν λύρᾳ τὸν ἀνωτάτω καὶ πρῶτον, ἐν δ' αὐλοῖς τὸν κάτω καὶ τὸν τελευταῖον ἐπέχουσαν, ἔτι δὲ τὴν μέσην ἐν ᾧ τις ἂν χωρίῳ τῆς λύρας θέμενος ὡσαύτως ἁρμόσηται, φθεγγομένην ὀξύτερον μὲν ὑπάτης, βαρύτερον δὲ νήτης.*'

[3] It is, of course, not necessary to postulate a Kithara of five strings for the practical use in melody of the seven octave species, forming the double octave of the Tonos known as the P.I.S. The second octave from Mese to Nete Hyper-

had been obtained ; this modal sequence of two octaves was then recognized by the Theorists as a standard scale of definite form.

The correlated Tonoi or Tropoi were shown to be curtailed Modes taken on a common fundamental or string-note, and comprising merely the Dorian Species, taken in each of those Modes as a transposition scale, and retaining the tonality proper to each Mode. The four terms, Harmonia, Tropos, Tonos and Species, are all expressive, therefore, of different phases of modality ; the order followed by the first three of these is that of their Mese, according to its position as a member of the Harmonic Series, whereas the order of the species is that of its Tonic, as it occurs in the Tonos, and according to the ratio it bears therein.

Plutarch [1] several times refers to the Harmonia, Tropos and Tonos as meaning the same thing, for instance : 'We must call the first five either Tonoi or Tropoi or Harmoniai. . . .' Elsewhere, again, Plutarch [2] has something to say on this subject ('An seni sit gerenda', Cap. 18) :

> For if it were fitting for them to continue singing, it was necessary, since there are many *Tonoi* and *Tropoi* of the voice, which the musicians call Harmoniai, not to pursue the high (ὄξυ) and the syntonon together, being old men, but to continue in that which is easy and has the fitting ethos.

Besides the comprehensive statement concerning Tonoi, Tropoi and Harmoniai as meaning the same thing, Plutarch thus makes it clear that *Syntonon* when qualifying a scale or species is not synonymous with high-pitched. It may, however, signify (1) a scale in which the characteristic interval has been raised—i.e. is more tense, as will be seen in the case of the Spondeiasmos ; or (2) the syntonos may refer to the straining or tense effect of a Modal Scale having a keynote situated on a degree at the acute end of the scale, such as the Lydian on the 6th degree above the Tonic, and the Mixolydian on the 7th degree. An *a priori* statement has been supplied in Chapter i of the Modal System, as still in general use in the days of Plato and Aristotle, a detailed exposition of which, however, is lacking in classical and later sources. The theoretical knowledge of this system seems to have been mainly possessed by the Harmonists—custodians of the Harmonia—and to have been taught and practised in their schools. For evidence of the correctness of this statement, the present writer depends

bolaion was probably obtained by octave harmonics of the strings from Proslambanomenos or Hypate Hypaton—a practice known as magadizing. Soph., Fr. 228 ; Ps-Arist., *Probl.*, xix, 39*b* (1) ; Gev., p. 20 ; 18, p. 22 ; and 9, p. 76. This device is, of course, purely melodic. If the finger producing the harmonic octave be immediately withdrawn, by an elastic movement, after the pressure on plucking the string, there is very little perceptible difference in the timbre of the note so magadized, when played in sequence with notes given by the whole string.

[1] *De E Apud Delphos*, Cap. 10 (Didot, p. 475) : '*πέντε τοὺς πρώτους εἴτε τόνους ἢ τρόπους εἴθ' ἁρμονίας χρὴ καλεῖν ὧν ἐπιτάσει καὶ ὑφέσει τρεπομένων κατὰ τὸ μᾶλλον καὶ ἧττον αἱ λοιπαὶ βαρύτητες εἰσι καὶ ὀξύτητες.*'

[2] *An Seni sit gerenda*, Cap. 18 (Didot, p. 968), and Reinach, *op. cit.*, p. lix and lx. '*"Ωσπερ γὰρ, εἰ καθῆκον ἦν ᾄδοντας διατελεῖν, ἔδει, πολλῶν τόνων καὶ τρόπων ὑποκειμένων φωνῆς, οὓς ἁρμονίας οἱ μουσικοὶ καλοῦσι, μὴ τὸν ὀξὺν ἅμα καὶ σύντονον διώκειν γέροντας γενομένους, ἀλλ' ἐν ᾧ τὸ ῥᾴδιον ἔπεστι μετὰ τοῦ πρέποντος ἤθους,*' &c.

partly upon the light thrown on the subject by the following twelve Polemics of Aristoxenus against these Harmonists.

THE TWELVE POLEMICS OF ARISTOXENUS AGAINST THE HARMONISTS

In his Elements, Aristoxenus, burning with enthusiasm for an entirely different musical and non-Modal Scale, of which he seems to have been the first exponent in writing, provides by his polemics against the Harmonists unique and valuable information concerning the principal features of the musical system studied in their schools, which are found to be identical with those of the Modal System.

These Harmonists were a recognized body of professors of Harmonic (ἁρμονική), the name given by them to that branch of the theory of music that treats of the Harmonia, or Modal Scale—not as Aristoxenus erroneously states of the Enharmonic Genus—of its natural basis and of all the implications thereof: intervals, ratios, scales, genera, modality, tonality, &c. Among the Harmonists were to be found the leading exponents of the science and philosophy of music initiated in Greece by Pythagoras, such as Lasos [1(1)] of Hermione, the music-master of Pindar, Agathocles,[1(2)] Pythokleides [1(3)] of Ceos, an Aulete with whom studied Damon [1(4)] of Athens; Archytas [1(5)] of Tarentum—the last two are known as the music masters of Plato—Lamprokles [1(6)] of Athens, Epigoneios,[1(7)] the inventor of an instrument of many strings, probably required for the study of the Harmoniai and for the practical realization of the diagrams used in the schools of Harmonic. Eratocles,[1(8)] the expounder of the different figures of the modal octave, later known as the species (εἴδη), who was singled out by Aristoxenus from among his predecessors for a special and undeserved polemic. Pythagoras [1(9)] of Zacynthus and Agenor [1(10)] of Mitylene, leaders of the school in which the Modal Scales were expounded. Hippasus [1(11)]

[1] (1) LASOS, see Plut., *de Mus.* (ed. Weil and Reinach), p. 115, notes 292, 293, 294; Gev., *Probl.*, p. 103 and Note; *Aristoxenus*, p. 3, trans. Macr., p. 167, Suidas, s.v.; Athen., x, 455*c*, xiv, 624*c*; Herod., vii, 6.

(2) AGATHOCLES, Plut., *op. cit.*, p. 64, Note 156; p. 66, Note 157; Plato, *Laches*, p. 180D; Monro, p. 104.

(3) PYTHOKLEIDES, Osk. Paul., *Die absolute Harmonik d. Griechen*, p. 13, note 38; from *Schol. Plat. Alc.*, p. 118C. He seems to have had as music pupils not only Damon, but Agathocles and Lamprokles; Plut., *op. cit.*, p. 67.

(4) DAMON., Plut., *op. cit.*, p. 67; Plato, *Rep.*, iii, 400B; iv, p. 424C; Aristides Quint., *de Mus.*, ii, p. 95M.

(5) ARCHYTAS, Ptol., *Harm.*, I, xiii and *passim.*

(6) LAMPROKLES., O. Paul, *op. cit.*, p. 13, note 39; Son of Midon, *Schol. Aristoph.*, *Nub.* 967, composer of a Hymn to Pallas Athene mentioned by Aristophanes; Athen., xi, 491C; Plut., *de Mus.*, Cap. xvi, Weil and Reinach, pp. 64–5.

(7) EPIGONEIOS, Laloy, *Aristox.*, pp. xv and xxviii; Athen., iv, 183D; xiv, 637F; *Aristox.*, Macr., 3, p. 97, 6.

(8) ERATOCLES, *Aristox.*, Macr., 5, 6, pp. 168–9; Laloy, p. xvi. *Porph. ad. Ptol.*, Wallis, p. 189.

(9) PYTHAGORAS of Zacynthus; Laloy, p. xxix; *Aristox.*, Macr., 36–7; Athen., xiv, 637.

(10) AGENOR, ibid.

(11) HIPPASUS of Metapontum. *Theo of Smyrna*, ed. J. Dupuis, p. 97.

of Metapontus, a follower of Pythagoras, who tested musical ratios by means of vases of different capacities partly filled with water, which on being struck gave out notes of different pitch. He may perhaps be included among the early Harmonists on account of his speculative research in the ratios of the Harmonia; there were many others.

These Harmonists, who seem to have been the chief custodians and teachers of the mysteries of the Modal System, obviously had a large and influential following among musicians and theorists, as may be inferred from the untiring efforts of Aristoxenus to belittle their achievements.

Although the system of the Harmoniai had by inference been in general use for centuries in Ancient Greece, as elsewhere, its theoretical and scientific basis appears for the most part to have remained an impenetrable mystery not only to the Greeks, but likewise to every other nation with the possible exception of the Persians.[1]

The evidence afforded by these polemics of Aristoxenus, although inferential rather than positive, is the most comprehensive we possess upon the nature and scope of the Modal System. The chief indictments of Aristoxenus against the Harmonists may be recapitulated at this point thus (where not otherwise stated the references are to Macran's edition):

Polemics 1 and 2.—Concerning the Harmonia

POLEMICS 1 AND 2.—That their tables of scales exclusively represent the Enharmonic genus (p. 165).

> The early students of Harmonic [Aristoxenus states], (*a*) investigated the Harmonia [Enharmonic, Macran] alone (τῆς ἁρμονίας) without devoting any consideration to the other genera.[2] (*b*) *This may be inferred* from the fact that the tables of scales presented by them are always of Enharmonic scales, never in one solitary instance of Diatonic or Chromatic, and that too, although these very tables in which they confined themselves to the enumeration of Enharmonic octave scales, (2) *nevertheless exhibited the complete system of musical intervals.*

This passage has been quoted at length to show how little Aristoxenus is to be relied upon for the formation of any serious judgement on the music of his predecessors or rivals. His first statement (*a*) is categorical and unequivocal, and yet in (*b*) the assurance has in a measure disappeared, and what was stated in (*a*) as a fact, becomes an inference, based upon insufficient acquaintance with the tables of scales which he obviously failed to understand. The fact revealed (see 2) that these scales exhibited the complete system of musical intervals, at once makes the allusion clear.

[1] See Kiesewetter, *Musik der Araber und Perser*, 1842; Mahmud Schirazi Durret, ed. Tadsch, fourteenth-century Encyclopaedist, reproduced in abstract with comments by Dr. Hugo Riemann: *Studien zur Gesch. d. Notenschrift*, Leipzig, 1878. See pp. 77–85. See also Al-Fārābī, Rouanet, *op. cit.*, pp. 2685 sq. and 2711 sq. Safi ed-Din, *Le Traité des Rapports Musicaux ou l'Epître à Scharaf ed-Din*, par le Baron Carra de Vaux, Paris, 1891, pp. 16 sqq. The MSS. of Persia, anterior to the twelfth century, have nearly all perished, either taken or destroyed by the Arabs. See also Chaps. vii and ix.

[2] Cf. Plut., *de Mus.* (= Aristoxenus) (ed. Weil and Reinach), pp. 133–5, §§ 331–6 Macr., *op. cit.*, p. 169.

The tables of scales were apparently given in the form of Katapyknotic diagrams of the modal material available for the three genera in all seven Modes. It is by no means necessary to have separate diagrams for each genus, for the Enharmonic Katapyknosis comprises within itself also the modal material for the Chromatic and the Diatonic genera which are thus implicit in the Enharmonic. It is a simple matter to obtain the Chromatic and Diatonic genera through the division by 2 for each genus, of those numerators of the fractions indicating the lengths of string, which consist of even numbers only. For example, in the Dorian Mode

Enharmonic	44	43	42	41	40	39	38	37	36	
Chromatic	22		21		20		19		18	&c.
Diatonic	11				10				9	

Polemic 3.—The Close-packed Scales of the Harmonists

POLEMIC 3.—That the Harmonists in their pursuit of Katapyknosis or ' close-packed schemes of scales ' fail to demonstrate the interrelationship of keys for the purposes of modulation (p. 170). This statement is forthwith qualified by Aristoxenus, who adds that although some of the Harmonists have touched upon the subject, no single writer belonging to this school has taken up the general point of view. To this the Harmonists would very properly reply that the keys would be demonstrated in other diagrams, the Katapyknotic division being a Modal and not a Tonal concern.

The Modal System offers unique possibilities of modulating from key to key, or of merely changing the pitch location of a composition by transposition, which are equally unknown in the Aristoxenian and in the modern system, such as the following:

(*a*) by changing the species;

(*b*) by changing the Mode, whereby the compass of pitch remains the same, while through the modal shifting of the modal pivots and consequently of the *tessitura*, a different region of the voice comes into use. To change the species affects both the compass and the regional disposition of the melody.

(*c*) by changing the pitch of the species by the mere expedient of taking it as the species of a different Mode, which is what happens in the *Tropoi* or *Tonoi*. Ptolemy has demonstrated this, but without realizing that a change of Tonos for this purpose, expressed by means of the *Onomasia Kata Thesin*, becomes a change of modality.

(*d*) by tuning the monochord string to a different pitch note or Tasis (τάσις). All or any of these methods may well have been exhibited in the diagrams of the Harmonists without Aristoxenus being aware of the fact, since he was unacquainted with their theories, and would not understand the significance of the process indicated (Macran (37), p. 192).

Polemic 4.—Concerning the Tonoi

POLEMIC 4.—Aristoxenus says (Macran, pp. 128 and 192–3): ' The fifth of our parts is the one about the Tonoi, placed upon which the systems are melodized. Upon these matters no one has provided any explanation, neither what Tropos must be taken, nor upon what consideration their number must be assigned.' [1] There is, of course, a number (the Determinant) that belongs to each Tropos, or Tonos used in the P.I.S., which makes the passage perfectly clear.

Aristoxenus then continues his polemic: 'that in their account of the Keys τόνοι the Harmonists are not agreed among themselves . . . Others again having regard to the boring of fingerholes on the *Auloi* assume intervals of three quarter-tones between the three lowest Keys, the Hypophrygian, the Hypodorian and the Dorian, &c. . .' The order of the Tonoi and of the intervals between their Tonics is certainly slightly confused, for which Aristoxenus is probably to blame, not knowing that the Tonoi are curtailed Modes (see Chap. iv), and that therefore they were rightly referred in classification to their keynotes, as they occur in order in the Harmonic Series. The order of the Species, on the other hand, being that of their Tonics as they appear in the Tonos, is the reverse of that of the Modes. Because the order of progression in the Harmonic Series is reversed in the Modal Series, moreover, the fact that some of the Harmonists may have reckoned from the Conjunct forms of the Tonos, and others from the Disjunct, may have added to the bewilderment of Aristoxenus; the modal significance of this will be apparent further on. The passage concludes with the plaint: 'But they have not informed us on what principle they have persuaded themselves of this location of the Keys and that the close-packing of small intervals is unmelodious and of no practical value whatsoever, will be clear in the course of our discussion.' [2]

In the closing part of the polemic, Aristoxenus harps again on the Katapyknotic diagrams of the Harmonists, while failing to state to what use the Harmonists claimed to put them.

Polemic 5.—Notation as the Goal of the Science of Harmonic

POLEMIC 5.—'That some of the Harmonists find the goal of the science of Harmonic to be in the notation (παρασημαντική) of melodies, declaring this to be the ultimate limit of the apprehension of any given melody.'

The Harmonists rightly claimed that only through the science of Notation could the true limits, or, in other words, the exact intonation, of the melos be ascertained; a claim, the value and significance of which may be realized when the full modal scope of the alphabetical scheme of notation has been revealed (see brief outline in Appendix).

Polemic 6.—The Theory of the Aulos and of the Pipe-scales

POLEMIC 6.—Aristoxenus continues thus: 'Others again find it in the theory of the Auloi, and in the ability to tell the method by which the pipe-scales are produced and their provenance,' [3] meaning generally that

[1] Translation by E. J. considered more literal and more correct than Macran's. —K. S.

[2] Macr., pp. 129 and 193.

[3] Translation kindly supplied by Prof. J. F. Mountford, Nov., 1920. Macr., p. 130.

some lay more stress on the study of notation, others on the theory based on the structure of the Aulos, and the position and measurements of the lateral holes to be stopped by the fingers. Here, then, is the crux of the matter. That these claims on behalf of the Harmonists should have been made by Aristoxenus, even in an attitude of scepticism, is of utmost importance, for they are precisely the claims made on behalf of the Modal System. ' If a man notes down the Phrygian scale,' said Aristoxenus, ' it does not follow that he must know the essence of the Phrygian Melos.' It depends, of course, on the man, but it is evident that the essence of the Phrygian Melos is there in the Notation, to be had for the asking: just as the Mode is inherent in the boring of the pipe, as demonstrated in Chapter i.

The system of Greek Notation, as used by the Harmonists, was by no means entirely adequate, but given a knowledge of its modal basis, through the use of the alphabetical symbols in sequence in the Harmonia, the exact intonation of any note was ascertainable; such subtlety cannot be claimed for our modern Notation. Later the system received additions in respect of time values.

Polemic 7.—The Aulos as the Foundation of the Order of Harmony

Polemic 7.—That according to the Harmonists, the foundation for the order of Harmony (*τῆς τοῦ ἡρμοσμένου τάξεως*) is supplied by the Aulos (pp. 196–7). This important and revealing polemic has been given in full in Chapter ii (q.v. with comments and elucidation of obscure points). Polemics 4, 5 and 6 show that a considerable body of the Harmonists used the *Aulos* as criterion, regarding it rightly as the original source of man's practical knowledge of the Modes. The vehemence of the polemic against the Aulos indicates how widespread was ' this fatal error '. What is born on the Aulos is the Harmonia (or its part) from grave to acute, corresponding on the monochord to the whole string, plus all the notes obtainable through the equal division, by stopping one segment or more, and plucking the remainder until the octave is reached at the half of the string. The workable number of holes on a pipe is limited by technical exigencies.

Aristoxenus betrays a curious lack of comprehension of the significance of the actual structure of an instrument as an embodiment of a natural law, quite independently of any knowledge of that law and of skill, or lack of it, in the executant musician who is using the instrument. The Aulos, by virtue of its own essential nature, constitutes in fact, as long as it survives, an imperishable record of a melodic scale which, in spite of the statement of Aristoxenus to the contrary, is indeed found on the Aulos ' fixed unerring and correct '.

For the Aulos, in spite of its aberrations from the point of view of the mere executant, holds a unique position among all instruments, not only as an infallible recorder—a distinction not shared even by the flute, for reasons given elsewhere—but as creator of the Harmonia. Our chief concern, however, is to know from these polemics that the Harmonists, on the other hand, were not only aware of this, but that they actually proclaimed the fact that the Aulos was the basis of the Harmonia through its embodiment of the natural law of proportional ratios known as Hermosmenos, brought into operation, as we now know, through the division by equal measure as recorded by Aristotle.

Aristoxenus is referring to an Aulete playing—not the Modal Scales proper to the instrument, but struggling to win from it the intervals of the scale of tones and semitones, which Aristoxenus calls the exact order of melody, but which no followers of his, forgoing the use of ratios, could ever bring to birth on a reed-blown pipe.

Polemic 8.—Eratocles and the Harmonists in General Treat only of the Octave

Polemic 8.—To return to our points, Aristoxenus complains that the Harmonists and specifically Eratocles, select for exclusive treatment a single magnitude, namely the octave (p. 169).

Polemic 9.—Concerning Systems

Polemic 9.—On his part, Aristoxenus states:

> That the fourth section of his Elements will be devoted to a consideration of the systems, (*a*) how many they are and of what kind; (*b*) how they are put together from intervals and notes. For neither have the Tropoi [1] been treated with regard to this part of the subject by the previous writers; nor has it been considered whether the systems are put together from the intervals in any way, or whether any of the syntheses are contrary to nature; nor have all the differences of the systems been thoroughly enumerated by any one author [Translation by E. J.].

The point marked (*b*) is purely Aristoxenian. There was no likelihood that the predecessors, i.e. Harmonists, would describe the construction of scales from intervals and notes, for the Harmonia is not put together arbitrarily interval by interval; that is the method of Aristoxenus and Ptolemy; the Harmonia comes to birth according to natural laws through its Arche, as described in detail in Chapter i.

Aristoxenus continues:

> They made (i.e. the Harmonists) no attempt at enumeration of scale-distinctions, confining their attention to the seven octave-scales which they called Harmoniai; or if they made the attempt, they fell very short of completeness, like the school of Pythagoras of Zacynthus and Agenor of Mitylene [p. 192, 37].

There is no need at this point to establish the identity of the seven Modes with the Harmoniai, but only to bear in mind the fact that it was the Harmonists who called the seven octave scales by that name. The two schools mentioned add to our knowledge of the extent of the Harmonists movement.

[1] Macr., *op. cit.*, p. 127, lines 14 and 15, pp. 36, 191. Tropos = the Tonos in the P.I.S. and is so called by Alypius. Tropos has been used by Pindar in the sense of Harmonia or Mode.

Polemic 10.—Eratocles determines the Species by the Recurrence of the Intervals

POLEMIC 10.—That Eratocles, one of the Harmonists, was the only one of the predecessors to give a partial enumeration of the magnitudes and figures of scales.

The other systems, [Aristoxenus complains] no one has dealt with by a general method, but Eratocles has attempted in the case of one system, in one genus, to enumerate the forms or species of the octave, and to determine them mathematically (?) by the periodic recurrence of the intervals: not perceiving that unless we have first demonstrated the forms of the Fifth and Fourth and the manner of their melodious combination, the forms of the Octave will come to be many more than seven.[1]

This clearly refers to the species of the Harmonia in the Enharmonic Genus as it occurs in the P.I.S., in which the unit is the octave, for the inference is that Eratocles, leader of a school, was a Harmonist. In dealing with the Species the following interesting contingency may present itself: the tonic of each Enharmonic modal species, being characterized by the superparticular ratios of its Determinant, it follows that with a Lichanos at a distance of a major or a minor 3rd from Mese, a difficulty arises in respect of the Modal Species claiming the ratio included in the undivided 3rd, characteristic of the Enharmonic and Chromatic genera. The difference between a system determined by ratios from the Arche, and the Aristoxenian system in which the function of the note is paramount, is exhibited here.

FIG. 39.—Required a Phrygian Enharmonic Species of the P.I.S.

HYPATON				MESON			
Hyp.	Parh.	Lich. Chr.		Hyp.	Parh.	Lich. Chr.	Mese
28	27	26	*	22	21	20	16

* *N.B.*—24, the characteristic ratio of the Phrygian species is absent.

According to the Harmonists, then, no Phrygian *Species* could exist in this Chromatic or Enharmonic genus of the P.I.S., for as is patent, ratio 24 should occur in the leap of the 3rd between 26 and 22, and has, therefore, been omitted. There can be no Phrygian Chromatic Species in the P.I.S., for the Phrygian Species cannot begin on any other number but 24. The Enharmonic-Chromatic Harmonia (not species) has, however, been given quite correctly in the Notation of the Lydian Tonos by Aristides Quintilianus.[2] The same difficulty occurs with regard to Lichanos Meson, which in the Enharmonic genus of the P.I.S., excludes the Hypophrygian Species.

Polemic 11.—The Harmonists assert that points of Pitch consist of Ratios and Rates of Vibration

POLEMIC 11.—Presents the highly significant statement that some of his predecessors, the Harmonists, ' introduced extraneous reasoning, and rejecting the senses as inaccurate, fabricated rational principles, asserting

[1] Monro, *op. cit.*, p. 50; Macr., *op. cit.*, p. 169.
[2] *Op. cit.*, M., pp. 21 and 22.

that height and depth of pitch consist in certain numerical ratios and relative rates of vibration—a theory utterly extraneous to the subject [1] and quite at variance with the phenomena' (Macr., pp. 188–9).

We are not concerned here with the errors of Aristoxenus—which are many—the important fact is the definite statement that the Harmonists bent on accuracy, as true followers of Pythagoras,[2] used mathematical ratios in exposition of the basis of their theories, i.e. those of the Harmonia, and that they computed differences of pitch by rates of vibration, which also formed the criteria for the differentiation and co-ordination of the species of the Harmonia.

Polemic 12.—On the Twenty-eight Consecutive Dieses

Polemic 12.—This polemic on the twenty-eight consecutive dieses will be discussed further on together with other references to that number.

The report of the use, for the purposes of demonstration, by Harmonists whose teaching was based on the Harmonia, the Tonos and the Species, of a series of 28 consecutive dieses is in itself illuminating. It affords confirmation of the structure of the modal Tonos at the period when the scale began on Hypate Hypaton in the Mixolydian Species, which was in effect produced by the division by Determinant 28 for the Chromatic Genus, and by Determinant 56 for the Enharmonic, which yields precisely 28 notes, neither more nor less, to the octave. It is, as shown in previous chapters, from these 28 dieses that the standard Tonos is formed in the three Genera; the tetrachord Hypaton corresponds with the formula of Archytas beginning with $\frac{28}{27}$.

The polemics of Aristoxenus, in the aggregate, thus provide invaluable data upon the aims and methods of the Harmonists. It is found that the instruction given in their schools, for instance, was mainly concerned with the theory and practice of the Harmonia; that the teaching was imparted scientifically by means of mathematical ratios and rates of vibration—the only exact method—and not empirically as recommended by Aristoxenus. Diagrams and tables were freely used for the purpose of combining theoretical with practical exposition. Taken in connexion with the injunction of Pythagoras to his disciples that they should study their monochords (see fn. [2]), we can come to no other conclusion than this: that the practical application of the theory was demonstrated by means of the monochord, for there was no other known agency for calculating the equivalents of ratios in rates of vibration, which could be used in practice.

Finally, although the monochord is not mentioned by Aristoxenus in any connexion, it is evident that for a system founded upon superparticular ratios, entailing in theory minute divisions of the string (upon which

[1] The subject was 'Harmonic and its Parts' (θεωρίαν περὶ μέλους παντός), p. 188, and the natural laws according to which the voice in ascending and descending places the intervals.

[2] Cf. Arist. Quint., M., p. 116.

Aristoxenus harps), the monochord was indispensable if these intervals were to find their way into the practice of music. The ability to differentiate and recognize, for the purpose of teaching, a collection of 28 dieses, implying accurate gradations of seven octave scales within the same octave, and, therefore, requiring seven differentiations of each degree, e.g. 7 different *D*'s bearing to *C* the ratios $\frac{14}{13}$, Mixolydian; $\frac{13}{12}$, Lydian; $\frac{12}{11}$, Phrygian; $\frac{11}{10}$, Dorian; $\frac{10}{9}$, Hypolydian; $\frac{9}{8}$, Hypophrygian; $\frac{16}{15}$, Hypodorian, and so on with the *E*'s and other notes, would be impossible for a teacher without the use of a monochord. To a musician, they constituted his language of music, and would dwell in his consciousness and memory, but for the training of students and for the theorist, the monochord was, as we have said, a *sine qua non.*

It is of the utmost importance that the mathematically exact basis of proportional ratios, upon which it is claimed that the Modal System rests, should be demonstrated to have a foundation in fact, if it is to be accepted by Greek scholars. This mathematical basis to which the whole system must be referred is that of an arithmetical progression, best known in musical theory and practice as displayed (1) in the Harmonic Series constituting the physical basis of sound, (2) in the natural scale of our brass wind instruments. The geometrical progression occurs only in the octave relation inherent in the arithmetical progression itself. The ratios of the degrees of the Tonos of which the P.I.S. consists—whether considered as (1) the Harmonia proceeding from Hypate Meson to Nete Diezeugmenon, or (2) as the full compass of two octaves from Hypate Hypaton to Nete Hyperbolaion—with the added Proslambanomenos and the alternative tetrachord Synemmenon—must first be established. They have been presented in Chapter iv.

THE CHARACTERISTIC RATIO (11/10) OF THE DORIAN HARMONIA ASCENDING FROM THE TONIC: HYPATE TO PARHYPATE MESON

The principal ratio in the Tonos is the ascending one from Hypate Meson, characteristic of the Dorian Harmonia, viz. 22 : 20 or 11 : 10. It is known on the authority of Plutarch [1] that the Dorian Ethos was dependent on this tetrachord as beginning and end of the melos, and that in the Dorian Tonos, the tetrachord Hypaton was left unused, that the Ethos of the Mode might not suffer: the significance of this ostracism is grasped at once on hearing the Modal Tonos.

It will be granted that in order to establish a basis for the system of correlated Modes known as the P.I.S., the Dorian Harmonia as central link must be shown to have its Mese on the 4th degree above the Tonic (i.e. Hypate Meson); a conception of Tonality, or rather of Modality so far removed from our modern one, cannot be accepted as merely fortuitous, or as an example of arbitrary selection: some definite and adequate reason

[1] *de Mus.* (ed. Weil and Reinach), p. 78, Chap. 19 (end), § 183.

must be adduced that shall make a keynote on the 4th degree [1] of the scale a fact, uncontrovertible and inevitable.

ELEVEN, THE ONLY DETERMINANT NUMBER THAT COULD PLACE MESE UPON THE FOURTH DEGREE OF THE SCALE

Such a fact exists in the use of Determinant 11, the only number which, with the principle of equal measure or division, could place Mese on the 4th degree. The result of the working of the law underlying equal measure likewise fixes the ratio of the Tonic interval for Determinant 11, as 11 : 10. But it is a matter of absolute necessity to produce satisfactory evidence that the Dorian Harmonia, as nucleus of the Tonos, actually did begin on Hypate Meson with an interval of the ratio 11 : 10 in the Diatonic Genus, during the pure modal period in Ancient Greece ; otherwise the whole foundation of the Modal System, as presented in this work, must remain insecure. It is claimed that this may be done. On the other hand, once this step has been duly established, and the part played in the Modal System by the arithmetical progression $\frac{n+1}{n}$, understood by the terms *Emmeleia* (ἐμμελεῖα) and *Hermosmenon* (τὸ ἡρμοσμένον) is realized as a practical factor in the structure of the Modal System, the remaining ratios offer no difficulty, and may be established by a similar procedure with equal certainty.

THE RATIO 11/10 AS FIRST DIESIS ON THE TONIC CONFIRMED BY ARIST. QUINTILIANUS (p. 123 M.)

There is a definite record of the use of the ratio 11 : 10 in the Tonos by Aristides Quintilianus,[2] a Greek theorist who flourished perhaps at Ephesus towards the beginning of the second century A.D. From his work it is evident that he had access to many of the ancient Greek works on Music which have since perished.

During a discussion on the musical values and relations of numbers based on the theory and practice of the Ancients, Aristides,[3] having worked up from unity, had just dealt with number ten :

> Of the number eleven, he continues, there is also something to be said : for the Tonos (transposition scale), if we rise as far as the first diesis, clearly exhibits, the ratio of that name (i.e. 11 to 10). The number 12 is the most musical of all. For none of those which precede it, taken with as many as possible of the available numbers, demonstrates the harmonic consonances ; though those available numbers

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[1] Such, for instance, as the compelling Tonic relation of the 3rd to the 4th Harmonic in the Harmonic Series. This tempting solution is merely specific, and not due to any modal principle ; however, it must be rejected as failing to provide equally satisfying explanations for the position of the keynote in the other six Modes.

[2] Cf. Ch. Em. Ruelle, *Le Musicographe Aristide Quintilien*, Intern. Mus. Ges. Sbd. xi, Heft. 3, 1910, pp. 313–24.

[3] *de Mus.*, M., p. 123, lines 16 sqq. : ‘ *ἔστι δέ τις καὶ περὶ τὸν ἕνδεκα λόγος. ὁ γὰρ τόνος πρὸς τὴν πρώτην δίεσιν, ἐὰν ἐπίτασιν ποιῶμεν* * *τὸν ἐπώνυμον αὐτοῦ λογοῦ ἔχων ἀναφανήσεται.*

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* The text is Albert Jahn's (Berlin, 1882); Mb. omits ποιῶμεν, ἔχων and has αὐτὸν. See his Notes pp. 319–20.

divided up as far as possible, do show certain other ratios with the other breadths of the parts (i.e. considered as lengths of string divided by means of a constant denominator). This number alone bears in relation to the number 9 the ratio 4 : 3 ; in relation to 8 the ratio 3 : 2, &c. . . .

A full context has been quoted to prove that the number eleven does indeed signify the ratio 11 : 10 and not 11 : 12.

This clear and unequivocal statement is of the utmost importance, and would of itself suffice as evidence in favour of the interpretation of the ratios of the Tonos and of the Harmoniai presented in this work. There is, however, yet more evidence available to confirm these ratios, and—which is equally important—by implication, the whole system ; for in dealing with an arithmetical progression by *one* such as that of the Harmonic Series (which forms the basis of the Modal System), to be able to locate one number on a particular degree of the Tonos is to provide a clue to all the others. As, moreover, the Tonos consists, as was seen in Chapter iv, of a chain formed from an octave of each of the original Harmoniai, taken in their order as species, there should be no difficulty in admitting the claims of the other modal ratios, since the order in which both Modes and species occur is known from literary sources.[1] It will not, however, be necessary to rely entirely on this mathematical implication, for the whole Tonos may be reconstructed from the material furnished by Ptolemy.

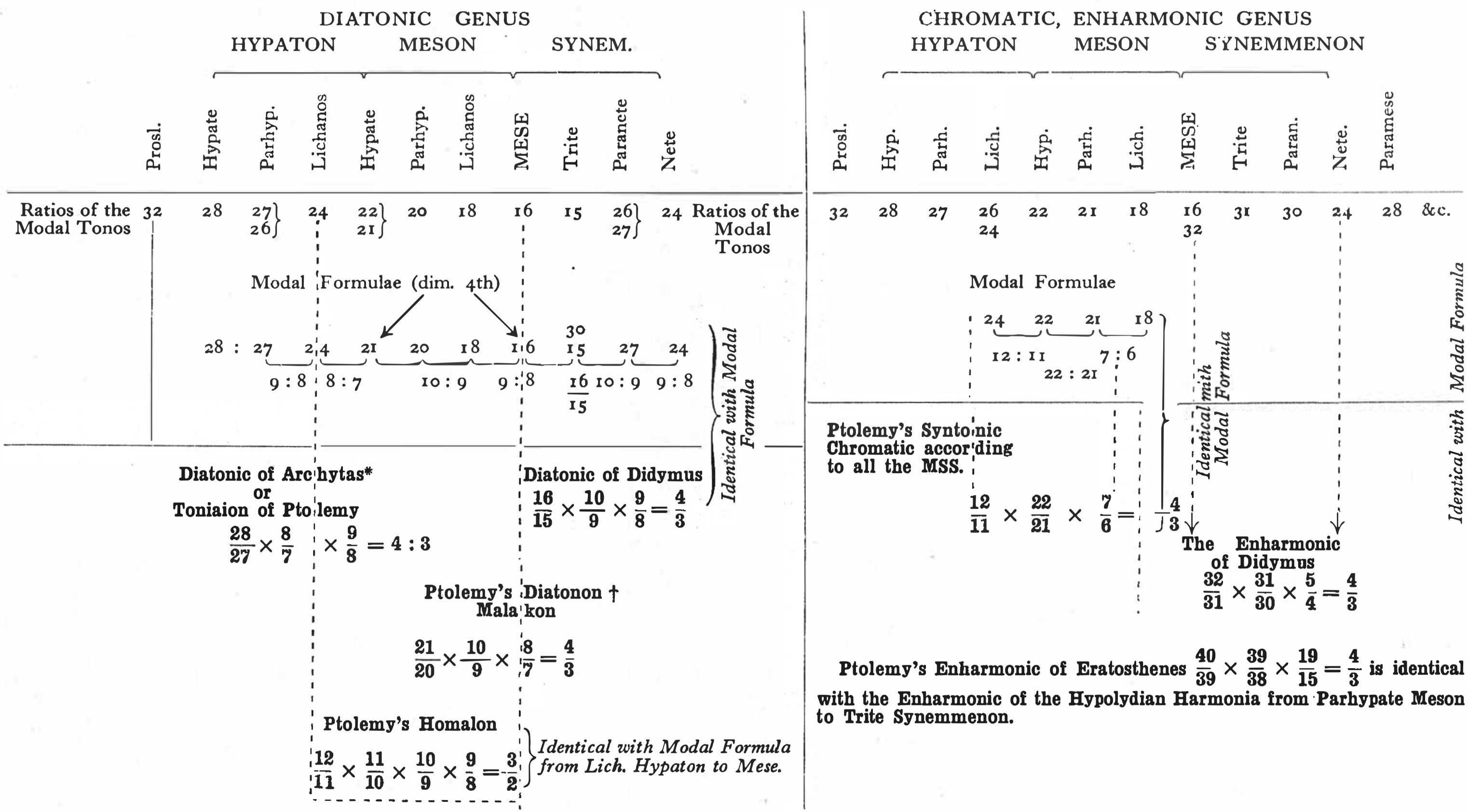
FURTHER SUPPORT FOR THE USE OF RATIO 11/10 IN THE TONOS FROM PTOLEMY

Among the shades of the Genera, according to Ptolemy's [2] tables there are actually to be found sections of the Modal Tonos, unrecognized as such, which he has introduced as formulae for the divisions of the tetrachord, to to be repeated as units, or mixed with others at will to form the octave. Three of these shades, viz. (1) the Diatonic of Archytas—identical with Ptolemy's own Tonic Diatonic, (2) the *Diatonon Malakon*, and (3) the Diatonic of Didymus, when co-ordinated, form a complete diagram of the Modal P.I.S. in the Diatonic Genus. But one slight transposition and one plausible modification of the modal ratios need be made. These are necessitated by Ptolemy's failure to bring these divisions of the tetrachord into line with his entirely sound conception of the *Emmeleia* and of the *Hermosmenon* as a whole. He destroyed the ideal of the inevitable and infinite arithmetical succession, by making it subservient to his self-imposed rules for the division of the tetrachord, instead of the division of the octave. He failed to see that the tetrachord, taken as a structural unit, leads to no recurring cycle as does the octave. Otherwise, as an aggregate, these three formulae may be recognized as the Modal P.I.S. in the late form in which it is implied in the tables of Alypius. It will be seen from Fig. 40 that certain of the formulae for the Genera recorded by Ptolemy and ascribed

[1] Aristides, *op. cit.*, pp. 17–18 ; Bacchius, M., pp. 12–13, *Tropoi* ; pp. 18–19 ; εἴδη ; Ptol., *Harm.*, ii, Cap. xi.

[2] *Op. cit.*, ii, Cap. xiv, pp. 170–2 (Wallis, 1682).

FIG. 40.—The Modal Ratios of the P.I.S. identified with Ptolemy's Formulae for the Chroai (*op. cit.*, ii, 14)



* Transposition of ratios $\frac{8}{7}$ and $\frac{9}{8}$.

† Ptolemy, in order to obtain a perfect 4th, has increased the third ratio from $\frac{9}{8}$ to $\frac{8}{7}$.

by him to such musicians and theorists as Archytas, Didymus and Eratosthenes, are actually sections of the Modal P.I.S., every tetrachord of which is thus shown to be in agreement with the testimony of Ptolemy. The figure shows that the modal ratios for the tetrachord Hypaton correspond with the Diatonic of Archytas and the Tonic Diatonic of Ptolemy (*τονιαῖον διατονικόν*), except for the transposition of ratios $\frac{9}{8}$ and $\frac{8}{7}$ displaced by Ptolemy to satisfy his idiosyncrasies concerning his subdivision of the Symphoniai into superparticular ratios. Both Archytas and Ptolemy make use of the ratio 27 instead of 26 for Parhypate Hypaton. There is evidence that the characteristic Lydian ratio 13 (26) was at some early date replaced in the Theorists' scale by the more commensurate 27, and this is unmistakably implied also by the Tables of Alypius. The modification of ratio 26 to 27 produces with the 24 of Lichanos Hypaton, a major tone $\left(27 : 24 = \frac{9}{8}\right)$, which was probably at the root of the change, since it indicates an approximation to the teachings of Aristoxenus.

A curious confirmation of this practice may be traced in the interval of the Lute Scale of the Arabs, indicated by the fret termed *Wosta of Zālzāl*, recorded as having the ratio 27 : 22 [1] on the open string; the first fret, known as *Sābbābā*, of ratio 27 : 24 gave the major second; the next frets provided different minor 3rds as modal alternatives; the *Wosta of Zālzāl* at 27 : 22 of the open string, was thus at an interval of 12 : 11 from Sābbābā. It is interesting to note in this context that the Arabian chronicler ***Ispāhāni*** relates that many Arabian Lutenists modified the Modes as used by *Ishāq-al-Mauṣili*, nephew and pupil of Zālzāl, according to the usage of the Christian Monks of Syria,[2] and they were, therefore, of Greek origin. The theoretical works of Arab musical writers were likewise based upon Greek treatises, translated through the Syriac by the College of Translators in Bagdad, inaugurated by Haroun al-Raschid. The *Wosta of Zālzāl* was, in fact, the interval that would occur on the 3rd degree of the Lydian Species, either in the Dorian or the Hypodorian Tonos, viz. having ratios 27 : 24 : 22 : 20. *Binṣir*, the fret of the major 3rd on the open string, according to the Scholiasts, would fall on 21 (to be accurate on $21\frac{3}{5}$), the Chromatic Parhypate Meson; and *Khinṣir*, the fret played by the little finger, the Lichanos Meson, on 20, which with 27 forms a tetrachord, a comma in excess of the perfect 4th (27/20 = 520 *cents*. $\frac{27}{20} \times \frac{3}{4} = \frac{81}{80}$ = 22 *cents*). This piece of evidence may be circumstantial, but nevertheless it supports the fact recorded by Ptolemy. The advantage that may be

[1] The interval $\frac{27}{22}$ = 355 *cents* exceeds the minor 3rd $\left(\frac{6}{5}\right)$ = 316° by the diesis $\left(\frac{45}{44}\right)$ = 39°. Jules Rouanet, ' La Musique Arabe ', *Encycl. de la Musique* (Lavignac, Paris, 1922, pp. 2713, 2702, 2694*a* and *passim*.

[2] Kosegarten, *Alii Ispahanensis Liber Cantilenarum* (Gripeswoldiae, 1840), pp. 199–200.

gained from a comparative study of the Classical and Graeco-Roman theories of Greek Music, taken respectively from Greek and Arab sources, has perhaps hardly yet been realized. If it may be assumed that Ptolemy was correct in the equivalents he gives of the principal tetrachord taught by Archytas of Tarentum (fl. *c.* 400 B.C.), then the alteration of the Lydian Species, as used in the P.I.S., must have become general already in the day of Archytas.

The tetrachord Meson was originally constituted by the Ancients from ratios 22, 20, 18, 16, as implied by the statement of Aristides ; the 22 falling on Hypate Meson survived in the Dorian and Hypodorian Tonoi, in the Ionian group, and in the Hyperphrygian Tonos. This ratio was in course of time modified in the other Tonoi, the Syntonic interval 11 : 10 being relaxed to 21 : 20—hence the qualifying term Diatonon *Malakon* of the formula. As this did not satisfy the growing requirements of Theorists, such as Ptolemy, that every tetrachord should be in the ratio 4 : 3, and since the Modal Meson tetrachord 21, 20, 18, 16, fell short of this interval by the small diesis $\frac{64}{63}$, it would seem that Ptolemy substituted the $\frac{8}{7}$ septimal Tone for the $\frac{9}{8}$, in order to place Mese at a perfect 4th from Hypate, regardless of the fact that this obliteration of the 16 of Mese as Arche entirely destroyed the modality of that tetrachord.

We now pass on from the Meson to the Synemmenon tetrachord, the true modal form of which must be sought in the first tetrachord of the pure Hypodorian Harmonia of ratios 16, 15, 13, 12. It may be traced with those ratios in the notation of the Synemmenon tetrachord of the Homonym Tonoi of the original Harmoniai in the Diatonic Genus. But in the Synemmenon tetrachord of the Aeolian group, and in the Ionian Tonos, the formula given by Ptolemy as that of the Diatonic of Didymus, viz. $\frac{16}{15} \times \frac{10}{9} \times \frac{9}{8} = \frac{4}{3}$ may be identified, for it corresponds with the modal formula 32, 30, 27, 24. It may be surmised that the acceptance by the theorists of the ratio 27 instead of 26 for Parhypate Hypaton of the Lydian Species was also likely to affect the use of that ratio, in the Hypodorian tetrachord 16, 15, 13, 12, when figuring as Synemmenon, more especially during the Graeco-Roman period. To what extent this actually occurred may be ascertained through the allocation of the symbols of notation in the Tonoi. The modal chromatic Pyknon of Diezeugmenon would, as in the Hypaton, be 28, 27, 26. If, therefore, in any Tonos, the same symbol occurs for Paranete Chromatic Diezeugmenon as for Paranete Diatonic Synemmenon, it is clear that the ratio must be the same, viz. 26 : this is, in fact, the case in the Dorian, Hypodorian, Phrygian, Hypophrygian and Mixolydian Tonoi (see under 'Notation', Appendix i).

The Diatonon Homalon of Ptolemy, of the formula $\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8}$ corresponds exactly with the section of the modal P.I.S. from Lichanos Hypaton to Mese in the Phrygian Species. It is a scale still in use in the

East in some of the Greek Churches in Asia Minor, according to Tzetzes,[1] writing in the 'seventies of last century.

The Syntonic Chromatic of Ptolemy[2] (a form of the same Phrygian Species) has the formula $\frac{12}{11} \times \frac{22}{21} \times \frac{7}{6} = \frac{4}{3}$ or, in modal ratios, 24/24; 22, 21, 18, according to all the codices of Ptolemy's Harmonics collated by Wallis, and to many others consulted by the present writer in the Vatican Library and in the Biblioteca Laurenziana Medicea of Florence. It is obvious that the corrected Syntonic Chromatic tetrachord fits into the modal P.I.S., as does the Homalon Diatonic between Lichanos Hypaton and Mese.

It will be noticed that the second ratio 11 : 10 of the Homalon Diatonic thus gives an implicit confirmation of the statement of Aristides Quintilianus, concerning ratio 11 : 10, and that the Syntonic Chromatic also emphasizes the 22 as Hypate Meson, which is the Modal Determinant of the Dorian Harmonia.

There is still much to be said concerning the use of the modal ratio $\frac{11}{10}$, characteristic of the Dorian Harmonia. It may be traced again in the two intervals known as Spondeiasmos and Eklysis, described thus by Aristides Quintilianus[3]:

> We must now speak of the Eklysis, the Spondeiasmos and the Ekbole. Use was made of these intervals by the Ancients for the specific differences of their Harmoniai. The uncomposed interval of three dieses descending was called Eklysis. The same interval ascending, Spondeiasmos. The interval of five dieses ascending, Ekbole.

DEFINITION OF THE DIESIS BY ARISTIDES QUINTILIANUS (p. 123M.)

Aristides' definition of these intervals expressing specific differences in the Harmoniai of the Ancients appears vague in respect of magnitude, since the word diesis has been applied to a variety of intervals differing in magnitude (for instance, this very interval $\frac{11}{10}$ is spoken of by him as a diesis). The etymology of the word from διΐημι = to thrust through, supports the meaning as the piercing through that occurs in Katapyknosis, when passing from an interval in the Diatonic Genus, to the constituent dieses of the same interval when split up in the Chromatic or the Enharmonic Genus, as shown by the arrows in the accompanying diagram (Fig. 41).

This thrusting through typifies the division that takes place on the rule of the Monochord. The magnitude does not seem to count so much as the fact that the diesis may be recognized as a legitimate intermediate

[1] Dr. Joh. Tzetzes, *Über die Altgriech. Musik in der griech.* Kirche (Munich, 1874), p. 77 and *passim*.

[2] This reading, contested by Wallis, was inserted out of its proper place at the end of Lib. ii, Cap. xv, p. 186. See note on p. 185. And he added on p. 87 a version which he calculated according to Ptolemy's text, as given in detail in Chap. iv.

[3] *Op. cit.*, p. 28.

note, obtained by the process of Katapyknosis which produces the genera; the primary meaning being a *note*, not an interval.

FIG. 41.—The Definition of the Diesis. The Diesis of Aristides (p. 123)

11 . . . . 10 Diatonic Genus

22 . . . . 21 . . . . 20 Chromatic Genus

44 - - 43 - - - 42 - - - 41 - - - 40 Enharmonic Genus

The intermediate note, marked with an arrow as diesis, is obtained in changing from the Diatonic to the Chromatic, and from the Chromatic to the Enharmonic Genera.

THE RATIO 11/10 AS SPONDEIASMOS AND EKLYSIS

In a passage from Aristoxenus quoted by Plutarch,[1] the Spondeiasmos is discussed with the same lack of grip that distinguishes that theorist's treatment of everything connected with the Harmoniai. The following facts may be extracted from Plutarch's circumstantial account; they must be judged by the Canons of the Modal System.

(1) The *Spondeiasmos*, so named because it was the characteristic interval of the *Spondeiakos Tropos* or *Spondeion*, the Ancient Libation Mode or Hymn sung to the Spondaic metre, which I have identified with the Harmonia of Terpander, both are reported as being in the Dorian Mode and lacking Trite Diezeugmenon, for which a perfectly simple explanation, based upon the Genesis of the Mode itself, has already been supplied.[2]

(2) The interval to which reference is made here is qualified as *Syntonoteros*, 'the more highly-strung Spondeiasmos', thus implying two such intervals. The more tense of these may be identified with the interval of ratio 11 : 10, characteristic of the Dorian Harmonia, while the less tense may well have been the 22 : 21, a later version used in the Tonos, and introduced probably at the same time as the ratio $\frac{28}{27}$ of the Hypaton diesis, the first use of which in all three genera is ascribed by Ptolemy to Archytas.

(3) The exact degree of the P.I.S. on which the Spondeiasmos occurs is further definitely implied in the passage in question (§§ 115–17) as that of the Pyknon of the Meson tetrachord (*τὸ γὰρ ἐν ταῖς μέσαις ἐναρμόνιον πύκνον*, Plut., p. 48).

(4) The evaluation of the Spondeiasmos as an interval of three dieses,[3] according to Aristides, is further defined by Aristoxenus (in Plut., § 112,

[1] *de Mus.*, Chap. 11, p. 1135 (ed. Weil and Reinach), pp. 46–51, and Chap. 19, pp. 74–7.

[2] See Chap. i; also Aristotle, *Probl.*, xix, 32; Gevaert, p. 33; Nicom., M. p. 7; Plut., *de Mus.*, Cap. 28, p. 105, § 270; and Cap. 18, pp. 73–9, § 168 sqq.; p. 119, § 299; p. 31, § 47.

[3] Arist. Quint., *op. cit.*, p. 28; Plut., *de Mus.* (Weil and Reinach), Chap. 38, p. 1145C, W. & R., p. 152, § 399, quoting Aristoxenus and evidently alluding to the Spondeiasmos, the Eklysis and the Ekbole.

ed. Weil and Reinach) as 'of one diesis less than the Tone near the Hegemon', which as seen above is the designation of Mese, that suggests the downward progression, the Katapyknosis. The Tone implied is, therefore, the one between Mese and Lichanos, the real Aristoxenian Tone of ratio 9 : 8, and was merely cited as a basis for the comparison of magnitudes—since Aristoxenus does not use ratios; it also carries with it a kind of protest against assigning to the Diatonic Genus a scale which rises thus from the Tonic through an interval valued at one diesis only less than a whole Tone.[1]

(5) Aristoxenus (in Plut., § 113) urges that such a scale would be inharmonious (*ἐκμελής*), for to regard the more highly strung Spondeiasmos as a Tone would be to admit of two successive diatones, one composite, the other undivided. The word *διάτονα* with the same meaning as Hyperhypate occurs in the treatise of Theo of Smyrna [2] as a substitute for Lichanos Hypaton. The objection raised by Aristoxenus may be interpreted modally thus:

The manuscripts all have *διάτονα* in this passage (in Plutarch) which has been corrected by Meziriac to *δίτονα* and accepted by Reinach. The correction assumes the scale with the more highly strung Spondeiasmos to have been in the Enharmonic Genus; this may also be explained within the Modal System thus:

It is clear that, whether Aristoxenus was referring to a succession of two ditones or of two diatones, as being by him considered inharmonious, and therefore, not to be tolerated, either of these words makes sense in the Modal System, Diatones implying the Diatonic Genus and Ditones the Enharmonic.

(6) Then follows (§ 115) a confirmatory statement intended to bring conviction—

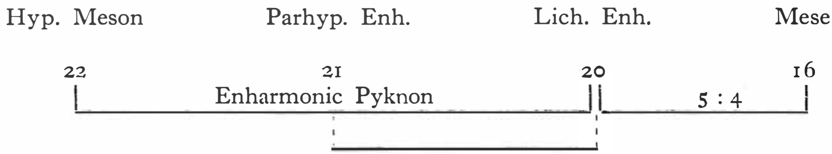
> For the Enharmonic Pyknon of the Meson Tetrachord [Plutarch reports further] actually in use at the present day does not seem to have been practised by the composer of old. It is easy to be convinced of this in listening to an Aulete playing in the archaic style, for he keeps even the Meson semitone incomposite. Such was the nature of the earliest Enharmonic.

This passage clears up any possible doubt as to the degree of the octave on which the Spondeiasmos occurred. The Aulete keeps undivided even the semitone of the Meson (ratio 21 : 20), i.e. the Hypate to Parhypate of present practice, which is heard undivided in the ancient Enharmonic Pyknon, viz.

[1] If the Spondeiasmos valued at 11 : 10 be deducted from a $\frac{9}{8}$ tone, the excess of the latter over the former is $\frac{45}{44}\left(\frac{9}{8} \times \frac{10}{11} = \frac{45}{44}\right)$ which is the next ratio in order of progression to the one with which the Enharmonic form of the Spondeion would start, i.e. 44.

[2] See Théon de Smyrne, *Exposition de Connaissances Mathématiques utiles pour la lecture de Platon*, traduite de Grec en Français par J. Dupuis, Paris, 1892, pp. 145, xxxv, and pp. 150 and 152, xxxvi.

FIG. 42.—The Enharmonic Pyknon of the Ancients (Plut. [Weil and Reinach], *de Mus.*, pp. 48–51)



The Meson semitone of Plutarch's day identified as forming part of the Enharmonic Pyknon of the Ancients playing in the Archaic style

The ratio $\frac{21}{20}$ was, it will be remembered, the Modal Meson diesis in use in several of the Tonoi, which were most frequently used in Plutarch's day, and this diesis $\frac{21}{20}$ approximates to the semitone recognized by Aristoxenus. It is also the diesis of the formula given by Ptolemy as that of the Malakon Diatonic.

In connexion with the evidence afforded by the Aulete playing in the ancient style, may be mentioned the fact ascertained through the research of the present writer that this Spondeion or scale of Terpander has been embodied through the boring of the lateral holes upon one of the Elgin pipes preserved in the Graeco-Roman Department at the British Museum (see Chap. x, ' Record of Elgin Aulos ').

This precious and unique relic of Greek music of the age of Pericles thus hands down to posterity a record of one of the ritual Libation Hymns, and of the scale historically established by numerous references as that of Terpander.

To return to our evidence of the use of the characteristic Dorian modal interval of ratio 11 : 10, a consideration of the references to Eklysis by Plutarch and Bacchius serves to identify that interval also as of the ratio 11 : 10, quite independently of the fact that it was stated to be of the same value as the Spondeiasmos, i.e. of three dieses, but used in the descending (ἄνεσις) instead of in the ascending scale (ἐπίτασις). Plutarch [1] writes as follows : ' To Polymnestos[2] is attributed the Mode known to-day as Hypolydian, and it is said also that he created the *Eklysis* and the *Ekbole*.'

The scheme of the Hypolydian is given on page 208.

The interval named Eklysis, of modal ratio $\frac{11}{10}$, is seen actually installed as the first characteristic descending interval of the Hypolydian Harmonia (just as the Spondeiasmos of the same ratio appears as the Tonic interval of the ascending Dorian Harmonia), both Mode and interval being ascribed to Polymnestos. The Modal Hypolydian Harmonia likewise contains the other interval credited to Polymnestos, viz. the Ekbole of five dieses, occurring as the

[1] *de Mus.*, Chap. 29, pp. 112–13, §§ 286–7.

[2] According to Weil and Reinach, § 286, p. 112, Polymnestos was an Ionian composer who flourished *c.* 600 B.C. (cf. §§ 40, 56, 57, p. 1133B), Son of Meles of Colophon and mentioned by Pindar and Alcman.

FIG. 43.—Scheme of the Hypolydian Harmonia extended to two Octaves, Kata Thesin and Kata Dynamin, with Ratios of the Ekbole and Eklysis. This is the scheme implied in the Canon of Florence by 'A' (lines 1 to 3)

	HYPATON				MESON				DIEZEUGMENON				HYPERBOLAION			
	Prosl.	Hyp.	Parh.	Lich.	Hyp.	Parh.	Lich.	MESE	Paramese	Trite	Paranete	Nete	Trite	Paranete	Nete	
Onomasia Kata Dynamin	28	26	24	22 21	20	18	16	14	13	12	11	10	9	8	7	A new tuning
Onomasia Kata Thesin	32	28	26 27	24	22 21	20	18	16	14	13 27	12	11	10	9	8	Tuning of the Classical period The Hypolydian as species of the Dorian

Conjunct — Hypolydian Harmonia — Conjunct;

Lydian Plagal

The Hypolydian Species

40, 36, 35
Ekbole
$\frac{40}{35} = \frac{8}{7}$
= 5 dieses

11 10
Eklysis
3 dieses descending

Plutarch ascribes to Polymnestos the Hypolydian Mode and the Ekbole and Eklysis

24 21/22 20 18 16 15 13 12 11 10 9 8/16 15 13 12

Hypodorian Harmonia

Dorian Plagal

The scheme implied by Scribe 'B' in the Canon of Florence, with Proslambanomenos on Segment 24 and Hypate Meson on 16 and Mese on Segment 12 (*v. ante*).

Septimal Tone $\frac{40}{35}$ between Trite and Paranete Hyperbolaion, and on the corresponding lower octave between Parhypate and Lichanos Meson (in the Kata Dynamin nomenclature), as the Tonic or ascending interval $\frac{40}{35}$ of the first tetrachord of the Harmonia, but having the values belonging to the later form of the P.I.S. There is, of course, an Ekbole interval $= \frac{8}{7}$ in the Hypolydian, as in every Tonos, between Mese and Paramese —when it adds the last note of the notorious Tritone—and again between Proslambanomenos and Hypate Hypaton.

THE EKBOLE, INTERVAL OF FIVE DIESES

Both these intervals, Eklysis and Ekbole, thus occur, as stated by Plutarch, in the Hypolydian Harmonia, and are used, as Aristides avers, for the specific differentiation of the Harmoniai.

From Bacchius, an entirely independent confirmation is obtained. He has been using the Notation of the Lydian Tonos to illustrate his exposition of the various features of the P.I.S. (pp. 3 sqq.); but when he arrives at the Eklysis and Ekbole, without giving any reason for the change, he selects the Notation of the Hypolydian Tonos: 'What is an Eklysis?'[1] he asks. 'It is the descending from a certain note of the Harmonia through three dieses [to the oxypyknon].' These last words, which appear in one of the two chief manuscripts, Meibomius (note, p. 30) takes to be a gloss which has slipped into the text by mistake, and he omits them in his Latin translation, since he observes they are not in Aristides (p. 28); von Jan also omits the words. Bacchius is more precise than either Plutarch or Aristoxenus; he indicates the Eklysis and Ekbole in symbols of Notation, so that there may be no possibility of error. Meibomius, however, finding that the Eklysis from $\frac{\text{E}}{\sqcup}$ to $\frac{\Theta}{\text{V}}$ in the Lydian Tonos (he has not noticed the change to the Hypolydian) contained between those letters, gives an interval of a Tone, instead of one of three dieses, has without ado altered Theta (Θ) given in all the manuscripts, to Eta (H) and V to >. It is quite evident, however, that the manuscripts contain a correct transcription of the passage, but that the original illustration referred to the Hypolydian Tonos, and not to the Lydian; in spite also of the fact that the Tables of Alypius do not give Theta among the symbols of that Tonos. There is, however, in the Royal Library at Munich, a Greek manuscript (No. 104, fol. 289) containing a complete scheme of all the notes in the Hypolydian Tonos, with their Notation (Vocal and Instrumental) for the Diatonic Genus, and several extra notes interposed, which are not included by Alypius.[2]

[1] *Introd. Artis. Mus.*, p. 11M.

[2] The scheme of the Hypolydian Tonos in the Munich MS. has been reproduced by A. J. H. Vincent, *Notices et Extraits des MSS. de la Bibl. du Roi*, Tome xvi, Pt. ii, Paris, 1847, pp. 254 and 258; see also J. Tzetzes, *op. cit.*, pp. 100 sqq. The versions differ in a few particulars.

Some of these extra notes are marked with a *Chi* (*X*), i.e. obviously for Chromatic, others with *Phi*, for *Phthorai* perhaps, as supplementary modal notes. One of these additional notes marked with *Phi* has been read by Vincent as *Theta*,[1] which would fall upon ratio 22 as a lower Nete Diezeugmenon, whereas Alypius places *Zeta* upon that degree corresponding to ratio 21, while *Epsilon* is 20, viz. :

Θ	H	Z	E
V	>	⊏	⊔
44 } 22	43	42 } 21	40 } 20

Theta and *Zeta* in the Hypolydian Tonos, therefore, would indicate two alternative notes bearing ratios 22 and 21 respectively, both for use as Nete Diezeugmenon. It is possible that the original manuscript contain-

[1] See *Mon. Cod.*, gr. 104, fol. 289*v*. The reading given by Vincent as Theta, for the note in the seventh space, as an alternative Nete Diezeugmenon is debatable but not impossible, it is a reading with which I am in agreement. Tzetzes, *op. cit.*, p. 101, has given a drawing of the scheme which he claims to be accurate ; it contains but few inaccuracies.

There is a sufficiently satisfying corroboration of Vincent's assumption that the symbol for an alternative lower Nete Diez.  interposed between I and Z in the Canon drawn in *Cod. Mon.* 104, is intended for *Theta*, as indicated by the alphabetical order of symbols, and not for *Psi*, which could only appear nine places further to the right. The alphabetical order of the letters used in Vocal Notation is, in fact, the unfailing guide in unravelling the mysteries of Greek Notation as it appears in the Codices.

It is obvious that the Scribes were sorely puzzled in copying tables of Greek Notation, and found it difficult to distinguish the identity of the letters in majuscule capitals and uncials, minuscules and cursive script, which were frequently used indiscriminately in the same MS., during periods of transition. Research among the early medieval Codices of Boethius (*de Mus.*, Lib. iv, Cap. xiv and xv) in the Tables of the Tonoi and of the species, reveals the origin of the confusion in *Mon.* 104 of the inverted angular Psi for Theta. This same form of the symbol appears over instrumental Epsilon for Half-Theta, in the lower range of mutilated or inverted symbols, in the Dorian Tonos on Parhypate Hypaton, and in the Hypodorian Tonos on Parhypate Meson, thus :  in the following Codices of Boethius, and doubtless in many others :

Eleventh century, Brit. Mus. Arundel 77, fol. 53, 54 and 54*v*.
Eleventh century, Cologne Codex of Boethius and Hucbald (see Oskar Paul, *Die Absolute Harmonik d. Griechen*, Leipz., 1866, Tables ii and iii).
Twelfth to thirteenth century, Brit. Mus. Royal MS. 15B, ix ; fols. 43 and 44.
Twelfth to thirteenth centuries, Wolfenbüttel, *ap.* Osk. Paul, *op. cit.*, Tab. iv ; also in the Glareanus edition of Boethius (Basel, 1546), pp. 1158–9. Kircher's *Musurgia* (Rome, 1650), Vol. 1, Tables, p. 540.

It is easy to understand how Half-Theta came to be transformed into *Psi* ; in passing from Scribe to Scribe, the half-circle or oval gradually assumed an angular shape pierced by a straight line, and indistinguishable from inverted angular *Psi*. It was then frequently read as a capital *Alpha*, as in Arundel 77.

It may be added that the Munich MS. gives as secondary Chromatic note, in this position, *Delta*, which falls to ratio 36 or 18, the correct one for the 2nd degree of the older Modal Scale, thus again pointing to an older tradition. The Appendix on Notation may also be consulted at this point.

ing this scheme of the Hypolydian Tonos emanated from a period antecedent to that of Alypius, and that Bacchius likewise had access to some treatises belonging to the older Modal period, or to the Schools of the Harmonists, from which he obtained his illustration of the Eklysis, used in the Notation of the Hypolydian Tonos, where in effect it should be found. Bacchius also gives an illustration of the *Ekbole*, as from $\substack{\text{E} \\ \sqcup}$ to $\substack{\text{℧} \\ \text{Z}}$ which is likewise to be identified in the Notation of the Hypolydian Tonos as the Septimal Tone 40/35 of 5 dieses

E	Δ	Γ	B	A	℧
⊔	⊐	N	/	\	Z
40	39	38	37	36	35

ascending on the Hypolydian Tonic, Trite Hyperbolaion, where one would expect to find it. This seems a more rational course to pursue than the one followed by Meibomius: since neither of the intervals illustrated by Bacchius can be found in the Lydian Tonos with the correct value in dieses.

Since Spondeiasmos and Eklysis are intervals of the same value but occurring on different degrees of the Harmonia, these illustrations, chosen by Bacchius for both Eklysis and Ekbole from the Hypolydian Mode—where they occupy the exact position demanded by the Modal System of the Tonoi, and where, according to Plutarch, they belong by tradition—add one more link to the chain of evidence corroborating the statement of Aristides concerning the ratio $\frac{11}{10}$, between the 1st and 2nd degree of the Tonos (Hypate to Parhypate Meson), which is also the first descending ratio of the Hypolydian Harmonia and Species (see Fig. 43). If additional testimony be required for the correspondence of the modal ratios of the P.I.S. with existing evidence, it is only necessary to consider the Six Harmoniai of Plato, as explained and illustrated by Aristides Quintilianus (M., pp. 21 and 22). It will be well, however, not to fall into the same error as Meibomius, who did not notice that the Notation of the Lydian Tonos had been used only for the Dorian (β), the Phrygian (γ), the Mixolydian (ε) and probably also for the Syntono-Lydisti (ς), whereas that of the Hypolydian Tonos appears in the schemes of the Lydian (α) and of the Iasti (δ). We may recall the fact that in both of these modal Tonoi the ratio 21 has, according to the evidence of the Tables of Alypius, replaced the ratio 22 characteristic of the Dorian Harmonia, and also that in the latter the use of an extra note below Hypate Meson, variously known as *Diatona*,[1] *Hyperhypate* [1] or *Diapemptos* [1] was allowed in the Melos, probably in order to compensate for the rejected lower conjunct tetrachord, to

[1] 'Théon. de Smyrne', ed. J. Dupuis (Paris, 1892), pp. 144 and 150, where these two terms are given as synonyms. *Diapemptos*, Carolus Janus, *Musici Script. Graeci*, Lips., 1895; *Excerpta Neapolitana* (Neap., iii, Cap. 2), § 27, pp. 400, 421; also Vincent, *op. cit.*, p. 254. *Diapemptos* from διαπέμπω = to send across, i.e. the note across the border, i.e. Hypate Meson, in analogy with Proslambanomenos.

which objection had been raised on account of the prejudicial effect of that Mixolydian tetrachord (28, 26, 24, 22) on the Ethos [1] of the Dorian Harmonia.

If these points connected with the theoretical equivalents of the degrees of the Tonos be borne in mind, then the Modes as stated by Aristides are, on the whole, in agreement with the schemes given in this work. The P.I.S., indeed, in its triple function as the vehicle used for Keys, Tonoi (*τόνοι*), Modes (*ἁρμονίαι*), and Species (*εἴδη*) was still elastic enough in its Graeco-Roman form to allow of the restitution of the correct ratio, even if not indicated by the Notation; or alternatively, of the substitution of another adjacent ratio for the one indicated by the Symbol of Notation.

This, then, constitutes the sum of the fivefold evidence brought forward in support of the claim that according to the modal principle of equal division, the only Determinant which can place Mese or Arche on the 4th degree above the Tonic is, in its simplest expression, the prime number eleven: that Mese in this position is the characteristic of the Dorian Harmonia and Species alone. The implication is, therefore, that the ratio 11 : 10 on the Tonic is the only possible first step for the Diatonic Genus in the type scale known as Tonos. To recapitulate briefly this fivefold evidence concerning the interval of ratio 11 : 10: (*1*) Aristides Quintilianus clearly states this as a recognized fact in the Music of the Ancients, i.e. in the Harmonia. (*2*) The interval occurs among the formulae of Ptolemy. (*3*) There are ample grounds for identifying the interval of ratio 11 : 10 with the *Spondeiasmos*, as rising interval on Hypate Meson in the Dorian Harmonia, and with the *Eklysis*, as descending interval of the same ratio, in the Hypolydian Harmonia. (*4*) Through Bacchius, further confirmation is obtained from his illustration of the Eklysis in symbols of Notation, derived from the scheme of the Hypolydian Tonos; through the Katapyknotic apparatus (explained in Appendix i, 'Notation'); the Eklysis may be identified with the same ratio 11 : 10. (*5*) Among the six ancient Harmoniai of Plato, noted by Aristides, four in the Lydian Tonos and two in the Hypolydian, the species called by him Lydisti, given in Hypolydian Notation, falls on the degree proper to that Harmonia, and is, therefore, characterized by the series of ratios obtained through Determinant 20. This is also in agreement with the rest of the evidence.

THE 28 DIESES OF THE HARMONISTS ACCORDING TO ARISTOXENUS AND ARISTIDES QUINTILIANUS

We may now proceed in like manner to introduce certain evidence in support of the claim that Hypate Hypaton, Tonic of the Mixolydian Harmonia and species, must bear the value conferred by Determinant 28. As an amplification of the evidence, derived as a whole from the Polemics of Aristoxenus against the Harmonists, the following may be selected in support of our contention. Aristoxenus states: [2]

[1] Plut. *de Mus.* (ed. Weil and Reinach), pp. 78–9, and *supra*.

[2] *Op. cit.*, M. (28); Macr., pp. 119 and 185 and notes, p. 252. See also Monro, *op. cit.*, p. 52.

In inquiring into continuity we must avoid the example set by the Harmonists in their condensed diagrams (ἐν ταῖς τῶν διαγραμμάτων καταπυκνώσεσιν) where they mark, as consecutive notes, those that are separated from one another by the smallest interval. For so far is the voice from being able to produce 28 consecutive dieses, that it can by no effort produce three dieses in succession, &c.

What was the nature of this diagram used in the schools of the Harmonists, and how is it that Aristoxenus, who prides himself so constantly upon the thoroughness of his own methods of exposition, should be so singularly indefinite and vague in his condemnatory references to the teaching of his rivals and predecessors? After all, a diagram is a definite thing, even when faulty, and had Aristoxenus recorded what the Harmonists claimed to illustrate by their diagrams, we should not be reduced to hypotheses, all of which, however, afford satisfactory explanations. Examples of these will now be submitted:

(1) The sequence of 28 consecutive dieses to which Aristoxenus takes exception may have been the result of a Katapyknosis by Determinant 28—in which case there would be 28 intervals, not all dieses, and yielding 14 notes to the octave, a chromatic division. Or alternatively, and more probably, of an Enharmonic Katapyknosis by 56 giving 28 notes to the octave, such as was required (e.g. in the Lydian and Hypolydian Tonoi) in order to provide the requisite differentiations for the three genera in the P.I.S. This division is the one demanded by the structure of the Tonos in the P.I.S., which begins on Hypate Hypaton in the Mixolydian Species with 28, of which tetrachord the formula ascribed to Archytas is a statement. A division by Determinant 28 or 14 is the only one which places Mese on the 7th degree above the Tonic as in the P.I.S. during the second stage of development, when the scale was considered by the Theorists to begin on Hypate Hypaton.

When Nicomachus,[1] relying for his information on Pythagoras and Plato, asserts that the Egyptians ascribed 28 sounds to the universe, calling it 'The Twenty-eight sounding Universe', the allusion may be to the Mixolydian Harmonia.

(2) A second diagram, which would have been indispensable to the Harmonists and may well have been one of those to which Aristoxenus refers, consists of a table representing the two dieses of each Pyknon for each of the seven ancient Harmoniai, four for each Mode, in which case the diagram would serve to demonstrate to students all the possible different magnitudes of a modal diesis (see Fig. 44).

(3) Or again, the 28 dieses in such a table might be used to illustrate the Pykna of the seven Tonoi bearing the tribal names of the Ancient Harmoniai, and in this diagram, during the pure-key period, the magnitudes of the dieses should be the same for all the Tonoi, but the intervals would be at a different pitch indicated by the groups of alphabetical symbols used in Notation. A monochord would enable the Harmonists to translate each example immediately into its practical sound-equivalent. This, of

[1] Nicom., *Excerpta*, p. 278, Jan.

course, also provides the best means of realizing the scope and melodic value of the Modal System of the Ancients.

Such a diagram does actually appear among the musical documents compiled by Aristides Quintilianus (p. 15M.). This diagram purports to give the 24 dieses of an octave of the Harmonia of the Ancients; but this is mere guesswork on the part of Aristides, or rather Meibomius, who has misunderstood the symbols and their meaning.

FIG. 44.—The Twenty-eight Dieses of the Harmonists

	Nomenclature according to Position *								
	MESON				DIEZEUG.				
The Harmoniai with their modal Determinants Enharmonic and Chromatic	Hypate	Parhypate	Lichanus	Thetic Mese	Paramese	Trite	Paranete	Nete	
Mixolydian Harmonia 56 or 28	56	55	54	44	40	39	38	28	Enharmonic
	28	27	26	22	20	19	18	14	Chromatic
		1 2				3 4			the dieses
Lydian Harmonia 52 or 26	52	51	50	40	36	35	34	26	Enharmonic
	26	25	24	20	18	17	16	13	Chromatic
		5 6				7 8			the dieses
Phrygian Harmonia 48 or 24	48	47	46	36	32	31	30	24	Enharmonic
	24	23	22	18	16	15	14	12	Chromatic
		9 10				11 12			the dieses
Dorian Harmonia 44 or 22	44	43	42	32	28	27	26	22	Enharmonic
	22	21	20	16	28	27	26	22	Chromatic
		13 14				15 16			the dieses
Hypolydian Harmonia 40 or 20	40	39	38	28} 30}	26	25	24	20	Enharmonic
	20	19	18	14} 15}	26	25	24	20	Chromatic
		17 18				19 20			the dieses
Hypophrygian Harmonia 36	36	35	34	26	24	23	22	18	Enharmonic-Chromatic
		21 22				23 24			the dieses
Hypodorian Harmonia 32	32	31	30	24	22	21	20	16	Enharmonic-Chromatic
		25 26				27 28			the dieses

A Suggestion. (Two dieses in each Pyknum—four in each Octave—for the seven Harmoniai.) Total: Twenty-eight

* This table is expressed in the Nomenclature according to position of the strings on the Kithara, for here Mese is purely one of position, the 4th note of the Meson tetrachord; in the Dorian Harmonia the two nomenclatures are unified. The real Mese occurs as 32 or 16 on the proper degree of the scale in each Harmonia if not omitted to form the skips characterizing the Enharmonic and Chromatic Genera; e.g., in the Mixolydian, Hypolydian Hypophrygian.

As Meibomius states them, the pairs of letters actually represent the intervals of the Hypaton Pyknon $\left[\text{of ratios } \frac{28}{27}, \frac{27}{26}\right]$ for each of eleven of the Tonoi taken in order of pitch from the Hypodorian upwards, therefore, a total 21, not 24, to the octave. Had Meibomius faithfully reproduced the diagram as it stood in the codices he collated for his edition of Aristides, the document would form a very valuable piece of evidence indeed. Unfortunately, however, Meibomius explains in his Preface and in the notes (pp. 224–5) that he has entirely reconstituted the diagram 'according to the clear and lucid words of the author', for in all the manuscripts the letter symbols were corrupt. Independent research by the present writer establishes the fact that some such diagram must indeed have been given in the original manuscript of Aristides, but not in the form represented by Meibomius, for the diagram (in thirteen manuscripts examined) consists entirely of the letters of the Instrumental Notation, so that each pair represents two separate notes forming the interval of a diesis, whereas in Meibomius each pair consists of two different signs for the same note, and it takes two consecutive pairs to form one diesis. Moreover, at least 25 of the letters he prints cannot be traced at all in any of the manuscripts. Therefore, the diagram as given by Meibomius has no value as evidence. The identification of the pairs of notes in Instrumental Notation actually given in the manuscripts examined, indicate that the diagram was intended as an example of the characteristic first diesis in each of the tetrachords, when the pairs of symbols are referred to different Tonoi, in which they bear their individual ratios.

To return to our discussion; one might easily be tempted to see in this reference to the 28 dieses of the Harmonists merely an allusion to the 28 notes required for the P.I.S. in the 3 genera, as minutely described by several of the Theorists.[1]

This, however, could not be regarded as a satisfactory solution of the meaning of the diagram of the Harmonists, for these presented tables of *28 dieses*, i.e. intervals, not notes, and 28 notes only produce 27 dieses; Aristoxenus clearly meant intervals, equal to quarter-tones, or thirds of tones, for he speaks of the Pyknon in this passage (Macr., pp. 185 and 204) in question as consisting of 2 dieses. Moreover, the 28 notes of the P.I.S. enumerated above are required for a double octave system in the 3 genera and do not all produce dieses nor are they used consecutively. Such a scheme would only produce 14 notes to the octave. The Katapyknotic diagrams, on the other hand, are such as might have been used by Lasos of Hermione, by Damon the Athenian, or by Epigonus of Ambracia to illustrate their lectures in the schools of the Harmonists.

As bearing upon the significance of Determinant 28 in the Modal System, reference may be made to certain inferences to be drawn from the

[1] Nicom., Lib. ii, pp. 38–40; Eucl., *Intr. Harm.*, pp. 5, 6; Arist. Quint., pp. 9, 10, &c. Of the new translation: of 'Aristeides Quintilianus', *Von der Musik*, Eingeleitet, übersetzt und erläutert von Rudolf Schäfke, pp. xii, 366 (M. Hesse: Berlin-Schöneberg, 1937) I cannot speak, not having yet received my copy.

statements of Plutarch concerning the Mixolydian Harmonia and Species, which, as claimed in this work, result from a division by the constant Determinant 28. An instance of this division has survived in the Canon of Florence, as seen earlier in this chapter.

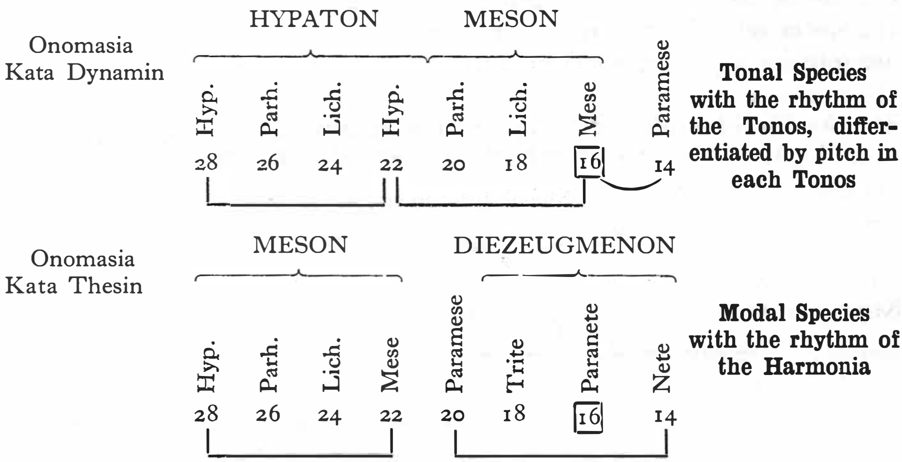
From Plutarch's various references [1] the following facts are elicited: In Plutarch's day the Mixolydian Species extended from Hypate Hypaton to Paramese; the invention of the Mode was attributed by Aristoxenus to Sappho, through whom it was introduced into Tragedy; in adopting this Mode, the Tragic Poets associated the Mixolydian with the Dorian Modal Species; that is how Aristoxenus (*apud* Plutarch) records his discovery that the extended Dorian Harmonia was linked up through conjunction with the Mixolydian Harmonia, and that the combined Ethos of these two Modes was of the essence of Tragedy. Then the theorist Lamprokles, the Athenian, stepped in and discovered that the Mixolydian disjunction had been displaced by its association with the Dorian Harmonia, and was found at the acute end of the scale, i.e. between Mese and Paramese. This passage becomes at once intelligible if we remember that the modal octave consisted of two tetrachords kept distinct through disjunction; it was not a case of the mixing of tetrachords, as some have thought, but that the *Hermosmenon* itself, through its inevitable succession, brought this about as a first cause, the recognition of the two identities following later (see Fig. 45).

But the difficulty troubling Lamprokles was this very fact, that in Harmonia and species the position of the disjunction was a different one, and this, of course, in the species, carried with it the definition of the position of Mese on the 7th degree of the scale from Hypate Hypaton. The interesting point raised by Plutarch's record is what caused Lamprokles to become aware of the distinction, for a somewhat new factor is hereby introduced as may be seen from Fig. 45 below. A practical musician must judge this matter, either from a variation in pitch, or in rhythm, or in both: had it been a matter of pitch, it would have merely been a question of a different Tonos. As theorists we are able to express by the help of both nomenclatures, used simultaneously, what would prove a simple matter to the Kitharistes, for we may see from the diagrams that all that is required in playing is a change of rhythm. It is only in the Dorian Harmonia that the Tone of Disjunction follows Mese; in all the other Harmoniai the Disjunction—not by any means always consisting of a Tone—was invariably found between the two modal tetrachords. The failure of Aristoxenus to recognize the nature of the Harmonia prevented his grasping the significance of the fact, that whereas the *Onomasia Kata Thesin* represented the Modal Species enthralled by the Dorian Harmonia, which thereby imposed its own peculiar structural features and rhythm upon the other Modes, the *Onomasia Kata Dynamin* represented the emancipation from bondage and the restoration of the pure modal freedom. This could not be effected while every tetrachord remained Dorian in form, i.e. S.T.T., and while one species of tetrachord alone was in use. It will now be realized that what Ptolemy

[1] *de Mus.*, C.16 D and E, pp. 62–7 (Weil and Reinach).

was striving to expound was the double nature of the species within the P.I.S.[1]

FIG. 45.—The Mixolydian Harmonia as Tonal and Modal Species.



(*a*) THE TONAL SPECIES, differentiated in pitch through the Tonos and following the monotonous rhythm of the Tonos or Dorian Harmonia (*Kata Thesin*), but with the sequence of intervals belonging to the Mode.

(*b*) THE MODAL SPECIES, free of all shackles, following its own rhythm as a Mode, and merely making use of the P.I.S. Notation (*Kata Dynamin*).

The Modal Species might be further emphasized as to pitch, by being taken within the Homonym Tonos between the limits of the octave of *F* (Ω to Γ).

These two kinds of species might be identical in pitch, being expressed by the same ratios, but the essential differences were of rhythm and of Ethos and were melodic.

The *Kata Thesin* nomenclature always represents the Kithara strings occurring in the same order for every Harmonia and lying between Hypate Meson and Nete Diezeugmenon.

The *Kata Dynamin* nomenclature was the *tuning* of the strings which revealed the true Harmonia in use by the position of its tuned Mese in relation to the conventional position of that keynote as a string of the Kithara, or as a degree of the scale.

The Mixolydian Harmonia, as its name implies, included a Lydian element, and that, of course, is patent from the ratios 10, 9, 8, 7, of the upper tetrachord, which is Hypolydian. That fact recalls the next lines of Plutarch's quotation stating that the *Epaneimene Lydisti* is the opposite of the Mixolydisti but similar to the Iasti.

[1] Cf. Macran's exposition of the Mixolydian Mode with the double nomenclature (*op. cit.*, p. 63). Macran shows only the rhythm of the Tonos throughout the table.

BRIEF RECAPITULATION

The main points in the chain of evidence proving the existence of the Modal System, which have been provided by this long chapter, may, in conclusion, be very briefly recapitulated. The basic principle to which the Harmonia must be referred is affirmed by Aristotle to be that of equal measure by a [determinant] number.

FIG. 46.—The Mixolydian as the opposite of the Hypolydian (Plut., *de Mus.*, Cap. 16, § 157 ed. Weil and Reinach, to be compared with Figs. 43 and 45)

Modal Interpretation.

Mixolydian Harmonia. Determinant 28, on *B* string

	B	*C*	*D*	*E*	*F*	*G*	*A*	*B*
Ratios	14 or 28	26	24	22	20	18	16	14

Hypolydian Harmonia, Determinant 20 on *F* string

F	*G*	*A*	*B*	*C*	*D*	*E*	*F*
20	18	16	14	13	12	11	10

The tetrachord 20, 18, 16, 14 is common to both Harmoniai; in the Mixolydian Harmonia it is the upper tetrachord; in the Hypolydian Harmonia, the ratios are those of the lower tetrachord. The first tetrachord of the Mixolydian Harmonia, moreover, 28, 26, 24, 22 (or 14, 13, 12, 11) is present as conjunct tetrachord in the Hypolydian Harmonia.

The Mixolydian Harmonia may, therefore, very properly be said to be the opposite of the Hypolydian.

The Mixolydian tetrachord 14, 13, 12, 11 is yoked to the Dorian by conjunction.

All the important features which have developed out of the operation and implications of this modal principle have been found supported by quotations from the Theorists; for instance, the Modal Tonic, common in respect of pitch to all seven Harmoniai; its power as differentiated unit, of becoming a starting-point of characteristic proportional value for each of the modal sequences in turn—a new idea in our Western musical system—is clearly described by Aristides Quintilianus (p. 18 M., lines 7 sqq.).

The inherent causative or formative power of Mese as Arche, which is so obvious in the genesis of the Harmonia, clears up all the difficulties raised in the Ps-Aristotelian Problems on that score.

The modal ratios of the P.I.S., clearly emphasized as Modal Species, are shown to form the aggregate from which Ptolemy has derived the tetrachordal formulae of the principal shades of the Genera, ascribed by him to various ancient musicians (see Fig. 40). The octaval structure of the P.I.S., unrecognized by him as modal, he regards as composed of 'mixed' tetrachords.

Of these modal ratios of the P.I.S., the supremely significant first step in the Tonos of the pure modal period (from Hypate to Parhypate Meson) is accurately described by Aristides Quintilianus (p. 123) as bearing ratio 11 : 10. Thus the identification of this ratio, as the first in the Dorian Harmonia and in the Tonos, is fully justified ; eleven, moreover, is the only Modal Determinant that places Mese, as an octave of Arche, upon the 4th degree above Hypate Meson. The establishment of this important interval carries with it the identification of the Spondeiasmos and Eklysis, as ascending and descending intervals of 3 dieses, ' used by the Ancients for the characterization of their Harmoniai ' as of ratio 11 : 10. Bacchius, who defines the Eklysis and Ekbole in symbols of Notation, is found to add his confirmation of the modal valuation—respectively 11 : 10 and 8 : 7—when the modal basis of Notation provides the clue.

The various allusions to the number 28 as significant in the P.I.S., by Nicomachus, Aristoxenus and others, are found to have an obvious explanation in the Modal System, and are supported in their various implications as actualities in ancient Greek music by the division of the Canon of Florence ' into 28 equal parts starting from Proslambanomenos '.

Finally, the six ancient Harmoniai of Plato, given by Aristides in ' Vocal and Instrumental Notation ' (p. 22 M.), are found to be on the whole in agreement with the Modal System described in this work, which also furnishes a modal explanation of the diagram given by Aristides (p. 15) of the 24 dieses of the Harmonia of the Ancients, when the version of the manuscripts is adopted instead of the discredited restitution provided by Meibomius.

CHAPTER VI

THE CYLINDRICAL MODAL FLUTE WITH EQUIDISTANT FINGERHOLES EMBODYING THE HARMONIA

The Harmonia installed on Aulos and Flute. The Significance of Diameter in the Acoustics of the Modal Flute. The Aulos alone gave birth to the Harmonia. The Harmonia on the Flute. Formula No. 1 (v.f. from Length). Formula No. 2 (Length from v.f.). Formula No. 3 (to find the Position of Hole 1). The Significance of the Increment of Distance (I.D.). The Four Aspects of the Increment of Distance. Nine Aspects of Allowance in respect of Diameter in Modal Flutes. The Classification of Modal Flutes. The Three Experimental Sensa Flutes. The Modal Sequence of the Hypophrygian Harmonia on Flute 'Sensa C'

THE HARMONIA INSTALLED ON AULOS AND FLUTE

NO investigation of the genesis of the Harmonia (ἁρμονία) upon the Aulos, or of the rise and development of Modality in Ancient Greece, can be deemed satisfactory or adequate that fails to provide, at the same time, a collateral study of the modal flute. It will, therefore, be conceded as an indispensable premise that a comparison of the properties, characteristics and implications of these two types of wind instruments should be immediately available, so that a considered judgement may be formed of the value of each of these, as embodiment of the Harmonia. Aulos has hitherto been used somewhat indiscriminately as a generic term [1] for a wind instrument and has usually been rendered 'flute', sometimes 'clarinet' or 'oboe'. It is preferable to translate Aulos by 'reed-blown pipe', bearing in mind that the same pipe may be played by means of (*a*) the double-reed mouthpiece, as primitive oboe; or (*b*) by means of the beating- or single-reed mouthpiece, as primitive clarinet. In each case the musical result has definite characteristics.

Although the Harmonia may be installed on both Aulos and flute, it can be claimed to have come to birth only on the Aulos when played by a double-reed mouthpiece, for reasons already explicitly given in Chapters ii and iii.

These are very cogent reasons, and it will be clearly shown besides, in this section, why it is that the Harmonia could not have originated on the flute, whether vertically or transversely blown, or through a fipple or

[1] Liddell and Scott, *Greek-Engl. Lexicon*, 8th ed., 1901, s.v. Aulos: 'any wind instrument, usually rendered flute, though it was more like a clarinet or oboe, for it was played by a mouthpiece (γλωσσίς)', Aeschin., 86, 29; the first mention of them in Iliad, 10, 13; 18, 495; Pind., *O.*, 5, 45—Λύδιος. Liddell and Scott, ed. by Sir Henry Stuart-Jones (9th ed.), Αὐλός,ὁ, *pipe, flute, clarinet*; Il., 10, 13; 18, 495, h. Merc. 452; Λύδιος, Pind., *O.*, 5, 19....

recorder mouthpiece. There is no historical or archaeological evidence to be adduced for this disqualification of the flute ; it is simply due to certain structural features and acoustic principles, and to the manner in which the sound is produced on the instrument.

THE SIGNIFICANCE OF DIAMETER IN THE ACOUSTICS OF THE CYLINDRICAL MODAL FLUTE

It is the factor of DIAMETER which accounts for this disqualification of the flute as a natural generator of the Harmonia ; and as the implications of diameter, as applied to the flute, will probably not be immediately apparent to those unversed in the technicalities of the instrument, this important feature must be considered in some detail. Diameter in this context refers either to (*a*) the width of the bore measured across the inner walls of the flute at the exit end ; or to (*b*) the width of the circular or oval embouchure (mouthhole) ; or to (*c*) the width of the fingerholes accurately measured by means of fine calipers. In relation to the flute, it will only be necessary to consider the significance of diameter as a lengthening factor, without entering into the acoustic principles involved.[1] The three implications of diameter at exit (Δ), at embouchure (d) and at fingerholes (δ) are of great importance in the determination of the pitch of the notes produced on the flute ; they are taken into account in most of the formulae to be used in this section. The diameter of the bore at exit is a simple factor ; at the embouchure, if the width is less than that of the bore, the difference must be considered as an additional item of length and be added to that of the bore ; and the diameter of the fingerholes is treated in the same way. Thus the narrowing of the aperture through which air in vibration makes its exit has a lengthening effect on the sound-wave, equal to the difference between the diameter of the bore and that of the fingerhole. Hence the somewhat paradoxical fact that reducing the diameter of a hole lowers the note and vice versa. This principle is often used on modal flutes in order to allow of a smaller hole being placed nearer the embouchure than is imposed by the increment of distance ; the device was frequently used by the ancient Aulos- and flute-makers (see Chap. ii). The smaller the fingerhole the greater the addition to the length of the sound-wave and to the allowance which has to be made in respect of diameter when making or investigating a modal flute.

This allowance for diameter is inherent in the flute itself ; it is only indirectly regulated by the flute-maker's selection of the calibre of the tube, and by his judgement in making the embouchure and fingerholes of any fancied diameter. The allowance is always included in the effective length of sound-wave, from which it may in calculations be eliminated as the remainder, after subtracting from the effective wave length the actual measurable length of the flute at any given point. The effective or sound-

[1] For the acoustics of the flute, the following work may be consulted : R. S. Rockstro, *A Treatise on the Construction, the History and the Practice of the Flute, including a Sketch of the elements of Acoustics* (London, Rudall Carte & Co.), revised edition, 1928.

wave length thus embraces two other significant lengths: (*a*) the actual measure of the flute, from the embouchure to the exit, or to the centre of a fingerhole, and (*b*) the allowance for diameter,[1] or extent of the latter's sphere of influence.

The allowance for diameter forces itself upon the notice of the craftsman when the position of the first hole has to be correctly determined. It is in this very weighty and significant aspect that the allowance for diameter makes its first appearance; it passes from an ethereal and abstract existence to emerge as a concrete and visible factor in the structure of the modal flute. It is evident, therefore, that however technical and complicated the whole subject may appear to the casual reader, a practical acquaintance with diameter, and a firm grasp of all its implications in the modal flute, are indispensable to the serious student and investigator—more especially as it is impossible to obtain assistance of this nature from existing text-books on the acoustics of the modern flute.

We are still on the outer fringe of the subject, the intricacy of which will gradually unfold, until we shall be left wondering how primitive musicians have contrived to deal successfully with the invisible factors upon which pure modal sequences depend.

THE AULOS ALONE GAVE BIRTH TO THE HARMONIA

The crux of our thesis: that the Aulos alone gave birth to the Harmoniai in their integrity, can now be revealed in one simple statement, viz. for the reed-blown pipe, diameter has no significance whatever as a lengthening factor or as a determinant of pitch. The Aulos was entirely free from that incubus and from the many problems it brings in its train.

As for the flute, the innate feeling for proportion common to all primitive races, to which they react in their decorative arts, as well as in those that appeal to the ear, guides the primitive musician in boring the fingerholes. This feeling is testified by the almost invariable rule of equidistant spacing found in flutes of all kinds and in all lands. But investigators, who have noticed this feature in flutes and pipes ancient and modern, have missed the significance of the one element which lies at the root of modality—i.e. the Modal Determinant—and they have failed to see that equal-spacing does not by itself solve the problem of the scale produced. For this equal-spacing is not merely prompted by a desire to please the eye: the flute is made for use, and the practical musician sees to it that the holes are placed where he can conveniently cover or uncover them. He therefore marks the centres of the holes by the lay of the fingers on the flute. But it is evident, from an investigation of the specimens, that the primitive musician also feels the necessity of satisfying his feelings of relationship between the parts and the whole; and this he brings about

[1] The term 'allowance for diameter' (All.) has deliberately been used throughout this work. Other authors prefer 'End-correction', which sounds slightly impertinent, since the diameter is a natural feature of the flute and its lengthening influence a reality. No note can be produced on the flute without diameter. See Chap. viii, Hornbostel.

by shifting his fingers above the emplacement for the top hole, as far as the embouchure (which he makes first of all), so that the length of the flute ends by being a multiple of the increment of distance ; he thus helps to bring about modality by setting up a Modal Determinant.

Some investigators having accounted for equal-spacing on flutes as a manifestation of a desire for equality, imagine further that the equidistant holes produce *equal intervals*—or approximately equal—with the implication that any slight difference is due merely to faulty intonation and may, therefore, be considered negligible. To regard intervals produced by equidistant holes as equal in intention—and the slight discrepancies as accidental—is an error in principle : the cycles of 5ths or 4ths, to which theoretical sequences of equal intervals are usually referred, belong to a genesis which is essentially non-modal and impracticable for any but highly sophisticated craftsmen to instal on a flute.

It is absolutely impossible for equal-spacing of fingerholes on pipes and flutes to produce equal intervals,[1] or even two consecutive intervals of equal magnitude. The point at issue is not so much the extent of the difference between any two intervals occurring in sequence, which may amount to a few *cents* only : the whole significance, as already stated above, resides in the principle at stake in the genesis of the Harmonia. In its operation this principle of genesis may, according to the Modal Determinant selected, produce widely differing intervals which could by no stretch of imagination be described as approximately equal. In this matter the ear is more discerning than the eye, for the auditory musical phenomena are one and all based upon proportion : upon the law of the physical basis of sound, and of its receptivity by the delicate mechanism of the inner ear, whether apprehended as pitch alone, or accompanied by the sensations of tone quality.

The primitive pipe-maker, plotting the fingerholes upon his flute, is guided at first, no doubt, by convenience in covering the holes with ease with his fingers—a natural motive which invariably brings about equidistance.

Numerous specimens from all the world over reveal the fact that an instinctive feeling for proportion guides him further to realize the great principle of the unity and harmony that exist between each part and the whole.

Evidence likewise establishes the fact that the ear does not lag behind the eye in its appreciation of proportion, for the more primitive the listener, the more violently he reacts against false proportions in musical sounds.

The effect of perfect beauty and satisfaction produced by a scale of intervals in successive proportional ratios must be experienced before it can be realized. Such is, for instance, the following octave, consisting of seven intervals, rising in delicately increasing proportions, from the major semitone to the major tone, as octave of the fundamental. The sequence is the result of the aliquot division of a string or pipe by Modal Deter-

[1] The subject of scales of so-called equal intervals comes up for discussion in Chap. viii.

minant 16, belonging to the Hypodorian Harmonia: viz. from grave to acute.

Cents		*112*		*119*		*128*		*138*		*151*		*165*		*182*		*204*	
Ratios	$\frac{16}{15}$		$\frac{15}{14}$		$\frac{14}{13}$		$\frac{13}{12}$		$\frac{12}{11}$		$\frac{11}{10}$		$\frac{10}{9}$		$\frac{9}{8}$		8

Thus, for instance, an Aulos having as Modal Determinant 11—a universal favourite seemingly—may confidently be provided with fingerholes distant from one another by exactly $\frac{1}{11}$ of the total length of pipe + mouthpiece. It will be seen further on that this statement only applies with reservations to the flute.

On the flute, however, the modal sequence will indeed under certain conditions follow, interval by interval, true to the operation of the Harmonic law of proportion which is stimulated into activity by the slight hollows on the inner surface of the flute. These hollows are formed by the fingers covering the holes, but not to the point of internal obturation; nodes are thus created at those points for the aliquot division of the column of air.[1]

It is easy to see that a Modal System was bound to develop in due course from these early experiments as, for instance, when the one Harmonia on the Aulos could so easily be made to give place to a second, by merely pulling out the mouthpiece a little further. These facilities for rich modal variations on the Aulos are due to the mouthpiece; they are not obtainable in the same way on the modal flute.

THE HARMONIA ON THE FLUTE

On the modal flute, the conditions are found to be entirely different, and unfavourable to the birth of the Harmonia at the hands of unsophisticated musicians.

We must now deal with the technicalities connected with the flute; but we shall restrict ourselves, as much as possible, to their practical rather than scientific application to the modal flute, as distinct from its modern non-modal progeny.

The application of diameter, as added length, to any problem connected with the pitch or with the modal proportions of flutes, is related either to: (*A*) the actual, visible flute length which, with the addition of diameter allowance, is equal to the half-wave length;

Or to (*B*) the effective, invisible complete wave length of any musical note, recognized by the ear, which may be produced on the flute. The epithet effective, applied to flute length, invariably implies the inclusion of the allowance made in respect of diameter.

When (*A*) refers to the half-wave length or ‘vibration simple’ of the French school, which is not audible, it includes the implications of diameter taken once only.

[1] This suggested explanation was already introduced in Chap. iii, in order to account for the proportional impulse given by a specific Modal Determinant which may be observed on the Aulos (see Chap. iii).

(A) as a half-wave length is an impulse which theoretically travels from embouchure to exit, once only, along the bore of the flute; it has no separate existence apart from the complete sound-wave; the half-sound-wave is inaudible and exists only on paper; it has, therefore, only a nominal vibration frequency, double that of (B). (B) refers to the effective length, appreciable by the ear, of the complete sound-wave, which has travelled forth through the tube and has been reflected back to the embouchure. The extent of the whole sound-wave is, therefore, *double* that of (A), and its vibration frequency consists of *half the number* of v.p.s., compared with that of (A).

With a specimen flute in front of him, the investigator can ascertain the exact vibration frequency of the fundamental note given out at the exit, or at any of the fingerholes, by using Formula No. 1.

FORMULA NO. 1

TO FIND THE V.F. FROM THE EFFECTIVE LENGTH

$$\frac{\text{340 metres per second}}{\text{(A)} = \text{effective length of flute} \times 2 \text{ (expressed in metres)}} = x \text{ v.p.s., frequency of the complete sound-wave.}$$

$$\text{e.g. } \frac{\text{340 m./sec.}}{\cdot 543 \times 2} = \frac{340}{\cdot 1086} = 313 \text{ v.p.s. (Sensa Flute A.)[1] frequency of the fundamental note of the flute.}$$

This simple formula is based upon the velocity, or rate, at which the complete sound-wave travels through air (at moderate temperature) until it impinges in due course upon the ear. The operation indicated in the formula implies the fact that the sound-wave, proceeding through air, advances a distance equal to its own length once in every second. Thus the velocity rate in air of 340 metres per second, divided by the length of any given sound-wave (stated in decimal fractions of a metre), will produce, on reaching the inner receptive mechanism of the ear, the number of periodical impacts per second which is known as the vibration frequency of a musical note: e.g. to state a hypothetical case: a half-sound-wave A, measuring ·332 (i.e. flute length = ·298 + diameter allowance) will produce a vibration frequency of

$$\frac{\text{340 m./sec.}}{\cdot 332 \times 2 \ (= \text{effective length (B)})} = 512 \text{ v.p.s.}$$

The problems with which a student investigating the musical values of specimen flutes is confronted depend upon many contingencies, viz.

(1) The judging of a specimen available and playable.

(2) Judging a specimen from measurements; these, to be of any use, must be absolutely accurate and expressed preferably in millimetres (e.g. see the efficient measurements of the Java flute supplied by Mrs. Elizabeth Ayres Kidd (see Chap. x, ' Java Flutes ').

[1] The formula, thus worked out as an example, actually forms part of the Record of Modal Flute ' Sensa A ', and therefore constitutes evidence of the correctness of the Formula. The effective wave length ·543 is that of ' Sensa A ', and produces a fundamental $E\flat$ of 313 v.p.s.

The only satisfactory proceeding in this contingency is to make a facsimile from the measurements, checking each item.

(3) The construction of a flute, of which only a photograph or drawing made to scale is available, a more difficult proposition, the results of which may only be offered with reserve.

(4) The making of experimental flutes in a specified Mode and tonality.

The importance of a thorough understanding of these allowances for diameter is realized as soon as it becomes necessary to make facsimiles, or to plot out experimental flutes in order to play in one or other of the Harmoniai, for which a predetermined fundamental is necessary. Several new formulae will then be required.

Wherever it is possible, the best course to adopt in all these contingencies is to ascertain first of all the pitch (vibration frequency) of the fundamental for exit and for Hole 1. The use of a monochord with one, or preferably two strings, with the rule marked as shown in Fig. 1, provides an easy method of ascertaining the vibration frequency, and those who do not relish endless calculations will find the tables of reciprocal lengths and vibration frequencies useful.[1] (See Table i, p. xlvi.)

FORMULA NO. 2: LENGTH FROM V.F.

To find the effective length of the half-sound-wave from the v.f.

$$\frac{340 \text{ m./sec.}}{\text{v.f.} \times 2} = x \text{ effective length of the half-sound-wave.}$$

e.g. for the same proposition as in No. 1 above (Flute Sensa A)
313 v.p.s. × 2 = 626 v.p.s.
then:

$$\frac{340 \text{ m./sec.}}{626 \text{ v.p.s.}} = \cdot 543 \text{ effective length of half-sound-wave}$$

$$- \cdot 465 \text{ actual length of flute}$$

$$\underline{\underline{\cdot 078}} \text{ Eff. allowance due to diameter.}$$

N.B.—A note of a given vibration frequency inevitably requires a half-wave of a fixed, unalterable length.

The variable factors are the actual length of the flute, the calibre of the bore and diameter of embouchure, all of which must be so accommodated the one with another that the theoretical allowance for diameter derived from the pitch agrees with the allowance from exit measured on the flute, taken twice. It is a foregone conclusion that the primitive flute-maker sets to work without any knowledge of these formulae. How, then, does he achieve his objective?

A study of the modal flutes of many lands and ages reveals the fact

[1] In these tables the reciprocals in juxtaposition may be taken to represent either v.f. or length. In every case the v.f. is stated in the 8 ft. or 4 ft. octave. The exact octave, in which the note occurs as reciprocal of a length, depends upon various contingencies; if this point is of importance and cannot be readily estimated, then the requisite formula must be worked out.

that, in the majority of cases, the exit fundamental does not bear to the note of the first hole any recognizable ratio belonging to the modal sequence of the flute; the inference is that this note was unused for that reason, and that the first hole, left always uncovered, served as vent, and as Tonic or initial note of the sequence.

In order to preserve the integrity and purity of the modal sequence from the exit, it is necessary to discover the correct position for Hole 1, at a distance from exit that takes into account the implications of diameter: this is, of course, the stumbling-block for the unsophisticated craftsman.

For this purpose, a special formula is required which introduces a new factor. Formula 3, compiled by the author, has been found to give correct results in practice.

In a few remarkable specimens such as the following: the analysis of the component factors of the aggregate allowance in Formula 3, corresponds to a millimetre with the actual position of Hole 1 on the flute, as placed by the primitive flute-maker; e.g. in the Graeco-Roman flute from the Bucheum at Armant, in Egypt, discovered by Sir Robert Mond during the excavations conducted under the direction of Mr. Oliver Myers.[1] (See Report on the Flute, by K. S., reproduced with a few additions in Chap. x, Record.) It will be seen from this report that this specimen—unfortunately unique up to the present—embodying the Dorian Spondaic Harmonia from the exit, has the first hole placed at ·058 from the exit, in the exact position demanded by Formula 3, for a flute of the given dimensions and modality. Whether this truly remarkable correspondence between theory and practice was due to a technical knowledge far in advance of ours in the twentieth century, or whether the methods employed were merely empirical, or again, as some may suggest, due to mere fortunate coincidence, must remain undetermined for lack of evidence. The fact, however, is incontrovertible.

FORMULA NO. 3: TO FIND THE POSITION OF HOLE 1

For the determination of the position of the centre of Hole 1:

$$\frac{\Delta \text{ (of bore)}^{2}}{2} + (\Delta - \text{d}) + (\Delta - \delta) + \text{I.D. (one or more)}$$
$$= x, \text{ distance of centre of Hole 1 from exit.}$$

The integrity of the modal sequence from exit cannot be preserved unless the full allowance due to diameter is carried out in actual length upon the flute between the exit and the centre of Hole 1, in addition to the I.D.

This fact constitutes concrete evidence of the necessity for taking into account the lengthening influence of diameter in all its implications, and the importance of this evidence should be duly noted for future reference.

[1] *The Bucheum*, by Sir Robert Mond and O. H. Myers, Vol. 1, 1934, pp. 103–4; p. 99, Pl. 88, Fig. 3.

[2] The Δ is halved in Formula 3, and the allowances are taken once because carried out actually on the flute, the length of which is comprised in the half-wave length.

The sequel will show that in the modal flute there is a double set of propositions to be considered, for which different formulae are required in order to solve problems connected with (1) the vibration frequency of notes and (2) the proportional laws operative in modality.

It might be imagined that once the fundamental note of the flute had been correctly diagnosed, and found to be in correspondence with the sound-wave length and the aggregate allowance; and that in addition, the centre of the first hole had been placed at the right distance from exit in order to produce the second note in the modal sequence, the rest would be plain sailing. Alas! our belief in the facile operation of aliquot division by a Modal Determinant resulting inevitably in the modal sequence of the selected Harmonia suffers a rude shock! The surprise may come during the examination of an original specimen: when the various items of measurement have been duly noted, the modal sequence identified by means of a monochord, and rising step by step in accordance with the ratios of the Harmonia. Then, when as a final test, the total length of the flute from exit to embouchure is divided by the I.D., it may be that the quotient falls short by several increments and indicates an entirely different Mode from the one actually played by the flute! What is the cause of this surprising fact? Such an impasse would be an impossibility with the Aulos, but it occurs frequently with the flute.

The result of numerous tests and experiments conducted with flutes reveals the perverse fact on the other hand that those modal flutes in which the I.D. is an exact divisor of the length of the flute, exhibit an unaccountable break in the modal sequence, which is not distorted, but only interrupted by the interpolation of a note intermediate between two degrees of the modal sequence. Moreover, the point at which the break occurs varies in the different specimens.

Following on this disturbing occurrence comes the still more agitating discovery that a flute the length of which may, for instance, contain 13 I.D., gives with absolute integrity the modal sequence belonging to 16 I.D., and yet that the fundamentals at exit, and at the first hole are in agreement in theory and practice with their reciprocal lengths (e.g. in Flute Sensa A). When assailed by these experiences, felt to be nothing short of disastrous to the whole foundation of modality; and felt, moreover, to invalidate the principles so confidently claimed as the basis of the Harmonia (because this claim has been absolutely vindicated in respect of the Aulos), the investigator is temporarily plunged into gloom. But this depression is short-lived; faith in the sound basis of modality, in the logical and infallible operation of the underlying law, gains the upper hand and a firm determination to get to the root of the trouble brings to light some interesting new indications.

The important fact that emerges is that instead of making the aliquot division on the tube of the flute—as may be successfully done on the Aulos—the aliquot division must be referred to a multiple length, part of which is not immediately apparent, for to the actual length must be added the allowance for diameter at exit, consisting of two items, viz. (1) the diameter of the bore, measured from the outer circumference of the flute to the opposite inner edge of the bore, thus including the depth [1] of the walls on one side. (2) The difference between the diameter of the embouchure and that of the bore (without depth) added to No. 1. This aggregate allowance, added to the actual length, is the multiple length of which the I.D. is an aliquot with the Modal Determinant as multiplier.

The Sensa Flute A will now be used in illustration of the incidence of the allowance in respect of diameter All. (Extract from the Record of the Flute, Chap x.)

Length of flute emb. to exit	·465
Diameter of bore	·023
Diameter of emb. (d)	·010

TO FIND THE MULTIPLE LENGTH FOR DIVISION BY THE I.D. (=·028)

(A)

Diameter of bore (Δ)	·023
Δ − diameter of emb. (d)	·013
Depth of wall (de).	·003
(A) Allowance on actual length	·039
Length of flute	+ ·465
Multiple length for aliquot division	·504

i.e. I.D. = ·028 × M.D. 18 = ·504

The fundamental note of the flute at exit is

$$\frac{E\ 18}{256} = 312{\cdot}8 \text{ v.p.s.}$$

BY FORMULA 2. To find the effective length of the half-sound-wave of that v.f.

$$\frac{340 \text{ m./sec.}}{312{\cdot}8 \text{ v.p.s.} \times 2} = {\cdot}543 \text{ effective length of half-sound-wave.}$$

·543 effective length of half-sound-wave.
− ·465 actual length of flute

·078 allowance on effective length
·078/2 = ·039 allowance on actual length of flute.

Thus, in proceeding by calculation from the effective length derived from the vibration frequency, instead of from the actual length plus allowance, the allowance is double that of the actual, as seen above.

(B) BY FORMULA NO. 3 in operation, the position of Hole 1 is thus determined.

Diameter/2$\left(\frac{\cdot 023}{2}\right)$	·0115
$\Delta - d$ (·023 − ·010)	·013
$\Delta - \delta$ (·023 − ·009)	·014
2 (I.D.)	·056
Space allowed between exit and centre of Hole 1	·0945

[1] It is not yet definitely proven that depth of walls needs to be taken into account in all computations of pitch, except at the embouchure and at the fingerholes. In item No. 1, it has been taken at exit *instead* of at embouchure, being more easily measured there.

The half diameter only is taken on the flute from exit to Hole 1, because its effect on the sound-wave is doubled.

·0945 − ·056 = ·0385 allowance transferred to actual flute between exit and Hole 1.

It will be noticed that in the Sensa A the allowances under (*A*) and (*B*) correspond: the half diameter taken in (*B*) compensated by the additional allowance for the diameter of the fingerhole, and by the omission of allowance for depth, balances the whole diameter + depth taken in (*A*).

The primitive flute-maker is not likely to discover by himself this remote process for the determination of modality, and consequently, after a few unsatisfactory trials, he probably discards the exit note or starts his scale from Hole 1, used as vent, and always left uncovered.

It will probably be granted that the process, by which these results are obtained, effectively rules out the possibility that a primitive flute-maker could bring the Harmonia to birth unaided. This could only be accomplished as a deliberate reproduction of the Aulos Harmonia by empirical means. It may be suggested that the primitive craftsman may have set about his objective in the following way:

He selects a suitable length of reed or bamboo, and pierces a passage through the natural knots, the even width of which he tests by the insertion of a rod. A mouth-hole of convenient size is then bored at one end, at a short distance from a natural knot, left for the purpose. Or alternatively a stopper is made by pouring in heated wax or resin, in order to fill up the space within the head, to a predetermined distance from the centre of the embouchure, sufficient to preserve the purity of the harmonic register. Next, as a skilled craftsman, the flute-maker would tentatively place what is destined to be the first hole at a fair distance from the end of the tube, so that the distance may be shortened between exit and first hole as desirable, when the rest of the holes have been plotted. Then comes the crux of the whole creation, the holes which are to sing for him the song of his desire. Carefully and lovingly he fingers the flute, and when six of the holes are plotted in position, conveniently placed—which will almost invariably be found at equal or approximately equal distances—he measures with his eye the remaining distance to the mouth-hole, and with his almost unerring instinct for proportion, sees to it that it is commensurate; may be trying the space first with the laying-on of fingers. Satisfied at length, he marks the centre of each hole to be pierced and rounds each smoothly by boring or burning. Testing his instrument, he whittles down the bamboo until the fundamental is to his liking, or gives the familiar note to follow the song below the first hole.

If our flute-maker is but a budding craftsman, as yet inexperienced in making a flute that will speak in a language familiar to him, he will probably begin by spacing his fingers from the exit upwards until he finds that something has gone wrong, as in flutes Nos. 8 and 8B, from North-west India. Then he tries again and again until he gets the right distance. Thus the flute comes into being without calculation, and it is left to the theorist to discover the laws upon which the embodiment of a modal sequence on a flute is based.

By virtue of the invariable and logical progression of ratios proper to each Mode, which is imposed by the characteristic Modal Determinant, operating in concert with the I.D. between the centres of fingerholes, it is possible to trace the workings of the effective length of the sound-wave, which includes the allowance due to diameter.

Step by step the Harmonia progresses—not merely on paper—through the scale embodied in the fingerholes of flutes. From these results (given in full in the Flute Records), a new light is shed upon the behaviour of the air-column within the flute, as soon as the initial vibratory impulse has been given through the mouthpiece; these point to certain helpful deductions concerning the harmonic register of flutes. By means of the mathematical apparatus supplied by the unique basis of the Harmonia, experts may now check, step by step, the operation of certain empirical rules and calculations upon which they have to rely *faute de mieux*, in plotting the scheme for the boring of fingerholes, when it is desired to introduce some improvement or new device for the modern flute.

THE SIGNIFICANCE OF THE INCREMENT OF DISTANCE (I.D.)

There is one factor in the production of a modal flute which has not yet received due consideration, and that is the equidistance left from centre to centre of the fingerholes, known as the Increment of Distance (I.D.) which, by the primitive flute-maker, is chosen haphazard. And yet, there is a chance that the increment may be an aliquot of the length from exit, or from vent; in which case, the Harmonia will be true to the Modal Determinant, but the sequence will be interrupted, because no provision has been made for the lengthening due to diameter; in fact, an erroneous impression may have been given that the factor was a matter of fortuitous allocation, of mere convenience for the spread of the fingers in covering the holes; yet this I.D. is the most noticeable indication of the modality of a flute, and its importance must now be stressed.

THE FOUR ASPECTS OF THE INCREMENT OF DISTANCE

It has been found that there are four important aspects of the I.D. to be recognized, for they form part of the inner workings of modality.

I.D. NO. 1.—The actual increment of distance, as seen on the flute itself, measurable and chosen without premeditation, therefore, may, or may not, be an exact aliquot of the whole length, although it represents equidistance.

I.D. NO. 2.—The proportional I.D. which is regular, and an exact aliquot of the length from embouchure to exit. To the primitive flute-maker, it may have come naturally, after realizing the implications of the first attempts with the lay of the fingers on the flute, to allow a definite number of these increments, even where no fingerholes were required. This obvious actual I.D., reckoned from embouchure to exit, however, brings about almost invariably an interruption in the modal sequence (as foreshadowed above), e.g. in the Sensa *B* and *C* flutes, in Nos. 8 and 8B, and in the Bali flute No. 1, where the length from exit is a multiple of the I.D. and the Modal Determinant. When the I.D. has been taken from Hole 1 (used as vent) which implies the rejection of the exit note, then the sequence frequently proceeds without interruption, as in the Java Soeling No. 6 and the Japanese Flute No. 13.

In a second contingency, somewhat rare in primitive flutes, when it happens that the first hole has been placed at a distance from exit which includes a sufficient allowance for diameter—sometimes equal to one or more I.D.—then also the Harmonia is produced from exit and vent in integrity, e.g. in the Java Soeling No. 5, in which the increments are exact to a millimetre throughout, and two of these have been allowed between exit and vent, but without extra allowance. The result is that the sequence proceeds from exit uninterrupted from ratio 11 to ratio 10 at vent, i.e. one I.D. only; the second increment serves as allowance, and the note of the vent, although too near embouchure by ·007 according to Formula No. 3, is forced into tune by the proportional impulse, favoured by the exact boring of holes at equidistance.

It is a noteworthy fact that this flute, so exact in measurements, displays agreement in the position of Hole 1 at ·0447 with the allowance for vent calculated independently of exit at ·045, and with the allowance on the sound-wave length, by elimination of actual length of flute, at ·0438, a triple confirmation of the operation of formulae.

Flute Java 6 'Soeling' is also a model of exactitude, giving the same scale as Java 5, but from vent. The allowances from the sound-wave length on the note of Hole 1, and the allowance calculated by Formula No. 2, correspond at ·0515 and ·051 respectively, while by Formula No. 3 the position of the vent with one I.D. only differs by 3 mm. at ·0485. These two flutes are a notable achievement of the musicianly Javanese.

I.D. NO. 3.—The third aspect of the I.D. proceeds directly from the effective wave length, and belongs to the realm of the theorist. Unsuspected, unrecognized, it is nevertheless always present in every flute, unobtrusively operative wherever a note is elicited, as

$$\frac{\text{effective wave length}}{\text{M.D.}} = \text{I.D. No. 3.}$$

This I.D. No. 3 comprises the actual I.D. No. 1, with the addition of the incremental allowance due to diameter, found by apportioning the allowance from exit, carried out on the flute between exit and Hole 1, among the number of increments denoted by the Modal Determinant. In fact, I.D. No. 3 inevitably dominates the situation, exacting its due, and acting as a compensating factor in irregularities or shortcomings in the boring of holes. It will probably be realized later that it acts like the gnomon in the scales, balancing the negative and positive excesses, where perfect intonation is achieved.

This I.D. No. 3 represents the effective sound-wave length, which has to be accounted for somehow in the subtle internal affairs of the modal flute. The remainder after the actual I.D. No. 1 has been eliminated (on paper) must be halved in order to realize its compensating effect on the actual flute measurements.

I.D. NO. 4.—When the incremental allowance has been subtracted from No. 3, then I.D. No. 4 emerges as the *ideal* increment (which may, therefore, differ from the actual). This ideal I.D. No. 4 is, however, alone capable of producing the modal sequence under perfect conditions in unbroken sequence and in correct intonation, as in Sensa A, made in 1917. For this no credit can be claimed, since at that date the adoption of its I.D. No. 4 (= ·028) was a matter probably equally due to chance and convenience, and not to foreknowledge. It is incontestably the ideal increment for this flute, as may be seen from the analysis, which follows in due course.

Just as there are several aspects of the I.D., so there are several contingent allowances due to diameter, viz.

NINE ASPECTS OF ALLOWANCE DUE IN RESPECT OF DIAMETER IN MODAL FLUTES

All. No. 1. Sound-wave Allowance.—The sound-wave length minus actual length. When measurements are taken from exit, if the pitch has been correctly estimated, All. No. 1 should be double the All. No. 2.

All. No. 2. Exit Allowance.—On the diameter at exit, i.e. by use of Formula No. 5 [Δ + (diameter − d. of embouchure) + de] = half No. 1.

All. No. 3. Actually on the Flute.—This is found actually represented in the length allowed between exit and Hole 1 − I.D. ; All. No. 3 (which is frequently approximately equal to All. No. 2) is obtained by use of Formula No. 3 $[\frac{\Delta}{2} + (\Delta - d) + (\Delta - \delta)]$.

All. No. 4. Vent or Standard Allowance.—The independent allowance for diameter when Hole 1 is used as vent, and is obtained by working out Formula No. 4 : $[(L + 2(\Delta + (\Delta - d) + (\Delta - \delta) + de)]$. All. No. 4 is used as Standard Allowance in apportioning from it the allowance per ratio at each fingerhole (termed Proportional Allowance), and in comparing this with the allowance obtained as x remainder (by use of Formula No. 2) from the length of the half-sound-wave minus actual length = x. There is another case in which All. No. 4 is of use, viz. in computing the exact values of cross-fingered notes on modal flutes (see Chap. vii).

All. No. 5. Proportional Allowance (Ex.).—Obtained from Exit All. No. 2 by the ratio of the fingerhole to that of exit, by use of Formula No. 5 $[\Delta + (\Delta - d) + de]$, divided by the ratio at the fingerhole.

All. No. 6. Proportional Allowance (Vt.).—Obtained from Vent All. No. 4 by ratio of fingerhole to that of vent.

All. No. 7. Incremental Allowance (= Inc. All. No. 7).—Additional cumulative allowance, not included in No. 5, derived from Exit All. No. 3 divided by the Modal Determinant, and multiplied by the number of increments accruing at any given hole (always one less than the number of the hole).

This cumulative allowance, progressing by increments, helps to determine the cause of the interrupted sequence by the interpolation of an intermediate ratio.

All. No. 8. The Allowance of the Ideal I.D. No. 4.—Obtained as complementary difference between I.D. No. 3 and I.D. No. 4; it then corresponds with Incremental All. No. 7.

All. No. 9. The Floating Allowance.—The floating allowance represents the ultimate pronouncement on the subject of allowance in respect of diameter in the modal flute. It is the irreducible concomitant of the proportional I.D.—which is a theoretical I.D. exclusive of any allowance, and of which the floating allowance represents the maximum. As the actual I.D. may equal the proportional, or lie anywhere between it and the I.D. of the effective half-wave length, the floating allowance and the proportional I.D. may be regarded as reciprocals. If the actual I.D. on a modal flute exceeds the proportional I.D., it is clear that it carries an allowance corresponding to the amount of that excess, and the floating allowance then consists of the remainder.

The ideal I.D. in a modal flute is the one that most nearly approximates to an aliquot of the total length of the flute plus the allowance from exit to the centre of Hole 1, taken twice for perfection—when it is equal to the floating allowance—or once for mere excellence.

N.B.—Whereas All. No. 7 includes the I.D. latent between exit and Hole 1, All. No. 6 ' proportional ' includes only those increments accruing from Hole 1. As an example, the record of the flute Sensa B may be cited (see Chap. x, Record).

The amount of the allowance due to diameter at any point on the flute is irrevocably settled through the difference between the sound-wave length and the actual flute length. The participation of one or more of the nine aspects of allowance, defined above, in the aggregate allowance is the affair of the theorist bent on analysis of the additional lengths accruing at a given measurable point. The aggregate allowance is always settled offhand by the pitch of the note; if, therefore, this aggregate cannot be balanced at all points, when the flute is tested, the vibration frequencies should be overhauled and carefully checked. Several methods have been tried in testing, and the one used in the Records is the most satisfactory yet found.

THE CLASSIFICATION OF MODAL FLUTES

Having cleared the way with these definitions of increments of distance, and of allowances due to diameter, suggestions for the classification of specimen flutes may now be offered, viz.:

CLASS I.—The most numerous consists of flutes in which the length from exit or vent to embouchure is a multiple within a few millimetres of the I.D. and the M.D. The flutes in this class are usually characterized by the interruption of the modal sequence at some point.

CLASS II.—Flutes on which this multiple length comprises the actual flute length from embouchure to exit (= Class ii Ex.) or from embouchure to vent (= Class ii Vt.) with the addition of All. No. 3; i.e. the one actually present in the length allowed between exit and the centre of Hole 1. The flutes of Class ii are characterized by a modal sequence free from irregularities. In this class it is the half-wave-length that is the multiple of the I.D.

CLASS III.—Flutes in which the actual length of the flute *less allowance* No. 3 is equal to the increment of distance × by the Modal Determinant, or conversely, in which the product of the I.D. × M.D. + allowance is equal to the actual length of the flute. Examples of Class iii are rare (see Chap. x).

This is a curious proposition which is found in flutes Nos. 8 and 8B, and in Sensa B. It has been found necessary to distinguish further in each class: (*A*) flutes having a normal or uninterrupted modal sequence, and (*B*) flutes having a modal sequence, interrupted at some unexpected point by the interpolation of a note bearing a ratio intermediate between two of the normal ratios of the modal sequence. After the interpolation, the modal sequence resumes its normal allure.

THE THREE EXPERIMENTAL SENSA [1] FLUTES

It was in order to obtain a solution of the unaccountable behaviour of the modal sequence, in some of the exotic specimens, that two companion flutes to Sensa A, viz. Sensa B and C, were made for purely experimental purposes.

When it was perceived that the only structural feature not directly determined by the pitch of fundamental or vent was the I.D. it became imperative to discover the effect of altering this I.D. while retaining the same length and diameters of bore, embouchure and fingerholes. Sensa B and C were besides plotted to play in the same Harmonia and at the same fundamental pitch as Sensa A. The only difference between these three flutes therefore, was the spacing between the fingerholes. The implication of this difference is one usually held to affect pitch. The following points may be noted:

(1) The larger the I.D. the nearer is the fingerhole to the embouchure, and therefore, the shorter is the actual length on the flute between these two points, and the higher the pitch and vice versa.

(2) It is expressly stated that the three flutes were plotted to give the same fundamental notes from exit and from vent; thus the vent must be centred at three different points from exit and from embouchure on the three specimens.

(3) If a note of the same vibration frequency issue from three different distances from embouchure, the wave length must be of the same length in each. The inference is that the differentiation is the affair of the allowance due to diameter.

(4) The diameters of bore, embouchure and fingerholes are identical in the three specimens.

[1] The ' Sensa ' flute was made for the incidental music composed by Elsie Hamilton for the play ' Sensa ' which deals with Ancient Egypt. The music composed in the species of the Harmonia demands a modal flute with equidistant holes: two specimens of this flute were played by professional flautists. (See Records, Chap. x.)

(5) It seems obvious that some unusual factor, some unexplained function is accountable.

(6) The dimensions of the three Sensa flutes being identical, except for the I.D. and the flutes having been plotted to play in the same Mode and at the same pitch, it follows that they cannot all belong to the same class. Since the class depends upon the I.D., Sensa A is in Class ii A; Sensa B and C are in Class iii B.

Sensa A has an I.D. (No. 4) which carries an allowance sufficient to ensure purity in intonation.

But in the Records of the flutes, the Proportional All. No. 5, calculated always on the standard allowance on Hole 1, does not include that part of the allowance which is represented by the difference between the actual I.D. and the proportional I.D., and this difference must be added cumulatively hole by hole to the proportional allowance in order to bring about a correspondence between the results of the two methods of presenting the analysis of the aggregate allowance exhibited in the Records (q.v.).

Sensa B was given an I.D. equal to the diameter of the bore, i.e. ·023; ·023 × 18 = ·414 and therefore ·465 − ·414 = ·051. This is clearly an instance of a flute belonging to Class iii, since the actual length of the flute exceeds the multiple length ·414 by an allowance of ·051. The result of a practical test of Sensa B revealed the fact that the intonation of the fundamental and of the notes of the first three holes was good, and the tone mellow. When compared with Sensa A, however, it was found that Hole 3 on Sensa A at ·3195 from embouchure and on Sensa B (owing to the smaller I.D.) at ·330—thus implying a lengthening of ·0105 in the distance of Hole 3 from embouchure—nevertheless produced a note of the same pitch from both holes. Thus this additional length of ·0105 on flute B, did not lower the pitch—a fact which seems to call for some explanation.

On Sensa B, the 4th fingerhole provided a further surprise, for instead of *A* 13—the next ratio in the modal sequence—the flute obstinately played *A* 27, a lower note, intermediate between 14 and 13, and therefore, due at a half-increment only, above *G* 14. Holes 5, 6 and 7 continued the modal sequence faultlessly, with notes of ratios *A* 13, *B* 12 and *C* 11, respectively, although each was produced at one whole increment nearer the embouchure than was normally to be expected from the effective length of the sound-wave, and from the order of the modal sequence.

This extraordinary experience on Sensa B does not, however strange, necessarily weaken one's reliance on the formulae, for if the actual positions of Holes 5, 6 and 7 on Sensa B be compared with those of Holes 4, 5 and 6 on Sensa A, the positive excess lengths on Flute A over B are respectively of ·007, ·0025 and ·002 only.

The onus for disturbing the sequence obviously falls upon the I.D., the only factor that has been changed in flute B. We are called upon here to witness the struggle of two forces thus brought into opposition; viz. the proportional impulse against the element of length in all its implications. Why does the proportional impulse break down, then, just at this point in the sequence?

The explanation of this phenomenon is to be found in the allowance for diameter from exit (see No. 2), which is actually carried out on the flute itself, when plotting the position of the centre of Hole 1—the allowance is the same for all three flutes, viz.

Diameter of bore (Δ)	= ·023
Δ − diameter of emb. + depth	= ·016
(i.e. ·023 − ·010 + ·003)	——
Exit allowance (No. 2)	= ·039

N.B.—This allowance is an actual length on the flute and, therefore, when used in connexion with the pitch of the sound-wave, it must be doubled. As this All. No. 2 is contributory in the determination of the position of Hole 1, and further, since the vibration frequency of Hole 1 determines the length of the sound-wave, and by elimination, of the aggregate allowance due at Hole 1—and thereafter by proportional ratio, at any of the fingerholes—so in an analysis of the aggregate, this proportional allowance must in addition bear its share of All. No. 7, which we have termed incremental; it consists here of ·039/18 = ·00216, or as effective, ·00216 × 2 = ·00432 per hole.

All. No. 7 is cumulative; it amounts on the flute only to a fraction over 2 mm. per hole, with a doubled effect of ·0043 per hole on the sound-wave; it is, therefore, negligible when considered as a single unit, but being cumulative with each additional hole or increment, it becomes sooner or later a disturbing element when a node or half-increment value is reached. This is what has in fact occurred at Hole 4 on the Sensa B flute, at 5 increments, viz. ·00216 × 5 = ·0108.

To this cumulative allowance must, in this instance, be added the difference between the actual I.D. ·023 and the proportional I.D. from Hole 1 as vent $\left(\frac{\cdot 376}{16 = \text{M.D.}} = \cdot 0235\right)$, viz. ·0005 × 3 = ·0015 for the three (I.D.) between Holes 1 and 4 thus:

Incremental All. No. 7 for 5 (I.D.) = ·00216 × 5 =	·0108
·0005 × 3 (I.D.) =	+ ·0015
Aggregate Incremental Allowance =	·0123

This aggregate allowance is equal to a fraction of 1 mm. more than the half I.D. at the node (viz. ·0115). The opening of Hole 4, therefore, produces the note due at the half-increment lower instead of its normal note.

N.B.—The proportional allowance is reckoned from All. No. 4, standard vent at Hole 1, and therefore, it is taken for three increments from Holes 1 to 4, whereas Incremental All. No. 7 is cumulative from exit for five (I.D.) inclusive of the two latent between exit and Hole 1, since no allowance for diameter is included in the I.D. of this flute.

This, then, is the secret cause of the interrupted sequence wherever it may occur.

The reason why the modal sequence of flute Sensa A is immune from disturbance may be demonstrated thus:

The flute has been given the ideal I.D., No. 4, measuring ·028, which exceeds the exact aliquot for these flutes (I.D. No. 2 = ·0235 as in Sensa C) by ·0045, an amount which represents the allowance for diameter per increment. It will be seen that this excess of ·0045 is sufficient to cover the Actual All. No. 3 = ·00216, plus the incremental, No. 7, amounting in these Sensa flutes also to ·00216 per increment. Since, therefore, these two disturbing factors are in Sensa flute A, included in the I.D., there can be no question of interruption in the modal sequence as far as Hole 7. This flute has 7 holes = 6 increments ; ·00216 × 6 = ·01296, or less than half the I.D. Thus the cumulative effect of All. No. 7 could only become operative at an 8th hole. This was, in fact, discovered in 1917, when a duplicate Sensa A was given an 8th hole, which was found to play *E* 19 —instead of *E* 9, octave of the fundamental—a note intermediate between *D* 10, and *E* 9, and therefore, due at a half-increment between Holes 7 and 8. The octave of the fundamental, viz. *E* 9, was obtained from a 9th hole at a distance of ·152 from the embouchure.[1] The flute Sensa C suffers from the same kind of interruption of the modal sequence as Sensa B, but not until Hole 6 has been reached. The notes of the first five holes give out a perfectly tuned sequence, and the results of the operation of the formulae correspond as they should according to ratio, exact to the tenth of a millimetre, without a hint of the necessity for the addition of an incremental allowance, which manifests itself suddenly at Hole 6 ; when the aggregate for the five increments is needed to balance the proportional No. 6 with All. No. 1 from the effective half-sound-wave length. In Sensa C, the I.D. (·0235) is the exact aliquot of the length from embouchure to vent, at which the M.D. is 16, viz. ·376/16 = ·0235. This I.D. is No. 2 in which no provision has been made for the allowance due to diameter.

Proceeding as for Sensa B, Alls. Nos. 2, 3 and 7 being the same for Sensa C—it is found that at Hole 6, at which *C* 11 should follow *B* 12, a note intermediate between the two, i.e. *B* 23 is given instead ; and *C* 11 is played in tune from Hole 7 and *D* 10 from Hole 9. Hole 8 produces the intermediate note *C* 21.

THE MODAL SEQUENCE OF THE HYPOPHRYGIAN HARMONIA ON THE FLUTE ‘ SENSA C ’

Holes	Exit	1	2	3	4	5	6	7	8	9
Ratios	*E* 18,	*F* 16,	*G* 15,	*G* 14,	*A* 13,	*B* 12,	*B* 23,	*C* 11,	*C* 21,	*D* 10
Actual Length	·465 ;	·376 ;	·3525 ;	·329 ;	·3055 ;	·282 ;	·2585 ;	·235 ;	·2115 ;	·188
Cents		*204 ;*	*112 ;*	*119·4 ;*	*128 ;*	*138·5 ;*	*73·66 ;*	*76·9 ;*	*80·5 ;*	*84·4.*

N.B.—The actual lengths given above are those at which the holes are bored in the flute, measured from the centre of the embouchure to the centre of the fingerholes.

[1] The cause of the interruption of the modal sequence in this flute was not understood at the time ; it was attributed to a different cause, viz. the lengthening effect of the same diameter for the second octave in a cylindrical flute—a matter which is discussed further on in connexion with a Chinese Formula for the determination

The Incremental All. No. 7—the same for the three Sensa flutes— $= \frac{\cdot039}{18} = \cdot00216$, and for five increments at Hole 6 = ·0108 (·011). The accumulated incremental allowance ·011 reaches parity with the length of the half-increment ·0235/2 = ·0117, forming a node, and the strong proportional impulse, reinforcing the factor of length, causes Hole 6 to speak the note that should issue from a hole half-way between 5 and 6, viz. having a ratio *B* 23, intermediate between 12 and 11. Hole 6 centres at ·2585 from embouchure, the v.f. of ratio *B* 23 $= \left(\frac{352 \text{ v.p.s.} \times 32}{23}\right) = 489{\cdot}7$ v.p.s.

$$\frac{340 \text{ m./sec.}}{489{\cdot}7 \times 2} = \frac{340 \text{ m./sec.}}{979{\cdot}4 \text{ v.p.s.}} = \cdot3471, \text{ effective length of half-sound-wave.}$$

The actual length of Hole 6 = ·2585, increased by the addition of the allowance for five increments.

$$\begin{array}{rl} \cdot2585, & \text{actual length;} \\ + \ \underline{\cdot0117}, & \text{half I.D.;} \\ \cdot2702, & \text{virtual length.} \end{array}$$

therefore :

$$\begin{array}{lrl} (A) & \cdot3471, & \text{effective length of half-sound-wave.} \\ & - \ \underline{\cdot2702}, & \text{virtual position of Hole 6 from embouchure;} \\ & \cdot0769, & \text{effective allowance in aggregate.} \end{array}$$

(*B*) Proportional All. No. 6.

$$\frac{\cdot107 \times 23}{32} = \cdot0769;$$

thus the results from (*A*) and (*B*) agree to a fraction of a millimetre.

N.B.—The incremental allowance at ·0108 is 1 mm. short of the half-increment at ·0117, but nevertheless the interpolation takes place. The dominance of the proportional impulse again makes itself felt at Hole 7, which plays the *C* 11 due at Hole 6.

The rest of the progress of the modal sequence through Holes 7, 8 and 9 may be traced in the Record of Sensa C. The role of the Proportional All. No. 6 is to apportion the Vent All. No. 4 in accordance with the pitch ratio at any given fingerhole, for the purpose of checking the correctness of the allowance. The incremental allowance needs to be added to the proportional, in order to balance, in flutes such as Sensa A, having the ideal increment of distance, in which the incremental allowance is included.

This explanation may sound complicated; it is, however, quite logical and it is, moreover, necessary to the understanding of the vexed question of allowance for diameter in modal flutes, and more especially in order to throw light upon the cause of the baffling interruptions in the modal sequence, occurring so frequently in primitive flutes.

of length and pitch in such a case. It is a curious fact that the working out of the Formula for All. No. 7 given above, and of the Chinese Formula, give identical results for a 9th hole on this flute.

The fact that the increments of distance progress from exit upwards, in a direction opposite to that pursued by the sound-wave, always measured from the embouchure, may be confusing. Thus to increase the I.D. is to decrease the actual length traversed by the sound-wave, with a consequent rise in pitch. When, therefore, the cumulative incremental allowance reaches parity with the half I.D., the effect appears to be a retrograde one, dragging down the pitch ratio of the hole to that of a half-increment lower.

It may, therefore, be noted that as the modal sequence rises by ever-widening intervals, so the proportional effect of the lengthening influence of diameter on the actual length diminishes (owing to the fact that ratios of pitch are superparticular, whereas those of length are fractional). Again, the cumulative Incremental All. No. 7, increasing with the number of increments, gradually gathers strength, until it reaches parity with the length value at a nodal point: the two factors being reciprocals in operation, or in inverse proportion. This is proved beyond doubt by the correspondence of the two results, based, the one upon the effective allowance from pitch (No. 1), the other upon the analysis, item by item, according to the formula. It is thus hoped that this somewhat lengthy investigation into the intricacies of allowances may be considered fully justified by the results.

A word or two may be added at this point to draw attention once more to the strong proportional and rhythmical impulse generated in modal flutes by the Modal Determinant. This remarkable phenomenon appears to afford a confirmation of the fact that this proportional impulse produces the notes of the modal sequence true to ratio, rather than to absolute length, and is strong enough to dominate the situation; at times, it even seems to control the periodical pulsations of the column of air within the flute at the instance of some stimulus. A suggestion has already been offered as to the nature of this stimulus observed in operation on the Aulos (Chap. iii). Under analogous conditions, proportional impulses initiated on a string produce various harmonic overtones, which are either collectively inherent in the pitch note of a string, or singly dominating —according to the stimulus—and substituting their aliquot vibrational impulses for those of the string as a whole.

The reactions to stimulus in wind instruments, and notably in flutes and pipes, are of a much more subtle nature than those of a string. The fact that the incremental allowance produces no definite effect on pitch, until the accumulation approaches the value in length of node or antinode, presents an analogy with the behaviour of strings.

CHAPTER VII

THE CYLINDRICAL MODAL FLUTE IN THEORY AND PRACTICE

EXTENSION OF COMPASS BY CROSS-FINGERING AND HALF-STOPPING

Introductory. Five Important Factors in the Acoustics of the Modal Flute Recapitulated. Extension of Compass by Half-stopping in the Modal Flute. Extension of Compass by Cross-fingering in the Modal Flute. Experiments on the Mond Flute in the Theory and Practice of Cross-fingering. Demonstration of the Significance of Formula No. 3 in the Pitch-determination of the Cross-fingered Note. Further Examples demonstrated on Flute ' Sensa C '. Cross-fingering as a Means of Transition from one Musical System to another. The Evidence of Virdung and Agricola on Cross-fingering. Agricola's Cross-fingering tested in Practice. Hypolydian Flutes cross-fingered give the *Authentus Protus*. The Diameter in Cylindrical Tubes in Octave Relation : the Chinese Formula. Purity of Intonation in Modal Flutes, in spite of Excess Diameter in Upper Half of Pipe. Types of Flutes other than Transverse. The Transverse Flute in Ancient India : Treatises by Bhārātā and Sarangdev. Implications inferred from Sarangdev's Table of 15 Modal Flutes with Equidistant Fingerholes. The Transverse Flute in Evolution in Europe. The Ditonal Scale adopted by the Arabs. The Duplication of the First Tetrachord in the Octave Scale. The Modal Flute in Syro-Arabic Sources : Al-Fārābī's Evidence. The Influence of Diameter recognized by Ptolemy. Al-Fārābī describes Flutes and Pipes with Equidistant Fingerholes. The Lute Accordance No. 3, introduced by Ishāq al-Mauṣili (Fourth, Fifth, Fourth). Ishāq's Classification by Courses (*Mājārī*) corresponding to the Modal Species. The *Wosta of Zālzāl* (= Ratio 27/22) implies the use of the Lydian Species of M.D. 27. Further Modal Implications of the *Wosta of Zālzāl*. Our Minor and Major Modes are akin to the *Mājāri* through *Wosta* (Min. Third), or through Binṣir (Maj. Third) of Arabian Lute Accordance. The Modal Scales of the Octoechos traced in Ishāq's Classification of Melodies, and in his Lute Accordance. Al-Fārābī and Al-Kindi stress the Ditonal Scale : Ishāq-al-Mauṣili the Modal Species of the Harmonia. The Modal Flute based on Proportional Modality breaks new Ground

INTRODUCTORY

THE somewhat forbidding array of factors, visible and invisible, which participate in the reactions of the air column in flutes, must be accepted as scientific facts. It is, however, necessary that these same technical and theoretical conditions, under which music is coaxed from the modal flutes, should be explicitly stated here, since the modal flute has remained until now an unknown quantity. The apparent simplicity of the instrument, devoid of keys or similar devices for increasing the compass in practice, is deceptive.

Nearly all the primitive flutes are modal—a fact which suggests that the rise of primitive music is almost universally modal—but as the Modes themselves are unknown to Western musicians, the flutes, when tested, are usually regarded as of slight interest and importance, capable only of producing inconsequent sequences, false in intonation according to Western criteria. How erroneous such a judgement is will be obvious when the records of modal flutes from many distant lands are examined and found to contain examples of scales unmistakably akin in modality, intonation and genesis.

Elsewhere it will be shown how these modal scales—born anew among all races and in all ages, have developed and formed the basis of various musical systems [1] in both East and West.

FIVE IMPORTANT FACTORS IN THE ACOUSTICS OF THE MODAL FLUTE RECAPITULATED

In flutes, the complicated relationship of actual and effective length with the diameters of the bore, embouchure and fingerholes, as well as with the I.D., need not act as a deterrent to any student. The vital points to be grasped are concerned with the following five factors, and it is hoped that the more practical aspect now to be given may prove helpful.

(1) The pitch of the fundamental, i.e. of the flute blown with all holes closed, accurately determined, from which is obtained No. 2.

(2) The effective half-wave length of the fundamental, corresponding to the actual length of the flute (No. 3) with the addition of the allowance for diameter at exit (No. 4).

This half-wave length is not audible; it exists in theory only. In order to represent the audible note of the fundamental, or complete sound-wave, the vibration frequency of the half-wave must be halved. The object of introducing the inaudible half-wave length in theory is that it applies to the *actual* length of the flute, visible and measurable, whereas the whole sound-wave involves a double transit along the flute, plus a double allowance for diameter.

(3) The actual length of the flute from the centre of the embouchure to the edge of the exit.

(4) The definite allowance in respect of diameter at exit, easily calculated by means of Formula No 4 (see Chap. vi) and confirmed by the elimination of No. 3 from No. 2.

(5) The Increment of Distance, the full significance of which is not immediately apparent.

The first four factors are interrelated and interdependent. By means of Formula No. 2, Factor No. 1 unerringly produces Factor No. 2, while Factor No. 3 deducted from No. 2 gives us No. 4. Obviously any one of these may be deduced from the other three. These first four factors are common to all flutes, modal and non-modal, and involve no difficulties or uncertainties. Nos. 3 and 4 of these factors, being instantly measurable and ascertainable, are decisive. If either of these fails to agree with the findings of No. 2, as derived from No. 1, then the error lies in the pitch of the fundamental which has been incorrectly estimated, and must be carefully revised. There is no alternative or loop-hole. No. 4 is an

[1] See Chap. iv and also Chaps. viii and ix on Folk Music.

integral element of the flute as it stands; therefore, if the exact number of millimetres, of which the exit allowance in respect of diameter consists, does not emerge from the half-wave length (as obtained by deduction of factor No. 3 from No. 2), the estimate of pitch which is the concern of the ear is clearly at fault.

No. 5, the increment of distance derived from the equal-spacing of fingerholes, is the outward visible sign of modality, implying aliquot division; it introduces an element of subtlety, the implications of which have necessitated a threefold classification of modal flutes.

The I.D. between the fingerholes in primitive flutes is adopted haphazard, or presumably to suit the spread of the fingers, but without an inkling of its significance in the determination of modality; and of its responsibility towards the maintenance of an uninterrupted modal sequence, proceeding fingerhole by fingerhole in correct intonation.

The actual I.D., present between the fingerholes on any specimen, may be an exact (or a mean) aliquot of the actual flute-length, from centre of embouchure to exit, or to centre of Hole 1.

The actual I.D., however, while preserving equidistance between fingerholes, may prove incommensurable with this total length, and therefore, baffle the inquirer into its modality, of which his trained ear nevertheless assured him (see 'Sensa A' Record).

If the position of Hole 1 has been even approximately correctly estimated, at a distance from exit considerably in excess of one I.D., it will probably be found that by adding the amount of the allowance for diameter at exit to the actual length, a workable multiple of the I.D. is obtained—e.g. as in 'Sensa A' and in flute No. 4 from North-west India—which reproduces a modal sequence free from complications or interruptions. In such a case the I.D. has been happily plotted, because each increment carries its own portion of the allowance for diameter, in addition to the purely proportional increment. This is reckoned from Hole 1 by dividing the length by the ratio number at that hole. In 'Sensa A', e.g. the length from the centre of Hole 1 $= \frac{\cdot 375}{16} = \cdot 0234$; $=$ the proportional I.D.

The actual I.D. $= \cdot 028$. The difference between these two, viz. ·0046 represents the amount of allowance for diameter carried by the actual I.D.; the *Floating Allowance* per I.D. (see Chap. vi) denotes the maximum allowance available. How this works out hole by hole may be seen from the records.

The student possessed of sufficient driving power to wish to penetrate the obscure reasons for the behaviour of the air column in modal flutes under pressure from the I.D., may study the foregoing technical details of the exposition (see Chap. vi) and the records of the three experimental Sensa flutes A, B and C. With dimensions identical, except for the length of the I.D., and preserving the same fundamental intonation and modality, these three flutes differ only in the resultant fact that 'Sensa A' gave an uninterrupted pure modal sequence, while in 'Sensa B' and 'C' the scale was interrupted by the interpolation of an unwanted chromatic note; the break in the sequence occurring in B at Hole 4, and in 'Sensa C' at

Hole 6. In view of such perplexing results of experiment it became imperative to discover the cause of this baffling occurrence, which was eventually found to be due to the cumulative 'Incremental All. No. 7' (see Chap. vi); this remains latent until the accumulated length reaches parity with a nodal point at a half I.D. The addition to the wave length, then, having become effective, forces the note due at the fingerhole to vibrate at the slower frequency due at the half-increment lower, in spite of there being no fingerhole at that point.

In the Sensa flute A, with seven fingerholes (six increments) the Incremental All. No. 7 = ·00216. At Hole 7 the accumulated allowance of ·00216 × 6 = ·01296 has not yet reached the nodal point of the half-increment $\frac{\cdot 028}{2} = \cdot 014$, and therefore no break in the sequence occurs. The nearness of the approach to the nodal point, however, makes itself felt by a slight tendency to flatness in the note of Hole 7 which may easily be overcome.

Since the Incremental All. No. 7 is based upon the diameters of bore and embouchure, and is independent of the I.D., it is clearly an advantage, in plotting a flute, not to space the fingerholes nearer together than convenience in fingering the holes requires. After the detailed technical exposition of the complicated reactions of the air column in modal flutes, and this recapitulation of the more practical application of the many factors involved, we may now turn to other considerations.

EXTENSION OF COMPASS BY HALF-STOPPING IN THE MODAL FLUTE

COMPASS.—The compass of the modal flute may, as in the modern concert flute, be extended by means of the harmonic register. When the notes of the fundamental scale are overblown an octave, both modality and tonality are preserved; but overblowing a 12th or 14th brings about—not an extension of compass—but a modulation into the tonality of dominant or 7th.

The pentatonic or diatonic compass of a modal flute may easily be supplemented by means of so-called chromatic notes,[1] obtained by half-

[1] Half-stopping does not always produce notes belonging to the chromatic compass in the modern sense of the word. The half-stopped hole produces the note intermediate in ratio between those of its own hole and of the one below it: e.g. if the 5th and 6th holes of a flute produce notes in the ratio 7/6—a septimal 3rd apart—to half-stop Hole 6 lowers the note from ratio 12 to 13, thus:

	Hole 5	*Hole 6*	
	●	○	◐
	○	○	○
	○	○	○
	○	○	○
	○	○	○
	○	○	○
Ratio	7 or 14	6 or 12	13

The result of half-stopping is, therefore, to lower the note of Hole 6 by an interval of ratio 13 : 12. If the ratio between Holes 5 and 6 were 5 : 4, half-stopping would lower Hole 6 to ratio 9 at a minor tone, instead of a major 3rd above Hole 5.

stopping a fingerhole and thus producing through that hole a note of lower pitch. This lowered note is identical in intonation with the note which would be produced by a fingerhole bored at a half-increment only. In the majority of instruments lacking the modern device of keys, this expedient is eminently satisfactory and easy to carry out.

To reduce the diameter of a fingerhole lowers the pitch proportionally, in accordance with the fact that the diameter of an exit for the sound-wave, which is less than that of the bore of the flute, has the effect of lengthening the issuing sound-wave by the amount of the difference between the two diameters taken twice, e.g. in a flute having a bore of ·020 and of fingerhole ·008 in diameter, the open fingerhole has an additional lengthening effect of $2(\cdot 020 - \cdot 008) = \cdot 024$.

Half-stopping the same hole increases the wave length thus: $2(\cdot 020 - \cdot 004) = \cdot 032$. The result of half-stopping is an addition to the original allowance computed for that hole, and by adding to the length of the sound-wave, the pitch is lowered.

In Oriental music, in which microtones are used melodically, quarter-stopping is also practised.

EXTENSION OF COMPASS IN THE MODAL FLUTE BY CROSS-FINGERING

On a bamboo flute having seven equidistant holes, Mr. P. Sambamoorthy, a highly skilled flautist from Madras, was able, in my presence, by half- and quarter-stopping and by cross-fingering, to obtain all the śrutis requisite for playing in a great variety of rāgs.

There is evidence in a Hindu treatise by Bhārātā [1] on the Drama, entitled 'Nātya Shāstra' dating from the fifth century A.D., that the practice of half- and quarter-stopping was in common use in India at that early date, as well as the expedient known as cross-fingering (French *doigté-fourchu*; Ger. *Gabelgriff*), still in use on wood-wind instruments at the present day.

Cross-fingering depends upon the fact that the open fingerhole nearest to the mouthpiece is the one that speaks, and in order to produce its own note in perfect intonation, all holes below it must likewise be uncovered. Every hole closed below that open one lowers the pitch of the note to some extent, and this device of cross-fingering gives scope for great skill. Not only is this expedient used in correcting and adjusting intonation, but also in providing alternative fingering for notes of the same pitch, but differing slightly in timbre, which are found to be awkwardly placed for fingering. Attempts have been made from time to time by scientists to explain the reactions of the air column upon which the practice of cross-fingering on the flute is based, but without providing definite authoritative data. The whole question is, however, usually treated from an abstract point of view, wrapped up in an algebraic formula, [2] and there left.

[1] Bhārātā, translation of Chap. xxx on Music by Joanny Grosset (Lyons, 1897); and see also Chap. ii.

[2] E.g. in a modern work, *The Acoustics of Orchestral Instruments*, by E. G. Richardson, pp. 48–53, and pp. 147–55. The statements there given, professing

EXPERIMENTS ON THE MOND FLUTE IN THE THEORY AND PRACTICE OF CROSS-FINGERING

We are concerned here, however, with the structure and actual performance of flutes; with the factor of length in its various applications, and with its reciprocals in vibration frequencies.

As already stated, the modal flute lends itself admirably to research on these questions. It is proposed, therefore, to leave the scientific explanation to the experts, and to give an example of the effect of cross-fingering on a modal flute [1] which will show whether the theoretical estimate is in agreement with the practical result, apprehended by the ear and confirmed by the monochord and the modally tuned piano.

The experiment was carried out on the Mond [2] flute from Sicily (see Record for details) with six fingerholes in front and the 7th at the back, covered by the thumb. The total length from the centre of the embouchure (diameter ·009) to exit = ·271; the diameter of the bore = ·020; of the fingerholes ·008 to ·0088; the I.D. is constant at ·025 with one exception between Holes 6 and 7, at rather less than a half-increment. The cross-fingering consisted in opening Hole 4 which plays an *F* of 704 v.p.s. at a Perfect 5th above the exit note *B* 12/256, the modal ratios being those of the Phrygian Harmonia, 12/12 : 8/12. Both notes, on being tested with normal fingering and blowing, by monochord and modally-tuned piano, were found to be true in intonation. Then Holes 1, 2, 3, below Hole 4 were closed, and the *F* hole now played *E* 9, of 625·6 v.p.s., which is the note normally sounded through Hole 3, when Holes 1 and 2 are open below it. Passing on the same breath from the cross-fingered *E* 9 of Hole 4, to the normal *E* 9 of Hole 3, the notes were found to be identical: no difference in intonation could be detected.

Thus, in this experiment the result of closing three holes below the speaking-hole, lowered the note of Hole 4 to the note of the next lower increment by a whole major tone (9/8), viz. as in the diagram on opposite page.

to explain the method of determining the position of note-holes, are vague and entirely inconclusive. To work out through area and circumference of the air column, which are not directly measurable on any specimen flute, without explaining their bearings on the factors of length on the flute, which must be accurately computed in order to arrive at the pitch frequencies, is to treat the subject in the abstract. In the plotting of a flute on p. 151, the particulars available are likewise totally inadequate and prove nothing. No dimensions of the flute are given. The holes are numbered from the embouchure, instead of from exit, and there is no clear statement of an underlying theory.

[1] See the critical review by K. S. of Dr. Richardson's *Handbook*, *op. cit.*, in *The Musical Standard*, 1930, issues of May 17th and 31st, and June 14th and 28th; see June 14th, p. 198.

[2] This distinctive name was given to the Sicilian peasant's flute in memory of the donor—the late Mrs. Ludwig Mond—and of the keen, unflagging interest she took in this work of research.

FIG. 47.—Scheme showing Crossfingering Performance and Results on the Mond Flute No. 2.

Normal fingering

(1)

●*
●
●
○
○ $= \dfrac{F\,8}{512} = 704$ v.p.s. ratio to fundamental 12 : 8 perfect 5th
○
○

* = Hole in the back of flute.

Cross-fingering

(2)

●
●
●
○ $= \dfrac{E\,9}{512} = 625{\cdot}6$ v.p.s. ratio to fundamental 12 : 9 perfect 4th.
●
●
●

This note is identical with the one produced by normal fingering.

(3)

●
●
●
●
○ $= \dfrac{E\,9}{512} = 625.6$ v.p.s.
○
○

DEMONSTRATION OF THE SIGNIFICANCE OF FORMULA NO. 3 IN THE PITCH-DETERMINATION OF THE CROSS-FINGERED NOTE

It is obvious that the closing of Holes 1, 2 and 3 (see Fig. 47, No. 2) and the opening of Hole 4 on a flute *confers temporarily upon the open hole the functions and implications of a first hole.* The operation of Formula No. 3 for the determination of the position of Hole 1 may, therefore, be used to confirm the evidence of the ear.

The working out of the formula gives the allowance for diameter at exit, actually carried out on the flute between exit and Hole 1, to which is added the sum of the increments of distance passed over between the two points.

On the Mond flute (see Record, Chap. x), therefore, from which our illustration of cross-fingering is taken, the dimensions are:

Length C. of emb. to exit	·271
Length C. of Hole 4 to emb.	·140
,, ,, ,, ,, to exit	·131
Diameter of bore (Δ)	·020
Diameter of emb. (d)	·009
Diameter of fingerhole (δ)	·008
I.D..	·025
Length C. of Hole 1 to exit	·056
Theory demands	·058

therefore, Hole 1 should be centred 2 mm. nearer embouchure and other fingerholes likewise.

Hole 4, considered as Hole 1, in consequence of cross-fingering, by closing Holes 1, 2 and 3, should be placed according to Formula No. 3 at ·133 from exit, but is actually at ·131.

ACCORDING TO FORMULA NO. 3

$\frac{\Delta}{2}$	·010	
$(\Delta - d)$	·011	
$(\Delta - \delta)$	·012	
	·033	Allowance at Hole 1.
I.D. × 4 (inclusive of the one between exit and Hole 1) = ·025 × 4 =	·100	
Aggregate	·133	theoretical position of Hole 4 considered as Hole 1.

It is, however, the actual position at ·131 from exit, or ·140 from embouchure, with which our experiment is concerned. This length from exit to Hole 1 (normally Hole 4) may now be considered as the effective allowance which, added to the actual length, gives the effective half-wave length, viz.

$$\cdot131 + \cdot140 = \cdot271.$$

BY FORMULA NO. 1

$$\frac{340 \text{ m./s.}}{\cdot271 \times 2} = \frac{340}{\cdot542} = \quad 627 \text{ v.p.s.}$$

$$\text{modal computation of } \frac{E\,9}{512} = \quad 625{\cdot}8 \text{ v.p.s.}$$

OR BY FORMULA NO. 2

$625{\cdot}6$ v.p.s. $\times 2 = 1{,}251{\cdot}2 =$ $E\,9/512$, note resulting from the cross-fingering of Hole 4

$$\frac{340 \text{ m./s.}}{625{\cdot}6 \times 2} = \frac{340 \text{ m./s.}}{1{,}251{\cdot}2} = \cdot2717 \text{ eff. } \tfrac{1}{2} \text{ wave length.}$$

The discrepancy is a negligible one. It is thus demonstrated that in the Mond flute, Hole 4 which normally plays $F\,8/512$ of 704 v.p.s., and $E\,9/512$ of 625·6 v.p.s. when cross-fingered, the expedient has the effect of lowering the note of Hole 4 by a major tone (9/8). The analysis of the effective allowance of ·131 discloses a curious feature of the Mond flute,[1] viz. a diameter abnormally wide for such a short flute: the length from embouchure to first hole is only $10\frac{1}{2}$ times the width of the diameter, and to exit $13\frac{1}{2}$ times, whereas in a modern flute the length is approximately 30 or more times the diameter of the bore. The practical conse-

[1] Consult Record Mond Flute, Chap. x.

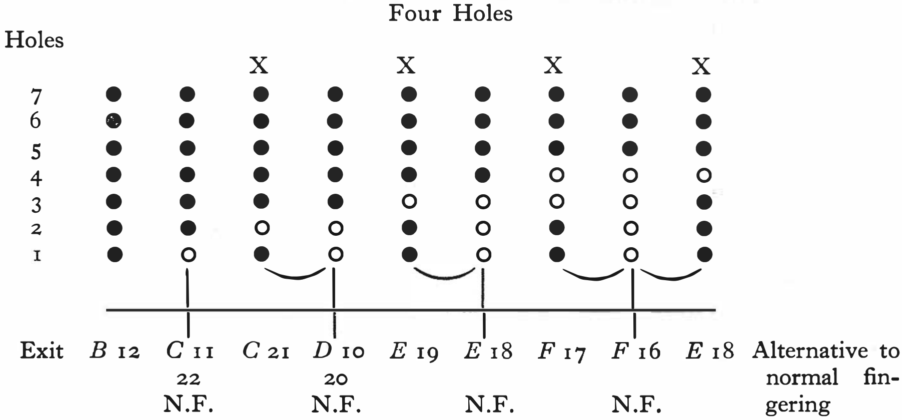
quence of this is that in the Mond flute the effective allowance in respect of pitch has diameter × 3 instead of diameter × 2.[1]

It will be noticed on perusing the Record of the Mond flute that the triple diameter allowance is only required for the effective or sound-wave length, used in the determination of pitch, and does not affect the calculations concerned with the actual dimensions of the flute, e.g. for the position of Hole 1. This example of the operation of cross-fingering on a modal flute will suffice to demonstrate the value of the formulae in checking results; and in drawing up a schedule of the fingering for an extended compass, such as the one given below in Fig. 48.

Cross-fingering on a modal flute may always be expected to produce one of the notes of the modal genesis. For instance, if Hole 4 be cross-fingered by closing Holes 1 and 2 and leaving 3 and 4 open, the result is computed in theory by considering Hole 3 as the virtual Hole 1, the position of which is ascertained and confirmed by the use of Formula No. 3 (with three increments); Hole 4 then sounds *F* 17/512 of 662·4 v.p.s., instead of its normal note of 704 v.p.s. *F* 17 is the next note of the sequence, one increment higher than our virtual Hole 1 (Hole 3) that normally gives *E* 18.

Similarly, if Hole 3 be cross-fingered by closing Holes 1 and 2 below it, so that its position and pitch are again those of a virtual Hole 1, its normal note of *E* 18/512 = 625·6 v.p.s. changes to that of the half-increment lower, viz. *E* 19/512 of 592 v.p.s., which may be justified by the use of the same three formulae.

FIG. 48.—The Compass of the Mond Flute with Cross-fingered Notes for the First Four Holes



N.B.—X indicates cross-fingering
N.F. ,, normal-fingering

It must be conceded that, compared with the forbidding graphs and equations propounded by Dr. Richardson (see fn. 2, pp. 245–6) in explanation

[1] A probable explanation of the discrepancy may be the faulty position of Hole 1 at ·056 instead of ·058 from exit, which adds 2 mm. to the length between emb. and c. of Hole 1; this may account for the slightly lower v.f. of the actual flute note compared with the pitch computed by formula.

of the results of cross-fingering on the flute, the very simple arithmetical calculations, by means of which definite and exact results—in agreement with practical tests—are worked out for the modal flute, have certain advantages.

FURTHER EXAMPLES DEMONSTRATED ON FLUTE ' SENSA C '

There is one more lesson to be learnt from cross-fingering on the modal flute, ' Sensa C ', which may justify the addition of the following example.

Reference to the Record of that flute shows that with normal blowing and fingering, the note *D* 10/512 played through Hole 7 on ' Sensa A ' required a 9th fingerhole bored at two I.D. nearer the embouchure to give that same note in pure intonation on ' Sensa C '. The reason for this astonishing fact has been given in the record of that flute.

If Hole 9 be cross-fingered by closing all holes below it, with the exception of Hole 4, then by the process of reasoning justified by the operation of our formulae, Hole 4 must be regarded, since it is the 1st open hole, as a virtual Hole 1. A new flute has in fact been called into being, with Hole 4 as exit or vent, and Hole 9 as Hole 1, at five increments nearer the embouchure.

The allowance in the determination of the position of Hole 1 was found to be ·0385, to which the five increments, viz. ·0235 × 5 = ·1175 are added = ·156. The formula (No. 4) thus gives the virtual position of Hole 9 as 1st hole from its exit (at Hole 4) as ·156, or ·1495 from embouchure, while the distance from the centre of Hole 9 to the centre of embouchure, measured on the flute, is actually ·188. When Hole 9 is cross-fingered thus :

○ . . . . the note $D\ \frac{10}{512}$, of 563·2 v.p.s. is
●
●
● . . . . lowered to $C\ \frac{21}{512}$, of which 536·4 v.p.s.
●
○ . . . by an interval of ratios $\frac{21}{20}$ = *85 cents*
●
●
●

the sequence now assumes the following ratios :

Modal Sequence of ' Sensa C ' Flute

(Results of cross-fingering marked X)

Holes	Exit	1	2	3	4	5	6	7	8	9
Ratios [1]	$\frac{18}{18}$	$\frac{16}{18}$	$\frac{15}{18}$	$\frac{14}{18}$	$\frac{13}{18}$	$\frac{12}{18}$	$\frac{23}{36}$	$\frac{22}{36}$	$\frac{21}{36}$	$\frac{20}{36}$
					$\frac{27X}{36}$					$\frac{21X}{36}$

[1] The ratios of notes given at the half-increment belong to a genesis by Modal Determinant 36.

On comparing these results with those obtained on the Mond flute by cross-fingering, it will at once be noticed that on this flute with Hole 4 open and three holes closed below it, the drop in pitch is of a major tone 9/8 (*204 cents*), whereas on ' Sensa C ' Hole 4 with three holes stopped, the drop in pitch is only of ratio 27/26 (*65 cents*, about $\frac{1}{3}$ tone).

Again on ' Sensa C ', the cross-fingering of Hole 9 as shown above drops the pitch down from *D* 10 to *C* 21 by an interval of 21/20 (*85 cents*) whereas on the Mond flute, tested under approximately similar conditions, the result is a much greater drop in pitch when the 7th or top hole is cross-fingered thus:

Hole 7	○	$=\frac{B\ 12}{512}\longleftarrow$	○	$\longrightarrow=\frac{G\ 14X}{512}$	●	
	○		●		●	
	○		●		●	
	○		○	$=\frac{E\ 18X}{512}\longleftarrow\!\!\longrightarrow$	○	$=\frac{F\ 16}{512}$ N.F.
	○		●		○	
	○		●		○	
	○		●		○	

The cross-fingered interval dropping from *B* 12 to *G* 14 is a Septimal 3rd 7/6 (= *267 cents*) on the Mond flute, compared with the drop of 21/20 (*85 cents*) on ' Sensa C '.

Why is this?

The cause of this wide divergence in the operation of cross-fingering on these two flutes may be assigned first of all to two structural features:

(1) The diameter of the bore in proportion to the length of the flute.
MOND FLUTE $\Delta = \cdot 020$; length ·271 from embouchure to exit, ·271/20 $= 13\frac{1}{2}$ times.
' SENSA C ' $\Delta = \cdot 023$; length ·466 from embouchure to exit, ·466/23 = 20 times.

(2) The increment of distance:

MOND FLUTE ·025 actual; Proportional I.D. = ·0225. The I.D. carries an allowance for diameter of ·0025 per hole. The floating allowance = ·0052.

' SENSA C ' increment of distance ·0235 both actual and proportional; carries *no allowance* per hole. This has to be made up from the floating allowance = ·0067.

With normal fingering the lowering effect of the allowance is for one hole only. With cross-fingering, the lowering effect is cumulative for each hole closed between the speaking-hole and the vent. With the ' Sensa C ' flute, each of the five I.D., which are concerned in the lengthening of the half-sound-wave, takes up its position on the face of the flute between exit and embouchure, without contributing an iota for allowance: the result is that all the accumulated incremental allowance, viz. $\cdot 00215 \times 5 = \cdot 01075$ (·011) becomes active, having practically reached a nodal point (within $\frac{7}{10}$ of a millimetre), and thus causes a drop in the pitch to that of a half-increment lower, i.e. from *D* 20 to *C* 21.

This operation of the formula is an entirely normal one which applies to all flutes of Class 1B, having an I.D. which is an exact aliquot of the length of the flute from embouchure to exit, or to Hole 1 if used as vent.

Not so on the Mond flute. Owing to the diameter being excessive in relation to the length of the flute, it has been found that in all computations of pitch, i.e. of the effective half-sound-wave, the diameter is taken three times instead of twice in the allowance, and this involves also the Incremental All. No. 7 which is of $\cdot 0028 \times 3 = \cdot 0084$ per I.D.: there are three increments between the open 4th hole and the open 7th hole; $\cdot 0084 \times 3 = \cdot 0252$ or one whole increment of distance; as the ratio of the 7th hole is 6/12, the drop of one increment is to ratio 7/12, an interval of a Septimal 3rd 7/6: which is an instance of the agreement of theory and practice.

On the modal flute, the device of cross-fingering passes from the empirical to the theoretical. The results may be predicted and confirmed, since both I.D. and the ratios of pitch they bear are known.

There is one fact that emerges from these computations of length, diameter and vibration frequencies to which we may pin our faith, and that is that each half-sound-wave length has a corresponding vibration frequency which is fixed and unalterable. These two factors, when they appear respectively as divisor of the rate of the velocity of sound in air = 340 metres per second—or as quotient in the division—are reciprocals (see Table i, p. xlvi).

That is a point from which all computations can safely proceed. Further, if it be understood that the effective half-wave length exceeds the actual measurable length of the flute, from the centre of the embouchure to the exit or vent, by the amount of the aggregate allowances in respect of diameter, all that remains to be done is to allocate the aggregate, according to the formulae set forth in these pages.

Our investigation of the operation of cross-fingering once more draws attention to the cardinal fact that the ultimate determinant of vibration frequency in cylindrical modal flutes is proportional, rather than absolute length, conditioned by the sequence of modal ratios borne by the increments. The note issuing from a given hole obeys the proportional impulse, stimulated by equidistant spacing of holes—which are present in the interior of the tube as well as on the surface—and are controlled from within by subtle conditional rulings.

CROSS-FINGERING AS A MEANS OF TRANSITION FROM ONE MUSICAL SYSTEM TO ANOTHER

The significance of the device of cross-fingering is not confined to the extension of compass on the modal flute. The importance of this technical expedient is best realized in relation to the evolution of music, when it is found used as a means of transition from one musical system to another, and thus indicates the contemporary existence in practice of both. Cross-fingering may then be regarded as the gnomon in the balance of tendencies prevalent at any special period in the evolution of music and of races.

In order to bear out the simile of the gnomon, it is necessary to bring forward something more convincing than mere vague allusions to cross-fingering given in theoretical or literary sources;[1] the fact must be demonstrated; the scheme must be seen in operation. Fortunately the necessary data are available in one of the earliest handbooks on musical instruments written in German verse by Martin Agricola[2] and illustrated by numerous drawings of instruments and tablatures, or schemes of fingering and cross-fingering for various pipes, shawms and flutes.

The holes, opened one by one in numerical order, denote the modal sequence actually embodied in the flute; the omission of one or more numbers from a sequence signifies a closed hole and therefore a lowering of the pitch through cross-fingering.

THE EVIDENCE OF VIRDUNG AND AGRICOLA ON CROSS-FINGERING

The precise evidence disclosed by such schemes for fingering flutes and pipes in Martin Agricola's handbook has an extremely important bearing upon the whole question of modality, both true and artificial, which forms the basis of our modern major and minor Modes, of the Ecclesiastical Modes, Gregorian Tones, and of other features which still exercise a vital influence on present-day music, both practical and theoretical, as a counterweight against atonality and its offshoots.

Up to the seventeenth century and even in the eighteenth, most of the surviving flutes, shawms, clarinets, oaten pipes, &c., without keys, which are preserved in national and private collections in England and abroad, have equidistant holes,[3] a slight variation from the norm being usually found compensated by a difference in the diameter of the hole. They are thus all designedly modal in origin, whatever use may have been made of them by cross-fingering, in approximation to the scale of tones and semitones.

[1] Such as the quotation from Quintilian, *Inst. Orator.*, i, 11, 6 and 7; see Chap. ii, for translation of the passage.

[2] The title of the first edition published in 1528 is: *Musica instrumētalis deudsch ynn welcher begriffen ist wie man nach dem gesange auff mancherley Pfeiffen lernen sol . . . und allerley Instrument u Seytenspiel nach der rechtgegründten Tabelthur sey abzusetzen.* (Mart. Agricola Wittemberg, 1528). The *Gesellschaft für Musikforschung* published a reprint facsimile of the 1st and 4th editions of 1528 and 1545 in one volume. Jahrg. 24, Bd. 20, of the *Publikation älterer praktischer u. theoretischer Musikwerke* (Leipzig: Breitkopf u. Härtel, 1896). Tablatures for flutes, pp. 25–30 and pp. 155–75.

[3] See descriptive *Catalogue of the Mus. Insts.*, Roy. Military Exhibition, London, 1890, compiled by Capt. C. R. Day (Eyre & Spottiswoode, 1891), Pl. i, p. 1; Fig. D, p. 28; Figs. E and F, pp. 27–8; Pl. iii, p. 53; Pl. iv, Figs. A, F, H.

As the plates are heliogravures from photographs of the originals the holes are accurately depicted; a slightly conical flute by Willis with one key, in my possession, may also be cited.

See also *Old English Insts. of Music*, by F. W. Galpin (Methuen, 1910), Pl. 29, p. 142, Figs. 1–4 and 5–8; Pl. 31, p. 154, Figs. 1, 2, 4, 7; Pl. 32, p. 164, Figs. 1–5; Pl. 34, p. 168, Figs. 1, 3, 4, 10, 11.

Many other examples exist.

FIG. 49.—Agricola's Scheme of Cross-fingering
(Schweizer, '*Pfeiffen des Discant's Scala und Fundament*')

Sequuntur tres irregulares harum Tibiarum scalae ad Epidiatess. transpositae

This Modal Flute has been bored for the Hypolydian Harmonia, by cross-fingering Authentus Protus

Normal Fingering of the Flute's Modal Scale. Ratios by K. S.	No. of Hole	Notes of Modal Scale	Agricola's Scheme for Cross-fingering Exit fundamental *D*		Effect of Cross-Fingering (by K.S.)
Holes 1 2 3 4 5 6		H a	r m o n i c	R e g i s t e r	
		C	*Stercker*	5 6	
		B♮		2 3 4 5	
		A		2 3 4	
	6	*D*		*dd* 6	
	6	*C*		2 3 4 5 6	
	5	*B*		♭♭ 5 4 5	
	4	*A*		4	
	3	*G*		§ 1 . 2 3	
	2	*F*♮		2	
	1	*E*	*Blas*	1	

Modal Ratios	Holes			Fundament	als	
10/20	1 2 3 4 5 6 ● ● ● ● ● ●	6	*D*		6	● ● ● ● ● ○
11/20	○ ○ ○ ○ ○ ○	6	♯ *C**	*C*	♮/♮ 2 3 4 6	● ○ ○ ○ ● ○
12/20	○ ○ ○ ○ ○ ●	5	*B*	25/40 *B*♭	2 3 5 | ♮ 2 3 4 5	● ○ ○ ● ○ ● *B*♮ ● ○ ○ ○ ○ ● 25/40 24/40
13/20	○ ○ ○ ○ ● ●	4	♯ *A*	27/40 *A*	2 3 4	● ○ ○ ○ ● ● 27/40
15/20	○ ○ ○ ● ● ●	3	*G*		1 2 3	○ ○ ○ ● ● ● 15/20
16/20	○ ○ ● ● ● ●	2	*F*♯	17/20 *F*♮ *204 cents*	2	● ○ ● ● ● ● 17/20
18/20	● ● ● ● ● ●	1	*E*	*Linde*	1	○ ● ● ● ● ● 18/20
20/20	● ● ● ● ● ●	Exit	*D*	*182 cents*	●	● ● ● ● ● ●

N.B.—*Holes 1, 3 and 6* (*for octave of* D *only*) *are normally fingered.*

This flute was bored for the Hypolydian Harmonia of Modal Determinant 20, the prototype of our major scale. By cross-fingering it has become the Authentus Protus or Phrygian.

* *The 6th hole which normally gives a sharpened* C (C.*11*) *is not used in the cross-fingered scale, which has the minor 7th.*

The last two pages in Virdung's [1] little volume (oiij recto and verso) contain schemes of cross-fingering for flutes: Bass, Tenor, Discant; on the last page verso the resulting scale is given for the Discant flute, viz. the keyboard scale on *G* obtained by cross-fingering from the Dorian Harmonia, in which the 4th, *C*, normally sharpened, is lowered by keeping Hole 2 covered and opening 1 and 3 (clearly indicated by '╳ 3, 2, 1' and the cross-fingered note by '3, 1'). Similarly, the *F*♯ is given the normal fingering 6, 5, 4, 3, 2, 1, which had been lowered in the tablature (to ratio 11/12) by closing Hole 5 and opening the others.

Virdung only prints these two schemes at the end of his little book, but they are valuable as evidence over a century earlier than Agricola's more complete tablatures which have been selected for reproduction here.

Agricola was no theorist, more especially so far as wind-instruments are concerned; the valuable information he has to impart is founded upon a knowledge of practical music, and of the technique of musical instruments, gained by empirical methods. From the nature of his schemes, we are bound to recognize that his ear was sensitive to niceties of intonation, as will be apparent from the ratios of intervals allowed or disallowed in his schemes, some of which are reproduced below with the addition of ratios and explanatory matter (by K. S.).

Agricola presents three groups of the primitive transverse, or side-blown flute, known in the early sixteenth century as 'Schweitzer Pfeiffen'.

They are represented in the drawings as cylindrical in bore, made in one piece, except for the head joint containing the cork; they have six equidistant fingerholes. Each group consists of bass, tenor or alto, and discant; their fundamentals rise in 5ths: in the 1st edition of 1528 (see pp. 25 sqq.) *D A E*, and in the 4th edition, 1545 (pp. 169 sqq.), *C G D*, and (pp. 180–2) *G D A*; a harmonic register is indicated, which gives them a compass of two octaves and part of a 3rd.

AGRICOLA'S CROSS-FINGERING TESTED IN PRACTICE

Although Agricola has furnished clear information as to the intonation and scales of these flutes, he has given no structural details whatever; no measurements, which would enable us to make facsimiles, and the drawings are not to scale, like those of Michael Praetorius, published a century later,[2] who, however, did not include schemes for cross-fingering. The fingerholes are inserted in the drawings at equal distances, which indeed, in itself constitutes no proof of modality. Nevertheless, it may confidently be asserted that the flutes which Agricola tested with infinite care and precision, as to intonation, were modal flutes, embodying one or other of the Harmoniai. Tests carried out (by K. S.) on modal flutes with the

[1] *Musica getutscht und auszgezogen*, durch Sebastianum Virdung, Priesters von Amberg (Basle, 1511). Reprint, ed. by Rob. Eitner, *Ges. f. Musikforschung* as Vol. xi of *Publikation älterer Praktischer u. Theor. Musikwerke*, xv and xvi Jahrh. (Berlin, 1882).

[2] *Syntagmatis Musici*, Teil ii, Organographia (Von den Instrumenten) (Wolffenbüttel, 1618), and Reprint by the *Publikation älterer praktischer u. theoretischer Musikwerke*, Bd. 13 (Breitkopf und Härtel, Leipzig, G, 1884 and 1894).

cross-fingering indicated by Agricola, give results identical with those recorded by him, but with a very few slight reservations due to the diameter of the bore; these have been duly noted on our Fig. 50. Flutes having a wide diameter in relation to length need less stopping to produce a given lowering of pitch than flutes having a narrow bore. Thus on a Dorian flute with the Modal Determinant 11, which we have assigned to Agricola's Discant (Fig. 50), the sharpened 4th 11/8 is lowered to the Perfect 4th by opening Holes 3 and 2, and closing Nos. 1, 4, 5, 6. That result is likewise obtained on the Mond flute with a diameter of ·020; but with Java i, having a bore of only ·0115, and with No. 4 from North-West India, of ·012, the Perfect 4th, instead of the 11/8, is obtained with Hole 3 alone open and all others closed: the narrower diameter requiring an extra hole to be stopped in order to produce the requisite drop in pitch. To urge that these transverse flutes were merely false in intonation, owing to bad workmanship, and that they therefore needed correction, would be a fallacy; for comparisons of certain exigencies—not fortuitous but constant—point to an underlying system, consistent in its implications but differing from the more usual one. Among the most significant of the implications of modality which call for correction by cross-fingering in the scale of tones and semitones, are the modal intervals of sharpened 4ths and 5ths, which have to be transformed into the perfect consonances of the 4/3 and the 3/2. There is only one Harmonia in which both perfect consonances are present in the scale, and that is the Phrygian. The occurrence of a sharpened 4th suggests either the Dorian 4th of ratio 11/8 (*551 cents*) or the Hypophrygian of 18/13 (*561 cents*). The Dorian Harmonia is distinguished from the Hypophrygian by having likewise a sharpened 5th of ratio 11/7 (*782·3 cents*), whereas the Hypophrygian 5th 18/12 is perfect (*702 cents*). The Hypolydian Tritone which does duty on occasions for a 4th has a ratio of 10/7 (*617·4 cents*).

These and other unusual intervals which characterize the Harmoniai, according to their modal function in relation to the Tonic, make it possible to diagnose the Mode from the schemes.

In the absence of measurements of the flutes, the readings given here of Agricola's fingerings are admittedly conjectures, but they are based upon certain unerring indications and facts, and they have been tested upon flutes bored for the Harmonia in question. For the first of the three groups (in the edition of 1528), Agricola used for his test flutes embodying the Dorian Harmonia, and the result of cross-fingering was to convert this Ancient Greek scale into the *Authentus Deuterus* or *e–e* keyboard scale, corresponding to the ditonal Dorian of the Graeco-Roman theorists. The fundamental of this Discant Flute was actually *E*.

HYPOLYDIAN FLUTES CROSS-FINGERED GIVE THE *AUTHENTUS PROTUS* OR *D* SCALE

For the 2nd group (pp. 169–75) in the edition of 1545, Agricola used three Hypolydian flutes of Modal Determinant 20, and by cross-fingering changed this scale—termed by him irregular—into the *Authentus Protus* or *D* scale, which is the fundamental note of the Discant flute of this group.

FIG. 50.—Agricola's Scheme of Cross-Fingering for the Schweizer Pfeiffen:
(1) 'Des Discants Scala' [ed. 1528, p. 30]; (2) 'Des Tenors Scala'

The Flute has been bored for the Dorian Harmonia which, by cross-fingering, has become the *Authentus Deuterus*

N.B.—The effect of cross-fingering has been tested on the Mond and Java Flutes

(1) THE FUNDAMENTAL OCTAVE

Normal Ratios	**Fingering of the Flute's Modal Scale by K. S.** (Holes 1 2 3 4 5 6)	**No. of Hole**	**Notes of Modal Scale**	**Agricola's Scheme for Cross-fingering Exit Fundamental is *E***		Holes	**Effect of Cross-fingering by K. S.** (1 2 3 4 5 6)
11/22	● ● ● ● ● ○	6	*E*	*E*	6	*mi*	● ● ● ● ● ○
6/11	○ ○ ○ ○ ○ ○	6	*D*♯	*D*	6 5	*re*	● ● ● ● ○ ○
13/22	○ ○ ○ ○ ○ ●	5	*C*♯	*C*	C 5 3 2	*fa*	● ○ ○ ● ○ ●
7/11 15/22	○ ○ ○ ○ ● ● ○ ○ ○ ◐ ● ●	4	*B*♯♯ *B*	*B*	♯* ♮ 4 3 2 ♭ 4 2 1	*mi*	● ○ ○ ○ ● ● ○ ○ ● ◐ ● ●
8/11	○ ○ ○ ● ● ●	3	*A*♯	*A*	3 2	*re*	● ○ ○ ● ● ●
9/11	○ ○ ● ● ● ●	2	*G*♯	*G*	2 1	*sol*	○ ○ ● ● ● ●
10/11	○ ● ● ● ● ●	1	*F*♯	*F*	♭ 1	*fa*	◐ ● ● ● ● ●
11/11	● ● ● ● ◐ ●	exit	*E*	*E*	●	*mi*	● ● ● ● ● ●

* *Hole 4 half closed.*

(2) SCHWEIZER PFEIFFE TENOR SKALA

Modal Ratios	Holes 1 2 3 4 5 6					Holes 1 2 3 4 5 6	
11/22	● ● ● ● ● ○	6	*a*	*γ*	*re*	● ● ● ● ● ○	*A*
6/11	○ ○ ○ ○ ○ ○	6	♯* *G*	6 5	*sol*	● ● ● ● ○ ○	*G*
13/22	○ ○ ○ ○ ◐ ●	5	♯ *F*	5 3 2	*fa*	● ○ ○ ● ○ ●	*F*
7/11 15/22†	○ ○ ○ ○ ● ● ○ ○ ○ ◐ ● ●	4	♯ *E* *E*	4 3 2 *ut fa* 4 2 1	*mi*	● ○ ○ ○ ● ● ○ ○ ● ◐ ● ●	*E* ♭*E*
8/11	○ ○ ○ ● ● ●	3	♯ *D*	3 2	*re*	● ○ ○ ● ● ●	*D*
9/11	○ ○ ● ● ● ●	2	♯ *C*	2	*fa*	● ○ ● ● ● ●	*C*
10/11 21/22	○ ● ● ● ● ● ◐ ● ● ● ● ●	1 1	♭ *B* *B*♭	1 1 *fa*	*mi*	○ ● ● ● ● ● ◐ ● ● ● ● ●	*B*♮ *B*♭
11/11	● ● ● ● ● ●	● Exit	*A*	●	*re*	● ● ● ● ● ●	*A*

* *The 7th is minor.* † *Hole 4 half-closed, 3, 2 and 1 open, 5 and 6 closed, gives ratio 15/22. On Java I Hole 4 open, 3 closed gives ratio 15/22.*

The *D* scale corresponds to the Phrygian Species of the Greek theorists. The 3rd group (see pp. 180–2) is distinguished by Agricola as having a 'regular' scale which, however, required a cross-fingering as drastic as that of the 'irregular scale' of Group ii.

The flutes used seem to have been Hypophrygian of M.D. 18, and when cross-fingered produced the Tonus Secundus.

The transition from one scale into the other may be followed in the interpretation of the Discant scale of each group, which is given in Fig. 50. The notes of the flute scale, when normally fingered, are seen on the left and the approximations on the right.

A few examples may now be given of cross-fingering in operation, e.g. on the Discant flute (Fig. 50 and Agricola, *op. cit.*, p. 172).

The Harmonia of the flute is Hypolydian, of Modal Determinant 20, from which our major scale gradually evolved. The exit fundamental of the flute is *D*, to which we add the ratio number 20 ; by cross-fingering the Hypolydian Modal Scale changes to that of the *Protus Authentus*—the conventional Phrygian of the theorists.

At Hole 2 the modal keynote of ratio 16 should normally sound *F* sharp, a Major 3rd above the tonic; but according to Agricola's instructions, Hole 1 is kept closed below it, so that the note is now *F* natural, of ratio 17 instead of 16 : this gives the Minor 3rd required for the *Protus Authentus.*

G now follows at Hole 3, normally fingered with Holes 1, 2, 3 uncovered, thus giving the Perfect 4th of ratio 20/15 on the Tonic, required in both Hypolydian and Phrygian Harmoniai.

Hole 4 produces, when fully fingered, the *A* sharpened of ratio 13, which Agricola lowers by cross-fingering (i.e. keeping Hole 1 closed) to *A* natural of ratio 27.

The note of Hole 5 on the flute is *B* 12, a Major 6th above the Tonic, but cross-fingering has lowered the *B* by a comma (ratio 81 : 80) to a minor tone above *A* instead of a major tone; this interval being required by the ear, between 5th and 6th in a scale in just intonation on *D*—which appears to be Agricola's intention.

The minute difference of a comma is obtained by closing Hole 1, and opening 2, 3, 4, 5.

Hole 5 is also used to produce a *B* flat, according to Agricola (in modal terms *A* 13 much sharpened), by closing Holes 1 and 4.

Hole 6, normally fingered, gives a sharpened *C* of ratio 20/11 which is lowered, by closing Holes 1 and 5, to *C* a Minor 3rd above *A*. The 6th hole, with all holes closed below it, produces *D*, an octave above the fundamental. The slight reservations mentioned above regarding the identity of results between Agricola's tests and those recorded here, refer to the reactions in the air column, due to the operation of certain allowances in respect of diameter, and more specifically to the Incremental All. No. 7 in its cumulative effect, which, taken in conjunction with the implications of the increments of distance actual, proportional and effective, account respectively for (1) the interpolation into the modal sequence of a ratio belonging to an intermediate lower half-increment, or (2) for the apparent overstepping of an I.D. by the ratio. Attention has been drawn in some detail to both of these contingencies in the first part of Chapter vi on the flute, with illustrations from the Records.

To any student, expecting always to find in a modal flute the same rigid progression of the modal sequence, hole by hole, as in the Aulos, such deviations might prove a stumbling-block; they might lead him to throw doubt upon the modal interpretations offered. It should, however, be recognized as a general principle in the identification of individual Modes that the casting vote is held by the characteristic modal interval between Tonic and keynote. In the results of cross-fingering, the decisive test may nearly always be based upon the Perfect 4th and 5th required by keyboard scales, whereas cross-fingering invariably reveals the modality of the flute, and the extent of the requisite cross-fingering acts as a guide in identifying the Mode actually embodied in the flute in question.

The importance attached to the characteristic modal interval is a distinctive feature of the ancient Harmoniai, which cannot exist in the keyboard scale, or in that within which the Ecclesiastical Modes have their being.[1]

Cross-fingering on the Fipple flute,[2] Recorder, notched flutes[3] of various types has the same significance as in the transverse flute, and is put in operation in the same way.

Agricola has provided schemes for the Fipple flutes (on pp. 17–19, 1528) in the Hypolydian Harmonia and in the Hypophrygian Harmonia (pp. 159–61, 1545) similar to those illustrating cross-fingering in the transverse flute. No further explanation of the operation of the device is therefore necessary.

RE DIAMETER IN CYLINDRICAL TUBES IN OCTAVE RELATION: THE CHINESE FORMULA

One more problem which has engaged the attention of many theorists and acousticians relates to the following fact: two cylindrical tubes intended to be in octave relation, that is, to produce notes whose v.p.s. are in the ratio 2 : 1, if the length of the one be doubled or halved, and the diameters remain equal. If the cylinders are of the same length, then the diameter of the one that is to give the higher octave must be of smaller calibre. Opinions differ as to the relative proportions of the two diameters. The present writer has found the estimate of the Ancient Chinese philosopher musician, Prince Tsai-Yu (1596) (as recorded by Hermann Smith),[4] works out correctly in modal flutes.

[1] Since the modality of the scale of the Harmonia, based upon the modal genesis, is either non-existent in the keyboard scale, or has been deliberately renounced in favour of a system of duplicated tetrachords, a comparison of the melodic development in the composition of chants in the Ecclesiastical Modes or Gregorian Tones with that of the extant Fragments of Greek Music based on the Harmonia emphasizes the strong contrast between the two systems. (See Chap. ix.)

[2] See the Records for Flute xi, Carpathian, and Bali, No. 20.

[3] The notched flute is a primitive form of the Fipple flute, but is not so easy to play. See Records of Flutes from the Soudan, presented by Dr. A. N. Tucker; see also The Inca Flute, No. xii, in Chap. x.

[4] *The World's Earliest Music*, p. 179 (William Reeves, London).

The diameters of the two cylinders, equal in length, should bear the ratio $\frac{3{\cdot}53}{5}$. In the case of the flute in which the octaves occur within the same cylinder, the diameter is fixed for the 2nd octave, and therefore the diameter in the upper half is inevitably in excess: it should be $\Delta \times \frac{3{\cdot}53}{5} = x$; therefore $\Delta - \left(\Delta \times \frac{3{\cdot}53}{5}\right) = x$ represents the excess diameter, significant as excess length, lowering the pitch, and even occasionally interrupting the modal sequence; in this respect it resembles the Incremental All. No. 7.

For this reason, the excess length must be taken into account in calculating or regulating the position of fingerholes placed in the second half of the flute.

The student may wish to know what happens in the case of a flute having equidistant fingerholes when brought up for a test; there is bound to be a lengthening of the half-wave due to the excess diameter, with its implication of a drop in pitch.

Is every modal flute then subject to an interruption of the modal sequence? The answer is a negative.

The two flutes 'Sensa A' and 'Sensa C', will serve to illustrate the operation of this law of diameters. Whereas (1) an excess in the diameter has a direct lengthening influence on the half-sound-wave: (2) an I.D. in excess of the proportional increment exercises a *shortening* influence on the sound-wave. Thus No. 2 frequently counteracts No. 1. Flutes belonging to Class i, having a proportional I.D. identical with the actual, and flutes in Class ii, having an I.D. in excess of the proportional, may be expected to react differently to this Chinese formula. Once the flute has been made, nothing can be done to modify the excess diameter which becomes active in the upper half of the flute.

PURITY OF INTONATION IN MODAL FLUTES, IN SPITE OF EXCESS DIAMETER IN UPPER HALF OF PIPE

Hitherto when a flute of Class ii, such as 'Sensa A', has holes bored beyond the half of the flute, and yet preserves a purity in intonation and an undisturbed modal sequence, the fact has been explained through the operation of the Incremental All. No. 7 which, at such points, has not attained the length corresponding to a nodal point and has thus remained inactive. Records show, in the case of 'Sensa C' of Class i, an interrupted sequence caused by the opposite occurrence, viz. an Incremental All. No. 7 brought into active function at a nodal point (see Hole 6).

Are these two contrasted influences—the one lengthening, the other shortening—then merely two attempts to explain an observed fact?

We shall endeavour to show that the respective activity or latency of the operation of the Chinese formula, and of the Incremental All. No. 7, are both controlled by the strong proportional impulse in the modal flute.

In 'Sensa C', the half length at $\cdot466/2 = \cdot233$ is reached at Hole 7, centred at $\cdot235$ from embouchure. The diameter $= \cdot023$ should be reduced in the upper half, viz. $\dfrac{\cdot023 \times 3\cdot53}{5} = \cdot0162$; but is unaltered; therefore, $\dfrac{\cdot023 - \cdot0162}{\cdot0068}$ = excess diameter in second half of the flute, which is doubled as a lengthening influence on the half-wave; therefore $\cdot0068 \times 2 = \cdot0136$.

The Incremental All. No. $7 = \cdot00215 \times 6 = \cdot0129$.

These are identical within $\frac{7}{10}$ mm.; both overstep the nodal point at the half-increment $\dfrac{\cdot0235}{2} = \cdot0117$. Therefore in 'Sensa C', either of these two factors explains the interruption of the modal sequence at Hole 7, but this apparent identity is merely a coincidence.

The Incremental All. No. 7, however, is cumulative, increment by increment, whereas the excess diameter is constant at $\cdot0136$ at each hole. The Record of 'Sensa C' for Hole 8 shows that the cumulative incremental allowance alone can account for the drop in pitch of the note, to the frequency of a ratio due at one and a half increment lower than that of Hole 8.

In the flute 'Sensa A', the conditions are different: the modal sequence from Hole 6 at the half-length is uninterrupted. What has become of the excess diameter with its lengthening influence in this flute? The diameter is the same in both flutes, $\cdot023$ and the excess likewise at $\cdot068$. But in 'Sensa A' (Class ii) the actual I.D., carrying its own allowance exceeds the proportional I.D. by $\cdot0045$ (cumulatively per increment).

But this lengthening of the I.D. from $\cdot0235$ to $\cdot028$ exercises a *shortening* influence, since it centres each fingerhole $\cdot0045$ nearer the embouchure, and thus serves to counteract the lengthening by excess diameter of the half-wave. There is a further reduction of length, which is due to the Incremental All. No. 7 latent at this point, therefore, the balance is completely restored and there is neither flattened pitch nor is the sequence interrupted at Holes 6 or 7, viz.

$\cdot0068$ = excess diameter at upper half of flute.

$\cdot0045$ = allowance borne by actual I.D., being its excess over the proportional I.D.

+ $\cdot00216$ = Incremental All. No. 7×1.

$\cdot00666$ = aggregate shortening factors.

Thus in 'Sensa A' the excess length due to the equal diameter in both halves of the flute is cancelled, or reduced to vanishing point at $\frac{1}{10}$ mm.

This illustration of the operation of the Chinese formula for dealing with the excess diameter present in the second half of the flute, which has been applied to specimens of the two most important classes of modal flutes, suffices as an indication of the nature of the technical considerations which enter into the successful plotting and making of a modal flute. *N.B.*—The most careful thought, therefore, should be given to the selection of the I.D. which should exceed the exact proportional increment by several millimetres, more especially when Hole 1 has been correctly centred according to formula.

* * *

TYPES OF FLUTES OTHER THAN THE TRANSVERSE

The formulae, introduced in these pages for dealing with the technical features brought into play in the making, testing and functioning of cylindrical modal flutes, apply equally to other types of flute, with certain reservations. In the case of the transverse flute all formulae used in the computation of the allowance in respect of diameter contain ($\Delta - d$) in addition to one whole diameter, the implication of which is that the mouth-hole deprives the diameter of the embouchure end, open as far as the stopper, of its full complement; therefore, in the vertical or obliquely held open flute, devoid of mouth-hole and merely having the edge thinned, two whole diameters are required instead of one.

Moreover, in measuring the length of the flute, the point at which the breath-stream strikes the inner wall in producing the note is the terminus at that end; the point varies at from 10 to 20 mm., according to the position of the flute and the calibre of the bore.

In the notched flute, such as the Inca, the horizontal line of notch is the terminal point, and the length of this horizontal part of the notch replaces the diameter of embouchure in the formula.

The same applies to the fipple flute and recorder.

THE TRANSVERSE FLUTE IN ANCIENT INDIA: TREATISES BY BHĀRĀTĀ AND SARANGDEV

A brief reference was made above to the use of cross-fingering in India as early as the fifth century A.D., not on the reed-blown pipe only, but also on the side-blown flute, the ancestor of our concert flute.

The earliest representation of the transverse flute known to the writer, moreover, occurs on an Indian bas-relief, dating from the first or second century A.D. This unique document on the ancestry of the flute is to be seen at the British Museum (on the grand staircase) on the sculptures of the rail of the Tope of Amarāvati [1] (Madras Presidency). In a scene depicting dancing to the music of flute, strings and drum, the flute is seen blown from left to right; it seems to have six or seven fingerholes. The side-blown flute does not appear to have been traced in Ancient Egypt, although the *Nay*, a cylindrical flute without mouthpiece, open at both ends, and merely thinned at the edge to facilitate blowing, is seen on many of the most ancient monuments: it is still popular at the present day. The Amarāvati specimen, therefore, represents for the present the prototype from the East of our flute; but at that stage—at the beginning of our era—all precise information as to its scale is lacking. A few centuries

[1] An outline reproduction of the bas-relief is given in the *Enc. Brit.*, xi ed., Vol. 10, p. 580, s.v. 'Flute', by Victor Mahillon and K. Schlesinger. A full documentation of other representations of the instrument in historical sequence is included in the article.

later, more definite indications of the structure, compass, modality and theory of the flute are found in two treatises, made available for investigators by Joanny Grosset, who published in 1897 a translation of Bhārātā's work on the Drama which includes a section on music and musical instruments. In his contribution to the *Encyclopédie de la Musique*,[1] Grosset has given considerable extracts from the Nātya-shāstra of Bhārātā, and from the Sangita-ratnākara of Sarangdev (thirteenth century A.D.) containing valuable information concerning the structure of Krishna's venerated instrument the *vamça*, mentioned already in the Vedic Hymns, and later known as the *murali* flute, and at the present day as *pillagovi* and *banṣuli* or *banṣri*. The measurements,[2] translated by Grosset into millimetres, are precise: Sarangdev used the angula or digit = ·019 as unit; he gives a table of 15 vamça flutes and 2 murali, graduated in length from 14 angulas = ·266 to 48 = ·914; this being the length of the whole bamboo. Included in this total length were (1) the head joint (not necessarily movable) containing the stopper, and (2) the space between the exit and the first hole, invariably used as vent, and left uncovered, thus avoiding the difficulties involved in allowances in respect of diameter. The head joint—often of considerable length in Eastern flutes, probably for the purpose of balance—is given by Grosset as of 2, 3 or 4 angulas for the smallest flute, measuring but 14 angulas = ·266; the ratios of increase for the longer instruments are not given and the same remark applies also to other measurements.

The distance from the embouchure to the top or eighth hole is given for each flute, but the distance from exit to Hole 1 given at 2 angulas = ·038 can only apply to the smallest flute of the set, as also the diameters of the bore (given as that of the little finger) of the embouchure (as having a circumference of one angula = *c.* ·006), an impossibility in long flutes, and of the fingerholes (of a small filbert or olive). The increment of distance given at ½ angula = ·0095 does not include the diameter of the fingerholes, which adds about ·0095, bringing the increment to a length of one angula.

It may at once be said that the table, as it stands, with its technical implications, as supplemented by Grosset's explanations derived from Bhārātā and Sarangdev, are contradictory and impracticable, as will be seen further on. As an instance, the number of angulas between embouchure and the eighth or top hole would thus correspond to the same number of I.D., to which must always be added the seven increments between the eight fingerholes; together these constitute the Modal Determinant. It is at once evident that the Modal Determinant thus indicated could only produce the diatonic scale given in the table, in flutes numbered 3 to 7;

[1] Edited by Alb. Lavignac, Librairie Delagrave, Paris, 1913–14. 'Inde' Fasc. 11, pp. 353 sqq.

[2] The measurements appear to be based upon those introduced by al-Mamun, son of Haroun al-Raschid, which are referred to the meridian mile, containing 4,000 cubits; each cubit = 24 digits; each digit = 6 barley-corns; each barley-corn = 6 hairs of a camel (*A Discourse of the Roman Foot and Denarius*, by John Greaves, Prof. of Astron., Oxf., 1647).

e.g. for No. 1 the M.D. = 8 and the holes would inevitably give the following sequence :

Holes	1	2	3	4	5	6	7	8
Modal ratios	$\frac{8}{8}$	$\frac{7}{8}$	$\frac{6}{8}$	$\frac{5}{8}$	$\frac{4}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$
Indian scale given	*sa*	*ri*	*ga*	*ma*	*pa*	*dha*	*ni*	(*sa*)

Thus the smallest interval is of a Septimal tone 8/7, after which come three 3rds, Septimal, Minor and Major ; a 4th, a 5th and an octave ; this is no diatonic scale. There is manifestly an error somewhere, probably in the evaluation of the I.D. In flutes 8 to 17, the scale normally obtained from the fingerholes would be chromatic or enharmonic ; the compass of the last half-dozen flutes could not exceed an Augmented 5th. It will now be shown, by working out the implications of the measurements given by Grosset, for flute No. 12, that the I.D. and the diameters given could only apply at most to the six shortest flutes.

IMPLICATIONS INFERRED FROM SARANGDEV'S TABLE OF 15 MODAL FLUTES WITH EQUIDISTANT FINGERHOLES

FIG. 51.—The Technical Implications of Sarangdev's Flute No. 12

To take flute No. 12 (aditya) as an example ; the dimensions are the following : as given by Grosset in italics ; their implications worked out by the present writer in ordinary type.

Total length of bamboo, 37 angulas	= *·704*	
Length from emb. to Hole 8, 12 angulas	= *·228*	
From vent to exit, 2 angulas	= *·038*	} = ·133
The head joint, 5 angulas. Grosset gives 4 for smallest flute	= *·095*	
{ *I.D. from Grosset,* $\frac{1}{2}$ *angula*	= *·0095*	} = ·019
{ + diameter of finger-hole estimated as of small filbert	= ·0095	(1 angula)

Length from emb. to Hole 1 as vent therefore ·704 − ·133
(head + exit to Hole 1) = ·571 modal length

There are 12 angulas from emb. to Hole 8	=	·228
7 increments of dist. or angulas from Holes 1 to 8	= +	·133
Distance between emb. and Hole 1 according to Grosset's data = ·361		·361

(See Grosset, *op. cit.*, pp. 353–4.)

Thus, this distance between embouchure and Hole 1 should be ·361 and not ·571, if Grosset's estimate of the I.D. at ·019 be correct ; and there would be 19 I.D., i.e. 12 + 7. The allowance for diameter might add one increment, 19 I.D. + 1 = 20 as Modal Determinant for a flute of Class ii.

But according to Grosset :

the length from embouchure to Hole 1 = ·571
+ I.D. for approximate allowance + ·019
·590

$\frac{\cdot 590}{20} = \cdot 0295$ I.D.

The implied M.D. = 20.

Grosset's table of lengths is more convincing than his estimate of the equivalent of the I.D.; moreover, the Modal Determinant 20 implied by Grosset's data gives the following scale which bears no resemblance to the diatonic scale of his Table.

FIG. 52.—Modal Sequence implied by Grosset's Increment of Distance. Modal Determinant 20

Ratios of notes produce a compass of a flattened minor 6th.

Holes.	1	2	3	4	5	6	7	8
Modal ratios	$\frac{20}{20}$	$\frac{19}{20}$	$\frac{18}{20}$	$\frac{17}{20}$	$\frac{16}{20}$	$\frac{15}{20}$	$\frac{14}{20}$	$\frac{13}{20}$

Cents *89*   *93·5*   *98·8*   *105*   *112*   *119·4*   *128*

= *Total of 746 cents*

maj. 3rd 5/4 = *386 cents*
perfect 4th 4/3 = *498 cents*
flattened minor 6th 20/13 = *746 cents*
the just minor 6th = *814 cents.*

It may, however, be suggested that Grosset's I.D. be read as 1 to 2 angulas (in any case the estimate based on ½ angula is impossible for any but the smallest of the flutes);

$$\begin{array}{rl} \text{then 2 angulas} & = \cdot 038 \\ \text{diameter of hole} & + \underline{\cdot 0105} \\ & \left.\begin{array}{r}\cdot 0485 \\ = \cdot 049\end{array}\right\} \text{new I.D. suggested by K.S.} \end{array}$$

Allowance for diameter $\Delta = \cdot 014$; diameter of emb. $= \cdot 009$
$= \text{diameter of fingerhole} = \cdot 009$.

$$\left.\begin{array}{rr} \Delta = & \cdot 014 \\ \Delta - \text{d of emb.} & \cdot 005 \\ \Delta - \delta \text{ of holes} & \underline{\cdot 005} \\ \text{Allowance} & \underline{\underline{\cdot 024}} \end{array}\right\}$$

$$\begin{array}{r} \cdot 571 \\ + \underline{\cdot 024} \\ \cdot 595 \end{array} \qquad \frac{\cdot 595}{\text{I.D. } 49} = 12 \ (+ \cdot 007) \text{ Modal Determinant.}$$

FIG. 53.—Modal Determinant 12

The Modal Determinant 12 may alternatively be ascertained thus:

Length from emb. to Hole 8 (Grosset)	·228
7 increments of distance holes 1 to 8 (K. S.) . . .	·343
·049 × 7	·571
diameter allowance as above	+ ·024
	·595

$$\frac{595}{\cdot 049} = 12(+ \cdot 007)$$

On long flutes played by men with long, slender fingers, an I.D. = ·049 would not be excessive.

FIG. 54.—Modal Sequence produced by I.D. of 2 Angulas + Diameter of Fingerhole (= ·049). Modal Determinant 12

Holes	1	2	3	4	5	6	7	8
Modal ratios	$\frac{12}{12}$	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{9}{12}$	$\frac{8}{12}$	$\frac{7}{12}$	$\frac{6}{12}$	$\frac{5}{12}$

Cents *151* *165* *182* *204* *231* *267* *316*

minor 3rd = *316 cents*

perfect 4th = *498 cents*

perfect 5th = *702 cents*

sharpened major 6th = *933 cents*

By half-stopping, the second half of the scale from Hole 5 would run thus:

Holes	5	6	7	8
Modal ratios	16	14	13–12	11–10

Holes 7 and 8 each giving two notes. The scale would then be that of the Phrygian Harmonia. The alternatives here offered are:

(1) The Modal Scale of Determinant 20 (Fig. 52) implied by a literal reading of Grosset's table, which yields a chromatic scale of semitones of graduated magnitudes instead of a diatonic scale.

(2) The Modal Scale of Determinant 12 (Fig. 54), based upon a conjectural reading by the present writer of Sarangdev's vaguely expressed measurements, supported by the inference that Grosset's dimensions and indications, supplied in the text, apply only to the smallest flutes of the set, and should be gradually increased with the length of the flutes, as practice demands.

The table of the set of 15 graduated flutes derived from Sarangdev's treatise on the Drama—even considered without its implications—is sufficient to establish the fact that the musical system of Hindustan, as used in the Drama at the opening of the thirteenth century, was still the ancient Modal System, uninfluenced by the influx of Mohammedan musicians and theorists with their non-modal system, to be discussed further on.

THE TRANSVERSE FLUTE IN EVOLUTION IN EUROPE

There is reason to believe that the transverse flute, although not a popular instrument, was not unknown in Central and Western Europe during the early centuries of our era. Representations of the instrument, dating from the ninth century, exist in illuminated manuscripts and other works of Art.[1] Information of a practical and theoretical nature concerning

[1] References to these will be found in the article 'Flute' in the *Enc. Brit.*, xi ed., Vol. 10, pp. 580 and 581, by V. M. and K. S.

the structure and scale of the flute are entirely lacking previous to Virdung [1] and Agricola's contributions to the subject.

The schemes of Agricola for cross-fingering on the flute provide an important confirmation of the survival of the two musical systems which we have already traced in contemporaneous operation in Ancient Greece, in Asia Minor and in North Africa. Evidence exists of the survival or rebirth of the genuine Modal System, on the one hand, based upon the ancient Aulos Harmoniai, and of the non-modal standard scale of tones and semitones on the other, to which, we have reason to believe, the early keyboard scale of hydraulic and pneumatic organs and later of clavichord, spinet, harpsichord, &c., was tuned.

There is convincing evidence, set forth in former chapters, that the inception and development of the P.I.S. in Ancient Greece, as well as the system of Greek musical notation, based upon modal ratios, originated with the Aulos Harmoniai. The exigencies of modality and of the Modal Species, however, determined the eventual formation of the P.I.S., as a system of degrees modelled upon those of the Dorian Harmonia. Since the nomenclature of the degrees, inspired by the use of the Dorian Kithara as chief exponent of the Harmonia, took no cognizance of ratios, or indeed of intervals, the P.I.S. became equally suited to the development of the ditonal scale of the theorists, or to the keyboard scale in just intonation. The fatal ease with which the ditonal scale could be explained and demonstrated in theory—even if there were frequent revolts against it in practice—fascinated the Graeco-Roman theorists and their medieval followers.[2]

The theory of the Aulos Harmoniai was never formulated, as far as can be ascertained, outside the schools of the Harmonists, so that only meagre shreds of evidence, disjointed or inferential, were available for contemporary theorists.[3]

Nevertheless, the Harmoniai have survived, preserved by reed-blown pipes and flutes, and fostered by the partiality of the Folk for those instruments and for melodic music.[4] Chief among the partisans of the ditonal scale were the Mohammedan Arabs. But the traditional music of the Arabs at the time of the Mohammedan conquests was undoubtedly based upon the Harmoniai—which are still in use among the Eastern Arabs and Persians at the present day.

[1] See Virdung's contribution and title of book, *supra*.

[2] A careful and comprehensive record of the long succession of expositions of this scale on the monochord is available in *Das Monochord als Instrument u. als System*, von Sigfrid Wantzloeben (Halle–a.-S., Max Niemeyer, 1911). The work is very fully documented with valuable references.

[3] Ptolemy has not furnished any theory of the genesis of the Harmoniai, although he has recorded the ratios of some of the modal tetrachords, e.g. the Diatonon Homalon $\frac{12}{11}\ \frac{11}{10}\ \frac{10}{9}\ \frac{9}{8}$, and the tetrachords of Didymus, Archytas (diatonic), Eratosthenes. Ptolemy has preserved a rigid silence on the Greek system of notation, which may indicate that he was not in touch with the teachings of the Harmonists.

[4] See Chaps. viii and ix, ' Survival or Rebirth of the Harmoniai in Folk Music '.

THE DITONAL SCALE ADOPTED BY THE ARABS

The zeal displayed by Arabian scholars—such as those gathered round him by Haroun al-Raschid at Bagdad—in the study of the Greek theorists, tempted them also to adopt the ditonal scale in theory, and possibly exceptionally in practice.

The scale was borne along with them in their victorious march through Persia to India, across Northern Africa, to Spain. In the Iberian peninsula this propagandist musical stream met the original ditonal stream flowing through Southern Europe, which may be traced in the pages of early medieval tracts, issuing from the monasteries, on the theory of music that generally included a section on the divisions of the monochord.[1] But although the ditonal scale flourished greatly in their theoretical treatises, it is doubtful whether it became firmly rooted in practice until some centuries had elapsed, if ever. The descriptions by Al-Fārābī[2] of the Auloi (Tibiae) having fingerholes at equal distances: and of the equidistant frets of the Tanbur of Bagdad, leave no room for doubt that the traditional music of the folk, and of the celebrated Arabian minstrels, still remained true to the Aulos Harmoniai. Before Al-Fārābī's data on flutes and pipes can be examined in some detail, a brief digression must be made on the origin of our major scale, for which, in common with many features of our theory of music, we are indebted to Ancient Greece.

THE DUPLICATION OF THE FIRST TETRACHORD IN THE OCTAVE SCALE

The first tetrachord of the Hypolydian Harmonia is identical with the second tetrachord (from *G* to *C*) of our major scale in just intonation; and it differs from the first (from *C* to *F*) only in the relative positions of the major (9/8) and the minor (10/9) tones.

The Harmonia in its natural genesis is composed of two entirely different tetrachords, each characteristic of one of the seven Harmoniai. Our major scale consists of duplicated tetrachords, similar if not identical, according to whether the reference is to just or to tempered intonation.

Thus, in contradistinction to the Harmonia with its octave unit, our major scale is based upon a tetrachordal unit. The duplication of the first tetrachord of the Harmonia has a twofold origin. In both instances the duplication arises from the operation of a technical device. The first occurred on the Aulos when the vibrating tongue of the beating-reed

[1] See Mart. Gerbert, *Script. Eccles. de Mus.*; 3 vols. with Indices (San Blasius, 1784).

[2] See Kosegarten, who has provided extensive extracts in Latin translation from Al-Fārābī's treatise ' Kitābu L–Mūsīqī al-Kabir, in the Introduction to ' Alii Ispahānensis Liber Cantilenarum . . . ex codicibus manu Scriptis Arabice editus . . . ab Joanne G. L. Kosegarten, Gripesvoldiae, 1840. (Latin and Arabic Tibiae, see pp. 95–105).

See also the French translation by Baron Carra de Vaux in Baron Rodolphe D'Erlanger's series *La Musique Arabe*, Tome 1*er*, Al-Fārābī; Paris, 1930, pp. 268 sqq.

Also Rouanet, ' La Musique Arabe ', *Encycl. de la Musique, Hist. de la Musique*, Paris, 1922, pp. 2696 and *passim.*

mouthpiece, shortened by pressure of the lips to the extent of a third of its length, raised to the dominant the pitch of the fundamental, and of each hole in succession as it was uncovered. The result was an octave scale consisting of two tetrachords, identical in structure but differing in pitch by a 5th: and this was achieved upon an Aulos with but three holes if the modal sequence began with the fundamental. When this proceeding was carried out upon a Hypolydian Aulos, the result was our major scale. The significance of this ingenious treatment of the beating-reed mouthpiece on the Aulos in Ancient Greece has already been described and illustrated in Chapter ii.

A somewhat similar result may have been obtained at an early date on the flute under certain conditions, e.g. on a flute with three or four holes and a low fundamental, sounded with difficulty, weak and veiled in quality, so that the piper used the harmonic register in preference. He obtained his scale by overblowing the fundamental and three holes an octave, e.g. *c*, *d*, *e*, *f*, then closing all holes and overblowing a 12th, he would, on a Hypolydian flute, such as the Graeco-Roman from the Bucheum (see Chap. x, Record), repeat the same tetrachord on the dominant, e.g. *g*, *a*, *b*, *c*, and thus play our major scale. Arabian theorists regard Modal Scales with duplicated tetrachords as the true Modes, and the octave Harmoniai, with their two different characteristic tetrachords, as mixed Modes; which, of course, is the natural viewpoint for theorists accustomed to computing in terms of tetrachordal units.

Attention is first drawn to the importance of the octave in the Greek musical system by the record that Pythagoras added an 8th string to the Kithara for the octave, and by the persistent recognition by the theorists of the Harmonia as an octave scale.

In a study of musical instruments of the Ancients our interest does not lie solely, or even mainly, in the instrument itself, but rather in its significance in relation to the evolution of musical systems and scales, and again not to these alone, but also to the stimulating psychological and scientific influence of each individual type of instrument.

Although the study of stringed instruments is usually considered more attractive than that of wind instruments, its value to the advancement of our knowledge of the foundations of the art and science of music is considerably less: with the possible qualified exception of instruments with fretted necks. There is nothing in the length of the strings, in their arrangement, that is revealing; moreover, since the tension of the strings, as ultimate determinant of pitch is by nature unstable, and liable to vary with temperature, the value of a stringed instrument as a record is nil.

On the other hand, it is impossible to stress too heavily the importance, as records, of flutes and reed-blown pipes with lateral fingerholes, as long as they remain intact from embouchure to exit. Such is the subtlety of the laws embodied in wood-wind instruments, upon which their scales and their capabilities in regard to sound production depend, that one may search in vain in the sources for any definite information on the subject of these instruments, as used from antiquity up to the fourteenth or fifteenth century. Even where some measurements or details of construction have been given, it is generally found that the essentials have been omitted.

As soon, however, as the significance and implications of the equal spacing of fingerholes or frets have been recognized, a working basis is available: modality has been established; and it may happen that the ratio of one single interval identified will give the clue to the Harmonia; a case in point is introduced further on.

In order to fill in the gaps in our practical and theoretical knowledge of the flute from Graeco-Roman sources (already discussed in previous chapters), we must turn to Syrio-Arabic sources.

THE MODAL FLUTE IN SYRIO-ARABIC SOURCES: AL-FĀRĀBĪ'S EVIDENCE

Here a curious situation arises: in default of any other known method of describing the production of sound, and the intervals of the scale on the wind instruments, Al-Fārābī, the most eminent Arabian authority on music, makes use for the purpose, of the accordance and fretting of the lute, which he has previously explained in detail. But in this experiment, Al-Fārābī has attempted to fuse two antitheses: (1) the ditonal scale of the Hellenic theorists applied to the lute, and (2) the unequivocally Modal System of the pipes in traditional use among the minstrels, singers and lutenists, but which was only dimly realized by Arabian theorists; and of which no attempt to explain the modality implicit in equal-spacing of frets on the Tanbur of Bagdad, or of the fingerholes in flutes, was made anterior to the Treatise of Safi-ed-Din [1] (*c.* 1292).

Equal-spacing, explained as to procedure, but not as to modality and Modal Determinant, and their implications, was attempted in terms of what has been called the *Messel Theory*, to which attention was drawn by Kiesewetter and von Hammer Purgstall,[2] and later by Hugo Riemann; [3] neither of these, however, draws a deduction from the theory which bears upon modality.

In Al-Fārābī's treatise there are sources of information concerning the reed-blown pipe and the flute which appear sound and reliable, as far as they go. They are surprisingly lucid when compared with the attempts of other writers, including Ptolemy. On the acoustics of flutes and cylindrical pipes, Al-Fārābī—amongst some few erroneous notions—has much to say that is confirmed by acousticians of the present day.

[1] 'Le Traité des Rapports Musicaux ou l'Epître à Scharaf ed-Dîn. par Safi ed-Din 'Abd El-Mumin Albaghdâdi', par M. le Baron Carra de Vaux (extrait du *Journal Asiatique*) (Paris, 1891). See also Paris, Bibl. Nat. Arab. MSS., No. 2479, dated 1492; Bodl. Lib. Oxf. MS., No. 992, fol. 30.

[2] *Musik der Araber u Perser*, by Kiesewetter and von Hammer-Purgstall, 1842. Contains translations of excerpts from Mahmud Schirazi's *Dürret ed Tadsh* (a Persian Encyclopaedist, died 1315), Al-Shirazi, Brit. Mus. Add. MSS. 7694, fol. 217*v*.

[3] Hugo Riemann gives a revised version of the excerpts dealing with the Messel Theory in *Studien z. Geschichte d. Notenschrift* (Leipz, 1878), pp. 77–85. His version is conjectural, as he had not access to the original Arabic source.

Riemann's *Gesch. d. Musiktheorie* (Berlin, 2nd ed.), 1898, p. 394, contains a reference to the Messel Theory and a suggestion that the perfected system of the Persians was that of the Messel.

THE INFLUENCE OF DIAMETER RECOGNIZED BY PTOLEMY

In the matter of the influence of diameter in cylindrical pipes, he goes one step further than Ptolemy [1] who recognized the significance of diameter, without, however, being ready with a formula for the practical definition of its influence on pitch. Al-Fārābī, relying upon his knowledge of the ratio of length in strings (which is in inverse proportion to the ratio of vibration in pitch frequencies), has fallen into an excusable error when he advances the theory that in cylindrical pipes of equal length, but differing proportionally in diameter—breath and other factors being equal—the notes of these several pipes will vary in inverse proportion to the ratios of their diameters (*op. cit.*, D'Erl., p. 266).

This is a cardinal error: the influence of diameter in such open pipes and flutes consists in an *addition*, by the amount of the diameter taken twice, to the length of the flute; and, therefore, diameter does not affect the pitch of the note proportionally, but only by addition, e.g.:

In two pipes, A and B, blown from one end, the dimensions are:

$$\left.\begin{array}{l}\text{A length } \cdot 270\text{; diameter } \cdot 014 \\ \text{B length } \cdot 270\text{; diameter } \cdot 021\end{array}\right\}\begin{array}{l}\text{ratio} \\ 2:3\end{array}$$

the influence of diameter by formula

$$\text{A length } \cdot 270 + (\Delta \times 2 = \cdot 028) = \cdot 298$$

$$\text{B length } \cdot 270 + (\Delta \times 2 = \cdot 042) = \cdot 312$$

$$\text{A} \qquad \frac{340}{\cdot 298 \times 2} = 582 \text{ v.p.s.} = D$$

$$\text{B} \qquad \frac{340}{\cdot 312 \times 2} = 544 \text{ v.p.s.} = C\sharp$$

[1] *Op. cit.*, Lib. i, Cap. 8, ed. Wallis, p. 33; but see ed. Ingemar Düring, vol. 1, Greek text, p. 17; Ptol., Lib. i, Cap. 8, Düring, p. 17, line 4. '*ἔτι καὶ τὰ πέρατα πρὸς ἃ δεῖ τὰ μήκη παραβάλλειν, ἐν πλάτει πως καθίσταται μετὰ τοῦ καὶ καθόλου τοῖς πλείστοις τῶν ἐμπνευστῶν ὀργάνων*, &c.

Düring's translation, vol. ii, p. 35: 'Sind noch dann die Endpünkte an denen man die Länge messen muss, nicht sicher festzustellen' [no mention of width or diameter]. *Commentary*, p. 182 (text), p. 17, 5: '*ἐν πλάτει πως*', vgl. 4, 20. 'Ptol. meint dass die Länge der Luftsäule zwischen Mundstück u. Bohrlöchern nicht so genau bestimmt werden kann.' [To this one might add: 'da sie auch vom Durchmesser der Luftsäule abhängt.'—K. S.]

The failure of the Translator to recognize in *πλάτει* the diameter of the pipe or flute leads to a confusion of facts and does less than justice to Ptolemy's acumen.

The implication of this statement by Ptolemy (i, 8)—which appears also in Porphyry's *Commentary* (see our Chap. iii, p. 130–1)—is that a given measurable length of pipe between embouchure and exit does not produce the expected note of exact pitch that the same length of string would yield: therefore another factor, viz. width (*πλάτος*), as yet undetermined, must also be taken into account. This factor of width can only refer to the diameter of the bore; it cannot apply to fingerholes (as has been suggested) for two cogent reasons: (1) Ptolemy's passage is addressed throughout to Syrinx and Aulos: the Syrinx has no fingerholes.

(2) Diameter in fingerholes takes effect solely on the individual note of any given fingerhole, and not at all on the fundamental of the flute which is the point at issue here. The passage reveals the fact that Ptolemy's main preoccupation was to discover an underlying principle of acoustics, of which practical use could be made in the determination of the sounds produced by musical instruments.

The difference in pitch due to diameter is that of a chromatic interval or approximate semitone, whereas the proportional ratio of the two diameters is that of a Perfect 5th. Further on, however, Al-Fārābī recognizes the necessity for further discussion on the question of ratios of length and frequencies of pitch in flutes and pipes,[1] reaching the following conclusions which pave the way for discrepancies between theory and practice. They are thus voiced by Al-Fārābī: 'Unde sit ut quantitates (sive potentiae) sonorum, qui ex foraminibus singulis prodeunt non semper respondeant spatiis quae inter foramina et locum inflandi interposita sunt' (p. 98, Koseg.).

In this passage the Arabian scholar reveals a definite advance on his Greek Sources, for although he has only apprehended a negative principle—and cannot supply the positive correlative—it is nevertheless true as far as it goes.

In describing the reed-blown pipes in use in his own country, Al-Fārābī gives more precise practical data concerning their structure and capabilities.

AL-FĀRĀBĪ DESCRIBES FLUTES AND PIPES WITH EQUIDISTANT FINGERHOLES

First and foremost comes the important fact that the seven fingerholes are disposed in a straight line between exit and embouchure *at equal distances from one another*: 'In ima parte orificiorum paratur aeri recta via abeunti; in dorso foramina septem terebrantur spatiis interiectis aequalibus.'[2]

In addition it is stated that two more fingerholes were bored in the back of the pipe, the first near exit, and the second between the last two holes near embouchure. The significance of the disposition of the fingerholes at equal distances, although duly observed, was not more clearly realized by the Arabian theorist than it was by his Greek predecessors.

[1] Kosegarten, *op. cit.*, p. 96, lines 8 to 5, from bottom. Al-Fārābī, fol. 76*v.* and 77*r*, 'Gravitas et acumen sonorum ab Tibiis editorum qua ratione longitudine et amplitudine (diameter), cavi atque orificiorum varia porro levitate et asperitate cavi orificiorumque, et denique spatiis inter foramina dorsi interpositis, rebusque aliis temperentur [fol. 76*v.* et 77*r.*] diligenter exponit' (Koseg., *loc. cit.*).

The translation of these pages from the Arabic MS. is not given nor can it be traced in D'Erlanger's edition but only a translation of the above, p. 268.

[2] This passage has been mistranslated in the French edition (*op. cit.*, p. 269, lines 3–4) thus: 'L'instrument comporte sept ouvertures transversales *d'égal diamètre* percées sur sa face supérieure.'

It is evident that if Kosegarten's Latin version is a correct translation of this passage, to apply the equality to the diameter of the fingerholes, instead of to the spacing between them, is both erroneous and unmeaning; it is, moreover, incompatible with the results of our research among flutes and reed-blown pipes of primitive folk in the East, e.g. the Zamr, or primitive oboe, still in use at the present day by the Arabs along the coast of Northern Africa (of which I possess a specimen), has the fingerholes bored at equal distances, whereas the diameter of the fingerholes may vary by a millimetre or two.

Equidistant fingerholes bear a highly significant implication, involving the whole of the Modal System of music, whereas equal diameter of fingerholes is an individual concern involving niceties of intonation, modification of timbre, or merely greater facility in fingering for the finger-hole in question alone, and in reed-blown instruments this makes only an infinitesimal difference, if any, in the pitch of the note.

Al-Fārābī's guarded statements concerning agreement between the notes of the pipes and lutes are strictly qualified : it is difficult, he acknowledges, to identify accurately the notes one hears from these wind instruments. He describes the scale of several varieties of pipes, relating the degrees of their sequence approximately to the notes given by the frets of the lute, but he adds that ' these flutes frequently play notes for which the frets of the lute provide no equivalent ; these must be found on the strings between the frets ' (D'Erl., *op. cit.*, p. 276).

The musicians who use these pipes only very exceptionally make use of the notes corresponding to both Wosta (the fret for the middle finger) and Binṣir (for the ring finger) on the lute (see figs. of the accordance of the Lute) ; ' in fact, in most of the pipes,' he adds, ' if there is a hole for Wosta, there is none for Binṣir '.

One fact emerges clearly from Al-Fārābī's statements on flutes and pipes. They were bored with equidistant holes, and, therefore, with normal fingering could only give one or other of the Harmoniai. No disjunct octave scale with duplicated tetrachords could, therefore, under any conditions whatever, have originated upon them.

Skilled pipers no doubt succeeded in eliciting from their pipes a semblance of the lute scale when used to accompany that instrument, by employing one of the several devices, such as cross-fingering, of which a somewhat confused and vague attempt at explanation is made (D'Erl., p. 271) ; by half-stopping, or by means of certain manipulations of the straw or reed-mouthpiece.

At this point some reference to the accordance and fretting of the lute seems to be called for. Al Fārābī describes three principal accordances for the lute :

(1) The ancient traditional Persian and Arabian accordance for the lute of four strings tuned in 4ths, e.g. *A*, *D*, *G*, *C* (Koseg., pp. 87–9).

(2) The accordance for the lute of four strings tuned in 5ths (D'Erl., p. 207).

(3) An accordance for the lute of four strings tuned to 4th, 5th, 4th.[1]

The 1st bass string	BAMM	= a 4th	octave	*A*
The 2nd string	MATLĀT (octave)	2nd and 3rd = a 5th		*D*
The 3rd string	MATNĀ			*A*
The 4th string	ZIR	= a 4th		*D*

The lute had frets extending over a tetrachord, which at different times

[1] D'Erl., p. 213 ; Yāhyā ibn Ali ibn Yaḥyā al-Munajjam an Nadim ; Brit. Mus. Or. MS., No. 823, fol. 236*v.* to 238 *v.*, *ap.* D'Erl., p. xxvii, or Or. MS. 2361, fol. 237, *Al-Kindi* ; Jaqūb Ibn *Ishāq* Al-Kindi, *Risālā fi hubr ta ' lif al-alhan* ', ' Über die Komposition der melodien Ṇach. d. Hds., Brit. Mus. Or. MS., No. 2361 ' ; *Herausgeg.*, von Robert Lachmann und Māhmud el-Hefni (Leipzig, 1931), Kap. 1, § 3, p. 22, and Einleitung, p. 7 ; see also fn. 8.

varied in number, and included alternative fingerings. The four basic frets were named after the fingers thus:

Open string, MOTLĀQ.
1st fret, forefinger, *Sābbābā* = tone, 9/8.
2nd fret, middle finger, *Wosta* = minor 3rd 6/5, or in the ditonal scale 32/27.
3rd fret, ring finger, *Binṣir* = ditone 81/64, or major 3rd 5/4.
4th fret, little finger, *Khinṣir* perfect 4th 4/3.

This is the earliest and simplest arrangement of the frets on the Arabian lute to which, in time, several others were added, the most important of all for our subject being known as the *Wosta of Zālzāl*, the ratio of which is given by Al-Fārābī as 27/22; its position is higher than that of the ordinary *Wosta*. The note of this fret at a ratio of 27/22 with the open string, bears an implication of modality and is examined in detail further on.

The frets are stretched right across the neck of the lute in parallel lines, giving the same intervals on all the strings. The note of the *Khinṣir* fret in Accordance No. 1, was the same as that of the next higher open string, consequently the scale consisted of a series of conjunct tetrachords, identical in structure.

An octave of identical duplicated tetrachords disjunct, such as our major scale, was unobtainable with normal fingering on the frets.

For such a scale, a lute with Accordance No. 2 in 5ths, or No. 3 in 4th, 5th, 4th, would have to be used (see Figs. 55, 57, 58).

THE LUTE ACCORDANCE NO. 3, INTRODUCED BY ISHĀQ AL-MAUṢILI (4TH, 5TH, 4TH)

If, as Al-Fārābī states, the pipes and flutes were used to accompany both voice and lute, it will be a matter of some interest to find an avenue of reconciliation between the Harmonia of the wind instruments and the predilection of the theorists for a ditonal scale on the lute.

With regard to our No. 3, Al-Fārābī states (D'Erl., *op. cit.*, pp. 213–14) that this unusual accordance is obtained by increasing by a tone the ratio between the 2nd and 3rd open strings (i.e. MATLĀT and MATNĀ) so that the third string MATNĀ gives the higher octave of the first open string (BAMM).

This is clear and unequivocal; and the accordance is confirmed by Al-Kindi.[1] Neither Al-Fārābī nor Al-Kindi mentions Ishāq in connexion with the accordance.

[1] Ja'qūb Ibn Ishāq Al-Kindi, *c.* 790 to *c.* 874, Risālā. On the composition of melodies, according to Brit. Mus. Or. MS. 2361, with translation and commentary and English Intro., edited by Robert Lachmann and Māhmud el-Hefni (Leipzig, 1931). There is no possible doubt that the accordance described by Al-Kindi in Cap. 1, § 3, is the one in which the two middle strings are tuned a 5th apart, i.e. if the open string of MATLĀT = *D* (*re*) then the open string of MATNĀ = *A* (*la*). This is described in the usual roundabout phraseology, but Al-Kindi clearly states that *the open string of* MATNĀ *would not sound the same note as that of the little finger* (Khinṣir) *on MATLĀT* (which invariably occurs when the strings are tuned in 4ths) *for the* MATNĀ *would be two 3rds the length of MATLĀT* (i.e. the ratio 3 : 2 of the 5th).

But, as the translators point out on p. 7 of the introduction, Al-Kindi, while

ISHĀQ'S CLASSIFICATION BY COURSES (*MAJĀRI*) CORRESPONDING TO THE MODAL SPECIES

Thus two pairs of strings each yield two conjunct tetrachords, while the two middle strings, tuned a 5th apart, provide the disjunct octave, as in the P.I.S. of the Greek system, when the tetrachord synemmenon is omitted; an extra note on *Zir*, a tone above the last fret for *Khinṣir*, gives Nete Hyperbolaion. Since the Arabian scholars are known to have been working on translations of the Greek theorists, Euclid, Ptolemy, &c., the source of Ishāq's inspiration is obvious, if not directly from those works, then through contact with the Christian Monks of Syria.[1]

This accordance of the lute is the one ascribed to Ishāq [2] ben Ibrahim al-Mauṣili in the fragmentary treatise of Yaḥyā (*op. cit.*), in which some details are given of the innovations introduced by Ishāq: e.g. he maintained that there were ten notes to the octave, whereas the *theorists* pretended that there were eighteen (this number corresponds to the eighteen diatonic notes of the double octave of the Greek P.I.S. inclusive of the tetrachord Synemmenon). Yaḥyā plainly states that the first note called 'IMĀD (base or fundamental: the Greek ἀρχή) is the open string of MATNĀ, and that the octave of 'imād is Ishāq's 10th note, an extra note on the 4th string ZIR, found without fret by slipping the little finger along, beyond the last fret, till a note at an interval equal to that of *Sābbābā* (9/8 tone) on the open string has been sounded. Yaḥyā names the ten notes as they occur on the strings

acknowledging the accordance as practised, is opposed to such tuning. This conjecture is confirmed by Al-Kindi himself further on (in § 3, p. 22) when he directs that the tension of MATNĀ must therefore be relaxed so that the *K* of MATNĀ (open string) shall become equal to the *K* of MATLĀT (i.e. = Khinṣir, 4th fret).

Al-Kindi does not refer this accordance to his namesake Ishāq al-Mauṣili (A.D. 767–850), who nevertheless has been named as the innovator of this system by Yaḥyā, who studied with him.

[1] In Al-Ispāhāni's Al-Aghāni, amongst many incidents in Ishāq's career, it is mentioned that he had not read any of the works of the Greek theorists, but had come of himself upon his theories. See Rouanet, *op. cit.*, p. 2694*b*; Al-Aghāni, I, 338; *vo.* 339; Koseg., *op. cit.*, pp. 199–200.

[2] Ishāq ibn Ibrahim al-Mauṣili (of Mosul) shared the wanderings of his father in Persia and then settled down with him in Bagdad. From the renowned Chronicler of Arabian singers, pipers, and lutenists, Al-Ispāhāni (see Rouanet, *op. cit.*, p. 2702 and *passim*), much is learnt concerning Ishāq and his innovations. Special praise is accorded to Ishāq's treatise on the *Mājārī* or courses, which lead either through the *Wosta*, or through the *Binṣir* fret, and which he used in his classification of songs. The *Mājārī* of Ishāq correspond with the modal species of the Greek theorists as they occur in the P.I.S., with the difference that the Arabian musician had already adopted the classification of scales through *Binṣir*, the major 3rd, or through *Wosta*, the minor 3rd, known in the West as the major and minor Modes. Taken in conjunction with Ishāq's accordance of the lute and his use of the *Wosta of Zālzāl*, these *Mājāri* afford valuable clues to the Octoechos of the early Greek Church in Syria, a subject discussed in the Appendix to the present volume dealing with the origin of the Ecclesiastical Modes. A brief reference to my conclusions on the significance and importance of Ishāq's innovations appeared in the Introductory Vol. of the Oxf. Hist. of Music, pp. 103–4, 1929 (London, Milford, Oxf. Univ. Press).

MATNĀ and ZIR, adding that the notes of the other two strings BAMM and MATLĀT are the same as those of MATNĀ and ZIR (i.e. an octave lower).[1]

Ishāq thus made a special point in his teaching of observing the functional value of the octave as unit, and he insisted on the number of notes being ten: 'more than that number were not to be found on lutes or wood-wind instruments'. The fact was further emphasized by his notation for the ten notes, which consisted of the first ten letters of the Arabian Alphabet, reduced numerically by the omission of *Dal*, the 4th letter, the inference being that since the 4th fret on the first string MATNĀ was the same note as the open string of ZIR, *Dal* was not required; and the lower octave would be noted with the same symbols as the first. It is obviously impossible for anyone familiar with the tuning and fretting of the early Arabian lute—who will take the trouble to go into the matter—to entertain any doubt of this being the accordance of Ishāq shown in the figure below.

FIG. 55.—Lute Accordance of Ishāq al-Mauṣīli by 4th, 5th, 4th in the Hypolydian Species in the MĀJRA through *Binṣir*

Open string Motlāq of	4/3 BAMM		30/20 = 3/2 MATLĀT		MATNĀ	4/3 ZIR	
Sābbābā	20/20 } 10/9	5/4	15/20 = 30/40 } = 10/9	5/4	20/20	15/20 = 30/40	
Wosta	18/20		27/40 = 4/3		18/20	27/40	
Binṣir	17/20 †		26/40 †		17/20	26/40	
Khinṣir	16/20		24/40		16/20	24/40	
	15/20		22/40*		15/20	22/40 20/40	The double 8ve extra position =on ZIR.

* This sharpened 4th = *498 cents* + *39 cents* = *537 cents*

† Ratios $\frac{20}{17} \times \frac{13}{15}$ (26/30) $= \frac{52}{51}$ very slight difference = + *34 cents.*

The accordance of Ishāq for the Lute may be compared with Fig. 59, showing the sequence of the Hypolydian Species on the Aulos. (Ratios by K. S.) It will be noticed how easily the scale of the pipe or flute may be accommodated to that of the lute (thus tuned) for two octaves of duplicated tetrachords by means of slight cross-fingering of *Khinṣir* (normally gives a sharpened 4th).

The Dorian Spondaic Harmonia, or Terpander, Scale of ratios 11, 10, 9, 8, 7, 6, 11, will be found on this lute, as species, by using the frets, thus:

KH. MATL.	MATNĀ MOTLĀQ.	*Sābb.*	*Binṣir.*	*SABB.* ZIR.	*Binṣir*	*Khinṣir*
22/40	20/40	18/40	$\boxed{16}$/40	14/40	12/40	11/40

This is the Harmonia on the Elgin Aulos at the Brit. Mus., on the Graeco-Roman Bucheum flute, the Inca, Java i and ii, and many others.

[1] Rouanet, *op. cit.*, pp. 2701–2.

Ishāq's contribution to the early theory of Arabian music is of great significance in the development of modality, with its insistence on the value of the octave unit, emphasized by his accordance, and by his classification of melodies by MĀJRA (course, similar to the Greek Modal Species) (see fn. 2, p. 277, on Ishāq).

THE *WOSTA OF ZĀLZĀL* (= RATIO 27/22) IMPLIES THE USE OF THE LYDIAN SPECIES OF M.D. 27

Although the present writer has not found it definitely stated, the probability is that Ishāq used the *Wosta of Zālzāl*, named after his uncle, a musician of renown resident in Bagdad. This *Wosta*, as mentioned above, is stated by Al-Fārābī to have stood in the ratio 27/22 to the open string on the lute.

The implication of this ratio is of great importance in any attempt to reconcile the Harmonia of pipe and flute with the scale of the lute; it furnishes a clue to the modality of the scale in use on those instruments.

The ratio 27/22 of the *Wosta of Zālzāl*, compared with the Minor 3rd (6/5), is sharper by an interval of 45/44 or *39 cents*.[1] This signifies that 27 is the number representing the open string, or the fundamental or exit note of pipe or flute. On the monochord it would be marked 27/27, instead of 26/26, as the modal tonic of the modified Lydian Harmonia, actually occurring in the modal P.I.S. as Parhypate Hypaton. The full octave sequence, therefore, is as follows:

Hypaton		Meson				Synem.	Diezeug.	
Parh.	Lich.	Hyp.	Parh.	Lich.	Mese	Tr. or	PM.	Tr.
$\frac{27}{27}$	$\frac{24}{27}$	$\frac{22 \text{ or } 21}{27}$	$\frac{20}{27}$	$\frac{18}{27}$	$\frac{16}{27}$	$\frac{15}{27}$	$\frac{14}{27}$	$\frac{27}{54}$

How this Harmonia is embodied on the pipe and incorporated in the frets of the lute may be seen from Fig. 57. The *Wosta of Zālzāl* suggests:

(1) A use by the Arabian musicians, pipers, lutenists and singers, of the modified Lydian Harmonia of Modal Determinant 27, known to us through the Greek sources from the tetrachords ascribed by Ptolemy[2] to Archytas.

(2) The use of the ancient Lydian Harmonia of Modal Determinant 26 (or 13) on the pipe from Hole 1 (see Fig. 58).

(3) The *Wosta of Zālzāl* further implies the use of the Phrygian and Dorian Species beginning in the P.I.S., respectively, on Lichanos Hypaton with ratio 24, and on the lute with *Sābbābā* of ratio (27/24 (= 9/8 tone

[1] i.e. $\frac{27}{22} \times \frac{5}{6} = \frac{45}{44} = 39$ *cents*,
therefore, $6/5 = 316$ *cents* + *39 cents* = *355 cents*, *Wosta of Zālzāl*.

[2] *Op. cit.*, ed. Wallis, 1682, pp. 170–2. Ptolemy records the tetrachords ascribed to Archytas for the three genera, in each of which the first step is given the ratio 28/27, so that Parhypate in each of the genera bears the ratio 27, being at an interval of 28/27 from Hypate. The consequence of this is that the Tonic of the Lydian species which falls on Parhypate Hypaton now has ratio 27 instead of 26, the Modal Determinant of the Lydian Harmonia. The first tetrachord of the Lydian species, therefore, is as the ratios 27, 24, 21, 20, in the Diatonic Genus (cf. with Fig. 57).

from the open string), and on Hypate Meson for the Dorian Species with 27/22 for the *Wosta of Zālzāl* on the lute (see Fig. 57), from the 3rd hole on the pipe (Fig. 58); both species take the MĀJRA through *Wosta.* The schemes in Figs. 58 (1), (2), and 57 and 55, show how the Phrygian and Dorian Species lie on the frets of the lute, and it is obvious that the accordance of Ishāq is preferable; for it accommodates with ease a double octave of the Lydian Species (M.D. 27), an octave of the disjunct Phrygian Harmonia, and an octave of duplicated conjunct Phrygian tetrachords of ratios 12, 11, 10, 9 (the Diatonon Homalon of Ptolemy) on the following frets, a feat which is only possible on the lute of Ishāq.

FIG. 56.—The Phrygian Species with Duplicated Conjunct Tetrachords on the Lute, with the Accordance No. 3 of Ishāq

BAMM			MATLĀT			MATNĀ	
Sābb.	W. of Z.	Khin.	Sābb.	W. of Z.	Khin.	MOTLĀQ	Sābb.
24	22	20	18 (36)	33	30	27	24
			= 12	11	10	9	8

The Phrygian disjunct tetrachords of the Harmonia on the lute with Al-Fārābī's Accordance (No. 1) in 4ths

BAMM			MATLĀT			MATNĀ
Sābb.	W. of Z.	Khin.	Sābb.	Bins.	Kh.	Sābb.
24	22	20	18	16	(7) 15	12
12	11	10	9	8	(7) 14	6

The string is slightly shortened on the fret of *Khinṣir* to provide a rise in pitch to 14.

As stated before in this work, the scale has been traced independently in Syria, and, according to Tzetzes,[1] it was still in use in that form in many of the Greek Churches in Asia Minor in 1870.

FURTHER MODAL IMPLICATIONS OF THE *WOSTA OF ZĀLZĀL*

This digression on the tuning and frets of the lute was rendered necessary because Al-Fārābī has omitted to give any adequate, precise indications of the scales proper to pipes having fingerholes bored at equal distances. To state merely that these modal pipes and flutes played approximately the same scale as the lute is tantamount to saying that all pipes produced the same scale, an erroneous conclusion, due perhaps to their having equidistant holes.

There is, nevertheless, nothing seriously amiss in Al-Fārābī's theory on the subject, as far as it goes, except that it does not go far enough.

Since no measurements of the pipes are given, and therefore no facsimiles can be made, we are reduced to conjectures; but these are not a pure effort of the imagination: they are well grounded within the range of the modality of the Harmonia, embodied in the boring of the fingerholes, i.e. of the legitimate scale of the pipe. What it is possible to produce in addition by cross-fingering and half-stopping is also known, and therefore the suggestions are based on practical, reasoned knowledge.

[1] Dr. Johannes Tzetzes, *Über die Altgriechische Musik in der griechischen Kirche* (München, 1874), pp. 77 and *passim.*

To effect a reconciliation between the scales of modal flutes and of certain lute scales has not proved such a difficult matter after all.

The *Wosta of Zālzāl* gives the clue : it cannot obviously have originated in the ditonal system of fretting on the lute ; its appearance among the other *Wostas* is altogether incongruous ; it clearly implies a modal origin, which has already been indicated above, and in Fig. 57.

The Harmoniai which yield a possible *Sābbābā* as first fret, at one tone above the note of the open string, are (1) the modified Lydian Species of Modal Determinant 27, *Sābbābā* 27/24 (9/8 tone), (2) the Hypophrygian 18 : 16, (3) the Hypolydian with the minor tone 10 : 9. The Hypodorian and the Phrygian can only provide the septimal tone 8/7, too sharp by *27 cents.*

FIG. 57.—Al-Fārābī's Accordance of the Lute of 4 Strings tuned in 4ths

(*See Kosegarten, op. cit., p.* 45.)

Strings MOTLĀQ = open string	BAMM.	MATLĀT	MATNĀ	ZIR
	27/27 = *A*	27/27 = *D*	27/27 = *G*	27/27 = *C*
Sābbābā **forefinger**	24/27 = *B*	24/27 = *E*	24/27 = *A*	24/27 = *D*
Wosta **middle finger**	23/27 = *C*	23/27 = *F*	23/27 = *B*♭	23/27 = *E*♭
Wosta of Zālzāl* **middle finger**	22/27 $\overset{\sharp}{C}$	22/27 $\overset{\sharp}{F}$	22/27 $\overset{\flat}{B}$	22/27 $\overset{\flat}{E}$
Binṣir†	21/27 *C*♯	21/27 *F*♯	21/27 *B*♮	21/27 *E*♮
Khinṣir **little finger**	20/27 *D*	20/27 *G*	20/27 *C*	20/27 *F*

The 6 frets with the ratios, implied by the use of the *Wosta of Zālzāl* (added by K. S.) are those of the first tetrachord of the modified Lydian species in the P.I.S., they form a sequence of conjunct tetrachords identical in structure.

* *The two* Wostas *are alternatives.*

† Binṣir *and* Wosta *are alternatives, major or minor 3rds.*

Binṣir of ratio 27/21 is a quarter-tone too sharp for the major 3rd $\left(\frac{27}{21} \times \frac{4}{5} = \frac{64}{35}\right.$ $\left.= 49 \textit{ cents}\right)$. *27 cents* too sharp for a ditone. $\left(\frac{27}{21} \times \frac{64}{81} = \frac{64}{63} = 27 \textit{ cents}\right)$.

The nearest to the major 3rd is a note intermediate between the *Wosta of Zālzāl* and the *Binṣir*, viz. 43/54: $\left(\frac{54}{43} \times \frac{4}{5} = \frac{216}{215} = 8 \textit{ cents}\right)$.

To produce these ratios the frets from *Sābbābā* to *Khinṣir* must be equidistant, aliquots of M.D. 27.

The scheme represents the effect of frets carried rigidly across the neck of the lute with the ratios of the intervals they produce on each of the open strings, In brackets are the ratios required on a pipe bored to the same Lydian Harmonia, in order to play with the lute. The piper would naturally not concern himself with ratios, being sufficiently guided by his musical ear to enable him to force the holes of his pipe into acquiescence with the notes of the lute, a result which would be effected *tant bien que mal*, according to his musicianship. In Fig. 57 the extent of the approximation required is indicated, and made possible by cross-fingering and other manipulations of the normal scale given by the fingerholes.

OUR MINOR AND MAJOR MODES ARE AKIN TO THE *MĀJĀRI* THROUGH *WOSTA* (MINOR 3RD) OR THROUGH *BINṢIR* (MAJOR 3RD) OF ARABIAN LUTE ACCORDANCE

Fig. 57 shows the first tetrachord of the Lydian Species of the P.I.S.—in use in Plato's day—in place on the frets of the lute tuned in 4ths, which, according to Al-Fārābī, was the traditional accordance of the old Persian and Arabian lutes. This tuning favours equally the *mājārī* through the *Wosta of Zālzāl* and through *Binṣir*, and yields conjunct duplicated tetrachords. The *Binṣir* fret, of ratio 21/27, compared with the major 3rd is *49 cents* too sharp, but only *27 cents* sharper than the ditone; the 4th of *Khinṣir* is a comma (*22 cents*) sharper than the perfect 4th.

In Fig. 58, the pipe or flute, bored to give the same Lydian Species as on the lute of Fig. 57, displays the normal scale of the pipe, together with the disturbance produced therein by the cross-fingering rendered necessary in order to allow of pipe and lute being played in concert. The approximation is excellent: the two conjunct tetrachords favour a *majra*, either through *Wosta* or through *Binṣir*, which on the MATLĀT string is the Mese or keynote of the Harmonia.

It is obvious from Fig. 58 that if Al-Fārābī's pipe played the Lydian Species depicted above, as the normal scale of the instrument, there is not much fault to find with his statement that the pipe played the scale of the lute. If such be the case an interesting light would be thrown upon the rise of the major scale in Western Europe during the early Middle Ages, for the source was twofold. The only notes that need to be modified on the pipe are:

(1) *Binṣir* on BAMM of ratio 54/43, which looks forbidding but is simple enough to produce at Hole 4 by cross-fingering (closing two or three holes). The *Binṣir* of ratio 54/43 approximates with a negligible difference to the major 3rd, being but *8 cents* sharper, whereas the ditone is a comma sharper—*22 cents*.

(2) The *Wosta of Zālzāl* on MATLĀT, of ratio 33, is obtained at Hole 6 on the pipe by lowering the Mese of ratio 32 by a quarter-tone to 33.

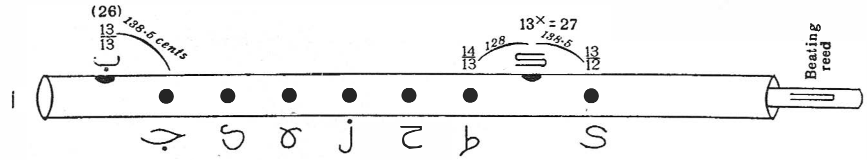
(3) *Khinṣir* on MATLĀT of ratio 20/15, a perfect 4th, is obtained by cross-fingering Hole 7 of ratio 14, easily lowered to 15.

Are we, then, entitled to conclude that the Arabian Modal Scale, used by singers and pipers—in contradistinction to the ditonal of the theorists—was the Lydian? It would be rash to come to such a conclusion without first examining the claims of other Harmoniai.

FIG. 58.—Al-Fārābī's Pipe [Aulos] with his Arabian Notation. Scale of Pipe with Approximation to Lute Scale (Fig. 57)

This pipe plays normally the Lydian Harmonia of M.D. 13 or 26 from Hole 1, and from exit, the modified Lydian Species of M.D. 27 of the P.I.S. The pipe is suitable for playing through the MĀJRA of *Wosta* and through the MĀJRA of *Binṣir*, when approximated to the two conjunct tetrachords of the lute with accordance in 4ths.

Fig. 58 (1) .



$\frac{27}{27}$	$\frac{12}{13}$	$\frac{11}{13}$	$\frac{10}{13}$	$\frac{9}{13}$	$\frac{8}{13}$	$\frac{7}{13}$	$\frac{6}{13}$	Pipe's normal scale.

Cents 204   151   165   182   204   231   267

MOTLĀQ OF BAMM (see Fig. 59).   MOTLĀQ OF MATNĀ

Fig. 58 (2)

The scale of the pipe, approximated by cross-fingering to the notes given by the frets of the lute, may be stated thus : (Ratios by K. S.). The accordance of the lute may be in 4ths or Ishāq's 4th, 5th, 4th.

BAMM MOTLĀQ.	27/27		MATLĀT MOTLĀQ.	20/27	
		9 : 8 = *204 cents*			10 : 9 = *182 cents*
SĀBBĀBĀ	24/27		*SĀBBĀBĀ*	18/27 (36/54)	
		12 : 11 = *151 cents*			12 : 11 = *151 cents*
WOSTA OF ZĀLZĀL	22/27		*WOSTA OF ZĀLZĀL*	33/54	
		22 : 21 = *80·5 cents*			33 : 32 = *53 cents*
BINṢIR	21/27		*BINṢIR*	32/54	
		21 : 20 = *85 cents*			16 : 15 = *112 cents*
KHINṢIR	20/27		*KHINṢIR*	30/54	

(Brackets at left: 22/27 spanning Bamm Motlāq to Wosta of Zālzāl; 21/27 spanning to Binṣir; 20/27 spanning to Khinṣir.)

The discrepancies in ratios and *cents* :

Re *Binṣir* $\frac{27}{21} \times \frac{4}{5} = \frac{36}{35} = 49$ *cents* : $\frac{27}{21} \times \frac{64}{81}$ ditone $= \frac{64}{63} = 27$ *cents* : $\frac{54}{43} \times \frac{4}{5} = \frac{216}{215} = 8$ *cents.*

N.B.—43/54 is intermediate between 22 : 21, i.e. at the half-increment.

Re *Khinṣir* $\frac{27}{20} \times \frac{3}{4} = \frac{81}{80} = 22$ *cents.*

The Phrygian, for instance, as seen above, in two alternative scales, disjunct and duplicated:

(1) The true octave Harmonia of M.D. 12, playable on the lute with the accordance of Al-Fārābī in 4ths, and on the Aulos (Fig. 58).

(2) The octave formed by the duplication of tetrachord 12, 11, 10, 9, which can be played only on the lute with the accordance of Ishāq, and not at all on the pipe or flute with equidistant holes.

(3) Next comes the Hypolydian Harmonia—the origin of our major scale.

In order to trace this scale on lute and pipe, we turn to Figs. 55 and 59.

THE MODAL SCALES OF THE OCTOECHOS TRACED IN ISHĀQ'S CLASSIFICATION OF MELODIES AND IN HIS LUTE ACCORDANCE (NO. 3)

This is not the old Arabian lute of Al-Fārābī; it is the one ascribed to Ishāq, described as unusual. Fig. 55 displays the Hypolydian modal sequence starting from the open string (MOTLĀQ) of BAMM and MATNĀ—known to Greek theorists as the Diatonon Syntonon of Didymus—which is thoroughly at ease on this lute, in a compass of two octaves, ending at an extra shift of the little finger on the treble string (ZIR). The *Sābbābā* in this Harmonia is a minor tone above the open string; the only note that needs to be modified is the *Khinṣir* of MATLĀT, of ratio 22 in the Harmonia, giving a 4th too sharp by *39 cents*, which is easily adjusted. This lute thus yields either the two disjunct tetrachords of the Hypolydian Harmonia—omitting the open string of MATLĀT and ending on the open string of MATNĀ; or the two conjunct duplicated tetrachords, also obtainable.

Fig. 59 gives the scheme on the pipe or flute for the Hypolydian Harmonia of M.D. 20. The normal scale of the instrument needs but little modification, and this is carried out without difficulty.

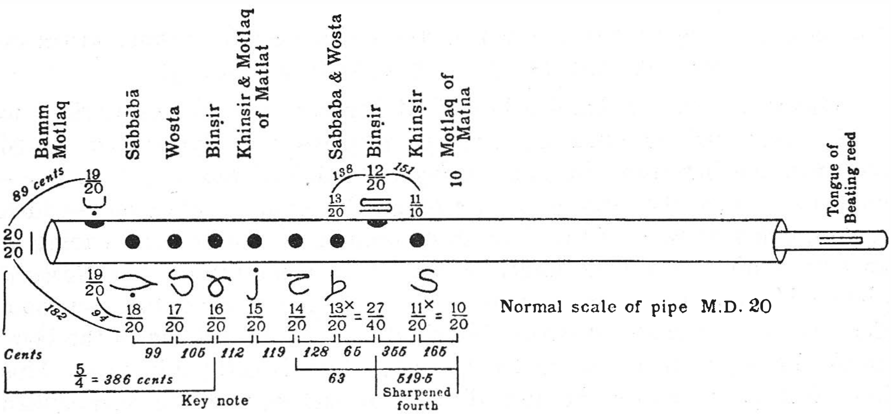
A glance at the double octave scheme provided by Ishāq's accordance [1] (Fig. 55), of the lute reveals its intimate connexion with the *mājārī* or species also introduced by Ishāq in his classification of melodies. Only on a lute, so tuned that a complete double octave with *Wosta* and *Binṣir* was available, could the species be carried out. This accordance explains the ease with which Ishāq obtained all the requisite notes for the accompaniment of his songs, set in different Modes.

In Fig. 55 the Modal Determinants of the Harmoniai of the Ancient

[1] Concerning the use of this accordance by the Arabian musicians, there can be no reasonable doubt—as already shown above; for its ascription to Ishāq al-Mauṣili there is Yaḥyā's testimony that after installing the scale of ten notes on MATNĀ and ZIR, placing 'IMĀD—the base or fundamental—on the open string of MATNĀ, the two lower strings BAMM and MATLĀT had no notes that were not already on MATNĀ and ZIR, i.e. merely a roundabout way of saying *that the notes of BAMM and MATLĀT were the same as those of MATNĀ and ZIR at a pitch an octave lower*. The importance of this statement will be realized in connexion with the implications of the MĀJĀRI of Ishāq (see Appendix, Eccles. Modes) described by Al-Ispāhāni, Kosegarten, *op. cit.*, pp. 179–82, and by Yaḥyā, Rouanet, *op. cit.*, pp. 2701–2, and Brit. Mus. Or. MS., No. 2361, fol. 236*v*. to 238*v*.

Greek System stand revealed as Tonics of the species of the Hypolydian Harmonia, e.g. Hypophrygian 18 ; Hypodorian 16 ; Lydian 13 ; Phrygian 12 ; Dorian 11 ; an octave of each of these is obtainable. The Mixolydian may be added to the list on the flute, by cross-fingering ratio 13 to 14, and using the note of that fingerhole twice, once for each ratio, the rest of the Mixolydian octave is obtained in the Harmonic register by overblowing an octave.

FIG. 59.—Al-Fārābī's Aulos (as in Fig. 58), with ratios of the Hypolydian Harmonia (by K. S.), compared with those of the Lute with Ishāq's Accordance (*cf. Fig.* 55)



N.B.—Normally, a pipe having equidistant holes must yield a modal sequence according to the arithmetical progression of ratios from the Modal Determinant, here 20. By cross-fingering the note of Hole 7—ratio 13 drops to 27 ; Hole 8 yields normally a note of ratio 12, for *Binṣir* at a major 3rd above the open string of MATLĀT. The 9th hole of ratio 11, half-stopped or cross-fingered, yields 10 for the open string of MATNĀ tuned to a 5th above the open string of MATLĀT.

N.B.—A cross at the side of a ratio denotes a cross-fingered note

In concert with the lute, a piper could play on a Dorian Aulos of Modal Determinant 11, which gives the simplest expression of the Harmonia mentioned in connexion with Terpander. This scale is characterized by producing an octave with seven notes on a Dorian Kithara of seven strings, a feat which is rendered possible by the omission of Trite Diezeugmenon according to the theorists, which is merely their explanation of the simple arithmetical progression from M.D. 11 which, after Mese as ratio 8/11, produces the undivided septimal 3rd 7/11 to 6/11, reaching the octave at the 7th note.

This digression through the Arabian sources not only brings to an end the discussion of the modal flute, but also actually forms at the same time a supplement to the chapters on the Aulos (Chaps. ii and iii), as well as an introduction to the subject of the ever-recurrent rebirth of the Harmonia through the ages, in the music of the Folk of many nations.

Al-Kindi, born at Kufa (*c.* A.D. 790–874), where his father was governor under Haroun al-Raschid, studied at Basra and Bagdad. He, therefore, enjoyed musical advantages and influences of a similar nature to those in the midst of which Ishāq al-Mauṣili had flourished some twenty-five years earlier as singer, lutenist and theorist, while Al-Kindi is chiefly famous as a Philosopher. They both describe the same accordance, although, as we have seen, Al-Kindi seemed to prefer the older one in 4ths; and both describe the *mājārī*. But although the influence of the Hellenic theorists is sensed in the writings and teachings of both Arabian musicians, and in the application of the MĀJRA to the species of the Harmonia, and to the Octoechos of the Greek Church, yet the Arabian musicians led the way in the distinction of the major and minor Modes of Western Music. This was attained through the emphasis laid upon the *Wosta* as minor 3rd, and *Binṣir* as major 3rd in the diatonic scale.

AL-FĀRĀBĪ AND AL-KINDI STRESS THE DITONAL SCALE: ISHĀQ AL-MAUṢILI THE MODAL SPECIES OF THE HARMONIA

Thus while a theorist such as Al-Fārābī laid special stress upon the ditonal scale on the lute—admitting the *Wosta of Zālzāl* and the Persian *Wosta* as more unusual than the 32/27 of the ditonal scale—yet we are inclined to suggest that the practice of music favoured the Hypolydian and Phrygian Harmoniai with the addition of the Hypophrygian, in all of which the wood-wind instruments could play in concert with the lute (see Figs. 55, 59 and 60). The Hypodorian tetrachord of M.D. 16, i.e. 16, 15, 14, 13, 12, which may be traced in Fig. 60 as species of the Hypophrygian Harmonia, bears the Persian name of *Ispāhān* in Arabian treatises.

These two chapters devoted to the lengthy discussion of the modal flute may, it is hoped, be recognized as germane to the subject of the establishment of true modality, i.e. of the modality of the Harmoniai through the Aulos, although the flute cannot be accorded a share in the origin of the Harmoniai, but only in the propagation of the Modal Scales—a function of signal importance nevertheless.

The writer knows of no positive evidence, either for or against, of the use of the flute as distinct from the reed-blown pipe in Ancient Greece. Yet there is a strong presumption that the vertical flute—the *nay* of Egypt and Arabia and the *Syrinx Monocalamos* of Hellas—was in use in certain parts of Greece, although it did not share the claims of the Aulos as national instrument, or enjoy its popularity. The feat of Midas of Agrigentum, celebrated by Pindar in his 12th Pythian Ode, whereby he gained the victory in a contest for the Aulos, is a case in point, which has been discussed at length in Chapter ii.

The scholion records the occurrence thus: 'The little tongue of the Aulos [therefore the beating-reed—K. S.] being accidentally broken by cleaving to the roof of his mouth, Midas played upon the reeds alone in the manner of the Syrinx and so delighted the audience and won the victory.'

The reasoned practical inference of what happened when Midas played on the reeds alone (*καλάμοις*, i.e. resonators) in the manner of the Syrinx, i.e. blowing across the end of the resonator, has already been discussed in detail. The conjecture introduced by the present writer is based upon

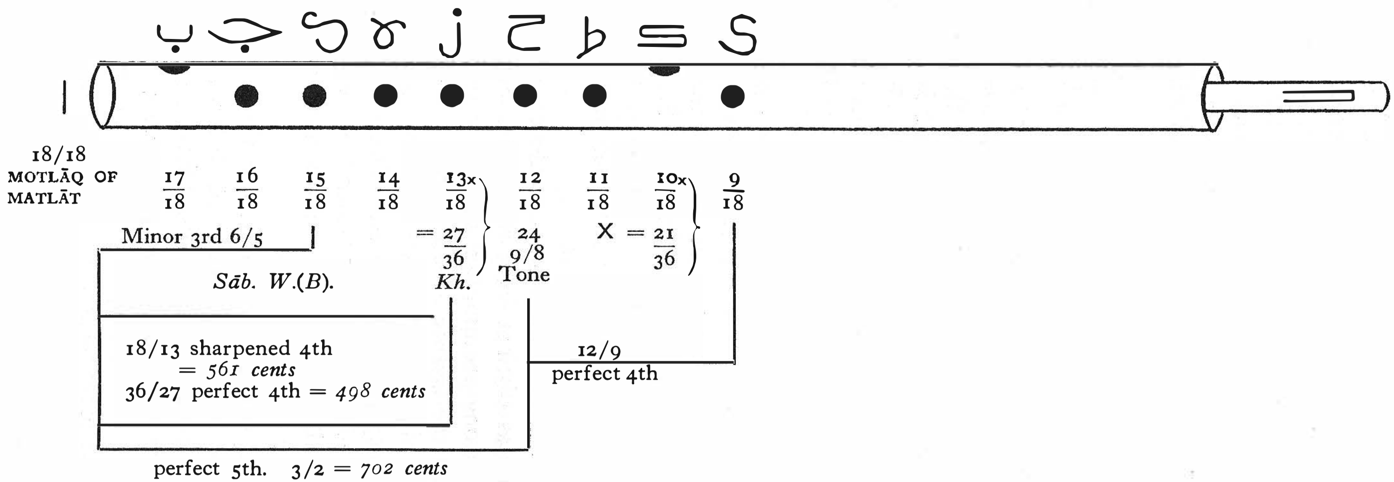
FIG. 60.—Al-Fārābī's Pipe, with Nine Fingerholes, bored for the Hypophrygian Harmonia from Exit or for the Hypodorian from Hole 1 (Ratios by K. S.)

(*cf. with Figs.* 58 *and* 59)

The MAJRA is through *Wosta*

N.B.—The finger-holes not required for the modal octave may be plugged beforehand, viz. Holes 1 and 4

To play with the four-stringed Lute with Ishāq's accordance on the 2nd and 3rd strings MATLĀT and MATNĀ



The Hypophrygian Harmonia is obtained by blocking Holes 1 and 4 (of ratios 17 and 14) and playing all others normally in succession. In concert with the lute, a conventional Modal Scale with disjunct tetrachords may be obtained by blocking out Hole 7 (ratio 11) and using Hole 8 twice: first cross-fingered to lower ratio 10 (20) to 21, then normally for ratio 10. Ratio 9 is obtained in the usual manner as the octave of the fundamental by closing all holes below it.

FIG. 61.

The Accordance of the Lute by Ishāq, tuned in Octaves from MATHNĀ and ZIR

Names of the Strings →	ZIR	MATHNĀ	MITHLĀTH	BAMM	
Notes of the MOTLĀQS → or open strings	*D*	Mese *A*	*D*	*A*	(PROSL.)
The Frets ↓ Sābbābā $\}=\frac{9}{8}$	*E*	(a'mad) *B*	*E*	*B*	HYP.
Wosta $\}=\frac{32}{27}$	*F*	*C*	*F*	*C*	PARH.
Binṣir $=\frac{81}{64}$	*F*♯	*C*♯	*F*♯	*C*♯	LICH. CHROM.
Khinṣir	*G*	*D*	*G*	*D*	LICH. DIAT. ETC.
Little Finger's extra note on ZIR only	*A* nete				

It is suggested that this accordance of the Lute by Ishāq represents the teaching of the Christian Monks

The Usual Accordance according to Yahia and Al-Fārābī

Names of the Strings →	ZIR	MATHNĀ	MITHLĀTH	BAMM	
Notes of the MOTLĀQS → or open strings	*C*	*G*	*D*	*A*	PROSL.
The Frets ↓ Sābbābā $\}=\frac{9}{8}$	*D*	Mese *A*	*E*	*B*	HYP.
Wosta $\}=\frac{32}{27}$ $\}=\frac{81}{64}$	*E*♭	(a'mad) *B*♭	*F*	*C*	PARH.
Binṣir	*E*♮	*B*♮	*F*♯	*C*♯	LICH. CHROM.
Khinṣir $=4:3$	*F*	*C*	*G*	*D*	LICH. DIAT. ETC.

Two notes are missing from the upper octave, viz. Paranete and Nete Hyperbolaion; Mese is on a fret not on MOTLĀQ.

The accordance of the Lute and the Frets according to the Scholiasts technical and practical knowledge of both instruments in question, of their characteristic qualities and capabilities, and of the implications of the sudden transformation of Aulos into *Syrinx Monocalamos*. The *Syrinx Monocalamos*, however, differed from the Panpipe in having fingerholes.

FIG. 62.—Transformations of the Dorian Aulos with the Ratios and *Cents* of the intervals of the three modal scales (conjecture by K. S.).

		Exit	H. 1	2	3	4	5	6
Dorian Aulos	Ratios	11/11	10	9	8	7	6	$5\frac{1}{2}$ (11)
	Cents	*165*	*182*	*204*	*231*	*267*	*151*	
Hypolydian Syrinx	Ratios	20/20	18	16	14	12	11	10
	Cents	*182*	*204*	*231*	*267*	*151*	*165*	
Hypophrygian Syrinx	Ratios	18/18	16	15	14	13	12	11
	Cents	*204*	*112*	*119*	*128*	*138*	*151*	

The triumph of Midas was by no means due to a mere fluke, for the removal of the mouthpiece, which had conferred upon the Aulos its characteristic Harmonia, deprived the piper suddenly, without warning, not only of the carefully prepared instrument, but also debarred him from playing the modal melody selected by him for the contest. He had on the spur of the moment to find another in a different Mode. The Harmonia was bound to be changed, for the Aulos had, with its mouthpiece, lost one or more increments of distance; e.g. if it had originally been a Dorian Aulos of M.D. 11 or 22, it would now be reduced in length, but under certain favourable conditions explained above, it might still be played as a Hypolydian Syrinx of M.D. 10 or 20; or as a Hypophrygian of M.D. 18. Midas was evidently not at a loss, and improvised without hesitation in the Harmonia thus imposed upon him through his unfortunate accident. Thus the feat of Midas recorded in the scholion deserves full credit from posterity. Moreover, it illustrates and emphasizes in a truly remarkable manner a principle as yet unrecognized in the acoustics of wind instruments: e.g. that fingerholes once bored in a pipe carry no absolute equivalent in pitch values. For instance, the ratios of the intervals of the Dorian Aulos, and of the Hypolydian or Hypophrygian Syrinx into which it was hypothetically transformed, by the mere act of depriving it of its mouthpiece, are illustrated in the figure above. The nature of the mishap to the mouthpiece informs us that the latter was of the beating-reed type—the tongue of which is very fragile. (See also Chap. ii.)

The one pipe, in its three transformations, would thus produce from the same fingerholes three different sets of intervals; the difference for these intervals is shown in *cents*.

The implication is that in all pipes having equidistant fingerholes the law of proportional ratios in arithmetical progression dominates and overrides the generally accepted reciprocals of length and vibration frequency: the ratios of the intervals are changed from hole to hole.

THE MODAL FLUTE BASED ON PROPORTIONAL MODALITY BREAKS NEW GROUND

A review of the field covered by these two chapters on the flute reveals the inherent influence of proportional modality (in the sense understood in this work) in breaking new ground and in providing new formulae to be given a practical test. It may, therefore, perhaps be conceded that an inquiry into the inner, subtle reactions of the air column in flutes was a necessary one ; a curious light has thus been cast upon the problem of the lengthening allowance due to diameter of bore, mouth-hole and fingerhole, which has to be made in order to reconcile the otherwise discrepant factors of actual measurable length of the flute, and of the length of the sound-wave producing the fundamental note of the instrument.

The Aulos, once the mastery of the mouthpiece is realized and understood, is a comparatively simple, straightforward proposition. Together with our study of the flute, it enables us to realize how it is that the troublesome influence of diameter is non-existent in the reed-blown pipe : that so long as nothing is allowed to impede or break up the proportional impulse, the Harmoniai give themselves unreservedly to the piper for his pure delight, as experience in the music of the Folk testifies.

With the flute, the whole question assumes a different aspect, on account of the different conditions induced by the two diametrically opposed methods of insufflation and sound production (into which we cannot enter in this study) : that of the Aulos propulsive directly from mouthpiece to exit ; that of the flute merely propelled in a thin, highly compressed stream, obliquely across the mouth-hole, thus providing a necessary stimulus for the initiation of a sound-wave.

In the Aulos, the timbre partakes in a measure, through the dominance of the mouthpiece and the action of the glottis, of the quality and individuality of the piper. This psychological element is communicated partly through the conformation of his vocal and respiratory organs, but mainly through his subconscious spiritual inspiration.

In the flute these influences upon the tone are to a large extent lacking or minimized, since the flute player's breath merely acts as an air-reed striking the air column within the flute at the open embouchure, and then vanishing into ambient air. What he contributes in emotional expression is more detached and objective ; but it nevertheless possesses a significant beauty of its own.

What the modality of the Harmonia owes to the flute will be clearly realized. The Harmonia has in most instances been protected from faulty intonation and the difficulty of finding the correct position for the first hole has been avoided by using that hole as vent always left uncovered, and thus abandoning the exit fundamental. The eminently solid and durable structure of the flute, moreover, ensured the survival of the Harmonia under difficult conditions ; without the flute the story of Music's development might have been very different. In this respect the flute forms a complete contrast to the more delicate Aulos, the mouthpiece of which has almost invariably perished or disappeared. An illustration is afforded by the adventure of Midas ; less gifted musicians came to grief frequently, owing to the fragility of this indispensable adjunct. The paramount importance of these two wind instruments in the initiation and development of the musical systems, which have culminated in the Octoechos of the early Greek Church, the Ecclesiastical Modes and Plain-song, and finally in our own modern system—now once more undergoing subtle transitions—has hitherto hardly received adequate acknowledgement.

CHAPTER VIII

SURVEY OF THE SCALES AND SYSTEMS FORMING THE BASIS OF FOLK MUSIC

Introductory. Brief Inquiry into the Origins of Folk Music. The Origins of Scales. The Evidence of Vedda Music. The Evidence from Greenland. Brief Survey of Scales or Musical Systems forming the Basis of Primitive and Folk Music. Scales derived from a Cycle of Ascending Fourths or Descending Fifths, on a given Fundamental C = 128 v.p.s. Dorian Harmonia or Pentatonic in Transition with a Sixth Step ? Ditonal Scale from Cycle of Fifths or Hypolydian Harmonia ? A Separate Origin for the Pentatonic and Heptatonic. The Myth of so-called Equal-stepped Scales. Dr. Erich M. von Hornbostel. Hornbostel's Cycle of Blown Fifths (Blasquintenzirkel). The Basis of Hornbostel's Theory of the ' Blasquintenzirkel ' in his own words (Engl. and Germ.). Entails the Falsification of an Acoustic Principle. The Lay View of the Influence of Diameter on Pitch. The Alleged Flatness of Overtones from Closed Pipes repudiated by Acoustic Law. The Records of Brazilian Panpipes blown by Hornbostel actually exhibit Overtones, **pure, sharp** and **flat** where all are alleged to be Flat. Correct and Faulty Methods of Blowing Panpipes. Analysis of Results given in Fig. 1. Even the Results obtained by Wrong Methods of Blowing suggest the Harmonia as Origin. The Antique Peruvian Flute ' San Ramon ' is in a different Category from the Panpipes. Dr. Manfred Bukofzer's Strictures on the Blasquintenzirkel. Evidence from an Agariche from Bolivia : all Overblown Fifths, **pure.** Is there in Closed Pipes a Natural Balance between the Interior Length + Diameter and the Exterior Length omitting Diameter. The Blowing of the Panpipes has proved to be the Undoing of the Theory of Blown Fifths. Unequivocal Rejection of the Theory of Blasquintenzirkel. The Contribution of Dr. Jaap Kunst from the Music of Java and Bali. The Harmoniai, identified from Dr. Kunst's Records as Origin of the Slendro and Pélog Scales of Java and Bali. The Flutes from Java and Bali embody the Harmoniai in their Equidistant Fingerholes. Duplication of the First Tetrachord on the Fourth or Fifth Degree in Evidence in some Javanese and Balinese Scales. The Music of the Folk in South Africa. The Music of the Bantu Folk of South Africa based upon the Harmonic Series. The South African Natives have discovered the Different Reactions of Open and Closed Pipes, closing them at will to increase the Compass. A kind of Transverse Flute in use by the Venda, Swazi, Pedi, and Other Tribes

INTRODUCTORY

THE avowed purpose of this chapter is the quest for the Harmonia of Ancient Greece, as a Survival or a Rebirth, in the music of the Folk of many nations. In order to find this search rewarded with an ultimate measure of success, it will be necessary to explore many avenues of approach, and to test all results with the greatest care by every known standard.

In order to form a helpful conception of Folk Music, it is necessary first of all to realize that in spite of the essential verity that ' Music is a universal language ' there has been at some obscure period a *musical Babel* : the music of the Folk does not spring from one single fount, e.g. the one in common use by modern musicians. The music of primitive folk, on the contrary, is in most cases found based upon a system, unfamiliar or unknown to the unsuspecting investigator, but none the less firmly established among the Folk, and assimilated by usage and tradition. The crux is, of course, to discover the particular system in use.

It is proposed to give an account, suggestive rather than exhaustive, of a few of these underlying systems, with examples of their use, and also to draw attention to the characteristics and landmarks by which they may be distinguished from other systems and scales and which reveal their identity. Although the discrediting of some theories may be involved thereby, there is no intention to disparage or belittle the immense permanent value of the records procured by indefatigable workers : the one reaps what another has sown.

BRIEF INQUIRY INTO THE ORIGINS OF FOLK MUSIC

The song that springs spontaneously from the heart and soul of the people is expressed in the language of music in which, as a child, the individual was born and grew up. This axiom, with which most musicians are in agreement, is just as true of primitive folk in remote ages as at the present day, and of the rural folk at any and every stage of evolution, even at the height of our own sophisticated civilization.

There seems to be no doubt that in the evolution of Music, Song was first in the field, unless it appeared simultaneously with the music of Speech : the latter induced by action with all its retrospective and prospective implications ; the former as the unburdening of feeling with its varied rhythms, short and sharp in the first surprise of anguish, fear or joy ; halting, soaring aloft, gliding onward, or descending step by step, or by leap to finality, as the mind intervenes with counsels or imaginings, leading to despair, resignation or supreme content. So far all is instinctive ; the language of this music is innate in man, and in tune with Nature's music : the humming of bees, the songs of the birds, the sighing or wild notes of the wind in the trees ; man is living in Music, not making it, it is part of his very being.

But let him once blow through a wheat or oat straw and make it sound, husky at first until he finds out how to make it speak, then mellow, reedy, high notes and low notes. He tries more stalks, he is lost in wonder : he has found a new world and revels in his newly acquired power ; he is a music-maker. His interest aroused, he becomes inventive, and when he discovers the effect of a hole in the side of the stalk, his joy knows no bounds.

He may perhaps, like Siegfried, try to imitate the bird's song on a pipe, but sooner or later he is disillusioned and learns that it is one thing to reproduce Nature's sounds with his own voice, but quite another matter to make a straw or reed obey his commands. It will only speak at its own sweet will. This is how he begins to build his Tower of Babel—he has to learn a new language for his pains. The evolution of Music is the sum of such experiences and of many others ; many ages elapse before Man realizes the wonders of his own voice, and it takes all his ingenuity and skill to reproduce a sound at will on any other instrument.

Strings are more tractable than wind instruments, but they are not creative; they do not originate scales and systems. The South African native upon his bow string, with a gourd attached as resonator, coaxes from it the harmonic overtones by a light touch with his finger at a node, plucking the string, or rubbing it with a stick.[1] This line of thought leads to the suggestion that the harps of Ancient Egypt, the Kitharas of the Ancient Sumerians and Chaldeans, were at first tuned to a vocal scale derived from the Harmonic Series. The first four notes from the 8th Harmonic (ratios 9/8, 10/9, 11/10) resemble the first tetrachord of the cycle of perfect 5ths born on the Panpipe.[2]

But this is anticipating the course of the early development of Music as an Art. Before man had reached the point when he could tune his harp to any desired sequence, he had to discover how to make strings that would sound when stretched, and explore the practical facts of resonance in order to reinforce those faint vibrations, besides many other structural and technical make-shifts.

Progress with the pipes was a simple matter. Our children, passing nowadays along wheat or oat fields, may be seen cutting a stalk between the joints to make a ' squeaker ' similar to the mouthpieces of the Ancients and of the primitives. If they happened to close their lips on the green or yellow ripened stalk at about $2\frac{1}{2}$ inches from the tip (as explained in Chap. iii), blowing steadily the while, they would be rewarded by hearing an *F*. It is from these simple beginnings that the favourite instrument of the Folk—the oaten pipe of Shakespeare,[3] the shawm, popular all the world over and at all times, grew and developed. Man thus learns a new language of music from his pipes, flutes, horns and trumpets, and since he finds it beyond his powers to reproduce on these instruments the notes of the voice, the voice readily learns the language of the pipes and that is how it comes about that the Folk in many regions use the traditional scales of flutes and shawms.

THE ORIGIN OF SCALES

Concerning the early stages in the development of Music, two opposed and debatable views as to the origin of scales hold the field.

(1) That the vocal elements of scale, from whatever source they sprang, (*a*) preceded the discovery of such scale elements on wind instruments;

[1] For a full description, see Percival R. Kirby, M.A., D.Litt., F.R.C.M., Prof. of Music, Univ. of Witwatersrand, Johannesburg: *The Musical Instruments of the Native Races of South Africa*, Oxf. Univ. Press, 1934.

[2] See further on for exact values of both, for comparison with the first tetrachord of the Hypolydian Harmonia.

[3] See Chap. ii:

' When Shepherds pipe on oaten straws '

Love's Labour's Lost, v, 2, line 913.

See also *A Midsummer Night's Dream*, ii, 2, 8, and *Hamlet*, iii, 2, 75; Chaucer, ' Pypes made of grene corne ' (played by shepherds). Virgil, *Ecl.*, i, 2; Virdung, *op. cit.*, fol. Diij *vo.* (Basel, 1511).

and (*b*) that in course of time, by empirical means, the vocal scale was transferred to the instrument.

(2) The view which is the subject of the present work, viz. that almost all scales originate on wind instruments, reed-blown pipes, flutes of various types, horns and trumpets. The first of these views (*a*) implies a certain appreciation of aesthetic values, and is, therefore, an attractive proposition; the sensibility of primitive folk to the harmonic constituents of sound is, moreover, an argument in its favour; (*b*) would be a difficult claim to substantiate; it can only originate with those who have not tried the feat of transference.

It may here be suggested that a clue to the origin of song and vocal scales may be found revealed in the early intonational stages of the growth of language.[1] It is not pitch alone that becomes highly significant here; the subconscious play of the emotions, the first glimmers of conscious thought, are expressed, not by a note, which is *sound at rest*, but through the interrelationships of sound in an infinite variety of intervals. The selection and control of these intervals through the agency and operations of the larynx, remain and are still retained at the present day as one of the secrets of man's subliminal self.

If the Harmonic Series, as the physical basis of sound, be recognized as the fount from which primitive folk derive the notes of their songs, then it might be assumed that there is in the subconsciousness a definite fundamental, to which the members of the series are related: for instance, *C* of 64 or 128 v.p.s. or the nature *F* of 176 v.p.s., the 11th Harmonic of *C*, and the standard note of the Ancient Chinese Musical System. The only safe criterion in our investigation, therefore, would seem to be the interval expressed by vibration frequencies, ratios or *cents*; it will reveal to us some curious facts. Recorded music of primitive tribes known to have possessed no musical instruments of any kind are rare. An examination of some of these phonographic records discloses the nature of the elements used in the early stages of vocal music by the voice in song: they may all be referred to intervals occurring in the Harmonic Series. Where the song has a compass of more than two notes, it is found in the majority of cases, in which no suspicion of civilized influence can be traced, that when the notes are arranged in line of pitch, *the intervals do not form any recognizable sequence from a given fundamental*; they seem to have been chosen haphazard, or in obedience to some instinctive urge. This suggests the inevitable hypothesis that the instinctive intervals habitually used by the individual in speech, in response to certain shades of emotion, rise up again spontaneously in song, and that each interval possesses an instinctive significance for the singer.

This conclusion is further strengthened by an investigation of the songs of the natives of East Greenland, who know of no other instrument than a shallow drum resembling a tambourine, and consisting of a substantial ring-

[1] I find that I was forestalled by Prof. Percival Kirby of the Witwatersrand Univ., Johannesburg; see his contribution, 'Some Problems of Primitive Harmony and Polyphony', &c., in *South African Jrnl. of Science* (Johannesburg, 1926), Vol. xxiii, pp. 951–70.

shaped wooden frame, on which is stretched a prepared skin of some animal. When held by a speaker, this instrument reproduces by sympathetic vibration the harmonics of the voice, emphasizing some of the intervals and prolonging them, thus illustrating the process of transition between speech and song.

Moreover, this highly sensitive instrument, when tapped at various points across the diameter, and round the circumference, gives out recognizable intervals of the Harmonic Series.

In my opinion these data point to the implication that the material on which primitive song is based *is the interval considered as a separate unit, connected in some strange way with fleeting emotional experience*; each unit having been impressed in the subconsciousness of the singer through constant use in speech.

The effect of the permutations of these detached independent units is one of fluctuation and impermanence, conditions which are entirely adverse to the development of a musical system.

The primitive elements of a scale only exist in a permanent state in the scales of pipes blown by means of a primitive double-reed mouthpiece, consisting of a wheat or oat stalk untreated and innocent of any fettering ligature; of flutes, horns, &c., which, when blown in a normal way, give out the same notes and intervals time after time and year after year (see Elgin Aulos (straight) Record; and Chaps. ii and iii). From these alone can sufficiently definite data be obtained to account for the gradual building up of a musical system. As will be seen in detail in the course of this chapter, the wind instruments form a reliable record of scales, under the following conditions.

These instruments embody certain natural laws operating by virtue of the dimensions of the instrument, and for these, formulae tested in theory and practice are available. Perfect correspondence between theory and practice exists when the piper produces from the instrument sounds identical with those demanded by theory, or when a pipe or flute is found by an investigator to give the exact results indicated by the formula, thus vindicating both lines of approach. This is the line of demarcation which remains unaffected by relative skill, or its absence in the performer. Such an instrument embodies for those who can read it, a true scale record as long as the pipe survives.

Neither continuity in the line of pitch, nor proportional interrelationship of notes and intervals exists as an idea in the singer's mind.

Where the recorded song appears to proceed—apart from niceties of intonation—along the line of pitch in our notation, this does not indicate that the singer is consciously following a sound-pattern; it probably occurs in a natural manner, owing to subtle changes in emotional feeling, which bring about the rise and fall of the voice in speech.

There is, moreover, no evidence of a conscious urgent desire for graduated steps or scales in primitive vocal music uninfluenced by musical instruments. The beginnings of scale or ordered elements in pitch sequence, though undoubtedly present at times in the vocal music of primitive folk, are generally more apparent than real when merely read in our notation, in the absence of the exact vibration frequencies obtained from phonographic records. The sound-pattern frequently presents little resemblance to the one suggested by notation.

THE EVIDENCE OF VEDDA MUSIC

Examples may now be given from two valuable contributions of this kind, made to the origins of music and more particularly to the scale, viz. : (*a*) from *Vedda Music* by Dr. C. S. Myers,[1] (*b*) *Eskimo Music in Greenland* by Hjalmar Thuren.[2]

(*a*) In this collection of songs by Dr. Myers the mean vibration frequency yielded by the record for each note is given. For the determination of pitch an Appun Tonometer was used which had a separate reed for every two v.p.s. between 256 and 512. A pitch-pipe of 256 v.p.s. was sounded into the recording phonograph before each record was taken by Dr. Seligmann in order to regulate the speed of the instrument. The margin of error is, therefore, 2 v.p.s. in the octave specified.

The songs in group (*a*) are the simplest, since their compass is of two notes only. The ratios of the intervals in three of the oldest of these is 11/10 ; of five other songs from tribes that have become semi-civilized through contact with the Sinhalese, the interval was a major tone (9/8) ; of one song of a minor tone (10/9) ; of two tunes of large semitones 14/13 and 15/14.

N.B.—The ratios that most nearly approximate to the vibration frequencies recorded have been given ; they are the nearest that can be considered useful.

In group (*b*) containing songs of 3 notes some interesting examples of detached units are found among the oldest songs.

Compass total

No. 37 v.f. $\underbrace{252 \quad 276}_{12/11\ =\ 151^\circ}$ $\underbrace{276 \quad 296}_{15/14\ =\ 119^\circ}$ 296 . . . . . . 7/6 = *270°* (*267°*)

No. 34 (2) v.f. $\underbrace{214 \quad 232}_{13/12\ =\ 138^\circ}$ $\underbrace{232 \quad 254}_{11/10\ =\ 165^\circ}$ 254 . . . . . . *303°* = 44/37

SONGS FROM VEDDA TRIBES INFLUENCED BY SINHALESE CULTURE

Compass total

No. 39 v.f. $\underbrace{198 \quad 220}_{10/9\ =\ 182^\circ}$ $\underbrace{220 \quad 232}_{19/18\ =\ 93{\cdot}6^\circ}$ 232 . . . . . . *275·6°* = 34/29

No. 14 (2) v.f. $\underbrace{226 \quad 264}_{7/6\ =\ 267^\circ}$ $\underbrace{264 \quad 296}_{9/8\ =\ 204^\circ}$ 296 . . . . . . *471°* = 21/16

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[1] Reprint from *The Veddas*, by C. G. Seligmann and Brenda Z. Seligmann, Camb. Univ. Press, 1909. See also 'A Short Account of Vedda Music by Max Wertheimer from four phonographs obtained by Frau M. Selenka', *Qu. Mag. Int. Mus. Soc.*, Year xi, Part 2, 1910.

[2] *Eskimo Music in Greenland*, by Hjalmar Thuren, and *Melodies from East Greenland*, by W. Thalbitzer and Hj. Thuren, 1 Kommission Hos., C. A. Reitzel, Bd. XL. (København, 1914). Ills. in text. Intro. in Engl. Words of songs in Eskimo.

This invocation, used by the Bandaraduwa Veddas, was sung by a Sinhalese, and thought to be of foreign origin. Consists mainly of leaps to a 4th or a 3rd upwards and downwards.

Nos. 27 and 36 (2) intervals of ratios 9/8, 16/15 = *112°*

In group (*c*), consisting of 9 tunes with a range of 4 notes and one of 5, my suggestion that the intervals formed by the notes used in the songs should be considered as independent units, not forming a sequence of ratios, is illustrated by the ratios occurring in the following songs, beginning with the oldest examples marked W.

								Compass total
W. No. 32 v.f.	230		252		268		284	
Ratios		11/10		17/16		18/17		= (26/21)
Cents		*165°*		*105°*		*99°*		= *369°*
W. No. 33 v.f.	222		226		260		276	
Ratios		49/48		15/13		19/18		= 5/4(= *386°*)
Cents		*35·7°*		*247°*		*93·6°*		= *376·3°*
H. No. 5 v.f.	224		246		272		320	
Ratios		11/10		21/19		7/6		= 44/31
Cents		*165°*		*173·4°*		*267°*		= *605·4°*
H. No. 44 v.f.	204		236		264		280	
Ratios		15/13		9/8		16/15		= 18/13
Cents		*247°*		*204°*		*112°*		= *563°*

There is in this last song a leap to a sharpened 4th of ratio 11/8 from v.f. 204 to 280·5.

No. 26 (1) v.f.	198		224		242		254	
Ratios		17/15		13/12		21/20		
Cents		*216·4°*		*138·5°*		*85°*		= *439·9°*
No. 53 (1) v.f.	210		232		254		276	
Ratios		10/9		11/10		12/11		= 4/3 =
Cents		*182°*		*165°*		*151°*		= *498°*

This is a Sinhalese song on a sequence from the Harmonic Series 9th, 10th, 11th, 12th Harmonics.

No. 41, a song from the Dambani Veddas semi-civilized by penetration from Sinhalese culture, has a range of the following five notes:

v.f.	170		192		206		236		256	
Ratios		9/8		19/18		22/19		12/11		3/2 =
Cents		*204°*		*93·6°*		*254°*		*151°*		= *702·6°*

No. 41 has as close the following:

compass $18/13 = 561^{\circ}$

v.f. 236 192 170
206

Ratios 22 19 18 16
$\overline{22}$

In this song the vibration frequencies for identical notes vary considerably, viz.:

F = *170*, 172, 174.
G = *192*.
A♭ = 206, 208, 210, 212, *205*.
B♭ = 234, 232, 230, *236*.
C = *256*, 254.

By selecting the frequencies *italicised* the following intervals belonging to a modal Hypophrygian scale are obtained. They indicate a flute or pipe scale as origin with $F = 170{\cdot}6$ as Tonic and $G = 192$ v.p.s. as keynote. The close now has the modal tetrachord of the Hypophrygian Harmonia. The $C = 256$ occurs many times elsewhere in the song and adds the perfect 5th above the Tonic.

No. 41.

v.f. *256* 205 170·6
236 921

Ratios $\frac{12}{18}$ $\frac{13}{18}$ $\frac{15}{18}$ $\frac{16}{18}$ $\frac{18}{18}$ Hypophrygian Harmonia

This sequence from the *C* to *F* with notes of different values occurs twice besides in lines 1 and 3.

As I have not heard the record, I cannot tell which of these two scales forms the basis of the song. The modal interpretation would imply Sinhalese influence, probably derived from flute or pipe.

From a general survey of the Vedda songs the impression created is mainly of speech sing-song. There are comparatively few leaps—not including mere repetitions from the same intervals. As exceptions may be cited No. 14 (2) a flattened 4th; No. 32 (ditto), an invocation; No. 44, Honey song; leap 11/8; No. 28*a*, leap 11/8; No. 34 (1) of a 4th; No. 51 leap of $18/13 = 561$ *cents*. The melodic line proceeds frequently by means of rapid articulations on the same or on a nearly related note, which suggests intonational speech and thus gives colour to my suggested origin of song, which is about to receive further confirmation.

THE EVIDENCE FROM GREENLAND

From East Greenland, as briefly mentioned above, valuable evidence is available which definitely turns the scale in favour of the hypothesis under discussion.

Hjalmar Thuren has collected 129 phonographic records of songs interesting in the highest degree for their tonality, and still more for their irregular rhythms. Each song is prefixed by the intervals of the scale actually occurring, and valued in *cents*, the songs are grouped according to subject matter. A few examples of the scale matter, when analysed from the record (not from the notation, which is frequently in disagreement with the intervals of the record), may now be given from Group 1. *Recitative songs of epic-lyric nature and children's songs* Nos. 1 to 40.

N.B.—In the following examples all ratios by K. S. and *cents* in brackets are exact equivalents of the ratios.

No. 14, p. 65 Scale 𝅗𝅥 𝅗𝅥 𝅝

Scale	*E*	*G*	*A*
Cents	40/29 = *554°* (*551°*)	*181°* (*182°*)	11/8 + 10/9
Ratio	11/8	10/9	

No. 17, p. 66 Scale ♩ 𝅗𝅥 𝅝 ♩

Scale	*D*	*F*	*G*	*A*
Cents	*312°* (*316°*)	*234°* (*231°*)	*159°* (*157°*)	6/5 + 8/7 + 23/21
Ratios	6/5	8/7	23/21	

No. 18, p. 66 Scale 𝅝𝄐 ♩ 𝅗𝅥

Scale	*G*	*A*	*C*
Cents	*167°* (*165°*)	*310°* (*316°*)	11/10 + 6/5
Ratios	11/10	6/5	

No. 19, p. 67 Scale ♩ 𝅝 𝅗𝅥

Scale	*G*	*C*	*D*
Cents	*460°* (*455°*)	*144°*	13/10 + 25/23
Ratios	13/10	25/23	

No. 21 (A), Variant of No. 19, p. 67

Scale 𝅗𝅥 𝅝 𝅗𝅥

(*a*)	*A*	*C*	*E*
Cents	*295°* (*297°*)	*337°* (*336°*)	19/16 + 17/14
Ratios	19/16	17/14	

No. 22, p. 68

Scale	𝅗𝅥		♩		𝅝		♩		♪	
(*a*, *b*)	*D*		*E*		*F*		*A*		*B*	
Cents		*251°*		*82°*		*438°*		*200°*		22/19 + 22/21 + 9/7 + 9/8
Ratios		22/19		22/21		9/7		9/8		

THEATRICAL DRUM SONGS, NOS. 49–66

No. 52, p. 75

Scale		𝅗𝅥		𝅝		♩		𝅗𝅥		
		E		*A*		*B*		*C*		
Cents	*543°*		*545°* (*551°*)		*155°* (*151°*)		*230°*			11/8 + 12/11 + 8/7 26/19
Ratios	26/19		11/8		12/11		8/7			

No. 57, p. 76

Scale	𝅗𝅥 (with fermata)		𝅝		♩	
	E		*A*		*B*	
Cents		*565°*		*165°*		18/13 + 11/10
Ratios		18/13		11/10		

DRUM SONG OF AMUSEMENT

No. 70, p. 81

Scale	𝅗𝅥		♪		𝅝		𝅗𝅥		♩	
	G		*A*		*B*		*D*		*E*	
(*a*) *Cents*			*362°* (*359°*)			*288°* (*289°*)		*180°* (*182°*)		16/13 + 13/11 + 10/9
Ratios			16/13			13/11		10/9		
Scale	𝅗𝅥		♪		𝅝		𝅗𝅥		♩	
No. 70 (*b*, *a*)	*G*		*A*		*B*		*D*		*E*	
Cents			*402°* (*409°*)			*248°* (*247°*)		*180°* (*182°*)		19/15 + 15/13 + 10/9
Ratios			19/15			15/13		10/9		
Scale	𝅗𝅥		[♭♩]		𝅝		𝅗𝅥		♩	
No. 70 (*c*, *d*)	*G*		*B*		*C*		*D*		*E*	
Cents			*452°* (*454°*)			*208°* (*204°*)		*180°* (*182°*)		13/10 + 9/8 + 10/9
Ratios			13/10			9/8		10/9		

These three sequences of ratios were used for the same staff notes in Song No. 70, of which the following few bars will give some idea:

The signs + and ÷ indicate that that note is higher or lower by less than a semitone.

In these songs from East Greenland, leaps to the 4th and to the 5th occur very frequently; in fact, in the majority of the songs the effect produced is of fullness of life, boundless energy and will power, in illustration of which an extract from the 'Juridical Drum Song', No. 79, is given below.

Tune No. 79, p. 89, lines 2 and 3

BRIEF SURVEY OF SCALES OR MUSICAL SYSTEMS FORMING THE BASIS OF PRIMITIVE AND FOLK MUSIC

It is proposed to examine the claims of the following seven scales or musical systems, which have at one time or another been suggested as an underlying basis of Primitive or of Folk Music, and to identify some of them in actual use.

(1) A cycle of ascending perfect 4ths or descending 5ths from a given fundamental.

(2) A cycle of ascending perfect 5ths or descending 4ths from a given fundamental.

The probable origin of 1 and 2 is the Panpipe: a series of 5 pipes yields a Pentatonic; a series of 7 pipes yields a Heptatonic; and a series of 12 pipes yields the whole tone scale, a chromatic, or the ancient Chinese scale of Yang and Yin.

(3) Scale systems said to consist of so-called equal intervals or steps, e.g.

5 equal intervals to the octave as in the Sléndro of the Javanese.

7 equal intervals to the octave as in the Pélog of the Javanese.

The chief protagonists of this system are: A. J. Ellis, C. Stumpf, E. M. von Hornbostel, O. Abrahams, Curt Sachs.

(4) Allied to No. 3 is Hornbostel's 'Blasquintenzirkel', or cycle of blown 5ths.

(5) The results of independent investigations by Dr. Jaap Kunst, corresponding approximately with the theoretical data of No. 4 and still better with the *Harmoniai* (see No. 7).

(6) The scales derived from parts of the Harmonic Series, whether as fundamentals or overtones.

These have been discovered in use by certain South African Tribes, Bantu and others, by Professor Percival Kirby of the Witwatersrand University, Johannesburg.

Sections of the Harmonic Series are also used by the peasants of the Bavarian, Austrian, Swiss, Rumanian Highlands in their Alphorns and long cylindrical pipes.

(7) The seven Harmoniai of Ancient Greek Music, which have survived or been reborn in the Folk Music of the World.

SCALES DERIVED FROM A CYCLE OF ASCENDING PERFECT 4THS, OR DESCENDING 5THS, ON A GIVEN FUNDAMENTAL. *C* = 128 V.P.S.

The scale derived from a cycle of seven ascending perfect 4ths—probably originating on Panpipes—was undoubtedly the origin of the ancient Dorian

FIG. 63.—Derivation of the Heptatonic Scale from a Cycle of 7 ascending perfect 4ths from *C* = 128 v.p.s.

	1		6		4		2		7		5		3		1
	c		*d*♭		*e*♭		*f*		*g*♭		*a*♭		*b*♭		*c*
v.f.	128		134·8		151·7		170·6		179·8		204·8		227·5		256
Ratios		256/243		9/8		9/8		256/243		9/8		9/8		9/8	
Cents		*90*		*204*		*204*		*90*		*204*		*204*		*204*	

ditone 81/64 — *408 cents*

ditone 81/64 — *408 cents*

tritone 729/512 = *612 cents*

4/3 = *498 cents* — Perfect 4th

3/2 = *702 cents* — Perfect 5th

THE CONJUNCT HEPTACHORD FROM HYPATE HYPATON TO MESE OF THE GRAECO-ROMAN THEORISTS:

Compared with the first Hypolydian tetrachord on *c* = 128 v.p.s. as a ditonal scale

	c	*d*♭	*e*♭	*f*	*g*				
Ratios	20/20	19/20	17/20	15/20	14/20	(13/20	12/20	11/20	10/20)
v.f.	128	134·7	150·6	170·6	182·8				
cents	*88·78°*	*192·3°*	*216·4°*	*119·3°*					

88·8°

281·1°

Cycle of 7 ascending 4ths on *G* = 96 v.p.s. grouped in a Heptatonic Scale on *G*

	g	*a*♭	*b*♭	*c*	*d*♭	*e*♭	*f*	*g*
v.f.	96	101·09	113·73	128	134·78	151·64	170·6	192
cents	*90°*	*204°*	*204°*	*90°*	*204°*	*204°*	*204°*	

Fig. 64.—The Parent Heptatonic and its Species derived from the Cycle of 4ths on *C* and *G*, the prototype of the Ditonal Dorian Species of the Graeco-Roman Theorists

THE PARENT SCALE ON *C*

	1	6	4	2	7	5	3	
	c	*d*♭	*e*♭	*f*	*g*♭	*a*♭	*b*♭	*c*
v.f.	128	134·8	151·7	170·6	179·8	204·8	227·5	256

THE PARENT SCALE ON *G* (TRANSPOSED)

	G	*a*♭	*b*♭	*c*	*d*♭	*e*♭	*f*	*g*
v.f.	96	101·09	113·73	128	134·78	151·64	170·6	192

by transposition same clef with 4 flats for all species

Species	On *C*	On *G*
1st Species LYDIAN	*d*♭ *e*♭ *f* *g*♭ — *a*♭ *b*♭ *c* *d*♭	*a*♭ *b*♭ *c* *d*♭ — *e*♭ *f* *g* *a*♭
2nd Species PHRYGIAN	*e*♭ *f* *g*♭ *a*♭ — *b*♭ *c* *d*♭ *e*♭	*b*♭ *c* *d*♭ *e*♭ — *f* *g* *a*♭ *b*♭
3rd Species DORIAN DISJUNCT	*f* *g*♭ *a*♭ *b*♭ — *c* *d*♭ *e*♭ *f*	*c* *d*♭ *e*♭ *f* — *g* *a*♭ *b*♭ *c*
4th Species Hypolydian	*g*♭ *a*♭ *b*♭ *c* (tritone) — *d*♭ *e*♭ *f* *g*♭	*d*♭ *e*♭ *f* *g* (tritone) — *a*♭ *b*♭ *c* *d*♭
5th Species LYDIAN + PHRYGIAN *Hypophrygian* (ditonal) or *Lydian conjunct*	*a*♭ *b*♭ *c* *d*♭ *e*♭ *f* *g*♭ *a*♭	*e*♭ *f* *g* *a*♭ *b*♭ *c* *d*♭ *e*♭
6th Species *Phrygian* + *Dorian* Hypodorian (ditonal) or *Phrygian conjunct.*	*b*♭ *c* *d*♭ *e*♭ *f* *g*♭ *a*♭ *b*♭	*f* *g* *a*♭ *b*♭ *c* *d*♭ *e*♭ *f*

FIG. 65.—Heptatonic Scale with its Species derived from a cycle of 12 ascending perfect 5ths (3/2) on $C = 64$ v.p.s.

Order of 5th in Cycle	1	2	3	4	5	6	7	8	9	10	11	12
	c	*g*	*d*	*a*	*e*	*b*	*f*♯	*c*♯	*g*♯	*D*♯	*A*♯	*E*♯
v.f.	64	96	72	108	81	243	91·1	136·69	205	157·7	230·6	172·29

Heptatonic scale of one tetrachordal unit
when duplicated on 4th = conjunct ; duplicated on dominant = disjunct

	Species								
		1	3	5	7	2	4	6	
HYPOLYDIAN DISJUNCT		*c*	*d*	*e*	*f*♯	*g*	*a*	*b*	*c*
			tritone						
LYDIAN CONJUNCT	1	*d*	*e*	*f*♯	*g*	*a*	*b*	*c*	*d*
PHRYGIAN CONJUNCT	2	*e*	*f*♯	*g*	*a*	*b*	*c*	*d*	*e*
DORIAN CONJUNCT	3	*f*♯	*g*	*a*	*b*	*c*	*d*	*e*	*f*♯
HYPOLYDIAN DISJUNCT	4	*g*	*a*	*b*	*c*	*d*	*e*	*f*♯	*g*
PHRYGIAN DISJUNCT	5	*a*	*b*	*c*	*d*	*e*	*f*♯	*g*	*a*
DORIAN DISJUNCT	6	*b*	*c*	*d*	*e*	*f*♯	*g*	*a*	*b*

ditonal scale, described by the Graeco-Roman theorists. The Parent scale of the cycle has two tetrachordal units of ST. T. T. structure, conjunct on the 4th degree ; and as characteristics on the Tonic, a perfect 4th, a diminished 5th, and a minor 3rd—slightly flattened in theory.

The disjunct scale of the P.I.S., displayed in the Meson and Diezeugmenon tetrachords, is obtained as 3rd species of the Parent scale. It will be convenient at this point to work out a cycle of 12 perfect 4ths on a fundamental *C* of 128 v.p.s. with their vibration frequencies. From this material may be derived pentatonic and heptatonic scales with their species.

The heptatonic scale derived from cycles of 4ths and 5ths with their theoretical values given on pages 303 and 304, serves also by elimination for the pentatonic.

DORIAN HARMONIA OR PENTATONIC IN TRANSITION WITH A 6TH STEP ?

A comparison of the species of pentatonic and heptatonic derived respectively from cycles of 4ths and 5ths reveals the fact that each species of the parent scale based upon one cycle corresponds with some other species based upon the parent scale of the other cycle—they are thus indistinguishable in practice when transposed. In tracing the basis of a folk tune which appears to be pentatonic, when it is found that the 6th step is absent, there is a contingency that the origin of the scale is the Dorian Harmonia, in its primitive hexatonic form of Modal Determinant 11, the scheme of which is given below. This missing 6th step may be noticed either in a pure pentatonic, e.g. in a scale corresponding to the 2nd or 4th species in the cycle of 4ths, and in the 1st or 4th species in the cycle of 5ths, or again, when the pentatonic is in transition between a five- and a seven-step scale.

DORIAN HARMONIA OF M.D. 11

	c		*d*		*e*		$\sharp$*f*		$\sharp$*g*		*b*		*c*
Modal ratios by K. S.	11/11		10		9		8		7		6		11/11
Denominator constant, v.f.	128		140·8		156·4		176		201·14		234·6		256
Cents		*165°*		*182°*		*204°*		*231°*		*267°*		*151°*	

The Septimal 3rd occurring in normal succession between the 5th and 7th steps, so that the 6th step appears to be missing, suggests a pentatonic in transition as a hexatonic scale. This is the scale found on many flutes and pipes, e.g. on the Elgin, the Java flutes Nos. 5 and 6, Bali flute No. 20, Mond Sicilian No. 2, &c. Other characteristics of the Dorian Harmonia are the 3rd on the Tonic, midway between major and minor, of ratio $9/11 = 347{\cdot}2°$ found noted with *G* transposition as $B\flat/2$, $\overset{\sharp}{B}\flat$, or $\overset{\flat}{B}\natural$ and the 7th, so frequently found below the Tonic with which it stands in the ratio, 12/11 = *151°* and forming a Perfect 5th 12/8 = *702°* with the key-note; there is finally the characteristic Dorian modal interval between Tonic and keynote, of ratio 11/8 = *551°*, which so often finds expression as an exultant leap, e.g. in the 'Hymn to the Sun' of the Incas (see Chap. ix) with which, repeated, the song opens and closes. This raised 4th is, however, not the unchallenged possession of the Dorian Harmonia, unless specifically indicated by v.f. or value in *cents*; it is not sufficient to find, with a *G* as final, a *C*♯/2 (Béla Bartok) or a $\overset{\sharp}{C}$ (A. H. Fox Strangways), or even *C*♯—for a glance at Fig. 85, Chap. ix, shows that there are rival claimants to this distinction. Vibration frequencies or *cents* soon settle the claim, however; hence the recurrent, insistent demand of musicologists, addressed to collectors of Folk Music for more and more precise values.

A comparison of the vibration frequencies of the notes obtained from a cycle of seven ascending 4ths, with the frequencies of ratios 20/20, 19/20, *17*/20, 15/20, 14/20, of the modal Hypolydian scale [1] reveals agreement—within a divergence of one vibration for one note only—which might easily lead to a confusion of origins, were it not for the characteristic feature of modality, viz. the keynote 16/20.

DITONAL SCALE FROM CYCLE OF 5THS OR HYPOLYDIAN HARMONIA ?

Henebry [2] provides an excellent illustration of the misleading resemblance of the parent heptatonic scale derived from the cycle of 5ths, with the modal Hypolydian. The tune is noted in the key of *G* with *F*♯ in the signature; all the *C*'s are raised by an accidental sharp. The tune runs up the first tetrachord three times to its sharpened 4th and only makes use of the second (duplicated) tetrachord, *G F*♯ *E D*, plagally below the first. This appears to be an entirely satisfying example of a cyclic 5th basis, until one discovers how well the scale fits the Hypolydian Harmonia, with tritone in the first tetrachord, which is then duplicated, on the dominant but with Perfect 4th on the octave of the Tonic, thus:

[1] See the Diatonic Syntonon of Aristoxenus as evaluated by Ptolemy (*Harm.*, ii, 14, Table iii) which corresponds with the following sequence of ratios: 20/19 × 19/17 × 17/15: the tetrachord is duplicated on the tone of disjunction. Nicom., Lib. 1, p. 24, line 11, speaking of the ditonal scale of the Pythagoreans, 'not as Eratosthenes wrongly interpreted it, or Thrasyllus, but as Timaeus the Locrian whom Plato followed.' The difference in values whether in ratios or *cents* between the ditonal and the tetrachord $\frac{20}{19} \times \frac{19}{17} \times \frac{17}{15}$ is very minute: $\frac{20}{19} \times \frac{243}{256} = \frac{1215}{1216}$; $\frac{19}{15} \times \frac{64}{81} = \frac{1216}{1215}$. Ptolemy, following the readings of the manuscripts, has thus given the modal values. From the strictures of Nicomachus, it appears that Eratosthenes was known to use both forms of the tetrachord.

[2] *Handbook of Irish Music*, No. 50, p. 207, Longmans Green & Co., Ltd. (Cork Univ. Press), 1928, Educational Co. of Ireland.

HYPOLYDIAN HARMONIA WITH TRITONE

Mese

Harmonia 20/20 18 16 15 14 13 12 11 10

(Scale by K. S.)
Ratios 20/20 18 16 14 20/10 18 16 15

Denominator constant Hypolydian (duplicated 1st tetrachord) *Cents* (K. S.) *182°* *204°* *231°* 3/2 $=$*702°* *182°* *204°* *112°* 4/3 $=$ *498°*

Tritone 10/7 $=$ *617·4°*

It is here that the touchstone of the Modal System comes into play to turn the scale: the characteristic modal interval of this Harmonia is the Major 3rd between keynote and Tonic, i.e. 16/20, and in tune No. 50 it is found strongly accented throughout—as would be expected in the Hypolydian Harmonia—whereas there is no definite reason for accentuating that note in the heptatonic derived from the cycle of 5ths. A simple tune given in Bartok's Hungarian Folk Music (No. 305, p. 84) is another clear example of the Hypolydian Harmonia; the tune opens with a run up through the 1st tetrachord to the Tritonic *C*♯, which occurs only that once, the Perfect 4th *C*♮ being used elsewhere. The duplicated form of the scale has *F*♯ alternating with the *F*♮ of the Minor 7th in imitation, perhaps, of the first tetrachord. The characteristic Major 3rd *B* falls twice with emphasis on the main and secondary caesurae, and is otherwise much in evidence.

The Hypophrygian Harmonia, which also has a sharpened 4th of ratio 18/13 $=$ *561°*, presents no other affinity with the parent scale or with the species derived from cyclic 5ths; from which it is besides excluded, on account of its orthodox Minor 3rd of *316°*.

Attention must be drawn once more to the duplication of the 1st tetrachord, which is of equal importance in all the systems under consideration, for it involves the obliteration of the 2nd tetrachord of the Harmonia. When the duplication occurs on the dominant, it also frequently introduces a 5th foreign to the Harmonia, a fact which is revealed on the flute by the spacing of the fingerholes. In the flute with three or four fingerholes, and of such proportions as make the fundamental note difficult to produce, so that the overblown octave serves as exit note, the notes of the four fingerholes are easily transposed on the dominant in the harmonic register.

The ingenious method of duplicating on the 5th in reed-blown pipes, played with a single- or beating-reed mouthpiece, has been described in Chapter ii.

Stringed instruments, such as the early lutes with fretted necks and strings tuned in 4ths, duplicate the tetrachord conjunct on the 4th.

The absence of duplication may carry an implication of the traditional use of the Harmonia, while a decided preference for the duplication of tetrachords suggests Folk Music developed under the influence of the instruments specified above.

Scales diagnosed as conjunct in the Greek sense, on the strength of an identical structure of the tetrachords will, when regarded as disjunct, display two tetrachords differing in structure. Whether this indicates a true Harmonia or a bastard is determined by the sequence of the ratios.

Béla Bartok[1] repudiates the duplication of tetrachords in Hungarian Folk Music, but only in respect of the Augmented 2nd (the modal interval $15/13 = 247$ *cents*) which, he states, only occurs a second time in the same octave in Rumanian Folk Music.

The interval 15/13 is an arresting interval, and it is possible that Bartok may not have noticed the duplication in the case of less striking intervals.

Examples may now be given of folk tunes which appear to be based upon the cycles of 4ths and 5ths.

At the head of these is the most modern of them all: the lovely first theme of the second movement of Sibelius' Second Symphony No. 2 in *D*, *op.* 43.

FIG. 66.—A Sibelius Theme from the 2nd Movement (Symphony No. 2 in *D*, 1902, *op.* 43)

In the DORIAN SPECIES (S.T.T.) of the Theorists, as it would appear in the Lydian Tonos, from Hypate Meson to Trite Diezeugmenon, with the ratios and symbols according to the interpretation of Greek Notation by K. S.

A SEPARATE ORIGIN FOR THE PENTATONIC AND HEPTATONIC

From the records of Folk Music, the view has been generally held that the heptatonic scale has evolved by accretion, through gradual stages, from the pentatonic; and the occasional occurrence of a hexatonic scale has strengthened that view.

A close examination of origins, however, leads to a different conclusion.

A primitive, having found expression in grouping together five pipes by their lengths, would not long remain content with so small a number; he would try the effect of seven or more. Or having succeeded in making a pipe or flute speak with three or four fingerholes, he would eventually add as many more as his fingers could conveniently cover.

It is by no means a convincing proposition that the prevalence of the pentatonic among primitives indicates an early stage in evolution, instead of being merely evidence of a partiality for the psychological effect of its plaintive Minor 3rds. There is a tendency to account for the appearance of a heptatonic scale in a district in which the pentatonic had previously been predominant, by the hypothetical expedient of bridging the gaps of

[1] *Hungarian Folk Music*, trans. by M. D. Calvocoressi, p. 55 (Oxford University Press, London, Humphrey Milford, 1931).

the 3rds by the interpolation of some intermediate note, consciously brought into play. In such a case the hexatonic scale is usually hailed as the first step.

The question of a separate origin for pentatonic and heptatonic, and incidentally for hexatonic scales, may be settled by the nature of the interpolations. If it should be found that the 3rds have not been divided arbitrarily by a variety of notes of uncertain values, but that the scale, on the contrary, exhibits a consistent invariable sequence of seven notes, then the argument tells in favour of a separate origin. There is no desire to be dogmatic on this issue, but only to bring to notice the contribution of wind instruments to the elucidation of the point. For instance, a flute embodying a Dorian or Phrygian Harmonia, may easily, through the position of the 1st of four fingerholes, appear as pentatonic or hexatonic; as heptatonic, if the piper half-stops the last hole of five.

It is, however, important to realize that a singer or a piper with a set of five Panpipes, wishing to bridge the Minor 3rds of the pentatonic, would be in a different category from the primitive Aulete feeling a similar urge to divide the Septimal 3rd sounding between the last two holes on his Dorian Aulos of M.D. 11. The Aulete, having bored the holes in his own pipe, would naturally come upon the expedient of boring an extra hole between the two, unless he had already found out that the same result could more easily be obtained by half-covering the top hole.

But whereas in the first case each individual singer or piper might interpolate a different note, and therefore produce a different modification of the scale each time, the Aulete would assuredly produce identical results every time, and on every Dorian pipe. The Aulete, through the permanent effect of his experiment, produces a record, and establishes a scale, from which basis a system may presently evolve. The interpolations of singer and Panpiper would be pronounced accidental or temperamental, and merely add to the confusion.

In fact, in all problems concerned with primitive and Folk Music, the French proverbial advice should be modified into *cherchez la flûte*, wherein the prospect of a correct solution would in all certainty be more assured.

A further illustration of the principle that guides the seeker unerringly to the goal, may be taken from the Panpipe; it will prove decisive further on in our investigation of Hornbostel's theories.

There is but one method of blowing the pipes of a set which inevitably produces the same intonation each time, no matter by whom the pipes are blown. It is the method, moreover, which gives identical results in practice and in theory, providing the requisite formula is used in computation, as shown in detail further on.

The correct method of blowing a Panpipe—which has been in use from the earliest times—is the following: [1] Hold the set of pipes ver-

[1] See *La Musique des Incas*, R. and M. d'Harcourt (Paris, 1925). Portfolio of Plates No. xxxvii (2), reproduction of a photograph of an Orchestra of 17 Panpipers, holding their pipes vertically in front of the mouth and blowing across the open ends.

tically in front of the mouth and rest the lower lip against—not over—the edge (as described fully later).

The wrong method is to blow *into* the pipe, instead of across the end, as in the Oriental *nay*. The note produced on the Panpipe by this incorrect kind of blowing may vary by a semitone or more, according (*a*) to the obtuseness of the angle formed by the breath-stream when striking the inner wall of the pipe, and (*b*) to the extent of the obturation of the opening by the lips of the player.

Blow the Panpipes correctly, and you have the musical facts which conduce to the birth of a system. Blow incorrectly, and the intonation of notes and the ratios of intervals are at the mercy of any and every player; there are no facts wherewith to build up a system.

These illustrations taken from the practical use of wind instruments throw a light on the part played in the evolution of musical systems by the principles and natural laws embodied in them.

Calculable results, under specified conditions, are obtainable from the interplay of such factors as length, proportion, diameter of bore and finger-holes. It is up to the player, therefore, to secure from the pipes the scale they embody. The question of origin thus ceases to be purely speculative and is based upon facts which may be tested and proven.

THE MYTH OF SO-CALLED EQUAL-STEPPED SCALES

The strangeness of the intervals identified by the analysis of records of Oriental and other exotic scales gave the impression that they consisted of equal intervals, five to the octave in the case of the Sléndro of Java, Indonesia and Melanesia, or of seven to the octave for the Pélog scales.[1]

The protagonist in the inception of this theory—which has proved to be an erroneous diagnosis—was Alexander John Ellis (formerly Sharpe, 1814–90), who published it in a paper on the 'Scales of all Nations'.[2]

Convinced by Ellis's firm adherence to this theory, and in the absence of any alternative explanation of the results of records, the protagonist was followed amongst others by Stumpf, Hornbostel and Abrahams.

During his examination of Siamese Music by the native orchestra of

[1] Known by various names in other parts of the world.

[2] Original title 'Tonometrical Observations on Existing Non-Harmonic Scales', *Proceedings of Roy. Soc.*, 1884; republished with extensive additions in *Jour. of the Soc. of Arts*, 1885, No. 1688, Vol. xxxiii, March 27th. Translated into German by Erich M. von Hornbostel, with comments, and published in the place of honour in the first vol. of the *Smb. f. Vergleichende Musikwissenschaft*, edited by Carl Stumpf and E. M. von Hornbostel, Vol. i (Munich, 1922), from which my refs. are taken.

In his introduction, Ellis gives a full and appreciative acknowledgement of his indebtedness to Mr. Alf. Jas. Hipkins, of the firm of Broadwood & Sons, who was gifted with an exceptionally sensitive musical ear, and was recognized as an authority on the History and Structure of Musical Instruments.

I gladly seize the opportunity of acknowledging once again with gratitude my own indebtedness to this genial, scholarly personality. He was the first to stimulate and direct into proper channels my budding love and zeal for musical research; he was never too busy to discuss, criticize and advise on matters connected with my work.—K. S.

the Siamese Embassy, at the head of which was Prince Prisdang, Envoy Extraordinary to the Courts of Great Britain, France and Germany, Ellis was assured by the Prince that all intervals from note to note were intended to be equal.

The implication is a division of the octave into either 5 or 7 equal steps expressed in *cents* as follows:

$\frac{1200}{7}$ *cents* = *171·43°* for the seven intervals.

	1	2	3	4	5	6	7	1	
Cents	*171°*	*343°*	*514°*	*686°*	*857°*	*1029°*	*171°*		= *1200°*

Ellis gives a list (*op. cit.*, p. 41) showing the margin of error he accepts as approximation of equal steps—an extremely generous one—which opens the door wide to a modal interpretation.

The 24 intervals cited by him between *160°* and *185°* 'represent (he adds) the normal interval (*171·43°*), while the divergencies show how easily the ear is satisfied with approximations when the intervals are non-harmonic.

Among the intervals included by Ellis within the specified limits (on p. 41) are found the first 6 of the modal Mixolydian of M.D. 14, viz. 14/13, 13/12, 12/11, 11/10, 10/9 and also 15/14, 27/25, 25/23.

These examples of equal intervals suffice to justify my rejection of equal-stepped scales, as a case of mistaken identity. This, however, is a conclusion formed without prejudice to the claim of the protagonist and his followers, who were faced with one of the hardest problems presented by the evolution of musical systems, and of the origin of folk and primitive music.

Ellis does not, of course, claim that the results of tested instruments actually show equal intervals, but that he considers that the intention was to produce cycles of equal intervals.

Anyone hearing such a sequence as the following, without knowledge of the identity of the individual steps, might be exonerated from blame for stating that the intervals were equal.

	12/11	11/10	10/9	12/11	11/10	10/9	9/8
Cents	*151°*	*165°*	*182°*	*151°*	*165°*	*182°*	*204°*

These intervals occurring in a scale of duplicated tetrachords in flowing proportional sequence—and unused in modern European music—are difficult to estimate. If they were taken in turn on the same fundamental and were heard in succession, no musician would fail to distinguish the differences in intonation.

Perspective presents the converse of this proposition. If we view the columns from the east end of the aisle in a cathedral, they produce the illusion of being placed at gradually diminishing distances, although we are well aware that these distances are all equal. This might be regarded as the architectural embodiment of the Harmonic Series ascending or descending according to the direction followed by the eye in its survey.

The succession of intervals given above will be recognized as a Modal Scale consisting of the 1st tetrachord of the Phrygian Harmonia, duplicated on the 4th and therefore conjunct. I have identified it from the vibration frequencies of records communicated to me by Dr. Jaap Kunst, of the scales of Gamelans (orchestras) of Java, Bali, Siam, and of Marimbas from Sierra Leone and Limpopo (see section ' Kunst ').

FROM PL. IV, KUNST
(B) Siamese Scale (Stumpf)
(C) Gamelan, Z. Bali.
(E) Gam. Moengang Jogja
(M) Sierra Leone, Marimba
(N) Limpopo, Marimba

FROM PL. Z., KUNST
No. 6 Kraton Jogja
No. 9 Gam. Madjakarta

With Hornbostel we enter upon a phase of the comparative method in investigation. His hypothetical Cycle of Blown 5ths is a culmination of the fixed idea of equal-stepped scales, which has dominated all his work on the music of primitive folk for several decades. When expressed in *cents*, the yield of intervals from the collected phonograms seemed to warrant the hope that a common basis might ultimately be discovered in a predilection for equal steps in the scale of sounds used by primitive musicians.

It must be conceded that a superficial consideration of the arithmetical progression which accounts for the Harmonic Series lent colour to the hypothesis of equal-stepped scales. But the recognition of the fact that in the Harmonic Series, the same numerical increment of vibrations, which separates any two adjoining members of the series, produces intervals diminishing proportionally in magnitude as the progression rises in pitch, might have given pause for thought, as far as the Art of Music is concerned.

Herein equality of increment in any one medium produces a proportionally graded inequality in another medium: interrelative equality in position among the members of a series is qualified by the individual relationship of each member to the fundamental, a principle which furnishes the key to the present inquiry.

The inequality in the musical effect of the equal increments is appreciated by the ear through the change in the periodicity of the sound-wave. This—as is known—advances through the air by its own length once in each of the complete vibrations of its periodicity.[1] Each advance of the sound-wave is announced by its impact on the ear, and the beat of a wave per second against the tympanum conditions our recognition of the vibration frequency of the note. The length of the sound-wave is thus determined not by that of the increment, but by the number of increments which separate it from the fundamental. A realization of the significance of the operation of the harmonic principle of the series might have prevented the false assumption that equidistant fingerholes could by any means produce intervals of equal magnitude.

A consideration of the respective significance of equal and approximately equal intervals—definitely recognized by those who use them—with differences of from *5* to *20* or more *cents* reveals the insidious effect

[1] *Sound and Music*, by Sedley Taylor (Macmillan & Co., 1883), pp. 20–32.

of a musical notation, consecrated by the sanction of centuries of musical progress, which is merely able to differentiate the notes of its sequences by a matter of *100 cents*.

DR. ERICH M. VON HORNBOSTEL

Appreciations of Erich M. von Hornbostel's life-work will be many: what we are mainly concerned with here is his voluminous output of musicological ideas and theories. He seems to have attacked the problems connected with the music of primitive folk from one shrewdly selected angle after another, with the persistent hope of finding a satisfactory solution. The extraordinary variety of the phonographic records, collected in the Archives of Berlin University, of the tunes of tribes and races from all parts of the world, some of which he investigated, deciphered and computed in vibration frequencies and *cents*, forms an invaluable contribution to our subject; many groups of such records with the addition of musicological and ethnological text have been published in the journals of learned societies. The material of our present chapter is intimately concerned with two of these. 'Musical Tonesystems'[1] and 'Standard Measures as a Means of Cultural Investigation', in both of which reliance is placed on the Cycle of Blown 5ths (Blasquintenzirkel) as a solution.

It is little short of a tragedy to find, on studying these two pamphlets, how nearly Hornbostel had reached the goal towards which he had so long and so persistently been striving. I refer to his general treatment of pipes and flutes with equidistant fingerholes. It is perhaps more than a little strange that this deep-seated idea of equal-spacing as motivation in the Arts, should not have led, in the case of music, to an investigation of the results of equal-spacing *in practice* on flutes and pipes. Hornbostel's efforts in this direction, e.g. in his examination of Loret's list of the surviving Ancient Egyptian pipes, with their measurements (Victor Loret, *op. cit.*), are a signal failure and entirely misleading, owing to the fact that his was merely paper work instead of practical. The intervals he attributes to individual specimens are all incorrect, for they do not take into account the all-important mouthpiece (see T. S., p. 442). Length of extrusion of the mouthpiece from the resonator—as already explained from many angles—sets the seal of modality for the Aulos, and determines the ratios of the intervals obtained through the fingerholes anew, each time the length of extrusion is changed for the same pipe.

Hornbostel evidently was not aware that the reed-blown pipe without its mouthpiece is merely a resonator, mute until the mouthpiece speaks and provokes a response.

None of his pipes has spoken.

[1] *Musikalische Tonsysteme, Handbuch d. Physik* (H. Geiger and Karl Scheel), Berlin, Bd. viii, 1927; Akustik, pp. 425 sqq. 'Die Massnorm als Kulturgeschichtliches Forschungsmittel', *Festschrift*, Publication d'Hommage offerte au P. W. Schmidt (offprints of both of which I owe to the courtesy of the author), published in *Anthropos*, Administration St. Gabriel (Mödling bei Wien, 1928), pp. 303 sqq.

Not one of Loret's pipes in Hornbostel's Table (p. 422) could by any possibility produce from exit or from its fingerholes the intervals assigned to it. As illustrated in detail in Aulos ii, the mouthpiece adds at the very least two or more increments of distance to the length of the pipe ; this combined length divided by the I.D. determines the Mode on the pipe by fixing the Modal Determinant. Hornbostel was on the verge of a discovery of the Modal System of the Ancients, but failed to find the clue. His conclusion on this subject is (*op. cit.*, 19) 'that the large group of pipes with fingerholes—flutes and oboes—yields scales which, in the aggregate, can hardly be said to constitute a system' ; elsewhere, he adds that the result is mere chaos.

I have found, on the contrary, that the scales they yield form the basis of a system in use in by far the greatest part of the world's Folk Music.

HORNBOSTEL'S CYCLE OF BLOWN 5THS (BLASQUINTENZIRKEL)

After these introductory remarks our interest is focused upon Hornbostel's hypothetical system, founded upon a Cycle of Blown 5ths. The essential points of this theory which claim our attention are the following :

(1) It is Hornbostel's unproven hypothesis.

(2) It applies solely to Panpipes consisting of cylindrical pipes closed at one end.

(3) It is claimed as an invariable rule that all cylindrical closed pipes overblow a flat 5th instead of a Perfect one.

Ergo : Since this claim impugns an acoustic law, it is clear that the overblowing of a single Perfect 5th from such a pipe (satisfactorily attested, or given by a phonographic record), would suffice to overthrow this hypothetical system. The sequel will show that this is what has actually occurred.

(4) The degree of flatness in the overblown 5th varies with pipes and players, but has been assumed as a working hypothesis to be constant, and Hornbostel has empirically fixed the flatness of the Blown 5th at *24 cents.* It will be seen further on that this estimate is entirely inadequate.

(5) The cycle is closed at the 23rd 5th with *− 6 cents.*

(6) The system of Blown 5ths which, he claims, has been found in almost universal use, is thus derived from a cycle of 23 5ths, each flattened from *702 cents* to *678 cents.* Hornbostel gives the vibration frequencies of the cycle, calculated upon a fundamental of 366 v.p.s., which he erroneously [1] attributes to the Ancient Chinese standard pipe measuring one Chinese foot of 230 mm.

(7) It must be clearly understood that this theory has not been based —as is usual in evidence of this kind—upon the playing of the Panpipes by the Indians of N.W. Brazil, in the presence of Hornbostel, nor on phonographic records of the music of the Indians.

The vibration frequency of the notes of the Brazilian Panpipes are

[1] Hornbostel admits that this standard frequency is not found in use on the Brazilian Panpipes.

those registered from Hornbostel's blowing of the pipes which, as he admits, did not always give the same result.

The method used in registering the vibration frequencies is not stated in the first account of the theory (K.-G., 1910); it was at that date probably made by means of an Appun Tonometer having a free reed for every two vibrations between 256 and 512.

(8) In the absence of phonographic records from Brazil, accurate measurements of the length and diameter of each separate pipe should have been provided.

There is, therefore, an entire absence of reliable evidence of the exact intonation of the pipes.

It is highly significant that although Hornbostel published his contribution on the Brazilian Panpipes as an appendix in Theodor Koch-Grünberg's book [1] and a provisional statement of his theory in 1920,[2] he should find it necessary to point out in 1927, with reference to the alleged flattening of the overtone, that ' this physical problem has not yet, to my knowledge, been worked out experimentally and theoretically ' (*Mus. Ton. Sys.*, p. 431, fn. 3). Since the Blown 5th of *678 cents* is the sole basis and *raison d'être* of Hornbostel's system, it is clear that a definite decision must first of all be reached by investigators as to the validity or unsoundness of the assumption.

THE BASIS OF HORNBOSTEL'S THEORY OF THE ' BLASQUINTENZIRKEL ' IN HIS OWN WORDS

The fairest way of presenting this theory is to allow the author of it to describe it in his own words. This is a translation of the passage from the Appendix to Koch-Grünberg's book in which the theory of the Blown 5ths was first launched in 1910 (p. 381, line 5, bottom).

§ 1. If one considers the coincidences of the fundamentals of our pipes with the 5th partial tones—not yet taken into account—one finds that every three of the intervals built up in accordance with the 3rd partials, e.g. i, iii, v, vii,[3] taken in sequence, lead exactly to the 5th partial of the fundamental of the first pipe of the set, or really to its lower octave. But if one proceeds upwards from any note, through three conjunct 4ths (e.g. *c*, *f*, *b♭*, *e♭*) [4] one arrives exactly at the minor 3rd (above

[1] *Zwei Jahre unter den Indianern*, Vol. ii, pp. 378 sqq., 1910. I quote from p. 381, line 5, bottom, to end of § 1, p. 382.

[2] ' Vorläufige Notiz über den Blasquintenzirkel ', in *Anthropos*, 1919–20, pp. 569 sqq.

[3] Hornbostel illustrates here from Table 1, *op. cit.*, p. 380, which gives the vibration frequencies of the Brazilian Panpipe No. 6322.

[4] Readers may be puzzled by these three conjunct 4ths *c*, *f*, *b♭*, *e♭*, said to be overblown flat 5ths transposed an octave lower (see (*e*)). Hornbostel has overlooked the fact that rising 5ths transposed to the lower octave (as 4ths) produce *descending* conjunct 4ths from a fundamental which must be common to both, else there can be no basis for comparison. A glance at Table 1, from which the example is taken, shows that Pipe 1 of 420 v.p.s. is the fundamental from which the three conjunct 4ths ascend; it is also the fundamental from which are overblown both 3rd and 5th Harmonics. The 5th Harmonic of Pipe 1 flat of 519 v.p.s. does indeed

the octave) of the starting-note, whereas the 5th partial is the major 3rd (above the double octave) of the fundamental ! (*sic*). How may this apparent contradiction of observed facts be explained in the light of the simplest acoustic laws ?

Hornbostel continues thus :

§ 2, (*a*) [1] **The pitch of a pipe does not depend solely upon the length, but also upon the width (diameter—K. S.) of the tube.** (*b*) **The note is actually somewhat lower than it should be, according to the simplest calculation, which takes into account the length only of the tube.** (*c*) This lowering of the pitch, in consequence of the influence of the width of the tube, also takes effect in the overtones. (*d*) The 3rd partials will, therefore, all be somewhat too low, the 5ths between them and the fundamentals too small, and if the overtones are transposed below the fundamentals, the 4ths will be too wide. (*e*) If three such 4ths are added together, the increases over the pure intervals—slight as they are in themselves—lead in the aggregate to a minor 3rd of the starting note, which is very considerably too large. (*f*) On the other hand, the 5th partial will naturally also be too low, in consequence of the influence of the width of the tube ; and the major 3rd it forms with its fundamental will be too narrow. That this diminished major 3rd should (almost) exactly coincide with that augmented minor 3rd, e.g. the 5th partial of (pipe) No. i with the fundamental of pipe No. vii obtained through (impure) 4ths, is in any case a remarkable and striking fact.[2]

This, then, constitutes in Hornbostel's own words the basis of his theory of the ' Blasquintenzirkel ' or Cycle of Blown 5ths (marked [2]) upon which he pinned his faith until the end.

Fact and fiction are strangely blended in this statement, which is called upon to bear the burden of proof of this ingenious but erroneous theory.

It must be recalled at the outset that the vibration frequencies of the Brazilian Panpipes, given in the various tables published in the Appendix in question, are those that were registered *from Hornbostel's own blowing of the pipes.* There are, to my knowledge, no available records whatever of the playing of any of these pipes by the Indians of North-west Brazil. Moreover, no measurements of the length and diameters of the pipes have been published : they would enable us to check the correspondence of the

almost coincide, as stated, with the note of Pipe vii of 516 v.p.s., whereas the true minor 3rd of Pipe I has a vibration frequency of 504 ; but why not ? they are not related. It is clear, therefore, that three conjunct 4ths *c f b♭ e♭* could not be transposed overblown 5ths. No basis for comparison exists.

[1] The lettering of the different points inserted by K. S. The original German version of (*a*), (*b*) and (*c*) above (*op. cit.*, p. 382, lines 4 sqq.) is as follows :

(*a*) ' Die Tonhöhe einer Pfeife hängt nicht bloss von der Länge sondern auch von der Weite des Rohrs ab. (*b*) Der Ton ist tatsächlich etwas tiefer als er der einfachsten, bloss die Rohrlänge berücksichtigender Berechnung nach sein müsste ; (*c*) diese Depression der Tonhöhe infolge des Einflusses der Rohrweite findet aber auch bei den Obertönen statt. Die dritten Teiltöne werden also alle etwas zu tief, die Quinten zwischen ihnen und den Grundtönen zu klein, und, wenn man die Obertöne unter die Grundtöne verlegt, die Quarten zu gross.'

[2] Hornbostel. *op. cit.*, p. 382, ' It is not the place here for a searching elucidation of the physical problem, which is not devoid of difficulties.'

(K. S.—Apparently there has, in all these years, been no suitable time or place found for such an elucidation.)

results given with the vibration frequencies that pipes of such dimensions would give if blown correctly.

We must turn our attention first to § No. 2. This is a thoroughly bad proposition and, as basis for any musical system, indefensible.

(*a*) is a correct statement as far as it goes but is entirely inadequate.

(*b*) contradicts (*a*): after stating that length and diameter are both necessary to the production of pitch, Hornbostel deliberately cuts out one of the two indispensable constituents of the note. To talk of a note from a pipe due to the factor of length alone is a purely fictitious conception. No such note can be produced on a Panpipe. What is here termed the 'simplest calculation' is merely error and an impossible proposition.

ENTAILS THE FALSIFICATION OF AN ACOUSTIC PRINCIPLE

(*c*) It cannot be said that the note is flattened owing to the influence of diameter or width, seeing that diameter is the co-partner with length, in the creation of the note. There is no note possible without diameter. Diameter is here falsely staged as the evil genius of the pipe. Fiction is treated as a reality and the fictitious flatness is passed on to the overtones.

It may here be categorically stated that there is no acoustic law or principle that supports the production of flat overtones from a stopped cylindrical pipe—providing, of course, that the pipe be correctly blown.

(*d*) The 3rd Harmonics are said to produce false 5ths in relation to their fundamentals: they are still endowed with the fictitious flatness incurred through the curious process of confronting the real note of the pipe, actually heard, with a fictitious note that can have no reality, since it cannot be produced; and of pronouncing the reality guilty of flatness by comparison with the fiction.

(*e and f*) Any results due to the alleged evil influence of diameter must be common to both fundamentals and Harmonics, therefore, the 5ths cannot be said to be flat 5ths, nor when transposed below the fundamentals to be sharpened 4ths—in relation to their fundamentals.

Such is the hazy origin, from hypothetical and erroneous notions, of the Blasquintenzirkel—the libellous Blown 5ths—and upon this false basis was built up the ingenious system which was to be given a world-wide application in solving the baffling problems of the scales used by the Folk.

THE LAY VIEW OF THE INFLUENCE OF DIAMETER ON PITCH

Dr. von Hornbostel is by no means singular in his misunderstanding of the incidence of diameter in the determination of pitch in pipes and flutes. It may, therefore, not be amiss at this point to give the lay view of the vexed subject of the so-called influence of diameter on pitch.

The science of Physics represents the inner reactions of cylindrical closed pipes by equations, in which the symbols R (radius) and π (= circumference) suggest solidity and cubic contents. But the sound-wave productive of pitch (and, therefore, considered from the standpoint of periodical vibrations) which emerges when the air column is set in vibration by some exciting agent, is conceived as having a purely linear progression from the pipe, through ambient air, to the ear. Diameter (or probably width right across the open end of the reed-pipe) also has a purely *linear* function in the sound-wave, i.e. the sound-wave in its pulse extends beyond the opening of the pipe, to a length equal to the width or diameter of the pipe, which is thus included in it as an inseparable and co-responsible factor of the length productive of pitch.

The sound-wave, initiated by blowing across the open end of the closed pipe, makes two journeys, along the pipe, for each complete vibration, reflected each time from the closed end, back to the opening—and beyond it to the extent of the diameter; the length of the sound-wave is, therefore, four times that of the Panpipe + twice that of the diameter.

If the velocity of sound in air at 340 metres per second (at a temperature of about 60° F.) be divided by the length, thus computed of the sound-wave, the quotient will give the vibration frequency of the note of the pipe.

FORMULA NO. 1 (B)

L = length $\quad \dfrac{340 \text{ metres/sec.}}{L \times 4 + (\Delta \times 2)} = \text{v.f.}$

Δ = diameter

or the converse

FORMULA NO. 2 (B)

$$\frac{340 \text{ metres/sec.}}{\text{v.f.}} = \text{L of sd.-wave}$$

and

$$\frac{\text{L of sd.-wave}}{4} = \text{L of pipe} + \frac{(\text{diameter})}{2}$$

The most satisfying test of the formula is to be able to make, by using it, a pipe which actually produces a note of requisite frequency given by a specified tuning fork.

The pipe must be cylindrical—the diameter at both ends equal—the pipe is then stopped at one end by a closely fitting cork. The length is that of the interior of the pipe.

THE ALLEGED FLATNESS OF OVERTONES FROM CLOSED PIPES REPUDIATED BY ACOUSTIC LAW

The immediate question at issue now is, therefore, (*g*) whether this alleged production of impure Harmonics by cylindrical closed pipes is inevitable, owing to an inherent property of Panpipes and, therefore, to be accepted as an indisputable fact supported by *Acoustic* Laws? The answer is no!

or (*h*) Whether the alleged flatness is categorically repudiated by acoustic law as the affair of the piper, due to a faulty method of blowing the pipes? The answer is yes!

The answer to (*g*) is an emphatic negative endorsed by Mr. D. J. Blaikley, a recognized authority on questions of Pitch and of Wind Instruments, who when questioned on this very point by Mr. A. H. Fox Strangways denied that this was the case.[1]

The answer to (*h*) is an equally unqualified affirmative about to be further substantiated.

THE RECORDS OF BRAZILIAN PANPIPES BLOWN BY HORNBOSTEL ACTUALLY EXHIBIT OVERTONES **PURE, SHARP** AND **FLAT**, WHERE ALL ARE ALLEGED TO BE FLAT

The question is absolutely settled once for all in the event of duly attested cases of pure Harmonics blown from any Panpipe, or alternatively of finding pure overtones indicated amongst the vibration frequencies of the notes of the Brazilian or other Panpipes.

The best of all attestations is that of Hornbostel against himself.

Among the published records of the Brazilian Panpipes, registered from the tests of the instruments while actually blown by Hornbostel, are to be found several Perfect 5ths, 4ths, and Major 3rds (5th Harmonic), the purity of which he evidently overlooked, both as practical overblown notes, and on paper from the vibration frequencies given in the tables.

The following instances may be given: In his exposition of the basis of his theory quoted above, he selects as illustration (from Table 1, K.-G., p. 380), pipes i, iii, v, vii, ' built up in accordance with the third partials which when transposed below the fundamentals give sharpened 4ths '.

But the vibration frequencies supplied by Hornbostel for these pipes show that the 4ths between Nos. i and iii and v are Perfect, which implies the overblowing of pure 5ths! (See Table 1.) The conclusion forced upon us by these few examples is that Hornbostel on his own evidence has destroyed the precarious assumption on which he founded his Cycle of Blown 5ths, while providing evidence of his integrity and sincerity. Even one single 3rd Harmonic overblown by himself as Perfect 5th would have sufficed. If Panpipes sometimes overblow pure Harmonics, a theory based on flat Harmonics, attributed solely to the evil effect of diameter —which nevertheless is an integral factor in every pipe and in every pipe note—can no longer hold its ground, it is once and for all discredited.

Acoustic theory does not allow the overblowing of impure Harmonics in cylindrical closed pipes: if such occur, they are due to infraction of the law, for which the faulty blowing of the piper is responsible. What,

[1] See *Proceedings of the Musical Association*, 62nd Session, 1935–6, p. 69.

It is not proposed, therefore, to refer the question of the purity of the harmonics of cylindrical stopped pipes to the written technicalities of acoustic law; the subject may be pursued from the text-books. My personal testimony may be added: in order to satisfy myself, I overblew perfect 5ths from my *Agariche* (Panpipes similar to those used by Hornbostel, but from N.W. Bolivia) on three occasions before competent witnesses, viz. an expert professional tuner, Mr. E. Anderson (twice), and Mr. A. H. Fox Strangways—the overblown 5ths were pronounced perfect—a little reluctantly by the latter, I fancied, owing to his long friendship with Hornbostel, but his sensitive musical ear would have winced at a flatness of less than *24 cents* even!

then, is the correct method of blowing the Panpipe ?[1] Clearly the one that ensures the overblowing of pure Harmonics, and which, moreover, produces a vibration frequency in the fundamental which is identical with the theoretical one computed by formula from the actual length and diameter of the pipe. The only method that fulfils these conditions is the following :

CORRECT AND FAULTY METHODS OF BLOWING PANPIPES

The Panpipes must be held vertically in front of the mouth—not obliquely—so that the lower lip rests against (not over) the edge, the upper lip compressed above the lower—neither encroaching over the edge. A narrow slit is thus left between the lips ; the breath-stream is directed across the diameter of the open end, and impinges on the sharp opposite edge, forming a node at that point for the sound-wave—as happens by analogy when a string is plucked.

By this means alone the full length of the tube comes into play with its true diameter, emphasized by the exciting breath-stream. This method of blowing is the same as that used on the concert flute.

The wrong method is to hold the Panpipe obliquely in front of the mouth and to blow *into* the pipe, as in the Oriental *nay*, so that the breath-stream is directed inwards at an angle, striking the inner wall at a variable distance below the edge of the aperture, and thus substituting a diagonal for a diameter, the result being a lengthening of the sound-wave by twice the amount of the difference between the two lengths, and a consequent lowering of the pitch. The note produced by this faulty method of blowing may vary by a semitone or more, according to the angle formed by the breath-stream when striking the inner wall of the pipe, and to the extent of the obturation of the opening of the pipe by the lips of the player. As both of these are variable and undefinable factors, it is obvious that the instrument, under such conditions, would be entirely useless for the building up of a musical system.[2]

There is, besides, a great difference experienced in the overblowing of Harmonics by these two methods. When blowing correctly across the open end, the pipe is held vertically in front of the mouth and the piper's larynx is in the normal position, equally favourable for the compression of the muscles of the glottis required to produce the overtones, and for the relaxation necessary to induce the fundamental tone of the pipe to speak.

In the faulty method of blowing into the pipe, the piper is at a great disadvantage : his chin sinks, the muscles controlling the glottis are thereby relaxed, whether he demands from his pipe a low fundamental or a high

[1] Something has already been said on the subject above.

[2] As an evidence of the variability of the pitch of notes obtained by this faulty method of blowing, attention may be drawn to Hornbostel's fn. 1 on p. 383 (Koch-Grünberg, *op. cit.*). He points out that the frequency of the fundamental of Pipe No. 1 in Table ii was determined at the 1st test as 420 v.p.s. ; and at a 2nd test as 409 ; he considers that according to calculation the mean 414·5 is the correct frequency !

The difference in the results of Tests i and ii amounts to a quarter-tone.

FIG. 67.—N.W. Brazilian Panpipes, Sets V.B. Nos. 6322/23, as tested and blown by E. M. von Hornbostel
N.B.—According to Hornbostel, all the 4ths are too sharp, being transpositions of the flat blown 5ths

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
v.fs. of fundamentals of Pipes	*420 414*	*481·6 480*	*560·5 559*	*651·3 648*	*374·4 378*	*439·5 439·5*	*516 510*	*598·5 592 594*	*699 690 684*	*397·4*	*461·6*	*538·5* Table II
Perfect 4ths theoretical	560 ↓	642	373·7 ↓	434·2	499·2	586	688	399	466	529·8	615·4	
Hornbostel's so-called sharp 4ths, from pipes	*560·5 III perfect 4th*	651·3 IV ♯ by 9·2 v.p.s.	*374·4* V *perfect 4th*	439·5 VI ♯ 4th by 5 v.p.s.	516 VII ♯ 4th by 17 v.p.s.	598·5 VIII ♯ 4th by 12 v.p.s.	699 IX ♯ 4th by 11 v.p.s.	397·4 X ♭ 4th by 1·6 v.p.s.	461·6 XI ♭ 4th by 4·4 v.p.s.	538·5 ♯ 4th by 8·7 v.p.s.	594·8 ♭ 4th by 10·6 v.p.s.	

COMPARISON OF THIRD HARMONICS (TABLE 1) REDUCED TO SAME OCTAVE
ACCORDING TO HORNBOSTEL'S THEORY THESE 5THS *MUST ALL BE FLAT*

Fundamentals	420 × 3/2 =	481·6 × 3/2 =	560·5 × 3/2 =	651·3 × 3/2 =	374·4 × 3/2 =	439·5 × 3/2 =	516 × 3/2 =	598·5 × 3/2 =	699 × 3/2 =			
Pure 5ths theoretical	630	361·2	420·3	488·4	561·6	659·2	387	448·8	524·2			
Flat 5ths overblown by Hornbostel	*622·5 ♭ by 7·5 v.p.s.*	*357·2 ♭ by 4 v.p.s.*	*414·3 ♭ by 6 v.p.s.*	*478·5 ♭ by 10 v.p.s.*	*553 ♭ by 8·6 v.p.s.*	*651·3 ♭ by 8 v.p.s.*	*376·5 ♭ by 11 v.p.s.*	*445·7 ♭ by 3 v.p.s.*	*517·5 ♭ by 6·7 v.p.s.*			

Comparison of 5th Harmonics Table reduced to same octave
All Flat according to Hornbostel

Fundamentals	420 × 5/4 =	481·6 × 5/4 =	560·5 × 5/4 =	651·3 × 5/4 =	374·4 × 5/4 =			The other pipes did not overblow readily the 5th Harmonics				
Pure major 3rds theoretical	525	602	700·6	407	468							
Hornbostel's 3rds (flat)	519 *♭ by 6 v.p.s.*	602 *Pure*	699 *Pure*	397·4 *♭ by 9·6 v.p.s.*	464 *♭ by 4 v.p.s.*							

Results of the Tests of the so-called sharp 4ths: 2 are perfect, therefore these 3rd Harmonics must have been blown pure; 3 are flat, therefore 3 must have been overblown sharp

Two of the 5th Harmonics were overblown pure by Hornbostel; all the 3rd Harmonics were actually blown flat by Hornbostel

v.fs. of Fundamentals, &c., left from Table 1, Hornbostel; v.fs. right, Hornbostel's, from a letter to Dr. Kunst, dated 1923

Roman, Nos. of the pipes as placed in the set

N.B.—Hornbostel's v.fs. are records of his own blowing of the pipes (Koch-Grünberg, *op. cit.*, Table 1).

FIG. 68.—Results of the comparisons of the Vibration Frequencies of Panpipes V.B. 6322/23 with the Ratios and the Vibration Frequencies of the Related Harmonia, supplied by K. S.

Fundamentals from Table 1, Koch-Gr., *op. cit.*; from Hornbostel's letter to Dr. Kunst, 1923, vibration frequencies according to theory, not practice

N.B.—The Pipes have been arranged in order of pitch. Orders Numbers in Roman

	IX*	V	I	VI	II	VII	III	VIII	IV	IX
Table 1. v.fs. (1909)	*174·7*	*187·4*	*210*	*219·7*	*240·8*	*258*	*280·2*	*299·2*	*325·6*	*349·5*
Hornbostel's letter (1923) v.fs.	*172·5*	*189·7*	*207*	*219·5*	*240*	*255*	*279·5*	*296·2*	*324·2*	*345*
Modal v.fs. (K. S.)	170·6	192	204·8	219·5	236·4	256	279·2	292·6	323·4	341·2
Ratios of Hypophrygian Species from Pipe IX or Hypodorian Species from Pipe V	18/18 Denominator Constant	16	15	14	13	12	11 22/36	21	19	18
		These results may be compared with those published by Hornbostel in *Akustisches Kriterium*, p. 613, for the Solomonen Inseln Brazilien, viz.								
		187·2	*414·5* 207·2	*219·5*	*240·8*	*258*	*280·5*	*299·2*	*325·6*	*349·5*

**N.B.*—Col. ix gives the vibration frequencies of the fundamentals which form the basis of the computations

When the pipes are grouped in order of pitch it is found that the v.f. of the notes of the Panpipes approximate more or less closely to those of the Hypophrygian Harmonia, the fundamental of which, however, is lower by 4 v.p.s. than that of the Panpipes. The results, although produced by Hornbostel's wrong blowing provide a clue to the Scale of Panpipes V.B. Nos. 6322/23, a pair tuned in unison, derived from Uanána on the R. Caiary-Uaupes, Brazil (see in this connexion my comments).

overtone, which it is an effort to produce in such a position, and which in consequence is frequently flattened for lack of sufficient compression.

Harmonics, which result from the spontaneous aliquot divisions of the air column into segments equal in length, and vibrating at the same frequency, are constituents of the fundamental tone, and are always in perfect tune. The constituent Perfect 5th heard simultaneously with the fundamental acts as a guide to the ear in isolating the 3rd Harmonic by overblowing. Distortion of the Harmonics in Panpipes is mainly due to the encroaching of the lips over the bore of the pipe, held obliquely; and to inadequate compression of the breath. A fixed idea of the inevitable flatness of overtones from Panpipes is, no doubt, a contributory cause, since the vocal organs endeavour to produce what the mind or imagination desires and suggests. The theory of the orientation by sharp 4ths, as reversed flat Blown 5ths has now completely broken down, for it relied for acceptance upon a travesty of the acoustic law.

It is, as a theory, devoid of any sound formative principle in scale-making, for the reason that the sharp 4ths are not calculable; they are due to a wrong method of blowing, which inevitably produces an unaccountable and uncontrollable variety in the extent of the flatness of the Blown 5ths—or by implication of the sharpness of the 4ths with which they stand in octave relation.

Such is the theory. What are the facts?

In Table 1 (*op. cit.*, p. 380) we are presented with the vibration frequencies of a duplicate set of Panpipes Nos. 6322/23, tuned in unison. The order of the pipes exhibits a double chain of rising 4ths as fundamentals; the one extending through pipes bearing odd numbers, the second through the evens. Under each fundamental is given the vibration frequency of its overblown 5th, which corresponds—with a variable margin of error—with the frequency of its lower conjunct 4th on the next lower odd or even pipe number, Pipe iii with Pipe i, and so on.

ANALYSIS OF THE RESULTS GIVEN IN FIGS. 67 AND 68

But those so-called sharp 4ths[1] displayed by the vibration frequency of the Brazilian Panpipes—to which the theory of the Cycle of Blown 5ths owes its conception—prove on inspection to be mixed.

It is astonishing to find this the case even in the three conjunct 4ths cited by Hornbostel in illustration of his theory (see footnote 4, p. 315, and *op. cit.*, pp. 381 and 382, and Table 1, p. 380) and drawn from the Panpipe Set V.B. 6322/23.

The examination of all the pipes of this set gives the following curious result:

[1] The sharp 4th observed by Hornbostel in Panpipe scales, which appear to be a confirmation of his theory of Blown flat 5ths, may instead be referred to characteristic modal intervals: either to the raised 4th of the Dorian (ratio 11/8); to the sharpened 4th of the Hypophrygian (ratio 18/13); or to the Tritonic form of the primitive Hypolydian of ratio 10/7 (see Table of values of these in Chap. ix, Fig. 85).

P denotes pure or perfect.

H denotes Hornbostel's vibration frequency for the sharp 4th.

Figures in brackets denote the differences in vibration frequency flat or sharp of the 4ths.

Roman numbers denote the order of the pipes.

DEDUCTIONS FROM HORNBOSTEL'S TABLES I AND II

Panpipe V.B. No. 6322/23

The 4ths which should all be sharp

Pure	SHARP	v.p.s. difference	FLAT	v.p.s. difference
i–iii	ii–iv	9·2	*viii–x	1·6*
iii–v	iv–vi	5	ix–xi	4·4
(viii–x ?)	v–vii	17 = *112°*	xi–xiii	10·6
	vi–viii	12		
	vii–ix	11		
	x–xii	8·7		

N.B.—Pipes xii and xiii are given on Table ii (Hornb.)

– This 4th should doubtless be reckoned as pure: an amount less than 2 v.p.s. might well be due to blowing or to its registration. The difference in these flat 4ths must be doubled in relation to a constant increment of sharpness in the 4ths.

A similar examination of Panpipes No. 6324/25, given by Hornbostel on Tables iii, iv, v and vii,[1] likewise reveals the insecure foundation of the theory, for 9 of the 4ths are flat: 5 of them by more than 20 v.p.s. (maximum = 74); the other 9 are sharp: 5 of them by more than 10 v.p.s. (maximum = 63). Both pure and flat 4ths testify that the overblown 5ths, from which they are said to derive, could not have been flat. In fact, the mixture of 4ths in the Panpipe scales revealed by analysis as pure, sharp, and flat, is entirely subversive of Hornbostel's theory.

On the other hand, it is suggestive of the Modal System of the Harmoniai, in which *pure 4ths* on the Tonic are found in the Hypodorian, Phrygian and Hypolydian (form ii); *flat 4ths* on the Tonic occur in the Mixolydian and Lydian and *sharp 4ths* on the Tonic in the Dorian, Hypophrygian and Hypolydian (form i) with Tritone. All of these Harmoniai would probably have been found represented in the scales of the Brazilian Panpipes, if correctly blown, for the reasons given in detail in the following pages. The series of interlaced 4ths in Panpipes No. 6322/23—a pair consisting of 17 pipes each, tuned in unison approximately, produces, when the pipes are taken in order, a sequence of split 4ths resembling the Sléndro of the Javanese in formation. This grouping of the pipes, however, merely represents an arbitrary selection from the musical elements of the underlying system, which is revealed when the fundamentals of the pipes of a set are taken in order of pitch; played in this order, the nature of the background is disclosed as a scale diatonic in character.

Since Hornbostel did not hear the Brazilians playing on these pipes, he

[1] See Tables vi and vii, *op. cit.*, 1910, and *Akustisches Kriterium*, 1919–20; Table ii, Salomonen–Brasilien (Panpfeifen). Communicated in correspondence by Hornbostel to Dr. Kunst, Oct., 1923.

could not explain—but only conjecture—the reason for the grouping presented in Fig. 67.

There exist three different readings of the frequencies of this pair of Panpipes in which some of the values differ by as much as 6 v.p.s., and in one case by 11 v.p.s. A test of the vibration frequencies of the overblown 5ths shows that they are all flat by amounts varying from 3 to 10. The overblown 3rds (5ths partials), given for 4 pipes only, reveal two pure major 3rds from Pipes ii and iii, the other two being duly flat.

There is obviously no advantage to be derived from a further inquiry into this sophisticated and discredited theory. The scales of the pipes, however—even if the vibration frequencies cannot be relied upon as precise data—are of great interest. When these frequencies are placed in order of pitch—instead of in the order of the pipes—they reveal results approximating to those of the modal sequences of one or other of the Harmoniai.

EVEN THE RESULTS OBTAINED BY WRONG METHODS OF BLOWING SUGGEST THE HARMONIA AS ORIGIN

It must here be definitely stated that in the frequencies of the notes of the Harmonia, differences exhibited between the vibration frequencies of two consecutive diatonic ratios may be as much as 51 v.p.s. in the octave beginning on $C = 512$ v.p.s. ($= 25$ on $C = 256$) for C 11 to D 10. Therefore, a difference of a few v.p.s. does not invalidate the evidence of the ratio in question, provided the number does not amount to a quarter of the increment. My practice is to write under the ratio the index frequency of the octave of C within which the note occurs, e.g. B 12/256 to C 11/512 or thus $\frac{B12}{256}$ (B being the limit of the index 256).

The margin of error in identifying the modal frequencies taken from records is seen to be a wide one, which varies with the ratio and the vibration frequency of the fundamental, and does not provide equal increments of vibration.

The fact that most of the sequences are those of the Harmonia as octave unit, and none of them duplicated, may probably be taken to imply the flute as origin of the scale. It was a foregone conclusion that the scales forming the basis of Folk Music in North-west Brazil, in Bolivia and in Peru, should prove to be those of the traditional Incasic musical system—the system of the Harmoniai.

That is to say that quite independently of the arrangement of individual pipes in the sets of Panpipes by the moderns, the underlying musical elements revealed by the pipes taken in order of pitch is suggestive of the Harmoniai, or of their species. It is not the mere correspondence or approximation in vibration frequencies between certain notes that is an implication of the modality of the Harmonia. It is the preservation of continuity in the progression of ratios, from step to step, in the modal sequence which is the decisive factor.

It is obvious that owing to Hornbostel's faulty blowing of the Brazilian

Panpipes, which was productive of variable and unpredictable margins of error, the results presented by him in his tables cannot be accepted as evidence of the survival or rebirth of the Harmonia, since we depend at present solely upon these results for our estimate of the scales. But, nevertheless, when it is found that vibration frequencies follow the order of a modal sequence, this at once suggests the Harmonia as origin of the scale to which the Panpipes were tuned at some undetermined time, probably from a flute having equidistant fingerholes.

Although the results of all the sets of Panpipes recorded by Hornbostel have been carefully analysed, and identification traced as far as possible with

FIG. 69.—The Antique Peruvian Clay Flute, San Ramon (No. V.A. 15901), from the Bolivar Collection. The Flute has 5 Fingerholes in front and one at the back.

The note of Exit and Fingerholes in order of pitch (taken from Table vii, p. 390, Koch-Grünberg, *op. cit.*)

v.fs. from Hornbostel's blowing of flute	*402*	*444·5*	*494* *488*	*(553)*	*614*	*707*	*(785)*
v.f. of the Dorian Harmonia calculated on the same fundamental, M.D. 22 and 44	402	442	491·3	552·7	609·9 (631·7)	707·5 (728·6)	804
Modal Ratios of Dorian Harmonia by K. S. (Denominator constant)	A11/11 A22/22	10 20	9 18	8 16	7 29/44 (14)	6 25/44 (12)	11
	165°	*182°*	*204°*	*171°*	29/25 = 7/6 = *257°*	* = *267°*	
v.fs. of corresponding notes of Panpipes recorded by Hornbostel's (No. 6324/25)	*402*	*442*	*487·8*	*545·7*	*598·4*	*704·3*	

$$* \frac{29}{25} \times \frac{6}{7} = \frac{174}{175}.$$

N.B.—The 1st tetrachord of the Dorian Harmonia is in close agreement with that of the Ancient Peruvian flute; the 2nd Tetrachord is distorted; this may be due to (*a*) insufficient compression of breath to produce the rise in pitch of the septimal 3rd; (*b*) to the influence of diameter on the reactions of the air column (see Records of the Kenas of K. S.), or (*c*) to an irregular increment of distance between the fingerholes.

(See Koch-Grünberg, op. cit., Table vii, p. 390)

the vibration frequencies and ratios of the Harmoniai, it is not considered that any useful purpose can be served by reproducing here more than one example (see Figs. 67 and 68 above). For the remaining sets, we must wait until tests have been made by other investigators and duly recorded by phonograph.

Since the flute is not open to the same objection as the Panpipes, the ancient clay flute recorded by Hornbostel (K.-G., *op. cit.*, Table vii, p. 389) as V.A. No. 15901, San Ramon, may be cited: it has five fingerholes in front and one at the back, a little above the 5th hole. The vibration frequencies of the notes produced from exit and fingerholes form intervals of the following ratios, which will be recognized as those of the Dorian Harmonia of M.D. 11 : Hornbostel compares the v.fs. of this flute with those of Panpipes V.B. Nos. 6324/25.

THE ANTIQUE PERUVIAN FLUTE 'SAN RAMON' IS IN A DIFFERENT CATEGORY FROM THE PANPIPES

RATIOS OF THE SCALES OF THE ANCIENT CLAY FLUTE OF PERU, SAN RAMON, V.A. NO. 15901 (SEE FIG. 69)

	11/11		10/11		9/11		8/11		*32/29	*29/25
Cents		*165°*		*182°*		*204°*		*171°*		
									= 7/11	6/11
										29/25 = 7/6
										= *257°*

***–* 32/29 evidently intended for 32/28 = 8/7 and 29/25 = 7/6 $\left(\frac{29}{25} \times \frac{6}{7} = \frac{174}{175}\right)$; the flattening of the last two notes may have been due to the blowing which requires increased compression as the scale rises. For v.f.s see Fig. 69.**

This early form of the Dorian Harmonia from M.D. 11 is the scale given by the Elgin pipe at the British Museum (Ancient Greek, *c.* 500 B.C.). The same scale is embodied in two Kenas (the Incasic notched flute), modern copies of ancient traditional Peruvian flutes (see Chap. x, Records).

If further evidence be required of the survival or rebirth of the Harmonia among the Incas, it is to be found in abundance in reproductions from photographs of their flutes in the Portfolio of Plates of *La Musique des Incas*.[1] The majority of these flutes have fingerholes bored at equal distances which must incontrovertibly produce the sequences of the Harmoniai. The authors have unfortunately not given phonographic records of scales or of Folk Music, nor have they attempted to indicate shades of intonation : they have merely assumed the pentatonic scale. There are also unmistakable signs of flute origin in the ratios of scales of Panpipes and marimbas given in Hornbostel's tables. These implications, as described and explained in detail (in Chaps. vi. and vii), are due to the inner reactions of the air column in respect of the Increment of Distance and of the allowances due to diameter. (See Chap. x, Records of Sensa Flutes A, B and C.) As the inner reactions of modal flutes having equidistant fingerholes form an entirely new proposition—not yet scientifically investigated—these reactions should prove of some interest and lead to much practical work on the subject.

From the evidence provided by the records of Xylophones from Burma, Siam and Africa, added by Hornbostel (see Table vii, and from *Akust. Kriterium*, Tables i and ii), it is found that all the seven Harmoniai are represented. In some of these the genesis to which the vibration frequencies are referred is from $C = 128$ v.p.s., and in others from $F = 176$ v.p.s. (the 11th Harmonic of C).

* * *

[1] By R. et M. D'Harcourt (Paris, 1925). See plates Nos. xxiii, xxiv, xxvi, xxvii.

DR. MANFRED BUKOFZER'S STRICTURES ON THE BLASQUINTENZIRKEL

An explicit rejection of Hornbostel's Blasquintenzirkel, on different grounds, has been published recently by Dr. Manfred Bukofzer [1] of Basel, working from the standpoint of Physics, as an excursion from the domain of Musicology. His paper provides an eminently lucid and reasoned exposition of his view, viz. that mouth-blowing on Panpipes, as he understands it, cannot produce from pipes of different dimensions played by the same or by other performers, a constant flattening, or even data susceptible of providing a working average. He admits in theory the inviolable purity of Harmonics in the reactions of the fundamental (so far we are in agreement). Yet, as far as his experience goes, mouth-blowing is ever an unmitigated culprit. Results of the working of mechanically blown pipes are also given.

A glance at his illustration, of what he assumes to be the *correct* blowing, shows the mouth of the player well over the edge of the pipe, and directing the breath-stream diagonally right into the pipe—thus failing to find any sharp edge to define the nodal point. The result of this method of blowing, which is quite correct when used in playing the *nay*, but not for the Panpipes, is to increase the lengthening influence of the allowance in respect of diameter, by the amount of the difference between diagonal and diameter taken twice, with the result of flattening the pitch.

Another diagram on the same page (655) shows a mechanically blown pipe in which the air-stream is directed across the end of the tube, and this (considered by me correct) is termed 'the false method of blowing'. In short, Dr. Bukofzer considers as wrong that blowing by mouth which has been indicated in this chapter as the only correct one, because it produces a frequency of the fundamental note which corresponds exactly with the one warranted by the dimensions of the pipe, inclusive of diameter. Moreover, it ensures the production at all times of a fundamental note of precisely the same pitch, and still more conclusively, allows the overblowing of Harmonics in perfect tune. Pure overtones the method allows, but does not guarantee, unless there is besides an adequate compression of breath, as already explained above. After all, this is likewise the method of blowing in use on the concert flute, and the one used by primitives, ancient and modern, in playing the Panpipes.

Although Dr. Bukofzer's conclusions on the correct method of blowing Panpipes are highly debatable and are disproved when submitted to practical tests by the comparison of fundamentals and overtones produced by this method, with the vibration frequencies computed by formula based upon the dimensions of individual pipes, yet with this reservation, the article should be studied, for it contains valuable suggestions, results of tests, &c., all stated lucidly and without unnecessary technicalities.

For the determination of pitch, an ingenious machine, the electrical

[1] 'Praezisionsmessungen an primitiven Musikinstrumenten', in *Zts. f. Physik*; Karl Scheel (Springer, Berlin, 1936), Band 99, Heft 9 and 10, pp. 643–65, with 6 illus.).

Tone generator [1] is described (p. 645, *op. cit.*), by which means the actual magnitude of intervals in *cents* is indicated by curves with a maximum error of 0·25/100 v.p.s., so that a tone of 800 v.p.s. might have a reading of 802 v.p.s.

It is now proposed to give an example of the correspondence between the pitch of a correctly blown Panpipe, and the vibration frequency estimated by formula.

EVIDENCE FROM AN AGARICHE FROM BOLIVIA: ALL OVERBLOWN 5THS **PURE**

The test was carried out (by the present writer) on a Panpipe 'Agariche', brought from North-west Bolivia for me by Miss Anita Berry.

My Agariche consists of seven cylindrical reed pipes, closed at one end by a natural knot (severed at the base of the knot); at the other open end the walls are thinned to about 1 mm., thus forming a sharp edge to facilitate blowing and purity of intonation. The dimensions of each of these pipes and their vibration frequencies are given below.

FIG. 70.—Tests carried out on an Agariche (Panpipe), showing Correspondence of Pitch with Formula: Difference in Vibration Frequency due to (A) Internal Measurement; (B) External. Δ = diameter

PIPE NO. 1

(A)		(B)
Int. L. = ·421 × 4 = 1·684		Ext. L. = ·428 × 4 = m. 1·712
Δ = ·014 × 2 = ·028		
1·712		
$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}712}$ = 198·5 v.p.s.	⟵⟶	$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}712}$ = 198·6 v.p.s.
Correspondence		*exact*

Nearest note on tuned piano = *G* 14 of 201 v.p.s.

PIPE NO. 2

Int. L. = ·333 × 4 = 1·332		Ext. L. = ·340 × 4 = 1·360
Δ = ·0135 × 2 = ·027		
1·359		
$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}359}$ = 250·1 v.p.s.	⟵⟶	$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}360}$ = 250 v.p.s.
Correspondence		*exact*

Nearest notes of Dorian Sequence = *B* 23 = 245 and *C* 11 256 v.p.s.

PIPE NO. 3

Int. L. = ·281 × 4 = 1·124		Ext. L. = ·288 × 4 = m. 1·152
Δ = ·012 × 2 = ·024		
m. 1·148		
$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}148}$ = 296 v.p.s.	⟵⟶	$\frac{340\text{ m./sec.}}{\text{m. }1{\cdot}152}$ = 295·1 v.p.s.
Difference		*1 v.p.s.*

Nearest note of Dorian Sequence = *E* 19 of 296 v.p.s.

[1] Described by Dr. Bukofzer, *op. cit.*, p. 645, referred to Werner Lehmann, *Helv. Phys. Acta*, 6, pp. 18 sqq. (1933).

PIPE NO. 4

Int. L. = ·240 × 4 = ·960 Ext. L. = ·245 × 4 = ·980
Δ = ·0115 × 2 = ·023

m. ·983

$\frac{340 \text{ m./sec.}}{\cdot 933} = 345{\cdot}8$ v.p.s. ⟷ $\frac{340 \text{ m./sec.}}{\cdot 980} = 347$ v.p.s.

Difference *1 v.p.s.*

Nearest note of Dorian Sequence = *F* 16 of 352 v.p.s.

PIPE NO. 5

Int. L. = ·190 × 4 = ·760 Ext. L. = ·196 × 4 = ·784
Δ = ·011 × 2 = ·022

·782

$\frac{340 \text{ m./sec.}}{\cdot 782} = 434{\cdot}7$ v.p.s. ⟷ $\frac{340 \text{ m./sec.}}{\cdot 784} = 433{\cdot}6$ v.p.s.

Difference *1 v.p.s.*

Nearest note of Dorian Sequence = *A* 13 of 433·2 v.p.s.

PIPE NO. 6

Int. L. = ·159 × 4 = ·636 Ext. L. = ·164 × 4 = m. ·656
Δ = ·0095 × 2 = ·019

·655

$\frac{340 \text{ m./sec.}}{\text{m. } \cdot 655} = 519$ v.p.s. ⟷ $\frac{340 \text{ m./sec.}}{\cdot 656} = 518{\cdot}2$ v.p.s.

Difference *4/5 of 1 v.p.s.*

Nearest note of Dorian Sequence = *C* 11 of 512 v.p.s.

PIPE NO. 7

Int. L. = ·126 × 4 = ·504 Ext. L. = ·130 × 4 = ·520
Δ = ·008 × 2 = ·016

·520

$\frac{340 \text{ m./sec.}}{\text{m. } \cdot 520} = 653{\cdot}8$ v.p.s. ⟷ $\frac{340 \text{ m./sec.}}{\cdot 520} = 653{\cdot}8$ v.p.s.
326·9(327)

Correspondence *exact*

Nearest note of Dorian Sequence = *F* 17 of 331·2 v.p.s.

The frequencies were tested by monochord and by piano tuned to Dorian Harmonia of M.D. 22 by an expert tuner from tuning-forks = *C* 256 v.p.s. and *F* = 352 v.p.s. and the modal monochord. The close approximation of the pitch of the pipes to the vibration frequency of the extended Dorian Harmonia will be noticed. The pipes rise in series by 3rds of various magnitudes; my Agariche is one of a pair intended to divide the extended scale between two pipers. To produce a sequence in order of pitch the pipes of the series are played alternately. The vibration frequencies denote the corresponding steps in the Dorian scale on the tuned piano, and the names of the notes with accompanying numerals, denote by the numerator the ratio of that note in the Dorian Harmonia, and by the denominator the octave of *C* within which it occurs.

IS THERE IN CLOSED PIPES A NATURAL BALANCE BETWEEN THE INTERIOR LENGTH + DIAMETER AND THE EXTERIOR LENGTH, OMITTING DIAMETER?

It will be seen that in this set of reed Panpipes, the thickness of the knot is equal to half the diameter. This result must not be regarded as establishing a theory of natural balance, but merely as a suggestion for further experiments.

The seven pipes actually speak at the frequencies worked out above. The knots in my Agariche have been closely severed, showing a depression in the centre of the closed end: in measurements of the external length of a pipe, this depression must be taken as the limit.

The vibration frequencies furnished by theory and practice in this set are identical: the fundamentals are either identical with the alternate notes of the Dorian Harmonia on $C = 256$ v.p.s., or else they approximate very closely to them.

The Harmonics overblown from the pipes are all pure: the 5th of pipe No. 1 gives the note of Pipe 3 slightly sharpened; the Harmonic of No. 2 corresponds exactly with *G* 15/512, which may have been a fundamental in the twin set.

The Harmonic of Pipe No. 3 (*E* 19/256) is sharper than *A* 13—as indeed it should be—since an interval of $19/13 = 656°$, whereas a true 5th $= 702°$. (This overtone has been tested before experts and pronounced *dead* true.)

The comparison of the results of computing the pitch of pipes from two different formulae, as given above for A and B, based respectively on internal and external length of the pipe:

(A)
$$\frac{340 \text{ m./sec.}}{4\left(\text{Int. L.} + \frac{\Delta}{2}\right)} = \text{pitch of pipe.}$$

(B) on the external length and no diameter, thus:
$$\frac{340 \text{ m./sec.}}{\text{Ext. L.} \times 4} = \text{pitch of pipe.}$$

If the suggestion, *re* internal and external length, should be found confirmed by the results of many tests, carried out with reference also to botanical and ethnographical considerations, the result would tend to establish on a sound practical basis the origin of scales derived from Cycles of 4ths and 5ths through the cutting of the reeds or Panpipes, each half as long as the last. The question arose out of Hornbostel's theory of blown 5ths: how to account for the inception of the idea in accord with proven steps in the evolution of Music? Hornbostel allows the proportional cutting of the reeds, but postulates a final tuning by ear to the flat overtones. If the cutting of the reeds proportionally as 2 : 3 or 3 : 4 was carried out instinctively, no tuning is necessarily implied as origin of the scales. The scales produced by graded pipes are accepted and assimilated by primitives.

But, on the contrary, the making of sets of pipes by grading the lengths, and then tuning the pipes by ear to a fine degree of flatness, implies preconceived notions of scales and sound-patterns, a sophistication that leaves origins far behind.

One would need to postulate many Hornbostels among the primitive folk of the nations, in order to produce prototypes of Panpipes, and to assume models handed down with the tradition through the ages, and passing from Ancient China, India, Sumer and Akkad [1] to other lands and races.

At what point, one may then inquire, did the consciousness of the significance of length, as a determinant of the sound issuing from the pipe, suggest to the pipe-maker the necessity for taking the internal instead of external measurement of length?

Unless Nature's compensation, suggested above, should prove to be a fact, the alternative of exterior versus interior length would always have to be clearly defined in estimating pitch, for the difference between the two measurements, which must be multiplied by four, is responsible for a considerable alteration in pitch.

THE BLOWING OF THE PANPIPES HAS PROVED TO BE THE UNDOING OF THE THEORY OF BLOWN 5THS

The only method of blowing Panpipes which could produce any semblance of reality has proved the undoing of the theory. Hornbostel's record of his own blowing of the Brazilian pipes exhibits some pure overtones, some sharp and some duly flat, where all should, according to his theory, be flat. On the other hand, if perfect Blown 5ths be admitted, the theory is nothing but the old, old one of the Cycle of 5ths, also known as Pythagorean and ditonal, in its resultant heptatonic scale.

Even so, there still remains this inevitable query, which must be satisfactorily answered: how is it, then, that the vibration frequencies of Hornbostel's Blown 5ths approximate in general fairly closely to those, for instance, of the Javanese and Siamese records of the Music of the Folk?

To this weighty question there is fortunately an answer forthcoming. It is this. The cycle of 23 flat 5ths, considered as an ideal theory, comporting a constant flatness,[2] empirically fixed at *24 cents*, presents when re-arranged in order of pitch certain revealing features. The results of the flatness are found to operate incrementally, growing greater as pitch rises through the octave: the flatness of the increments is cumulative, starting roughly with 6 v.p.s. then rising to 7, 8, 10, 11, in the same octave.

[1] On going to press with this work it was just possible to glance at *The Music of the Sumerians, Babylonians and Assyrians*, by Canon Francis W. Galpin, published by the Camb. Univ. Press. My comments upon some of the conjectural points within my province will be found at the close of Chap. ix.

[2] These theoretical values are built up on a basis of constant flatness, empirically fixed at *24 cents*, whereas the records of actual blowing of Panpipes by Hornbostel show differences of from 1 to 21 v.p.s. (not *cents*). If this flatness (due to false overblown 5ths) were really constant instead of being a mere assumption, some sort of parallel might be established with the frequencies of the Harmonic Series, but since the basis of the theory has been discredited this would serve no useful purpose.

This progression presents a certain analogy with the modal principle of genesis. If now we borrow the Phrygian Enharmonic genesis of M.D. 48, we shall get 24 intervals to the octave, none equal; but rising proportionally in arithmetical progression, the intervals become wider as pitch rises: the opposite, therefore, of what happens in the ascending Harmonic Series.

We now compare the vibration frequencies of the Phrygian—or of any other of the Harmoniai in Enharmonic genesis computed from the common fundamental selected by Hornbostel—with the theoretical values of the Blown 5ths: there are, indeed, a few coincidences here and there which are striking, but they do not occur on parallel degrees of a diatonic sequence. There is really no mystery in these accordances, they are the mere logical results of an excursion into minute Enharmonic subdivisions of intervals in which difference of frequency is minimized.

UNEQUIVOCAL REJECTION OF THE THEORY OF THE BLASQUINTENZIRKEL (BLOWN 5THS)

Finally, on gathering up the threads of this lengthy discussion, sad necessity compels us to make an unequivocal rejection on all points of this brilliant scholar's hypothetical theory of the Cycle of Blown 5ths, on the following grounds:

(1) An infraction of acoustic law is involved in the basis and prime motivation of the theory.

(2) The inception of the ingenious idea is due to a wrong method of blowing Panpipes, while testing them. It is a well-known fact that a fixed idea is very potent in its influence on the interpretation and reproduction of sound. So that in all good faith Hornbostel believed that the flat 5ths he produced on the pipes were due to some obscure acoustic principle inherent in the pipes themselves.

(3) Since it has been proved from several of Hornbostel's pitch values in vibration frequencies, that pure Harmonics can be and were produced by him on some occasions, these data, had they been observed by him, would have effectually ruled out his erroneous interpretation of the acoustic principles involved.

In spite of our rejection of the theory itself, there remains a genuine appreciation of the wealth of data which Musicology owes to the skilful labours which Erich M. von Hornbostel left as a legacy to other workers in the same field.

Where the scales of Brazilian, Bolivian and Peruvian Panpipes of the present day do not bear unimpeachable evidence of origin from the cycles of pure 4ths or 5ths—i.e. built up of tones and semitones as units—there seems to be no reasonable doubt that they have been tuned, like my specimen Agariche, to the scales of the *Kena* (flute), which are those of the pure octave Harmonia, as understood in this work. It is not too much to affirm that the system of the Harmonia was the common musical property of the whole of the Ancient East.

A list of identified Harmoniai is available and will be found at the close of the next chapter.

THE CONTRIBUTION OF DR. JAAP KUNST FROM THE MUSIC OF JAVA AND BALI

Dr. Jaap Kunst, for some years resident in an official musical capacity in Batavia, is justly recognized as the foremost authority on the Music of Indonesia, and more especially of Java and Bali. He had the finest opportunities for studying native music in all its activities and developments. His mental and artistic equipment as musicologist is exceptional, and he possesses in his wife a most efficient colleague. His harvest, therefore, published in several fine volumes, is a mine of wealth to which he is still making contributions. Some of these later unpublished data have, with great generosity, been placed at my disposal for unrestricted use in this work.

Dr. Kunst's records of the vibration frequencies and values of intervals in *cents*, of the Pélog (heptatonic) and Sléndro (pentatonic) scales of Indonesia, constitute evidence of an entirely original nature, derived from the actual practice of music by the natives in their gamelans (orchestras), of which there are in Java alone more than one thousand.

A long friendship with E. M. von Hornbostel and the discovery of the close approximation of these results, independently obtained, with those of the Blasquintenzirkel (theoretical assumptions), communicated in friendly correspondence during their protracted search for a common basis for primitive scales in general, led to the provisional adoption by Dr. Kunst of the Cycle of Blown 5ths as a welcome solution. Whether this clue will continue to hold sway with him remains to be seen. The choice lies between the Blown 5ths and the Harmonia of Hellas and of the Ancient East—in Survival or Rebirth—which are also the apanage of primitive musicians all the world over.

Had the Chinese standard proved all that was hoped from it, at 366 v.p.s. as fundamental, many arduous calculations would have been avoided, for nearly every set of records had its own basis, which in most cases differed again from the 366 v.p.s. of the Chinese Standard adopted by Hornbostel. For the purposes of comparison, therefore, the Harmoniai had to be transposed to that basis as well as to $C = 256$ viz. to $F = 176$ v.p.s., $G = 192$ v.p.s. and to many others used by A. J. Ellis, Karl Stumpf, O. Abrahams and J. P. N. Land.

After an examination of the data collected and classified by Dr. Kunst, it must be allowed that, in the absence of any knowledge of the modal system of the Harmonia, the approximation—frequently very close—of the vibration frequencies of Javanese music to those of the Cycle of Blown 5ths perhaps justified Dr. Kunst's Eureka.

But the data provided by him reveal overwhelming evidence of the Harmonia as origin of the musical system of Java, and also of Indonesia generally. The influence of the flute in the development of the Javanese scales of both Pélog and Sléndro is unmistakable and irrefutable; the equidistant fingerholes on the flutes constitute positive evidence of modality, confirmed by the ratios of the vibration frequencies recorded. There are but few records of solo performances of flute music; [1] although the pure

[1] Dr. Kunst supplies the following reference to a solo played on a Balinese flute, *Beka* (N.B. 15629). This record is unfortunately no longer obtainable.

FIG. 71.—**Vibration Frequencies of the Tonic of each Species in each Harmonia taken on** C = 128 ; F = 176 ; G = 192 v.p.s.

v.f. of the Tonic of each Species in each Harmonia taken on C = 128, F = 176, *G* = *192* v.p.s. as in Table XV.

	Hypodorian	Mixolydian	Lydian	Phrygian	Dorian	Ho. Lydian	Ho. Phrygian
Hypodorian **C** 32/32 F *64/64* *G*	**32 = 128** 32 = 176 *32 = 192*	**28 = 146·3** 28 = 201 *28 = 219·4*	**26 = 157·5** 26 = 216·6 *26 = 236·2*	**24 = 170·6** 24 = 234·6 *24 = 256*	**22 = 182** 22 = 256 *22 = 279·2*	**20 = 204** 20 = 281·6 *20 = 307·2*	**18 = 227·55** 18 = 312·8 *18 = 341·2*
Mixolydian **C** 28/28 F *56/56* *G*	**32 = 224** 32 = 154 *32 = 168*	**28 = 128** 28 = 178 *28 = 192*	**26 = 137·8** 26 = 189·5 *26 = 207·6*	**24 = 149·3** 24 = 205·3 *24 = 224*	**22 = 162·9** 22 = 224 *22 = 244·3*	**20 = 179·2** 20 = 246·4 *20 = 268·8*	**18 = 199·1** 18 = 273·6 *18 = 298·6*
Lydian **C** 26/26 F *52/52* *G*	**32 = 208** 32 = 286 *32 = 156*	**28 = 237·7** 28 = 326·8 *28 = 178·2*	**26 = 256** 26 = 352 *26 = 192*	**24 = 277·2** 24 = 381·2 *24 = 208*	**22 = 302·5** 22 = 416 *22 = 226·8*	**20 = {332·8, 166·4}** 20 = 457·6 *20 = 249·6*	**18 = 368·6** 18 = 508 *18 = 277·2*
Phrygian **C** 24/24 F *48/48* *G*	**32 = 192** 32 = 132 *32 = 144*	**28 = 219·4** 28 = 150·8 *28 = 164·5*	**26 = 236·4** 26 = 162·4 *26 = 177·2*	**24 = 256** 24 = 176 *24 = 192*	**22 = 279·2** 22 = 192 *22 = 209·4*	**20 = 307·2** 20 = 211·2 *20 = 230·4*	**18 = 341·3** 18 = 234·6 *18 = 256*
Dorian **C** 22/22 F *44/44* *G*	**32 = 176** 32 = 121 *32 = 132*	**28 = 201·1** 28 = 138·3 *28 = 150·8*	**26 = 216·6** 26 = 148·9 *26 = 162·4*	**24 = 234·6** 24 = 161·3 *24 = 176*	**22 = 256** 22 = 176 *22 = 192*	**20 = 140·8** 20 = 193·6 *20 = 211·2*	**18 = 312·8** 18 = 215·1 *18 = 234·6*
Ho. Lydian **C** 20/20 F *40/40* *G*	**32 = 160** 32 = 220 *32 = 120*	**28 = 182·8** 28 = 251·4 *28 = 137*	**26 = 196·9** 26 = 270·6 *26 = 147·7*	**24 = 213·3** 24 = 293·2 *24 = 160*	**22 = 232·7** 22 = 320 *22 = 174·5*	**20 = 256** 20 = 352 *20 = 192*	**18 = 284** 18 = 390·8 *18 = 213·2*
Ho. Phrygian **C** 18/18 on F *36/36* *G*	**32 = 144** 32 = 198 *32 = 108*	**28 = 164·6** 28 = 226·2 *28 = 123·4*	**26 = 177·2** 26 = 243·6 *26 = 132·9*	**24 = 192** 24 = 264 *24 = 144*	**22 = 209·5** 22 = 288 *22 = 157·1*	**20 = 230·4** 20 = 316·8 *20 = 172·8*	**18 = 256** 18 = 352 *18 = 192*

FIG. 72.—Identification of the Harmoniai in the Ancient Pélog (Heptatonic) Scale of Java and Bali. The Proto-Pélog and Pélog in transition, from the Records (unpublished) of Dr. Jaap Kunst *by kind permission*

Provenance	Kunst's Tables	v.f. of Kunst records in italics, arranged in pitch sequence ; v.f. and ratios of Harmoniai by K. S. in roman								
Proto-Pélog : Kjahi Bermårå (1) Kraton Jogjia	*No. i A unpublished* (U)	*v.f. Kunst*	*193·7*	*212·5*	*225*	*248*	*290*	*308*	*332·5*	*387·4*
		v.f. K. S.	193·7	211·3	221·4	244·7	290·5	309·9	332	387·4
		Ratios K. S.	24/24	24/22	24/21	24/19	24/16	24/15	24/14	24/12
		These are the ratios of the Phrygian Harmonia of M.D. 24								
Proto-Pélog : Kjahi Pengasih (2) Kraton Sålå	*No. i B* (U)	*v.f. Kunst*	*196*	*210·5*	*225*	*250*	*286*	*308*	*335·5*	*392*
		v.f., K. S.	196	209	222·9	250·9	285	313·6	330·1	392
		Ratios, K. S.	16/16	16/15	16/14	(16) 32/25 (261) (16/12)	16/11	16/10	32/19	32/16
		Or by duplication of 1st tetrachord on the true dominant of ratio 3/2. The ratios of the Hypodorian of M.D. 16.					294 3/2	313·6	336	392
							=16/16	16/15	16/14	16/12
							2nd tetrachord on dominant			
Proto Pélog : Gamelan Kjahi : Hanjoet (3) Mesem. (Mangkae Negaran Sålå) Half Kwarten Schaal in Overgang naar Pélog	*No. ii*	*Pélog in Transition*								
		v.f., Kunst	*212·5*	*230*	*257*	*294*	*314*	*340*	*425*	
		v.f., K. S.	213·5	232·9	256·2	301·4	320·2	341·6	427	
		Ratios, K. S.	24/24	24/22	24/20	24/17	24/16	24/15	24/12	
		The ratios are those of the Phrygian Harmonia of M.D. 24.								
Pélog : De Overgang voltooid. Gamelan Sekati Katjerbonan, (4) Cheribou.	*No. iii C*	*v.f. Kunst*	*193·5*	*210*	*230*	*254*	*292*	*313*	*347*	*(387)*
		v.f., K. S.	192·6	209	228	250·3	292·8	318·2	348·3	(385·2) octave
		Ratios, K. S.	13/13	13/12	13/11	13/10 13/13	13/12	13/11	13/10	(387) (10/9 tone)
		The scale of No. 4 starts one step lower than in No. 3 and is thus in a lower species, viz. the Lydian of M.D. 13. The first tetrachord is duplicated on the 4th degree ; the scale finishes at the octave with a minor tone (10/9). Whether Dr. Kunst will find himself in agreement with my identification of these Javanese Scales with those of the Harmoniai of Ancient Greece remains to be seen when he is acquainted with the basis of my thesis.								

liquid notes of the flute may be heard in the polyphonic music of the gamelans. The data we possess refer mainly to the various kinds of Xylo- and Metallophone instruments and gongs upon which no scale can be said to originate.

THE HARMONIAI, IDENTIFIED FROM DR. KUNST'S RECORDS, AS ORIGIN OF THE SLÉNDRO AND PÉLOG SCALES OF JAVA AND BALI

Dr. Kunst has collected evidence of affinity between the scales of the gamelans and those of the African Marimbas (many hundreds of which have been measured), which suggests a common origin. Hornbostel and others find this in the migrations of the earliest Chinese Panpipe scale. Hornbostel's idea is that this was the scale of the flat Blown 5ths, which antedated the scale of the perfect 5ths, still in use (known as Pythagorean or Ditonal), the origin of which he conceives as a correction of the flat 5ths achieved by the stringed instruments. This is an ingenious idea, dependent upon his mistaken practical experience in blowing the Panpipes, which led him to believe that perfect 5ths are not attainable on Panpipes.

There is, however, not much to choose between the theories involving flat 5ths and sharp 4ths of the Cycle of Blown 5ths, and the sharp 3rds and flat 6ths of the Ditonal Scale, in both cases to the extent of a comma = *24*°. The Ditonal Scale has the advantage of being firmly established historically as a theory in the literary and technical sources, however dubious its reality in musical practice may appear, whereas the scale of the Blown 5ths is purely speculative.

The earliest known traditional scales in use in the gamelans of Java and Bali, to which Dr. Kunst refers as Proto-Pélog and Proto-Sléndro, have a bearing on this question, since Hornbostel regarded these as evidence of the far-reaching dispersion of the System of Blown 5ths. The results are set out below from the data (not previously published) supplied to me by Dr. Kunst, to which I have added my identification of their origin as Harmoniai.

The first table (Kunst, v) exhibits recorded vibration frequencies of the Sléndro pentatonic scales, actually used in five of the Gamelans of Eastern Java. • The basis of the system of Sléndro consists of two conjunct 4ths, each divided approximately into halves.

Hornbostel's figures refer to the sharp 4ths obtained theoretically by transposing the flat 5ths below the fundamental *F* of 183 v.p.s. Dr. Kunst's are the frequencies recorded from actual performances by the Gamelans and untransposed. The modal ratios and frequencies by K. S. are taken from a genesis of the Harmoniai, calculated on certain specified fundamentals for the purpose of identification (see Fig. 63). There are occasional slight differences of a vibration or two, such as might be expected from the conditions under which the records were taken, for the Xylo- and Metallophone instruments give out powerful high Harmonics which render exact identification of pitch somewhat difficult at times.[1]

[1] For the identification of these v.fs., Dr. Kunst uses a specially marked monochord, and his highly sensitive musical ear.

Fig. 73.—Examples of the Scales—Sléndro (Pentatonic) and Pélog (Heptatonic) in use in Gamelans of Java and Bali. Identified with the Harmoniai from the Records—published and unpublished—of Dr. Jaap Kunst (*by kind permission*)

(1) Gambang van Batoe boelan Z. Bali.	IV D	*v.f., Kunst*	*183 (189)*	*207·7*	*232·5*	*254*	*278*	*315*	*343·5*	
		Modal v.fs. and ratios by K. S.	183	205·9	235·3	253·4	274·5	313	346	366
			18/18	16	14	13	12	21 (11)	19 (10)	9
		Cents	*204°*	*231°*	*128°*	*138°*	*231°*	*173°*	*93·6°*	
		Hypophrygian Harmonia slightly distorted at the upper end.								
(2) Gam. Moenggang u/d. Pakoe Alaman Jogja Dr. Kunst's frequencies arranged in order of pitch by K. S.	IV E	*v.f., Kunst*	*199·5*	*217*	*237*	*273*	*298*	*326*	*357*	
		Modal v.fs. and ratios by K. S.	198·2	216·2	237·9	271·8	297·3	328	352	
			12/12	11	10	9–12/12	11	10	9	
		Cents	*151°*	*165°*	*182°*	*151°*	*165°*	*182°*		
		Phrygian Harmonia duplicated on 4th.								
(3) Gamelan Pélog, No. 2 Kanjoet mèsem (Mangkoe Nagaran)	Z 2	*v.f., Kunst*	*295*	*317*	*345*	*399*	*439*	*465*	*512*	*590*
		Modal v.f. and ratios by K. S.	295	317·7	344	402·7	443	466	517	590
			14/14	13	12	11	10 } 20 }	19	17	15
			128°	*138°*	*151°*	*165°*	*89°*	*192·3°*	*216·4°*	
		Mixolydian Harmonia upper tetrachord slightly distorted (as occurs in the flute).								

(4) Gamelan Pélog. Ardjå Moeliå Kraton Jogja	Z 5	*v.f., Kunst*	*276*	*293*	*316·5*	*373*	*402·5*	*432·5*	*473·5*	*552*
		Modal v.f. and ratios by K. S.	274·5	292·8	313·7	374	399·2	428·2	474·8	549
			16/16	15	14	12	11	10	9	8
		Cents	*112°*	*119°*	*267°*	*151°*	*165°*	*182°*	*204°*	
		Hypodorian Harmonia uninterrupted modal ratios.								
(5) The Proto *Sléndro* on an evolutionary basis. Gam. miring van den Wedånå v. Nagempak Reg. Bodjånegårå	V D	*v.f., Kunst*	*202·5*	*227·5*	*266*	*310*	*348*	*366*		
		Modal v.fs. and ratios by K. S.	201·3	223·6	268·4	309·7	350	366		
			20/20	18	15	13	23/40 (12)	11		
		Cents	*182°*	*316°*	*247°*	*212°*	*77°*			
		Hypolydian Harmonia as basis, Hypophrygian from ratio 18.								
(6) Prae-*Sléndro* Schaal. Gam. miring uit Bodjånegårå	VI A	*v.f. Kunst in order of pitch*	*199·5*	*225*	*262*	*308*	*347*			
		Modal v.fs. and ratios by K. S.	198·2	226·5	264·3	307	352			
			24/24	21/24	18	15 } 30 }	27			
		Cents	*231°*	*267°*	*316°*	*182°*				
		From the Phrygian Harmonia.								

In view of the close correspondence between the independent records of Dr. Kunst and the frequencies and ratios of the Harmoniai, there can be little doubt that the origin of the musical system of Java and Bali is the Harmonia of Ancient Greece. These results and conclusions are confirmed by the records of my Java and Bali flutes [q.v.].—K. S.

FIG. 74.—Modal Schemes of 11 Sléndro Javanese Scales identified from Dr. Kunst's Tables V and VI in which v.fs. are given with the ratios of the intervals

Table V. A	Gamelan miring v.d. Regent v. Bodjånegårå East. Java	PHRYGIAN Species Intervals	24/36 21/36 18/36 31/36 27/36	8/7 7/6 7/6 8/7
B	ditto	HYPOPHRYGIAN intervals	36/36 32/36 27/36 #24/36 20/36	9/8 32/27 (6/5) 54/47 = 8/7 6/5
C	Gamelin Soendarèn v. Pak. Kasmaran by Pråbålinggå	DORIAN Species on Hypophrygian on *C*	22/36 19/36 17/36 15/36 13/36	7/6 9/8 9/8 15/13 or 7/6
D	Gamelin miring v.d. Reg. Bodjånegårå	HYPOLYDIAN Species on Dorian 44/44 on 183 v.p.s.	40/44 36/44 30/44 26/44 23/44	10/9 6/5 15/13 13/11
E	Other gamelans from the same district, East. Java	PHRYGIAN Species on Lydian 52/52	24/52 21/52 18/52 31/52 27/52	8/7 7/6 (36) 7/6 8/7
Table VI. A	Prae-Sléndro Scale Gamelan miring as in A, Table V	PHRYGIAN Species Lydian gen. 52/52	48/52 42/52 36/52 31/52 27/52	8/7 7/6 6/5 8/7 or 9/8
B	Transition of Prae-Sléndro into modern Sléndro from a Gendèr from S. Bali	PHRYGIAN Hypodorian genesis on *C*	(24) 21/32 18/32 16/32 14/32 12/32	(8/7) 7/6 9/8 8/7 7/6

C	Gendèr unearthed; series of slabs, Batavia Mus. Gen. No. 1051	LYDIAN gen. on 183 v.p.s. See G below	(52/52)	49/52	43/52	37/52	32/52	28/52
			13/12	8/7	7/6	8/7	8/7	
D	Kjahi Medarsih from the Kraton at Jogja	PHRYGIAN Species on Lydian 26/26 on 183 v.p.s.	24/26	21/26	18/26	16/26	14/26	
			8/7	7/6	9/8	8/7		
D	ditto	HYPODORIAN Species on Hypophrygian on F 176 v.p.s.	32/36	28/36	24/36	21/36	18/36	
			8/7	7/6	8/7	7/6 or 8/7		
E	Kjahi Pengravé Sari from Pakoe Alaman Jogja	HYPODORIAN on 183 v.p.s. 32/32	(32/32)	30/32	26/32	23/32	20/32	17/32
			16/15	15/13	26/23 =8/7	23/20 9/8	13/11	
F	Gamelan of Regent of Tjiandjver, West Java	DORIAN on 183 v.p.s. 44/44	(44/44)	42/44	36/44	31/44	27/44	24/44
			(22/21)	7/6	7/6	7/6	9/8	
G	Marimba N.W. Congo Mus. No. 35412	LYDIAN gen. on 183 v.p.s. 52/52 See C above	(52/52)	49/52	43/52	37/52	32/52	28/52
			(13/12)	8/7	7/6	8/7	8/7	
		HYPODORIAN Species on Phryg. gen. on C=128	16/24	14/24	12/24	21/24	18/24	
			8/7	7/6	8/7	7/6		

It has already been pointed out that the sharp 4ths regarded by Hornbostel as a confirmation of his theory of blown flat 5ths (which, transposed below the fundamental, become sharp 4ths) may be referred to definite modal scales, having sharpened 4ths on the Tonic, e.g. to the Dorian Harmonia (of ratio 11/8); to the Hypophrygian Harmonia (of ratio 18/13), or to the Hypolydian Harmonia (of ratio 10/7). Or, alternatively, to the Panpipe Ditonal Scale with Tritone consisting of three major Tones (of ratio $9/8 = 729/512$). See also Chap. viii, Fig. 72.

THE FLUTES FROM JAVA AND BALI EMBODY THE HARMONIAI IN THEIR EQUIDISTANT FINGERHOLES

Records from Kunst, Table v, of Sléndro Scales and our Fig. 74 (A) and (E) correspond with intervals of the Phrygian Species identified by their vibration frequencies. (B), more sophisticated, undoubtedly owes its origin to a Hypophrygian flute of M.D. 18, having seven fingerholes; the notes of the scale in B are obtained from exit, and Holes 1, 4, 5 and 7; intermediate intonations are obtained by half-covering the hole, by cross-fingering and by other ingenious devices which are practised by most primitive flute players. (C) gives a pentatonic derivation from the Dorian Species, which may also be obtained from a flute of M.D. 11; the four fingerholes, when opened in turn, produce notes of the following ratios:

Exit	Hole 1	Hole 2	Hole 3	Hole 4
11/11	9/11	8/11	7/11	6/11
	(19)	(17)	(15)	(13)

when normally fingered, and by half-stopping the notes in brackets.

N.B.—The ratio 22/19 between exit and Hole 1 is practically identical with the septimal 3rd 7/6 $\left(\frac{22}{19} \times \frac{6}{7} = \frac{132}{133}\right)$.

This Sléndro Scale may be produced with ease on the two Java flutes Nos. 5 and 6 and on the short flute from Bali[1] No. 20. The scale of (D) is of the Hypolydian Species, and may be played on a simple flute of M.D. 10, such as No. 8 of the Sudan flutes, used by the Acholi tribe, which I owe to the kindness of Dr. A. N. Tucker. The fingerholes of the Sudan flute give the following sequence:

Exit	Hole 1		Hole 2	Hole 3		Hole 4	
10/10	9/10		8/10	7/10		6/10	
●	○	●	●	●	●	◐	○
●	○	●	●	◐	○	○	○
●	○	●	○	○	○	○	○
●	◐	○	○	○	○	○	○
10/20	19/20	18/20		15/20	14/20	13/20	12/20

In his investigations Dr. Kunst has penetrated still further back towards the origin of the Sléndro type of scale: he has discovered as original source a still earlier scale—an extended sequence of smaller intervals—the Prae-

[1] Bali No. 20 is one of a set of six fine flutes with equidistant holes sent to me by Mr. Soekowati of Batavia to help me in my investigations.

Sléndro, the use of which antedates the Proto-Sléndro. In his Table vi he has sketched in outline the evolution of the modern Sléndro from this Prae-Sléndro material with its selective transitions, first into Proto, and finally into the Modern Sléndro pentatonic; comparing all his actual records in vibration frequencies with those of Hornbostel, theoretically computed. Kunst conceives this material as the result of the halved 4ths in overlapping sequences. An instance of this scale structure occurs on the *gendér* from which he compiled the material of the Sléndro genesis of nineteen different notes.

The Sléndro records exhibit some of the features of what is known from our Greek sources as εἴδη (species) occurring in natural course upon instruments having a range of two or more octaves.

The basis of the system, as it is found at the present time in Javanese music, is obviously modal in origin. The real Tonic of the species is not always immediately apparent in practice, and the keynote is frequently absent in the Sléndro. Regarded as pentatonic, the Sléndro in my opinion implies a transition from the Panpipe system to one based upon the modal flute (with equidistant holes); a deep-rooted predilection for the psychological effect of the 3rd, inherited from the Panpipe pentatonic, accounts for the selective use of the Modal Scale which may on occasion be observed, when the vibration frequencies indicate Harmoniai other than the Dorian M.D. 11; e.g. the Hypolydian M.D. 10; and Hypophrygian M.D. 9, which, in this simple form, provide natural pentatonics on a modal flute, combined with facilities for extension into hexatonic and heptatonic, which are non-existent on the Panpipe.

As already stated above, a true pentatonic form, i.e. a deliberate grouping of five notes within the octave, can only originate, according to my experience, from Panpipes so grouped in response to an instinctive urge for proportional measurements in cutting the reeds for the pipes, and not in obedience to the guidance of the ear; as though the piper were actuated by preconceived notions. This hypothesis leaves the way open for the derivation from cycles of Perfect 4ths and 5ths but not from Blown 5ths.

If the analysis given below of the Proto-Pélog records be added to those of Sléndro, it is at once evident that the origin of the Javanese musical system is based upon the Modal System of the Harmoniai.

Identified from the vibration frequencies supplied by Dr. Kunst's records of the Proto-Pélog, (*a*) as played by the gamelans of Kraton Jogja, may be recognized as derived from the Phrygian Harmonia of a flute scale. Whereas (*b*) the Proto-Pélog of the Kraton Sålå, derives from the Hypolydian Harmonia with tritone, wherein a truly remarkable correspondence with the original is revealed, the fundamental alone excepted. This, as exit note on the flute, is frequently more or less out of tune with the modal sequence beginning on Hole 1, owing to the difficulty experienced in finding the accurate position for that hole, which is of signal importance, as holding the key to the whole sequence of the modal flute.

Evidences of species are unmistakable in all these Javanese scales. It cannot be too strongly emphasized that when comparing and identifying results, the decisive factor is not mere identity in vibration frequency, or close approximation of intervals, selected here and there, from the scales, but is to be found more definitely in the evidence of a consistent development of the sequence of modal ratios occurring in the logical order of the Harmonia, whether conceived as pentatonic or heptatonic, and with an occasional admixture of intermediate ratios derived from the use of chromatic or enharmonic forms, or merely as alternative notes due largely to the structure and inner reactions of the air column in the flute.

DUPLICATION OF THE 1ST TETRACHORD ON THE 4TH OR 5TH DEGREE IN EVIDENCE ON SOME JAVANESE AND BALINESE SCALES

In the identification of these scales, there is a factor which frequently occurs, confusing the issues. This is the now familiar duplication of the 1st tetrachord of a Modal Scale on the 4th degree (if conjunct or plagal), or on the 5th (if disjunct or authentic). This device was widely used in Hellenistic Asia and by Arabs as well as in Hindustan. These duplicated forms are found in use in several of the Javanese scales.

From the selection of examples of Javanese scales given here—which is strictly limited by exigencies of space—the predilection of these musicianly Indonesian races in their remarkable and delightful polyphonic music seems to be for the Phrygian, Hypophrygian and Lydian Modes. Such preferences, however, will be determined on a wider range of evidence by later investigators.

Among some 50 Javanese scales analysed from Dr. Kunst's records, overwhelming evidence is found of all seven Harmoniai as origins of the scale, to which may be added those of the flute specimens actually tested and measured by me. The distribution of the different Harmoniai from Javanese sources may be seen in the following table:

Results of Analysis and Comparison of some 50 Javanese and other Scales from Dr. Kunst's Records, identified by their v.fs. with the Harmoniai (details of which are available in separate Tables)

Phrygian Harmonia	Total	18	
Lydian Harmonia	,,	10	
Hypophrygian Harmonia	,,	8	
Hypodorian Harmonia	,,	5	
Dorian Harmonia	,	5	: in Gamelans 2, Flutes 3
Mixolydian Harmonia	,,	3	
Hypolydian Harmonia	,,	3	
Grand Total		52	

N.B.—Less than a half-dozen of the records investigated gave unsatisfactory results.

THE MUSIC OF THE FOLK IN SOUTH AFRICA

Even the briefest of surveys of the Folk Music of the Bantu tribes of South Africa provides an amazing revelation of an entirely original development of music, when studied from the writings of Professor Percival R.

Kirby, M.A., D.Litt., F.R.C.M., of the University of the Witwatersrand, Johannesburg.[1]

Professor Kirby may be said to have reconstituted the musical history of the South African natives after long periods spent in their midst, observing, learning by immediate experience the methods adopted by the natives in choosing suitable materials, making their musical instruments and playing upon them; and attending their festivals and rites.

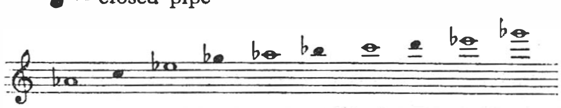
The musical systems in use among the various tribes, Venda, Korana, Swazi, Sotho, Xhosa, Zulu, Namaqua Hottentots, Bushmen, &c., although differing in detail, in practice and in aesthetic values, have a common origin in the overtones of the Harmonic Series, recognized by ear, and used as far as the 13th or even 15th partials. In their wind instruments, and more especially in the many varieties of reed flutes and pipes, the scale used consists entirely of the fundamental note and its overtones, in contradistinction to the scales of modal flutes obtained through fingerholes which yield a register of fundamental notes and from these by overblowing a harmonic register.

THE MUSIC OF THE BANTU FOLK OF SOUTH AFRICA, BASED UPON THE HARMONIC SERIES

The natives of South Africa have discovered for themselves the respective properties of open and closed pipes with their characteristic progressions of overtones, odd and even in numerical order from open pipes, and the odd numbers only from closed pipes.

By using an open pipe, and closing it at will by a finger, they combine both series. The significance of this device will be best realized from the scheme below, in which minims represent the notes of the open pipe, and crotchets the notes of the same pipe closed (*op. cit.*, p. 114).

𝅗𝅥 = open pipe ♩ = closed pipe



Number of each overtone 2 5 3 7 4 9 10 11 12 (14)

(The numbers of the overtones added by K. S.)

The advantages of using a combination of open and closed pipes are not only the possibility they afford of a continuous progression of intervals in the same octave, which, as in the first four notes above, could only occur on an open pipe as overtones 4, 5, 6, 7, and in the rest of the range as

[1] *The Musical Instruments of the Native Races of S. Africa* (Oxf. Univ. Press; London, Humphrey Milford, 1934); 'The Reed Flute Ensembles of S. Africa', *Jour. of Roy. Anthrop. Inst.* (London, 1933), Vol. lxiii, pp. 334–7; 'Some Problems of Primitive Harmony and Polyphony', *S. African Jour. of Science* (Johannesburg, 1926), Vol. xxiii, pp. 951 sqq.; 'A Study of Bushman Music', reprinted from *Bantu Studies*, Vol. x, No. 2, June, 1936; and many other contributions from the same source.

8, 9, 10, 11, 12. The implication may not occur to readers who are not proficient in flute playing; it is this: each Harmonic overblown requires a proportional increase in the compression of the breath and therefore, the higher the series number of the overtone, the greater the effort in compressing the breath required to produce it. The result of the combined use of the same flute as open or closed tube is shown graphically in staff notation.

Upon the Swazi flute of this kind called *umtshingosi* of reed, with a narrow bore, fundamental note *G*♭, an elaborate piece of music (one of many) was played, of which a few bars reproduced below will give some idea (*op. cit.*, pp. 115–16, No. 5).

FIG. 75.—Played with Swazi, narrow bore instrument in *G*♭.

10 9 6 10 6 10 8 9 6 8 6 12 11 6 10

Harmonics added by K. S.

(*Op. cit.*, *p.* 115)

These flutes, used by several other tribes under different names, sometimes have a conical bore; the embouchure consists of the end of the reed below a knot, cut off obliquely at an angle of 45° at the wider end of reed or cabbage-tree. The method of blowing is thus described by Professor Kirby: ‘The player then directs a stream of air across the embouchure towards its lower edge in a peculiar way, the tongue being shaped into a kind of channel, and the upper portion of the embouchure being largely blocked up by the gums. The tongue also serves to give an ictus to certain notes’ (*op. cit.*, pp. 113 and 145). The author expressly states that this method of blowing the flute differs from the one used on the Panpipe when the open end is laid against the lips.

THE SOUTH AFRICAN NATIVES HAVE DISCOVERED THE DIFFERENT REACTIONS OF OPEN AND CLOSED PIPES, CLOSING THEM AT WILL TO INCREASE THE COMPASS

These reed flutes without fingerholes are used in sets of from seven to nine or more, played by as many pipers forming a band or ensemble, and producing a kind of natural polyphony (see, for example, *op. cit.*, p. 160, &c.). It is a matter of great interest to find that the practice of these ingenious South Africans, who have discovered the different reactions of open and closed pipes, has a parallel in Central Europe among the Roumanians of the Maramures region in which similar flutes known as *Tilincă cu dup* with fipple, but no fingerholes used open at exit, and the *Tilincă fără dup* without fipple or fingerholes which is used both open and closed by a finger. Béla Bartok (*op. cit.*, p. xxvi and ex. No. 23 i) confesses that he could not discover either locally or from phonograms what was the result of such interference with the exit, nor for which notes the stopping was used. Another device is used on the *Tilincă* for obtaining at will, for temporary use, a new fundamental note from exit by partially closing this, only sufficiently to lower the note, but not to convert the open pipe into a closed one. Bartok prints several tunes played on the *Tilincă*, and in

15th Harmonic on C.

No. 23 i, an example occurs of the result of this partial closure, and the note on *C* used as appoggiatura is obtained instead as an overblown octave of the new fundamental.

In this way the additional length of the air column is procured by the narrowing of the exit, whereby the amount of the difference between the normal diameter of the bore and the one artificially created at exit becomes effective as lowered pitch of the flute.[1]

A KIND OF TRANSVERSE FLUTE IN USE BY THE VENDA, SWAZI, PEDI AND OTHER TRIBES

A kind of transverse flute is also used by the Venda, Swazi, Pedi and other tribes in South Africa who display remarkable ingenuity and a high degree of musical sensibility. They have performed the feat of making use in the same flute of members of the Harmonic Series in both ascending and descending geneses.

The reed or bamboo is closed at both ends by a natural knot; an embouchure is bored at one end with red-hot wire and near the other end are three fingerholes. Professor Kirby adds that the spacing being suited to the fingers of the maker, almost as many scales are produced as there are flutes made. As many as four fingerholes are occasionally found on these flutes.

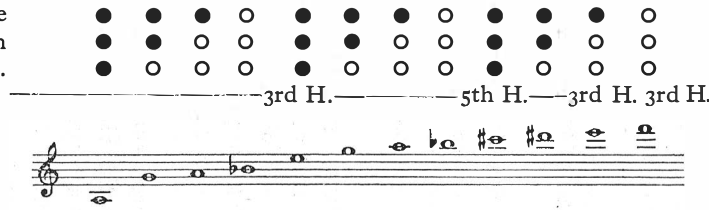
The cylindrical transverse flute has the properties of the closed pipe when all the fingerholes are closed, and yields the Harmonics bearing odd order numbers. But when one or all the holes are open the flute is a transverse flute, having Hole 1 as vent, and the others, if approximately equidistant, develop a few notes of the Modal Scale of their Modal Determinant, found by dividing the length from the centre of Hole 1 to the centre of the embouchure by the increment of distance between the fingerholes. When closed, the pipe plays an octave lower than the open flute.

Professor Kirby gives a scheme showing the fingering of such flutes with the extended scale obtained by the use of both systems (p. 124). In No. 1 the first four notes are produced by the fundamental *B*♭ with overtones, as closed pipe, followed by notes of the open flute which indicate a Dorian tetrachord of M.D. 11 from Holes 1, 2 and 3 = *A*, *B*♭, *C*; the tetrachord is completed by *D* as 5th Harmonic of the fundamental of the closed pipe *B*♭. This Dorian tetrachord is followed in scale No. 2 by a

[1] This practice tested upon a J. D. Coates flute of a length of ·369 and diameter ·014 gave as fundamental *A* 13/256 = 433 v.p.s. with end closed, and with exit 3/4 closed *A* 27/256 = 417 v.p.s.

FIG. 76.—Pedi Scale of Transverse Flute with Three Fingerholes, showing the Fingering (the exit is closed), but the Flute is converted into an open one when One Hole (or more) is left Open as Vent

Fingering of the holes overblown added by K. S.



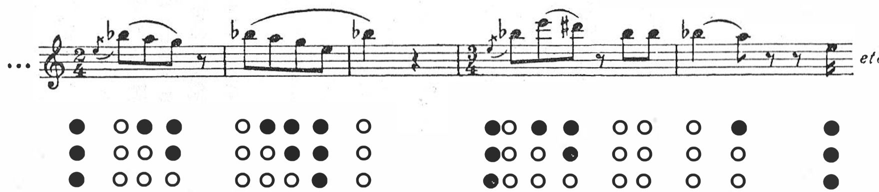
18/18 16 15 13 12 or 23 11 10

Hypophrygian Harmonia

Ratios of the notes given by the open holes (K. S.).

(*Normally, the* C♯ *should be sharpened and the* D *natural, or else noted* E♭)

Part of a tune played upon the flute by a Pedi boy (p. 347). For an illustration of the flute and Pedi boy (*op. cit.*, Plate 43, A, No. 8, and B)



N.B.—Where notes do not correspond with the fingering, Harmonics are indicated. On page 346, the scale of the Swazi transverse flute *umtschingosi* is given, which after the first four notes, continues in the Hypolydian octave, with the first tetrachord duplicated on the perfect 5th by overblowing. Ratios by K. S., 10/10 9/10 8/10 7/10 5th 10/10 9/10 8/10 7/10.

N.B.—The Swazi flute is open at the end opposite the embouchure.

A more ambitious tune follows, 24 bars in length: the fingering is indicated as above.

Hypophrygian tetrachord; in Nos. 3, 5 and 8 by a Hypolydian; in Nos. 4 and 6 by a Hypodorian and in No. 10 is again a Dorian (*op. cit.*, p. 124).

The fingering of the scale as given below for No. 1 is the same for all, but the difference in the length of the flutes and in the spacing of the fingerholes produces the variety of modalities.

The intonation of a modal tetrachord is thus obtained by a combination of two distinct musical systems: the one dependent on proportional lengths; the other on proportional compression of the breath.

It cannot be claimed on the strength of these data that the South African tribes used the Modal System of the Harmoniai, except incidentally in this fashion; the modal tetrachords may have been subconsciously familiar, since the flute players found the means of completing the tetrachord begun on the three fingerholes, by an overtone, correctly selected, on the fundamental of the closed pipe.

It is probable that there has been at some time contact with tribes using modal flutes with equidistant holes, for it is known that such are to be found at the present day in Africa; for instance among the *Acholi* Nilotic tribes of the South Sudan; eight specimens of which have been presented to me by Dr. A. N. Tucker, as mentioned and specified elsewhere (see Chap. x, Table at end).

A curious flute called *Khumbgwe*, used by the Venda, and *Ombgwe* by the Karanga of Southern Rhodesia, consists of a small kaffir orange pierced by two circular openings opposite each other, into the lower one of which is introduced the tip of a length of reed, the other end of which is closed by a natural knot. Near this knot are burned three fingerholes in the Venda instrument and two in the Karanga. The flute is blown through the top hole in the orange.

Professor Kirby has noted the scale and fingering of a well-made *ombgwe*, which gave the following sequence, that will be recognized as belonging to the parent scale of the Cycle of 4ths of Dorian Species (*op. cit.*, p. 129 and Plate 45A, Nos. 1, 2, 3, and 45B).

Scale of the Ombgwe of the Karanga

In the absence of vibration frequencies it is impossible to tell whether the notation represents the exact intonation of the notes—which would imply a Panpipe scale of the cycle of 4ths—or if the interval from *G* to *A* were of ratio 11/10 the first tetrachord of the Dorian Harmonia would be indicated. The Karanga performer, who sang into his flute while blowing it played a tune on this same *ombgwe* of which the following bars are a specimen (see page 350).

On the minim chords the instrument wavers between *G* and *C*, and seems at times to be able to yield both sounds simultaneously, when the

FIG. 77.—Played by a Karanga on the Ombgwe

Specimen Tune

(*op. cit.*, *p.* 130)

voice sings low *C*, but when the voice sings *F*, the instrument with the same fingering sounds *C* only.

It is impossible, owing to the exigencies of space, to do justice to the immense wealth of observed facts, and records of the music of the South African tribes contained in this fine volume, profusely illustrated by diagrams, music and by 73 fine plates, reproductions of photographs.

The only desideratum felt on closing the volume is for accurate measurements of flutes and pitch values, which are required in the interests of the comparative science of music.

The extraordinary development of music by the Bantu tribes of South Africa, based upon the Harmonic Series, seems to lend colour to the suggestion made earlier in this chapter of the rise of vocal music from the speech-song which develops naturally from intonational language. Interesting specimens of this speech-song may be heard on gramophone records of the Chipika Singers of Mashonaland, as indicated below.

Chakaya Wei (Burial Song) Sara Mu Gomo (War Song)	Regal G.R. 35, Columbia.
Mwana Wa Mangwende (Historical Song) Chimwe Chirombo Mandangwe (Dance or Clapping Song)	Regal G.R. 39, Columbia.

CHAPTER IX

QUEST FOR THE HARMONIA AS SURVIVAL OR REBIRTH IN FOLK MUSIC

Indications of the Survival or Rebirth of the Harmonia. Evidence of the Practical Use of the Harmonia in Ancient Greece. Fragment of Pindar's First Pythian Ode in the Hypophrygian Harmonia. The Fragment displays all the Hypophrygian Modal Characteristics, and five closes on the κῶλα. The Three Hymns attributed to Mesomedes: The First Hymn: 'To the Muse', in the Hypophrygian Mode. The Second Hymn: 'To the Sun', in the Hypophrygian Harmonia. The Third Hymn: 'To Nemesis', in the Hypophrygian Species. The Epitaph of Sicilus may be read in the Hypophrygian or Phrygian Species. Fragment from the Orestes of Euripides in the Hypophrygian Species. The Paean Fragment in the Berlin Papyrus (discovered by Dr. W. Schubart in 1918): Hypophrygian or Phrygian Species. Discovery of a Christian Hymn (in the Oxyrhynchus Papyri): Pseudo-Hypophrygian Species. The Two Delphic Hymns: The Delphic Hymn No. 1 in the Hypolydian Harmonia. The Delphic Hymn No. 2 in the Dorian Harmonia. The Prototypes of Greek Music all exhibit the Modal Characteristics of the Harmonia. The Characteristic Intervals of Folk Music defined by Ratios, Vibration Frequencies and Cents. The Modal Characteristics of the Harmonia in Early Medieval Chants. Hucbald's Evidence. Survival of Modal Pivots in early Medieval Liturgical Chants. The Era of Polyphonic Music heralds the Wane of the Harmonia in Liturgical Chants. Brief Reference to the Canon of Florence and to the Divisions of the Monochord. Cross-fingering on the Flute as Evidence of the Survival of the Harmonia in the Sixteenth Century. The Dorian Harmonia in Folk Music: Evidence from the Incas of Peru. The Scale of the Dorian Harmonia of M.D. 11 erroneously diagnosed as Pentatonic. Evidence from Hindostan, Hungary and Rumania. Evidence from New Mecklenburg, Turkey and the Jews of the Yemen. The Lydian Harmonia in Folk Music. The Phrygian Harmonia in Folk Music. Evidence from Synagogue Chants, from New Mecklenburg, Rumania, The Pawnees, Peru and Sumatra of the Hypophrygian Harmonia. The Hypodorian Harmonia in Folk Music. The Mixolydian Harmonia as Rāg Mālkos in Hindostan. The Hypolydian Harmonia in Folk-song: e.g. in East Greenland. The Closing of the Cycle: from Ancient to Modern Greece

INDICATIONS OF THE SURVIVAL OR REBIRTH OF THE HARMONIA

THE question of the survival or rebirth of the Harmoniai in the music of the Folk, although strictly speaking on the outer fringe of the subject of the Music of Ancient Greece, is yet highly significant and pregnant with aspects, some of which are entirely new and throw a retrospective light on the processes of the evolution of music.

What are the indications of survival or rebirth, regarded as the immediate cause of which the Harmonia identified in Folk Music is the effect?

Survival is in general attributed to a long line of tradition, some links of which at least remain as proof.

The links must be produced.

Rebirth points to the extraordinary nature of the origin of the Harmonia on the Aulos and flute, already discussed, whereby the embodiment of a natural law in the pipe creates through the boring of equidistant fingerholes, opened and closed at will, the requisite elements of the Harmonia. Nature exhibits no favouritism; she is prodigal in her gifts, and they are open alike to all.

It is evident that the Harmonia as origin of modality, of scales and systems, is a matter of rebirth, with the joys attendant on initial discovery free to all.

The Harmonia when once possessed and assimilated as the language of Music, survives among the Folk, forming the musical atmosphere into which music lovers are born. Thus the Harmonia progresses and thrives through environment so that rebirth and survival are the ever recurrent cause and effect.

Stringed instruments were undoubtedly one of the important means of perpetuating the Harmonia, but they could not originate it, nor can they leave permanent records—that is the sole apanage of wood-wind instruments with fingerholes.

Stringed instruments, however, performed the valuable functions of controlling and testing—by means of the monochord—the purity of intonation; of stabilizing the ratios of the intervals, of grouping them in order of pitch, and of welding the various elements into the system known as the P.I.S.[1] in Ancient Greece. The Kithara,[2] at first with its scale fixed at will, so far as individual performances were concerned, was the favourite and widely recognized instrument for impressing and carrying on the tradition. The Harmonia was enthroned by the Kithara in the heart and soul of the nation. It alone in Greece possessed the means of fusing the two elements, vocal and instrumental, of the rising and descending Harmonic Series. The strings plucked by the Kitharoedos while reciting an Ode, gave out the sound-pattern of the Harmonia in the melos: the individual strings, when plucked, liberated the intervals already familiar in modulated speech, as a still more ethereal sound-pattern or tissue of constituent Harmonics. Archilochus was the first, as recorded by Plutarch,[3] to introduce an accompaniment above the melos, whereas among the Ancients it was always in unison.

EVIDENCE OF THE PRACTICAL USE OF THE HARMONIA IN ANCIENT GREECE

The existence of the Harmonia through the centuries must now be briefly passed in review. Before any claim of survival can be substantiated, evidence must be produced of the practical use of the Harmonia in the melopoiia of Ancient Greece.

[1] The Systema Teleion Ametabolon—The Perfect Immutable System.

[2] For the Kithara as origin of the Violin, see fn. 1, p. 357.

[3] Plut., *de Mus.* (Weil and Reinach, pp. 107–11, §§ 277 sqq.) The accompaniment in unison favours the enjoyment of the ethereal development of constituent Harmonics; the accompaniment of Archilochus was an attempt to reproduce some of its beauty.

We are met at the outset by a serious difficulty : the few extant fragments of Greek music constituting the evidence, are written in the system of Greek Notation laid down for the 15 Tonoi (in the Diatonic and Chromatic Genera) in the Tables of Alypius.[1]

There is evidence that the system of Notation emanated from Pythagoras or his followers the Harmonists.[2] Similar interpretations of the symbols, made independently by F. Bellermann and Dr. C. Fortlage, both published in 1847, have been almost universally adopted, *faute de mieux*. This interpretation, based upon the ditonal system of the Graeco-Roman theorists, proves, however, on close examination to be entirely unsatisfactory, teeming with inconsistencies and errors ; it has, moreover, nothing in common with the system of the Harmonia, for which the system of Notation was originally conceived by the Ancient Greeks, but as the very nature of the Harmonia was an unknown quantity in Bellermann's day, no other explanation was to be expected. In the light of the Harmonia the scheme of Notation comes as an astonishing revelation of subtle ingenuity (see Appendix No i, on Notation, and Chap. v, Fig. 37, the Ratios of the Seven Harmoniai with their Notation).

With this scheme of Notation for use as reference, we may proceed to examine the prototypes in order to discover whether the Harmonia was a living basic element in the music of Ancient Greece, and to what extent it may be traced as an influence in composition.

FRAGMENT OF PINDAR'S 1ST PYTHIAN ODE

The symbols in use in most of the fragments are to be found in complete sequence in more than one Tonos, differentiated by their ratios, so

[1] Many reproductions of this weighty document exist in the codices, the principal of which are listed, with comments, by Karl v. Jan in *Musici Scriptores Graeci*, Teubner (Leipzig, 1895), p. xcii ; to which must be added No. 49 (q.v., *op. cit.*, p. xlii) ; Mus. Brit. Add. MS. No. 19353. fol. 237 ; No. 20, Esc. x, i, 12 ; and Nos. 16 and 17, *op. cit.*, p. xxxiv.

They were first made known for the seven Harmoniai to Western scholars by the Tables of Boethius, which are preserved in numerous codices. See ed. Glareanus (Basel, 1546), pp. 1158–9. The Lydian Tonos with description of symbols, p. 1132. This edition of Boethius, *de Musica*, is notorious for the misnaming by Glareanus of the Species in the Ecclesiastical Modes, i.e. calling the *D* Mode Dorian, and the *E* Mode Phrygian and so on ; an error which still appears in such works of reference as Grove's *Dictionary of Music and Musicians*. See for the origin of the error in Appendix, No. 2, 'The Ecclesiastical Modes '. The earliest printed versions of Alypius are (1) by *Vincentio Galilei*, Dialogo della Musica Antica e Moderna, 1581. Tables with description of symbols in Italian, contains Mesomedes' Hymns.

(2) By *Joannes* Meursius, *Aristoxenus Nicomachus*, *Alypius*, Nunc primus vulgavit ; Lugduni Batavorum, 1616. Symbols described but not printed in tables.

(3) *Athanasius Kircher*, Musurgia Universalis (Rome, 1650). Tables of Symbols in Diatonic and Chromatic genera, Icon., xiii, p. 541, Vol. i.

Meibomius, *Antiquae Musicae Auctores Septem*, Marcus Meibomius (Amsterdam, Elzevir, 1652), Vol. i, No. iv.

[2] Aristides Quintilianus, p. 28 M. Aristoxenus, *Elements of Harmony*, Macran (39), pp. 130, Greek, and p. 194, tr. See also the twelve Polemics of Aristoxenus against the Harmonists (by K. S.), Chap. v.

that each symbol may form with its neighbour intervals of at least two different ratios. The Tonos represents the elements of the musical material, thus in a sense approximating to the keys of modern music. The Mode is revealed by the functions (dynamis) of the various notes in relation to the melodic pivot or Tonic, on the one hand, and to the keynote, as Mese or Arche, which occupies a different degree of the scale in each Harmonia, and therefore, forms with the Tonic a distinctive interval.

The earliest fragment extant—supposing the musical setting to have been contemporaneous with the poem—is from Pindar's 1st Pythian Ode.

The authenticity of this musical fragment has long been regarded as doubtful by some scholars, and the genuineness of this piece of music has recently been again powerfully assailed. I propose to disregard the controversial aspects of the question, and to judge the fragment solely upon its musical merits.

I am firmly convinced that this setting of the few opening verses from Pindar's 1st Pythian Ode could not have emanated from any but a Greek source—not perhaps from Pindar himself, but possibly from one of the Harmonists, to whom the Harmoniai were a living reality, and who was familiar with their genesis.

Further, the grounds upon which the authenticity is attacked are mainly philological, and are largely due to the odium incurred by the writer of the *Musurgia*, Athanasius Kircher,[1] who was the first to introduce and publish the fragment, and the only one who claims to have seen the original manuscript, which has now disappeared, thus sharing the fate of many other valuable manuscripts. With few exceptions [2] the inherent significance and value of the fragment as a piece of Greek music has not been criticized or defended by competent authorities. It has yet to be satisfactorily proved that it could by any possibility have originated from Gregorian, Plainsong or any other sixteenth- or seventeenth-century source. Kircher introduced the fragment first (p. 541, Vol. i), as he claims to have found it in the original manuscript (as reproduced on Pl. 16) which is all that concerns us for the present. His interpretation in staff notation (Vol. i, p. 542, see above) is rhythmically grotesque and misleading; he had no pretensions to Greek scholarship; it contains glaring errors besides, which are not in the Greek version. If the music were a forgery, this would be the original.

Kircher's statement that the Ode is noted in the Lydian Tonos is obviously correct; he does not mention the species. Could he be sure of it?

The issue is by no means clear: for in line 4, the Greek symbols, and Kircher's interpretation in staff Notation, do not agree: as the music stands (on p. 542), there is a distinctly Hypolydian close on (ζο)-μέ-να ⅂ μ noted by Kircher as *A–F*, whereas according to the Greek symbols, a Hypophrygian close on *G–A* is indicated.

Was Kircher aware of this implication? Was he familiar enough with

[1] *Musurgia Universalis* (Rome, 1650), Tome 1, pp. 541–2.

[2] 'The Music of Pindar's "Golden Lyre"', by J. F. Mountford, Liverpool University, *Classical Philology*, Vol. xxxi, No. 2 (April, 1936).

the species to discern the correct position in the P.I.S. which, in the Lydian Tonos, would correspond in pitch and modality with the closes in the alleged forged document?

Had Kircher attached such importance to the species as would enable him to transcribe his chant correctly into Greek symbols; would he, avid of kudos, have allowed the opportunity to pass of mentioning the species as well as the Tonos? or would he have passed the proofs which jumbled up closes, modality and rhythms in line 4? No! each note being a movable type, it is, in my opinion, obvious that the printer dropped out the minim *A*, the 6th note in line 4, and inserted it further on by mistake,[1] as 2nd note after the bar line, on the syllable *τòν*. It is significant that the only recent defender of the authenticity of the fragment in this controversy has supported Kircher's blunder, in his interpretation.[2]

The range of the melos, consisting of six symbols, extends to a 5th above the Tonic, and below to a drop to the sub-Tonic on Parhypate Meson—the Tonic of the Hypolydian Species, thus:

Hypophrygian Species in the Notation of the Lydian Tonos

	Meson			Syn.		
	PH.	Lich.	Mese	Tr.	PN.	Nete
Vocal	P	M	I	Θ	Γ	Ʊ
Instr.	Ս	ꓶ	<	V	N	Z
	20	18	16	15	13	12

Ratios by K. S.

At a glance the Mode might be mistaken for Hypolydian with P Ս 20 as Tonic; or again, if the scale were read in the Hypophrygian Tonos, from Trite Diezeugmenon to Nete Hyperbolaion, the Mode would be Phrygian. It is neither of these.

What, then, is the decisive factor? The Tonic and the Mese or keynote are the cardinal points in the Harmonia, but the lowest of the range of notes used in the Ode is not the Tonic. The melos has merely been carried down to the sub-Tonic, Parhypate Meson, Kata Thesin—a practice

[1] The mistake is an obvious one, since the corresponding notes in staff notation for the same Greek symbols ꓶ ⱶ = *G A*, ratios 18 16 (end of line 3 and also in line 4, two notes before the bar line) and ꓶ ⱶ = *A F*, ratios 16 20 cannot be rendered by both *G*, *A*, and *A*, *F*. Neither should Kircher have repeated the Greek symbol ⱶ, for notes 5 and 6 in line 4, to represent two different pitch notes *A* and *B* flat.

Thus Kircher's alleged invention in staff notation (p. 542) exhibits errors and a false modal close, i.e. Hypolydian among five Hypophrygian orthodox closes. The Greek version (p. 541) is modally perfect and lacks the errors patent on p. 542.

[2] Paul Friedländer, *Ber. ü. d. Verh. d. Sächs. Akad. d. Wiss. zu Leipzig, Phil. Inst. Kl. 86*, Bd. 1934, Heft 4, p. 32, line 4*b* (Hirzel, Leipzig, 1934).

sanctioned in the classical period.[1] It is, moreover, known from Plutarch [2] that it was a recognized practice in Greek music, in all but the Dorian Tonos, to extend the scale to the whole, or part, of the lower conjunct tetrachord, Hypaton. The lowest note in question [P / ∪], Parhypate Meson, is the Tonic of the Hypolydian Species ; it occurs only twice in the second part of the Ode, without modal significance, and there are no Hypolydian closes.

THE FRAGMENT DISPLAYS ALL THE HYPOPHRYGIAN MODAL CHARACTERISTICS AND FIVE CLOSES ON THE κῶλα

The Harmonia is clearly Hypophrygian. The decisive features in the Ode are first of all the frequent use of the two cardinal modal pivots : Tonic = Lichanus Meson, and keynote = Mese, used in both directions, rising and falling. There are five of these closes, one on each of the κῶλα, denoted by the symbols [I M I / Ͱ ⊓ Ͱ / 16 18 16] or [M I M / ⊓ Ͱ ⊓ / 18 16 18], to which I have added the modal ratios, besides a 6th, differing in form, in line 4 thus : [⊓ Ͱ Ͱ / τεύ-χης-ἐ / 18 16 16]

The other characteristics of the Hypophrygian Harmonia are : its poignant raised 4th 18/13 (*561 cents* [M — Γ / ⊓ — N / 18 — 13]) used as leap or drop, from or to the Tonic ; and the augmented second of ratio 15/13 [Θ — Γ / V — N / 15 — 13] leading up to the fourth degree, from a Minor 3rd 18/15 on the Tonic, e.g. 18/18, 16, 15, 13, 12, the ratios of the first Hypophrygian tetrachord.

A leap from Tonic to 4th occurs in line 4 on [M — Γ / ⊓ — N / αἰ - χμα / 18 13].

It must thus be recognized that the modal characteristics of the Hypophrygian Harmonia all occur strongly emphasized in the Greek version of the Ode, but in Kircher's interpretation the errors in line 4 have obliterated the 4th close on ζο-μέ-να.

The ethos in the first part, owing to the descending motion from 5th to Tonic, is lyrical and poignant ; but in the second part of the Ode, with the allure of the melos ascending, the ethos proper to the Mode asserts itself : resolute, tense, poignant.

To find five modal closes in so short a fragment is remarkable, but what may be claimed as an exclusively Greek feature is the equal importance of the two pivots, *I* and *M*, both of which may be finals, as the melos swings up or down from one to the other *I.M.I.* or *M.I.M.*

[1] Gevaert, *Mel. Ant.*, p. 13, fn. 3. In quoting Plato, Aristides Quintilianus mentions his six ancient scales (p. 22M.) and indicates the additional degree in the Doristi which was known as Hyperhypate.

[2] *de Mus.*, Chap. 19, end, pp. 78–9, §§ 183–5 : ἐπὶ τῶν λοιπῶν τόνων.

FRAGMENT OF PINDAR'S FIRST PYTHIAN ODE

First published in Athanasius Kircher's 'Musurgia Universalis' Vol. I, pp. 541 and 542

That the Ode should be thus emphatically Hypophrygian is significant in relation to the text in praise of the Golden Kithara [1] of Apollo, for the

FIG. 78.—The Five Closes on the κῶλα. (Ratios by K. S.)

(1)		I πλο- 16	M κά- 18	I μιων 16		
	[(σύν) 15	I δι- 16	M κον 18	I Μου- 16	Θ σᾶν 15	] not a close, but used as a melodic figure
(2)	Θ ἀ- 15	Γ γλα- 13	M ἰας 18	I ἀρ 16	M χα 18	Note the drop from the Γ 13 to the Tonic M 18 which introduces the close

(3) In Part II ' Chorus Instrumentalis '

	M ⊓ φρωι- 18	I ꝑ μέ- 16	M ⊓ ον 18
(4)	I ꝑ ζο- 16	M ⊓ μὲ- 18	I ꝑ να 16
(5)	M ⊓ σβεν- 18	I ꝑ νύ- 16	M ⊓ εις 18

(4): *See* (6) *below.*

N.B.—The Orthography and accents are Kircher's.

(6) in the middle of the line a close differing in form thus :

M ⊓ τεύ- 18	I ꝑ χης 16	I ꝑ ἐ 16

[1] It is regrettable to find the Kithara so often mistranslated *Lyre* ; the lyre was the instrument of Hermes, the Kithara of Apollo. They are two different instruments ; differing widely in structure, resonance and functions. The Kithara, the instrument of the Kitharoedos, was the ancient prototype of the violin family, having a box-shaped soundchest with separate ribs. It evolved through the Rotta, the Guitar, both plucked and bowed, and the Viols. The Lyre had a hollow bowl-shaped back over which was stretched a skin, it was the instrument of the amateur and student, its resonance was feeble. From the lyre the noblest specimen evolved was the lute family. The rebab tribe, hybrids, to which the bow was applied in spite of the unsuitable structure of the soundchest of the rebab and rebec, had but a transitory existence in the West. For further details see *The Precursors of the Violin Family*, Vol. ii of *The Instruments of the Orchestra*, by Kathleen Schlesinger (W. Reeves, 1910) (Out of print).

Hypophrygian and Hypodorian Species were known to be favourite Harmoniai with the Kitharoedes.[1]

As the rhythmical element in the Ode is determined by the scansion, so there are several interpretations of the music of this fragment, viz. by Boeckh,[2] Fétis, Westphal, Gevaert and others, all differing widely from Kircher's.

Of these Gevaert's has been chosen (and I have added the symbols above the staff) on account of his special qualifications, not only as an authority on Greek music, but also on the music of the Liturgy of the West. Concerning Pindar's Ode, Gevaert states that a careful examination of the music revealed no peculiarity which could lead us to believe that the Ode was a seventeenth-century fabrication. Judged solely on its merits, this fragment of Ancient Greek music, inspired by the invocation to the Kithara of Apollo with which the Ode opens, is a perfect model—as it stands in the Greek version—(on p. 541) of composition based on the Harmonia. In the *editio princeps* of Kircher, the music is noted in the Lydian Tonos; the species as Hypophrygian correctly placed in the Tonos, with the Tonic on Lichanos Meson, and the keynote on Mese. Those two Hypophrygian pivots of melody, moreover, are both given the paramount importance they bear in the Harmonia, not only as final cadence, but occurring five times as closes on the κῶλα. All the characteristic elements of the Harmonia are exhibited in the melos—although they have gone wrong in lines 4 and 5 of the alleged forgery—and in spite of the fact that these cardinal points of the Harmonia had not found their way into the theory of music at that date or since.

Regarded dispassionately by an unprejudiced critic, familiar with the system of Greek music and its Notation, what would the verdict be on the respective merits, as original and interpretation, of versions *A* and *B*? (Kircher's Mus., pp. 541 and 542). My conviction that Pindar's Ode is a genuine piece of Greek music is further strengthened by the fact that the three Mesomedes Hymns all exhibit in their composition the characteristic features of the Hypophrygian Harmonia by closes, and in the melodic elements and figures. It is not a question here of borrowing or copying, nor is there actual resemblance between the melodies themselves; it is the basic principles in operation which are the same. In the Christian Hymn from the Treasure of the Oxyrhynchus and in the Schubart Berlin fragments, the Hypophrygian characteristics still dominate. In the Delphic Hymns the canons of the Harmonia still prevail, although the Mode here is Hypolydian in No. 1 and Dorian in No. 2. Extracts from all of these follow in due course to enable these melodic elements and modal features to tell their own story.

[1] Ps-Aristot., *Probl.*, xix, 30 and 48; J. Pollux, *Onom.*, iv. 64.

[2] *Boeckh, ap. Gesch. d. Mus.*, A. W. Ambros, Vol. i, 2nd edition (Leipzig, 1880), pp. 448–9; with Greek symbols and interpretation in staff notation. *Fétis*, ibid., p. 543, with scansion symbols but no Greek signs of notation. F. A. Gevaert, *La Mél. Ant.* (Gand, 1895), pp. 32–3, fn. 4, and pp. 48–9. Staff notation without symbols and Ode transposed to *A*, Westphal, *Griech. Metrik*, Rossbach and Westphal, Vol. i, pp. 626–47. Reproduced also by Carl Lang, *Kurzer Überblick über die Altgriech. Harmonik* (Heidelberg, 1872).

FIG. 79.—Fragment of Pindar's First Pythian Ode. (Symbols of Notation added by K. S., HYPOPHRYGIAN HARMONIA)

Range of Notes in the LYDIAN TONOS with the Symbols of Vocal and Instrumental Notation and the Modal Ratios by K.S.						
	P	M	I	Θ	Γ	⊔
	Ս	Π	<	V	N	Z
	20	18	16	15	13	12

(Adapted from Gevaert's version)

I see here, plainly exhibited evidences of the grip of the proportional sequences of the Harmonia upon the musical consciousness of the Greeks. What we retrospectively pronounce to be canons of composition, may after all be merely illustrations of the transmission, during inspiration, of the psychological effect of the *Ethos* of the Harmonia.

Before we pass on to consider other extant fragments of Greek music, the contribution made to the Ancient Greek foundations of Folk Music by the unique Elgin Auloi preserved in the Graeco-Roman Department of the British Museum may be recalled (see Chap. x, Plate No. 17), The two pipes of Sycamore, one straight, the other slightly curved, each have six fingerholes bored at approximately equal distances. When played by means of primitive, untreated double-reed mouthpieces, the straight specimen plays from the first hole, used as vent, the Dorian scale of M.D. 11 and of ratios 11/11, 10, 9, 8, 7, 6. Several mouthpieces, inserted at the same amount of extrusion from the resonator, have played the scale on a fundamental $C = 128$ v.p.s. over a period of many years without variation in intonation. Other mouthpieces of the same type, when the amount of extrusion is altered, play from the same fingerholes the Phrygian scale of M.D. 12; the Lydian of M.D. 13 and the Hypolydian of M.D. 10.

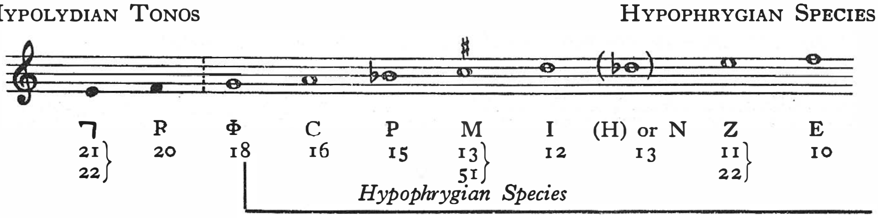
The curved Aulos plays with mouthpieces of the same type in the Lydian Harmonia of M.D. 13 and in the Mixolydian of M.D. 14. The performance of these Auloi, with dated tests, may be consulted in their Records.

THE THREE HYMNS ATTRIBUTED TO MESOMEDES: 1ST HYMN 'TO THE MUSE', IN THE HYPOPHRYGIAN MODE

The Hymns of Mesomedes,[1] a Cretan poet and musician living in the reigns of Hadrian and Antonine, are fully authenticated in several codices.[2]

The first hymn 'To the Muse' is noted in the Hypolydian Tonos in the following range of symbols. The species is Hypophrygian.

HYPOLYDIAN TONOS HYPOPHRYGIAN SPECIES



The same symbols appear also in the Lydian Tonos, in which they bear the ratios:

28	27	24	21	20	18	16	29(H)	28	27

of the Phrygian Species.

[1] C. von Jan, *Mus. Script Gr.* (Lips, 1895), pp. 455 sqq., who quotes Suidas, Eusebius ii, 2160, and Hieronymus, Antonini anno. vii.

[2] The Hymns with mus. notation are given in several codices; the earliest *Ven.*, vi, 10, §§ xii and xiii; in Neap., iii, Chap. 4; *Mon.*, 215; Par. 2532, 69. They are published in V. Galilei, *Dial. della Mus. Antica* (Fiorenza, 1581), with Greek mus. notation; see Jan, *op. cit.*, p. 457, for full list, Bellermann; Westphal, Gevaert, &c.

There are in this hymn five strictly Hypophrygian closes (familiar from Pindar's Ode) which would be meaningless in the Phrygian Species of the Lydian Tonos, viz.

FIG. 80.—From 'The Hymn to the Muse,' '*Εἰς Μοῦσαν*' by Mesomedes. Five Hypophrygian closes. (Ratios by K. S.)

line 1

Φ	Φ	Φ	C	C
μοῦ -	*σά*	*μοι*	*φί -*	*λη*
18	18	18	16	16

line 4

	Φ	C	PMΦ	C
φρέ -	*νας*	*δο —*	*νεί -*	*τω*
	18	16	15 13 18	16

(Use of modal tetrachord with close, see also ll. 5 and 7.)

line 5

C	P	M	C	C	Φ	C
καλ -	*λι -*	*ό -*	*πει -*	*α*	*σο -*	*φὰ*
16	15	13	16	16	18	16

line 6

Φ	C	C	C	C	C	⅂	R	Φ
μου -	*σῶν*	*προ -*	*καθ -*	*α -*	*γέ -*	*τι*	*τερ -*	*πνῶν*
18	16	16	16	16	16	21	20	18

line 9

M	Z	M	I	Φ	C	C
Εὐ -	*με -*	*νεῖς*	*πάρ -*	*ε -*	*στέ*	*μοι*
13	21	13	12	18	16	16

3/2 Final Cadence

This Hymn reveals once again, as in Pindar's Ode, the melodic structure based upon the Canons of the Harmonia. The characteristic pentachord ascending from the Tonic $\left|\begin{array}{ccccc}\Phi & C & P & M & I \\ 18 & 16 & 15 & 13 & 12\end{array}\right|$ in line 7, with the striking augmented 2nd 15/13 Greek symbols of Notation P and M (rho and mu) is seen in lines 4 and 8; the distinctive leap down from $\left|\begin{array}{c}M \\ 13\end{array}\right|$ to $\left|\begin{array}{c}\Phi \\ 18\end{array}\right|$ in line 9 leads to the final cadence.

THE SECOND HYMN: 'TO THE SUN', *Εἰς Ἥλιον*, IN THE HYPOPHRYGIAN SPECIES

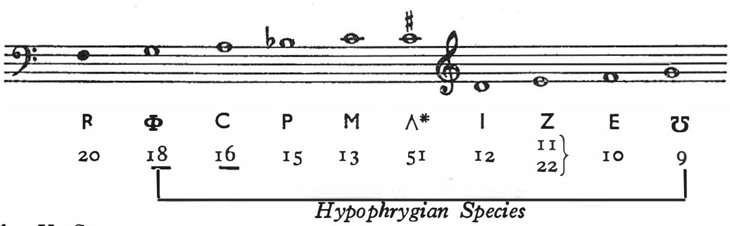
'The Hymn to the Sun' has the same range of notes and must also be read in the Hypolydian Tonos; the species is again Hypophrygian.

At line 7, the music sets in; it is of an ambitious and exalted character and makes frequent use of the second tetrachord, so that the *tessitura* is high almost throughout. Hypophrygian closes occur at the beginning, line 1; and at the end, line 25, as well as in various other parts of the Hymn. Full use is made melodically of the rise through the first tetrachord of the Harmonia to the raised 4th Φ 18, C 16, P 15, M 13 in lines 10, 11, 16, 23, 24, 25. When read in the Lydian Tonos in the Phrygian Species, a modulation into the Hypophrygian seems to be indicated, and it is not impossible that the choice of the Mode may have been left to the singer, when the Notation bears a double interpretation.

THE THIRD HYMN: 'TO NEMESIS', *Εἰς Νέμεσιν*, IN THE HYPOPHRYGIAN SPECIES

The Hymn to Nemesis has the following scale of notes read in the Hypolydian Tonos in the Hypophrygian Species.

HYPOLYDIAN TONOS HYPOPHRYGIAN SPECIES



Ratios by K. S.

* The symbol $\boxed{\text{Λ 51}}$, an enharmonic note, upward straining from $\boxed{\text{M 13}}$, occurs three times only in lines 3, 10 and 13.

As in the two preceding examples, the Hypophrygian closes on $\boxed{\text{Φ 18}}$ $\boxed{\text{C 16}}$ and the other characteristic intervals and figures proper to the Harmonia abound, more especially in lines 1, 2, 5, 6, 7, 9, 11, 12, 15.

The melos breaks off before the final cadence is reached. Thus the same principles seem to be active in the development of the melopoiia in all four examples.

THE EPITAPH OF SICILUS [1] IN THE HYPOPHRYGIAN OR PHRYGIAN SPECIES

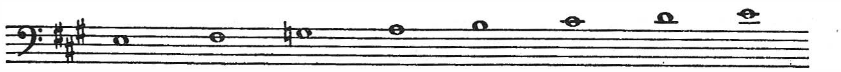
This little melody, discovered in the inscription to Seikilos by Mr. W. M. Ramsay at Tralles (Aidin), in Asia Minor, is a very short, lively song, which may be read in the Ionian and in the Hypoionian Tonos; the latter gives the better result in conformity with the close which is Hypophrygian. There are slight irregularities in the melos, due to the Tonoi of the Ionian group to which the Notation belongs. This group represents the newest additions to the system of the Tonoi—repudiated by Ptolemy as foreign to the seven original Harmoniai. The Hypophrygian Species of the song in the Hypoionian Tonos, has a Perfect 4th of ratio

[1] C. v. Jan, *op. cit.*, pp. 450–3, 'Sicili Epitaphium', song with symbols and interpretation in staff notation. *Bulletin de correspondence Hellénique*, vii, p. 277; Crusius in *Philologus*, lii (n.f. vi), p. 160; Bellermann, Monro, Gevaert, &c.

36/27 on Paranete Synemmenon, which has been substituted for the tense raised 4th of ratio 18/13, proper to the Harmonia. The melos bears evidence that the Perfect 4th on the Tonic was intended, thus indicating a weakening of the feeling for the Ethos of the Harmonia.

FIG. 81.—The Scale of the Epitaph to Sicilus

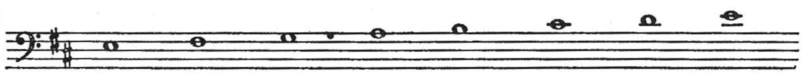
HYPOPHRYGIAN SPECIES — HYPOIONIAN TONOS



	⌉	X	Φ	C	O	K	I	Z
Denominator constant	36/36 TONIC	32 KEYNOTE	30	27	24	21	20	18

(27 to 18: opening leap of a 5th)

PHRYGIAN SPECIES — IONIAN TONOS



⌉	X	Φ	C	O	K	I	Z
24/24 TONIC	22 / 21	20	18	$\boxed{16}$ KEYNOTE	28 / 14	27/48	24 / 12

(18 to 24/12: opening leap of a 5th)

FRAGMENT FROM THE *ORESTES* OF EURIPIDES IN THE HYPOPHRYGIAN SPECIES, LINES 338–44

This fragment read in the Hypolydian Tonos has a Hypophrygian close used melodically on lines 1 and 8, viz.

		μαι	*μα-*	*τέ*	
line 1		C	P	Φ	
		32	30	36	
	τι-	*νά-(ξας)*		[*δαί-*	*μων*]
line 8	Φ	C	Π	P	C
	18	16	29	30	32 / 16

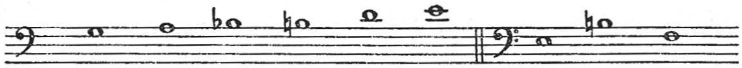
and possibly also in the final cadence, if Φ be admitted in the last line on

κύ —	*μα* —	*σιν*
Φ	Φ	Φ
18	18	18

(Crusius restoration)

Scale of the Notes used in the Orestes Fragment

Hypophrygian Species — Hypolydian Tonos



Φ C P Π I Z ˥ ȷ ⌐˥_ (vocal = Ꝛ)

36 32 30 29 24 21 (*Doubtful instrumental symbols*)

(Ꝛ)

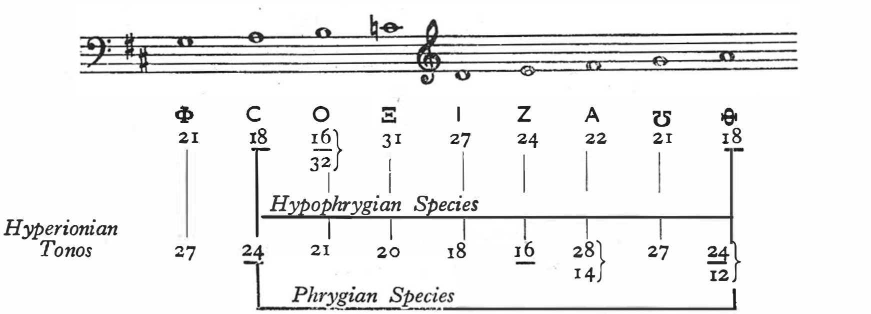
This form ⌐˥_ [1] may be the instrumental symbol L with the rhythmical ∸ added =⌐˥_ and misunderstood by a Scribe (K. S.)

THE PAEAN FRAGMENT NO. I IN THE HYPOPHRYGIAN OR PHRYGIAN SPECIES. BERLIN PAPYRUS DISCOVERED BY PROF. DR. W. SCHUBART, 1918

These Berlin fragments [2] (date *c*. A.D. 156) yield still more examples of the use of these favourite Hypophrygian and Phrygian Harmoniai ; the first, the Paean alone will be quoted owing to exigencies of space ; it occupies no less than 12 lines. Fragment 1, read in the Ionian Tonos, has Hypophrygian closes in lines 1–2 ; in lines 6, 7, 9 and in the final cadence, all descending from the keynote to the Tonic. Read in the Hyperionian

Range of the Paean

Ionian Tonos. — *Hypophrygian Species* — *The Scale.*



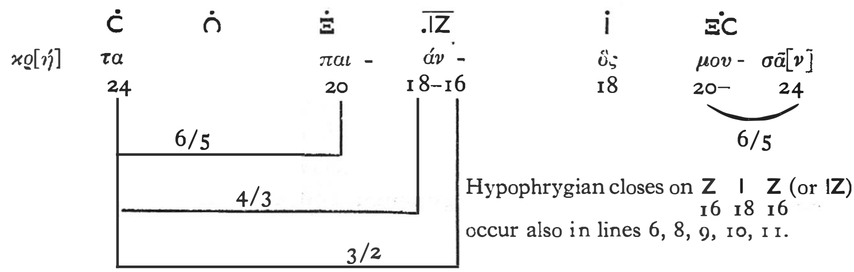
[1] C. v. Jan, *op. cit.*, pp. 427–31 ; Jan suggests that the symbols ȷ ˥ Ɔ placed in the text before and after δ(εινῶν πόνων) refer to the entry of the Auloi at this point. But with regard to the sign ⌐˥_ which occurs at the end of every dochmius in the papyrus (in lines 1, 3, 5, 7), it appears to me that the sign is meant for the instrumental (Ꝛ) L with ∸ added to the left of the vertical limb, viz. ⌐˥_.

For Crusius restorations, see *Philol.*, lii, p. 183, and liii, p. 148 ; J. F. Mountford, ‘ Greek Music in Papyri and Inscriptions ’, in *New Chapters in Greek Literature*, second series, 1930.

[2] *Sitz-berichte K. Preuss. Akad. d. Wiss.*, 1918, xxxvi, pp. 763 sqq. : also published with musical interpretation by Théodore Reinach, ‘ Nouveaux Fragments de Musique Grecque ’, *Rev. Archéol.*, 1919, Tome x, pp. 11–27. Hermann Abert, ‘ Der neue griech. Papyrus ’, *Arch. f. Mus. Wiss.* (Jany., 1919), Heft 2, Leipz. Breitk. & Härtel ; with facsimile of Papyrus and interpretation, pp. 313 sqq. *Der Berliner Notenpapyrus, nebst Untersuchungen z. rhythmischen Notierung und Theorie* ’, mit Tafel-Beilage, von Rudolf Wagner (Fürth). J.F.M. *op. cit.* in fn. 1.

Tonos, the Harmonia is Phrygian with cadences; some of the closes are Hypophrygian; the end of the Paean is missing. A typically Phrygian phrase

line 5



occurs, with the minor 3rd 6/5 on the Tonic, the 4th and 5th on the Tonic and the return by way of the minor 3rd. The Hypophrygian close 18, 16, 18, is here a natural outcome of the Phrygian Harmonia. The Phrygian close proper, an interrupted leap to the dominant, occurs as C IZ Z (24 18 16) on *τὸν Δά-λον* in line 2; and in line 11, Ż ϴ (32 24). Nevertheless the evidence, in favour of a reading in the Ionian Tonos, Hypophrygian Species, is the stronger: the alternative is the suggestion that the choice lay with the singer.

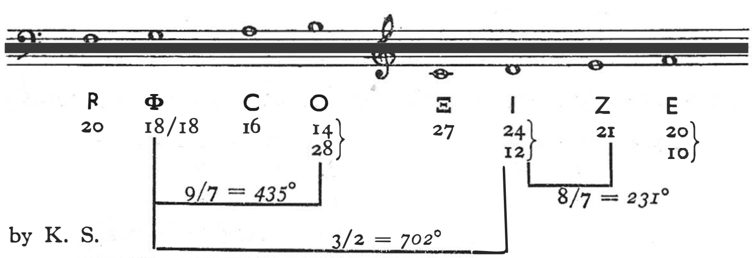
* * *

DISCOVERY OF A CHRISTIAN HYMN (IN OXYRHYNCHUS PAPYRI): PSEUDO-HYPOPHRYGIAN SPECIES

The latest accession [1] to our very meagre survivals of Ancient Greek musical composition (with the exception of one very small fragment [2]) is the Christian Hymn from the Treasure of the Oxyrhynchus, published

THE RANGE OF NOTES (Christian Hymn)

HYPOPHRYGIAN MODE HYPOLYDIAN TONOS



Ratios by K. S.

[1] *Oxyrhynchus Papyri*, by Prof. Bernard Grenfall and Prof. Arthur Hunt, Vol. xv, No. 1786 (Oxford, 1921). Interpretation of the music, and transcription in staff notation by Prof. H. Stuart Jones, photograph, facsimile.

Théodore Reinach,' Un Ancêtre de la Musique d'Eglise ', in *La Revue Musicale* (Paris, 1922), third year, No. 9, pp. 8–25, with musical transcription, but without symbols.

[2] ' A new Fragment of Greek Music in Cairo ', by Prof. J. F. Mountford, *Jour. of Hellenic Studies*, Vol. li, 1931, pp. 91–100, with facsimile photograph. This is an account of the most recent discovery, a mere scrap (No. 59533 in the *Catalogue Géneral* of the Cairo Museum), unfortunately too small to reveal the modality.

by Professor Arthur Hunt of Oxford ; the musical transcription by Professor H. Stuart Jones. The fragment of papyrus, 0·30 m. × 0·05 m., dates from the second half of the third century A.D. The symbols of notation (given below with scale) are from the Hypolydian Tonos.

The Mode is once again Hypophrygian, but of the bastard form of the Harmonia, beginning on Lichanos Meson, and omitting Synemmenon, it passes over from Mese to the Diezeugmenon tetrachord. This form denatures the Harmonia, giving it a Perfect 4th on the Tonic instead of the striking raised 4th of ratio 18/13 = *561 cents*; a Septimal tone 8/7 = *231 cents*, instead of the Augmented 2nd of ratio 15/13 = *247 cents* and a 9/7 3rd of *435 cents* instead of a Minor 3rd 6/5 of *316 cents.*

There are four Hypophrygian closes :

Φ	C	Φ	and	C	Φ	C
$\underline{18}$	$\underline{16}$	$\underline{18}$		$\underline{16}$	$\underline{18}$	$\underline{16}$

besides frequent melodic use of the intervals of the close. The three leaps to the 4th have lost their characteristic ethos, and one misses the interval 15/13. The melodic structure, however, still follows the canons of the Harmonia as displayed in the prototype : it is the scale that has been changed, as revealed by the Notation.

THE TWO DELPHIC HYMNS : DELPHIC HYMN NO. I

With the Delphic Hymns—the date of which has been assigned by MM. Weil and Reinach[1] to the end of the second century B.C.—the melos breaks fresh ground when regarded retrospectively from the limited range of extant fragments. The composition of the Hymn No. I is ambitious, one of the longest we possess, and the end has not survived. The Harmonia is Hypolydian noted in the Phrygian Tonos. There are in the Hymn some curious features ; a slight chromaticism is noticeable.

Whereas from bar 6 to bar 24 the tritonic 4th is greatly in evidence, e.g. in

Φ	I	M	I	M	Y	M
27	$\underline{28}$	$\boxed{32}$	28	32	40	$\boxed{32}$
			Tritone			

(Ratio 28 marks the upper note of the Tritone)

and in many other phrases, yet the perfect 4th, marked by ratio $\underline{\underline{30}}$, is introduced in the chromatic figures from bar 36 to bar 62 ; e.g. in

K	Λ	M	O
29	30	32	35

and the full tetrachord with perfect fourth is used melodically thus, bars 50–51, &c.

Y	O	M	Λ	M	O	Y
40	35	32	$\underline{\underline{30}}$	32	35	40

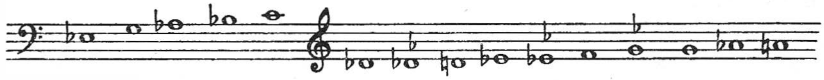
4/3 = *498°* 4/3

5/4 = *386°*

[1] *Bull. de Corresp. Hellénique*, xviii, ' La Musique du Nouvel Hymne de Delphes ', by H. Weil and Théodore Reinach, pp. 363 sqq., with plates.

THE SCALE OF HYMN NO. I [1]

PHRYGIAN TONOS | HYPOLYDIAN SPECIES



F	Φ	Y	O	M	Λ	K	I	Θ	H	Γ	(B)	Ʊ	Ѫ	Ж
27	21	20 / 40	35	32	30	29	28	27	26	24	22 / 23	21	20	39

Ratios by K. S.

There are no less than 10 Hypolydian closes of the major 3rd on the Tonic, Y M or M Y (20—16, 16—20). At a certain point towards the middle, a subtle Dorian atmosphere pervades the melos, heralded by the advent of three new symbols $\boxed{\begin{matrix}O\\35\end{matrix}}$ $\boxed{\begin{matrix}K\\29\end{matrix}}$ $\boxed{\begin{matrix}B\\22\\23\end{matrix}}$, used in various combinations (K and O from bar 38; B at bar 42); $\boxed{\begin{matrix}O\\35\end{matrix}}$ and $\boxed{\begin{matrix}B\\22\\23\end{matrix}}$ are not given by Alypius in the Phrygian Tonos, but the alphabetical sequence of the vocal notation gives the clue to the ratios alloted to those letters. $\boxed{\begin{matrix}B\\22\\23\end{matrix}}$ is used once only. An investigation of these ratios, as used in the Hymn, explains this reminiscent change of ethos. The symbol $\boxed{\begin{matrix}O\\35\end{matrix}}$ occurs always in company with the keynote $\boxed{\begin{matrix}M\\32\end{matrix}}$, and both this interval, and also $\boxed{\begin{matrix}M\\32\end{matrix}}$ $\boxed{\begin{matrix}K\\29\end{matrix}}$, are very close approximates of the characteristic Dorian ratio 11/10 on the Tonic [2]; which means that a singer familiar with the Dorian musical idiom, would probably sing all three intervals as 11/10. After the entry of these symbols, the Hypolydian closes are found to be *Dorianized* and now assume the forms

M O Y O M (*at bar 60 to 62 and bar 52*)
32 35 40 35 32

11/10 × 8/7 = 5/4   11/10 = *165°*

M Y O M (*at bar 56*)
32 40 35 32

5/4   8/7   11/10

[1] K. v. Jan, *op. cit.*, pp. 433 sqq., and *Musici Scriptores Graeci*, 'Supplementum: Melodiarum Reliquiae', Lips., 1899, pp. 12–19. The Hymn consists of 30 lines of text and 28 of music in staff notation in the new recension of the Supplement published in 1899.

[2] $$\frac{32}{29} \times \frac{10}{11} = \frac{320}{319} \text{ and } \frac{35}{32} \times \frac{10}{11} = \frac{175}{176}$$

In *cents* the differences are 11/10 = *165 cents*; 32/29 = *171 cents*; 35/32 = *155 cents*. A difference of *6 cents* = the difference between $\frac{286}{285}$ and of *10 cents* = $\frac{169}{168}$.

and these also occur thus :

(*bars 38 and 48*)			(*bar 60*)			
Λ	M	O	K	Λ	M	O
30	32	35 ;	29	30	32	35
16/15		11/10 ;		11/10		11/10

The extant fragments of Delphic Hymn No. 1 come to an abrupt end on bar 125 on note Ʊ (= *G*). There is no final close.

THE 2ND DELPHIC HYMN (IN THE DORIAN HARMONIA)

When compared with von Jan's[1] first publication in 1894, the 2nd Delphic Hymn, published in the Supplement of 1899, comprising the complete recension of all the Fragments, now consists of the old page 443 (bar 5 of Jan's No. 3) down to the last line of page 448, and includes the whole of pages 447 to 449. The revised arrangement numbers 168 bars with an epilogue (No. x, p. 32) of 14 bars, which are undoubtedly in the Dorian Harmonia. The Hymn is noted in the instrumental notation of the Lydian Tonos and the Harmonia is definitely Dorian.

The range of notes used in the Hymn is the following (with ratios added by K. S.) :

RANGE OF THE HYMN NOTED IN THE LYDIAN TONOS

DORIAN HARMONIA.

	HYPATON			MESON						DIEZEUG.			H'RBOL
	HYP.	LICH.	HYP.	PH.	LICH. ENH.	LICH. DIAT.	MESE	TR. SYN.	PM.	TRITE	PN.	NETE	TRITE
Original	Γ	F	C	U	C	[K*⊻]	<	V	⊏	⊔	Z	Ϻ	⊬
Harmonia	28	24	22	20	{39 / 44	{18 / 36 {35 / 44	16	15	14	{27 / 44	12	11	10
Later form in the P.I.S.	*28*	*24*	*21*	*20*	*39*		*16*	*15*	*14*	*27/44*	*12*	*21*	*10*

* *These two symbols* K *and* ⊻ *do not belong to the Lydian Tonos of Alypius.*

The Dorian Harmonia is used with the full range of an octave and one note, and with a descent to the Hypaton Tetrachord, and Trite Synemmenon and Trite Diezeugmenon both come into play. It will be noticed that owing to the extensive use of the upper Dorian tetrachord ⊏ 28 ⊔ 27 Z 24 Ϻ 22 the Hymn has a high *tessitura*.

Some scholars might feel inclined to diagnose the Mode as Mixolydian, but a cogent objection to that view is raised by the leap from the Dorian Tonic to the raised 4th of the keynote, which occurs four times in the last

[1] *C. Janus, Mus. Script. Gr.*: *Supplementum, Melodiarum Reliquae*, Teubner Series (Lips., 1899). This Supplement contains the complete revised arrangement of the Fragments, which agrees with the official version published in the *Fouilles de Delphes*, and in the *Bull. de Corresp. Hellénique*, xviii, 1894, p. 345.

fragment of the Hymn, p. 32, x, from bars 6 to 7; in bar 9; and in bars 12 to 14:

CLOSES IN THE SECOND DELPHIC HYMN

bars 6 to 7	C	∪	<	V	<
	κυ-	δίστα	καὶ	ναέτας	Δελφῶν
Ratios by K. S.	22/22	20	16	15	16

bar 9		C	<	∪	C
	(συμ)	βίοις	δώμα-	σιν	ἀ
Ratios		22/22	16	20	22

bars 12 to 14	<	∪	C	∪	F	∪	C	<
	Ρωμαίων		ἀρχὰν	αὔξετ'		ἀγη-	ράτῳ	θάλλ. . .
	16	20	22	20	24	20	22	16

Final close	. . .	∪	C
	(φερε)	νί-	καν
		20	22

Examples of the use of the upper tetrachord (Diezeugmenon) may also be cited: (p. 30, frags. ix.)

bar 144	⊔	C	⊏	⊔ C		C	bars *148–9*	(⊏)	⊔	<	⊏	⊔
	. . . ηρα		κατε-	κτ . . .		ος		(σ)	νύ	ριγμ'	ἀπ'	εὐ
	27/44	22	14	27/44	22	22		(28)	27/44	16	14	27/44

bar 158		(Z) Ι	Ϻ	⊦	Ϻ	⊔	Ι	Ϻ	⊦
	(βάρ)	βαρ-	ος	ἄ-	ρης	ὅτε(τε)	ὀμ	μαντός . . .	
		12	11	10	11	27/44	12	11	10

bar 164											
. . .	(<)	⊔	⊏	⊔	Ι	⊔	Ι	⊦	Ϻ	
θὲς	λη-	ζό-	με-	νος	ὤ-	λεθ	ὑγ	ρᾶ	χι(ονος)	
(28 or 14)	({32 16})	27/44	28/44	27/44	24/12	27/44	12	10	11	Tetr. Diez.

At the end of the Hymn (Fragment x, p. 32), there is a modulation to the Hypolydian, and back to a final close in the Dorian Harmonia (see above, bars 9 to 14) and preceded by:

HYPOLYDIAN CLOSES

p. 32, Fragment x, *bars 3 and 4*

∪	< ⊏		⊔	<	∪	<	V
τε	θεὰ		τόξ-	ων	δέσποτι κρησί-		ων
20	16	14	27/44	16	20	16	15

tritone *maj. 3rd* *perfect 4th*

bar 11

	<		∪	<
προς-	πόλοισι,	τάν	τε	δορί-
	16		20	16

maj. 3rd

continue from bars 12 to 14 as given above to close

Thus while the second Delphic Hymn is emphatically Dorian in conception, as revealed by the closes and the *tessitura*, the first Hymn exhibits the strongly marked Hypolydian characteristic features (pervaded at times by a suggestive Dorian influence), which later spread over Hellenistic Asia and the Alexandrian musical culture : witness the Enharmonic and Chromatic tetrachords of Eratosthenes

$$\frac{40}{39} \times \frac{39}{38} \times \frac{19}{15} \text{ and } \frac{20}{19} \times \frac{19}{18} \times \frac{18}{15}$$

The Hypolydian Harmonia is traced likewise in Arabian sources, as in the frets on the neck of Al-Fārābī's Tanbur of Bagdad and on his Aulos with nine equidistant fingerholes.

Moreover, the lute with the accordance of Ishāq al-Mausili also supports the Hypolydian modality (see Chap. vii, and Figs. 59 and 55).

From the same source may be quoted an Aulos with nine fingerholes, on which we have traced a Hypophrygian Species as a *mājra* through *Wosta.*

On another Aulos the Lydian Harmonia of M.D. 26 speaks from Hole 1 and from exit ratio 27 ; a lute having a fret for the *Wosta of Zālzāl*, an interval of ratio 27/22 = *355 cents*, is also illustrated by Al-Fārābī. The frets allow the playing of the Lydian scale implied by Zālzāl's ratios.

* * *

THE PROTOTYPES OF GREEK MUSIC ALL EXHIBIT THE MODAL CHARACTERISTICS OF THE HARMONIA

On reviewing our survey of the prototypes of Greek music, it is a remarkable circumstance to find such a large proportion of the Fragments in the Hypophrygian Harmonia ; of this there is no possible doubt, for they all exhibit in the melos the striking characteristics of that Harmonia.

It is not intended to suggest, however, that musical composition in the Harmonia was subject to rigid rules, but rather that since the intervals are integral parts of the Harmonia—which was the language of music used by the Harmonists—the melos became naturally imbued with these salient points.

In my opinion, the evidence is overwhelming in support of the authenticity of the Fragment of Pindar's First Pythian Ode, judged solely on the merits of the music, as a piece of Ancient Greek melopoiia. It is doubtful whether any chant, Gregorian or other, that might have been produced by Kircher from any contemporary source whatever, could be brought forward and proved to exhibit the unmistakable characteristics of the Hypophrygian Harmonia, which occur so abundantly in the Ode, and in the other fragments derived from fully authenticated sources. Not a single chant has actually been brought forward to my knowledge, by those who impugn the authenticity of the document, that presents the remotest resemblance to the style of composition exhibited in Pindar's Ode.

It is well to remember that the melopoiia of the Harmonia was unknown in Kircher's day, and that those who have contested the genuineness of the

Fragment are unacquainted with the very existence of the Harmonia in our sense of the word.

The question may now be raised whether the modal pivots and characteristic intervals of the ancient Harmoniai are not pure concerns of the

FIG. 82.—Second Step in the Heptatonic Scale of the Harmonia on C = 128 v.p.s.

Harmonia and Modal Determinant	v.p.s.	Ratio	*Cents*	
DORIAN M.D. 11	*C* to *D*♭ 140·8	11/10	*165°*	Suggested Spondeiasmos
PHRYGIAN M.D. 12	*C* to *D*♭ 139·63	12/11	*151°*	Three-quarter tone
LYDIAN (1) M.D. 13	*C* to *D*♭ 138·66	13/12	*138·5°*	The Harmonia
LYDIAN (2) M.D. 27	144	27/24 9/8	*204°*	The bastard Harmonia in the P.I.S. from Parh. Hypaton to Lich. Hyp.
MIXOLYDIAN M.D. 14	*C* to *D*♭♭ 137·84	14/13	*128°*	
HYPODORIAN (1) M.D. 16	*C* to *D*♭ 136·53	16/15	*112°*	The Harmonia from Mese through Synemmenon
HYPODORIAN (2) M.D. 8	146·28	8/7	*231°*	No. 2. The bastard Hypodorian from Proslambanomenos to Hyp. Hypaton; or the primitive modal flute with 3 holes; ratios: 8/8, 7, 6, 5
HYPOPHRYGIAN M.D. 18	*C* to *D* 144	18/16 (9/8)	*204°*	
HYPOLYDIAN M.D. 20	*C* to *D* 142·22	20/18 (10/9)	*182°*	

theorist, arising out of the systematized Harmonia of Greek music, and therefore consciously developed into canons of composition in the school of the Harmonists, and passed on thence with the Hellenistic culture to other lands and races; in fact, that where these modal characteristics are observed in the music of the Folk, it is a case of survival by tradition.

The alternative conception is that these modal distinctions are inherent in the essential nature of the Harmonia : that they manifest in the musician, who is alive to the potency of the Harmonia, as a subconscious compelling urge.

The answer to this question will be found in the analysis of the music

FIG. 83.—The Augmented Second of Ratio 15/13 = *247° Cents* in Folk Music, on Tonic *C* = 128 v.p.s. ; on Tonic *G* = 192 v.p.s.

	On Tonic *C*	On Tonic *G*	The Ratios of the Harmonia
DORIAN Conjunct M.D. 22	g to $\overset{\sharp}{a}$ 187·7 to 216·6 v.p.s.	$\overset{\flat}{d}$ to $\overset{\sharp}{e}$ 140·8 to 162·4 v.p.s.	22/22 20 18 16 15 13 12 11
PHRYGIAN Conjunct M.D. 24	$a\flat$ to $\overset{\flat}{b}$ 204·8 to 236·4 v.p.s.	$\overset{\flat}{e}$ to $f\sharp$ 153·6 to 177·2 v.p.s.	24/24 22 20 18 16 15 13 12
LYDIAN Disjunct M.D. 26	$\overset{\flat}{B}\flat$ to C 221·8 to 256 v.p.s.	$\overset{\flat}{F}$ to G 166·4 to 192 v.p.s.	26/26 24 22 20 18 16 15 13
HYPODORIAN M.D. 16	$D\flat$ to $\overset{\flat}{E}$ 136·53 to 157·5 v.p.s.	$A\flat$ to $\overset{\flat}{B}$ 204·8 to 236·2 v.p.s.	16/16 15 13 12 11 10 9 8
HYPOPHRYGIAN M.D. 18	$E\flat$ to $\overset{\sharp}{F}$ 153·6 to 177·23 v.p.s.	$B\flat$ to $\overset{\sharp}{C}$ 230·4 to 265·8 v.p.s.	18/18 16 15 13 12 11 10 9
HYPOLYDIAN M.D. 20	F to $\overset{\sharp}{G}$ 170·6 to 196·9 v.p.s.	C to $\overset{\sharp}{D}$ 128 to 147·7 v.p.s.	20/20 18 16 15 13 12 11 10

Obviously the Mixolydian Harmonia cannot have an interval of ratio 15/13, since its M.D. is 14 and that in the diatonic genus 15 and 14 are alternatives.

N.B.—In the ratios of the Harmonia, the denominator is constant, and according to the practice in this work, the numerators appear as ratios with the denominator understood. Each Harmonia has its own characteristic genesis by virtue of a principle common to the Seven Harmoniai. The vibration frequencies, ratios and *cents* result from the operation of the principle (see Chap. i).

of unsophisticated folk, who can have had no theoretical knowledge of the Harmonia, or of the traditions of the musical system of Ancient Greece. The most decisive of all answers is the finding of traces of the modal pivots in the music of primitive races.

THE CHARACTERISTIC INTERVALS FOUND IN FOLK MUSIC DEFINED BY RATIOS, VIBRATION FREQUENCIES AND *CENTS*

Before we proceed to trace the Harmonia in the music of the Folk, the following intervals that characterize the ancient Modes may now be given in tabulated form to assist investigators in their diagnosis.

FIG. 84.—The Interval of the 3rd on the Tonic in Folk Music. Tonics C = 128 v.p.s. and G = 192 v.p.s.

Derivation of 3rd	Tonic C v.f.	Tonic G v.f.	Ratios and *Cents*
Derived from Cy. 5ths	$\overset{\sharp}{E}$ = 162	$\overset{\sharp}{B}$ = 243	81/64 = *408°* ditone
Derived from Cy. 4ths	$E\flat$ = 151·7	$B\flat$ = 227·4	32/27 = *294°*
DORIAN HARMONIA M.D. 11	$\overset{\sharp}{E\flat}$ = 156·4	$\overset{\sharp}{B\flat}$ = 234·6	11/9 = *348°*
DORIAN HARMONIA M.D. 22	$\overset{\flat}{E\flat}$ = 148·2	$\overset{\flat}{B\flat}$ = 222·2	22/19 = *254°*
PHRYGIAN M.D. 12 HYPOPHRYGIAN M.D. 18 HARMONIAI	$E\flat$ = 153·6	$B\flat$ = 230·4	6/5 = *316°*
HYPODORIAN HARMONIA M.D. 16 between maj. and min.	$\overset{\flat}{E}$ = 157·5	$\overset{\flat}{B}$ = 236·2	16/13 = *359·18°*
LYDIAN HARMONIA M.D. 13 and 26	$E\flat$ = 151·26 $\overset{\flat}{E}$ = 158·5	$B\flat$ = 226·8 $\overset{\flat}{B}$ = 237·6	13/11 = *289·5°* 26/21 = *369·6°*
MIXOLYDIAN HARMONIA M.D. 14	$\overset{\flat}{E\flat}$ = 149·3	$\overset{\flat}{B\flat}$ = 224	14/12 (7/6) = *267°*
HYPOLYDIAN HARMONIA M.D. 10 and 20	E = 160 $E\flat$ = 150·6	B = 240 $B\flat$ = 225·8	5/4 = *386°* 20/17 = *281·3°*
NEO-LYDIAN M.D. 27	$\overset{\sharp}{E}$ = 164·57	$\overset{\sharp}{B}$ = 246·8	27/21 (9/7) = *435·5°* = 9/8 × 8/7
ARABIAN LUTE ZĀLZĀL'S WOSTA Neo-Lydian	$\overset{\flat}{E}$ = 157	$\overset{\flat}{B}$ = 235·6	27/22 = *355°*

There are on the Tonic at least eleven 3rds, major, minor and neuter, which may occur in Folk Music; they range between *254* and *435·5 cents*.

FIG. 85.—(A) Flat 4ths in Folk Music. (B) The Raised 4th in Folk Music

A

LYDIAN HARMONIA 13/10 on *C* 13/10 on *g* = 249·6 v.p.s.	♭ *F* 166·4 v.p.s.	*Cents* = *454·56*
MIXOLYDIAN HARMONIA 14/11 on *C* 14/11 on *G* = 244·36 v.p.s.	♭ *F* 162·9 v.p.s.	= *417·4*
LYDIAN TONOS } 21/16 on *C* Hyp. Mes. to Mese } on *G* 237·6 to 312 v.p.s.	*A* to *D* 158·5 to 208 v.p.s.	= *471*
DORIAN HARMONIA 22/17 on *C* 22/17 on *G* = 124·2 v.p.s.	♭ *F* 165·64 v.p.s.	= *446*

B

Origin and Genesis {Vibration frequencies in 4ths
Value in *cents* of interval on Tonic

Cycle of 5ths ascending: the 7th 5th from *C* = *f*♯ (or transposed to *g* = *C*♯)	*C* = 128 v.p.s. *f*♯ = 182·24 v.p.s.	*Cents* = *609·17*
DORIAN HARM. ratio 11/8 = ♯ on *G* = *C* of 132 v.p.s.	♯ *f* = 176 v.p.s.	= *551·26*
HYPOPHRYGIAN HARM. 18/13 18/13 on *G* = 132·9 v.p.s.	♯ *f* = 177·23 v.p.s.	= *561·23*
HYPOLYDIAN HARM. 10/7 } Tritone } on *G* 10/7 = 137 v.p.s.	*f*♯ = 182·85 v.p.s.	= *617·38*
{Diminished 5th } ♭ Cycle of 4ths } *g*♭, the 7th 4th The 4th on the Tonic is always perfect	♭ *g*♭ = 179·9* v.p.s.	= *596·90*

N.B.—In the absence of vibration frequencies in the notation of Folk Music, and of a statement of the method adopted in noting down the phonograms, it would be unsafe to attempt to determine the question of origin on this characteristic interval alone: the pentatonic and the extension to heptatonic should be consulted.

The modal keynote should be found strongly emphasized in modal tunes. The second tetrachord may provide the clue, e.g. in the Hypolydian Harmonia.

20 18 16 14 13 12 11 10

* In Phrygian 24/17 = 180·7.

Since the Ecclesiastical Modes derive from the later form of the Harmoniai one would expect to find in them some trace of the incidence of the two cardinal pivots.

The Tonic or Final, as distinctive feature of the species, with the dominant, i.e. the two pivots of the Phrygian Harmonia, loom large in the canons of the Ecclesiastical Modes; and rightly so, since the Modes owe their genesis to the change of modality which took place within the P.I.S., from Dorian to Phrygian (see Appendix), through the inclusion of Proslambanomenos [1] in the system of the tetrachords. This revolution came to pass in Hellenistic Asia, in the opening centuries of our era or shortly before.

How long the two melodic pivots continued to dominate the music of the Greek Church is a question that must be solved by more competent investigators.

THE MODAL CHARACTERISTICS OF THE HARMONIA IN EARLY MEDIEVAL CHANTS

Let us see how the modal characteristics enumerated above fare in early medieval chants. The few examples selected, regarded from the angle afforded by the Harmonia, throw a light upon the possibilities.

It is surprising to find in a ninth-century Latin treatise evidence of a practical knowledge of the Greek Musical System and its Notation applied to the music of the Liturgy. The writer, Hucbald, a travelled monk from the monastery of St. Amand, near Tournai, has not been at pains to furnish a mere *réchauffé* of the Ancient Greek lore of Boethius; his approach is from a different angle. His *De Harmonica Institutione* [2] is an exposition, according to his lights, of the theory underlying the Church music of his day, which he aptly illustrates, here and there, with superscript alphabetical Greek symbols belonging to the Lydian Tonos; or else in terms of the degrees of the P.I.S. In this way he establishes a correct relationship between the musical system of the Greek Theorists and the Ecclesiastical Modes in use in the Roman Liturgy, and he makes it quite clear besides that no other alternative musical system of Western origin had as yet emerged.

HUCBALD'S EVIDENCE

Hucbald, making a selection of Antiphons, groups them among the four authentic Modes, accompanied by the symbols of Greek Notation as seen below; this Notation has unfortunately only survived in the group of the Protus Authentus. The Tonic of this Mode (the *D* scale of the keyboard)

[1] Aristides Quintilianus is eloquent on the subject of the function of Proslambanomenos among the Ancients, as being outside the tetrachords of the P.I.S., and of having been included within them in his day. *Harm.*, p. 10 M, and note, p. 208 sqq.

[2] Published by Martin Gerbert in *Script. Eccles. De Musica Sacra*; printed at the monastery of San-Blasius, 1784, Tome 1, pp. 103–22. Gerbert gives as source of his printed version ' MSC Papyraceo bibliothecae Civicae Argentoratensis collato cum MSC biblioth. Cesenatensis ord. Minor Conventualium '.

marked by the sign $\boxed{\genfrac{}{}{0pt}{}{(\phi)}{\text{F}}}$ has been duly placed on Lichanos Hypaton, the Tonic of the Phrygian Species.

FIG. 86.—The 5ths in Folk Music

		Cents
Cycle of 5ths ascending the 5ths perfect of *702 cents*	on *C* = 192 v.p.s. for *G* on *G* = 288 v.p.s. for *D*	= *702°*
Cycle of 4ths ascending.	In the parent Scale (on *C*) the 5th degree has *G*♭ of 179·8 v.p.s. flatter than the *F*♯ 4th of the Cycle of 5ths	= *588°* = (*182·3°*)
DORIAN M.D. 11	the 5th degree of ratio 11/7 = *G*♯ 201 v.p.s. on *C* on *G* 11/7 = 301·6 v.p.s.	= *782·56°*
Dorian M.D. 22	on *C* the 5th step of ratio 22/15 = 187·7 v.p.s.	= *662·8°*
PHRYGIAN	Perfect 5th on Tonic on *C* = 192 v.p.s. on *g* = 288 v.p.s. (cf. Cycle of 5ths on *g*)	= *702°*
LYDIAN	5th degree, of ratio 13/9 flat on *C* = 184·3 v.p.s. on *g* = 13/9 = 277·2 v.p.s.	= *636·5°*
MIXOLYDIAN	5th degree, of ratio 14/10 very flat. Cf. 5th degree in Cycle of 4ths on *C* = 179·2 v.p.s. on *g* = 14/10 = 268·8 v.p.s.	= *582·3°*
HYPODORIAN	5th degree, of ratio 16/11 on *C* = 182 v.p.s. on *g* = 16/11 = 279·2 v.p.s.	= *649°*
HYPOPHRYGIAN	Perfect 5th on Tonic 18/12 on $\overset{\#}{C}$ = 192 v.p.s. on *G* = 18/12 = 288 v.p.s.	= *702°*
HYPOLYDIAN	Tritone = 10/7 on *C* = 182·9 v.p.s. on *G* = 10/7 = 274 v.p.s. 40/27 = on *C* = 189·6 v.p.s. on *G* = 40/27 = 284·4 v.p.s. 20/13 = on *C* = 196·9 v.p.s. on *G* = 20/13 = 295·4 v.p.s.	= *617·3°* = *680·8°* = *813·5°*

The group, as sketched, does actually function as Mode as well as Tonos, however; for the 5th, characteristic of the Authentus, starting from Lichanos Hypaton and descending to its Plagal on Proslambanomenos,

FIG. 87.—Characteristic Intervals and Features of the Seven Harmoniai

Harmonia	Tonic	Key-note	Characteristic interval	2nd step	3rd	4th	5th	Close	
DORIAN M.D. 11 or 22	*c*	*F*♯	*C–F*♯ 11/8 *551 cents*	*c–d*♭ 11/10 *165 cents*	*c–e*♭ 11/9 *348 cents* between maj. and min.	raised *c–f*♯ 11/8 *551 cents*	flat *c–g* 22/15 *663 cents*	*c–f*♯ *f*♯*–c* *c d*♭ *f*♯	The whole genesis of the Dorian Harmonia is sharp in relation to the Tonic
PHRYGIAN M.D. 12 or 24	*c*	*G*	*c–g* perfect 5th *702 cents*	three-quarter tone 12/11 *151 cents*	minor 3rd 6/5 *316 cents*	*c–f* perfect 4th *498 cents*	*c–g* perfect 5th *702 cents*	*c–g* *g–c* *c e g*	Frequently found duplicated on 4th or 5th 12 11 10 9 12 11 10 9 8 *or* 12 11 10 9 8 12 11 10 9
LYDIAN M.D. 13	*c*	*a*♭	*c–a*♭ 13/8 *840·5 cents*	*c d*♭♯ 13/12 *138·6 cents*	minor 3rd flattened 13/11 *289 cents*	flat 4th 13/10 *454 cents*	flat 5th 13/9 *636·5 cents*	*c–a*♭	The whole genesis of the Lydian Harmonia is flat in relation to the Tonic
MIXOLYDIAN M.D. 14	*c*	*b*♭♭	*c–B*♭♭ 14/8 *968·56 cents* flat minor 7th	raised ST. 14/13 *128·26 cents*	Septimal 3rd 7/6 *267 cents*	very flat 14/11 *417·38 cents* (perfect 4th 28/21)	very flat 14/10 *582·62 cents*	*c–B*♭♭ *B*♭♭*–c*	Genesis very flat in relation to Tonic
HYPODORIAN M.D. 16	*c*	*c*	Octave and perfect 4th, augmented 2nd, 15/13, 2nd to 3rd degree	Semitone 16/15 *112 cents*	between maj. and min. 16/13 *359·37 cents*	perfect 4th 16/12 *498 cents*	flat 5th 16/11 *648·5 cents*	*c–f c* *c e*♭ *f* 16 13 12	The Synemmenon tetrachord 16/16 15 13 12 passing to Hyperbolaeon
HYPOPHRYGIAN M.D. 18	*c*	*d*	maj. tone 9/8 *204 cents*	maj. tone 9/8 *204 cents*	minor 3rd 18/15 *316 cents*	very sharp 4th 18/13 *561 cents*	perfect 5th 18/12 *702 cents*	*c–d c* *d c d*	Origin of Harmonic minor in plagalized form 12 11 10 9 } 16 15 13 12 18 }
HYPOLYDIAN M.D. 20	*c*	*e*	maj. 3rd 20/16 *386 cents*	minor tone 10/9 *182 cents*	maj. 3rd 20/16 *386 cents*	perfect 4th 20/15 *498 cents* or Tritone 20/14 *617·38 cents*	very sharp 5th 20/13 *745·4 cents*	*c e c* *e c e* maj. 3rd	Frequently found duplicated on 4th 20 18 16 15 20 18 16 15 origin of our maj. scale

is given within its own octave, as was the case with the Dorian Harmonia in the Tonos of the Graeco-Roman Theorists.

The scheme, in short, bears the impress of the conversion of the P.I.S. from the Dorian Harmonia to the Phrygian. The letterpress (p. 119*a* and *b*) shows that Hucbald has accepted the revolution which acclaimed Lichanos Hypaton, instead of Hypate Meson, as Tonic of the Protus Authentus, and moreover, he has grasped the interrelation of authentic and plagal as inherent in the Tonos. '*Quatuor modis vel tropis, quos nunc Tonos dicunt*,' and that is why he introduces the scheme thus: 'Et autento quidem proto et plagiis ipsius hae aptantur octavae, a quibus mela ordiri hoc ordine pervidebis' (p. 120*a* and *b*).

Hucbald likewise recognizes, through the P.I.S., the double course of the *Tritus authentus* with Tonic on Parhypate Meson (Hypolydian), through Synemmenon with a perfect 4th, or through Paramese with the tritone; and he illustrates the mixture of the two tetrachords as used in the same Antiphons (pp. 113–14) and introits: a point which has puzzled many later Theorists.

Codex xxxi from the Library of the Ancient Spanish Cathedral of Vich (Auson), photographed by M. Sablayrolles,[1] yields two examples. The Codex contains a prose in honour of St. Thomas à Becket martyred in 1170, which limits the age of the manuscript to the end of the twelfth century. In the Epistola 'In Natale Domini' on a four-lined stave, *C* clef on 4th line, we find a long chant (30 lines). The modality is revealed by the figure *f*, *g*, *e*, *e*, *d*, used more than twenty times as close before a double bar. This figure comprises the first tetrachord of the Authentus Protus derived from the Phrygian Harmonia, moreover, the minor triad *d*, *f*, *a*, rises four times from the Tonic, as well as the octave leap *D–d*. There is, however, no special emphasis on the dominant *A* as keynote. Towards the end, the surprisingly modern phrase of the 7th *c*, *e*, *g*, *b*♭, occurs twice as though to mark the march of time, and the infusion of new ideas into the ancient canons of the Harmonia.

Another noticeable tendency, due to the character of this example as Epistola, is the repetition of notes of the same pitch and value, some 14, 10 or 7 times. This, then, is an example of the *D* scale or Phrygian *in which the second modal pivot—the keynote—is not stressed as such.*

A second chant from the same MS. xxxi, contained also in the older MS. cxi, of the eleventh century, is the Epistola, a Paschal Trope, 'In die Scō Paschae' (pp. 229–31). This chant presents innumerable closes in the Hypolydian Harmonia consisting of the major 3rd on the Tonic *F*, and this major 3rd *F–A* is used melodically besides in almost every phrase, as well as the perfect 4th *F–B*♭.

In MS. xxxi, folio 30, we note a Processional Trope with numerous closes on *A*, *G*, in orthodox Hypophrygian Style, but the 3rd on the Tonic is *B*♮, which denotes the bastard Harmonia 18/18, 16, $\left.\begin{matrix}14\\28\end{matrix}\right\}$27, with the

[1] 'A la Recherche des MSS. Grégoriens Espagnols', *I.M.G.*, Year xiii, Pt. ii, Jan.–March, 1912, pp. 235 sqq.

FIG. 88.—Hucbald's Scheme of Antiphons in the Protus Authentus with Greek Notation of the Lydian Tonos

			Ratios by K. S.		RATIOS OF THE LYDIAN TONOS
T	MESE	i	16	A. (Antiphon)	16 18 16 18 20 18 16 20 21 24 i m i m p m i p c f Erunt primi novissimi
T	LICHANOS MESON	m	18	A.	18 27 24 24 m b f f Ave Maria
T	PARHYPATE MESON	p	20	A.	20 21 24 24 p c f f vo-lo Pater
S	HYPATE MESON	C	21	A.	Ab hoc ordiri vix aliquid (f. solet) afferunt enim huius modi ex antiphona : Et minister meus
T	LICHANOS HYPATON	F (Φ)	24	A.	24 27 20 18 20 18 16 16 f f b p m p m i i Ecce nomen Domini
T	PARHYPATE HYPATON	B (Ɽ)	27	A.	27 24 24 24 24 b f f f f Ductus est Jesus
S	HYPATE HYPATON	Γ (⅂)	28	A.	Et ab hac fere nusquam : et est simile superiori antiphonae 20 21 24 24 24 27 24 26 24 21 21 24 24 24 p c f f f b f a f c c f f f circumdantur Vindicabor in eis
T	PROSLAMBANOMENOS	⊢ (7)	32	A.	24 27 24 27 20 20 21 f b f b p p c — — veni et ostende nobis

N.B.—Gevaert in *Melopée Antique* (Gand, 1895) gives references in Index to most of these Antiphons, of which he reproduces the musical notes in staff notation of the period. Gevaert interprets the Lydian Tonos of Hucbald as the keyboard scale of *A* from Proslambanomenos to Mese.

Hucbald uses indifferently symbols of Vocal and Instrumental Notation majuscule or minuscule. (Greek letters in brackets K.S.)

(p. 120, Gerb., op. cit., see Chap. x)

9/7 3rd = *435 cents*, and the perfect instead of raised 4th, all of which may be traced in the P.I.S., when the Hypophrygian Species passed from Mese to Diezeugmenon omitting Synenmenon. Since no precise indications of shades of intonation are given in these liturgical manuscripts, this reading is merely conjectural, but founded upon the basic system of the Harmonia.

SURVIVAL OF MODAL PIVOTS IN EARLY MEDIEVAL LITURGICAL CHANTS

It is interesting at this point to obtain confirmation of the survival of the modal pivots in liturgical chants of the thirteenth century from an unexpected source: viz. to observe through the ingenious diagnosis of Dr. W. H. Frere [1] how these unknown pivots of the ancient Harmonia strike a shrewd and learned observer, as he traces them in *statu quo*, and in transition in sacred, secular and Folk Music. In his interesting paper, he unfolds his theory of this musical phenomenon as an early manifestation of changes of related keys within the same chant.

Dr. Frere distinguishes the following types of key-relationships (i.e. modal pivots):

Dr. Frere	IDENTIFIED BY K. S. AS
(1) Tonic and Subdominant	PHRYGIAN AND HYPODORIAN HARMONIA.
(2) Maj. and Relative Min.	HYPOPHRYGIAN IN TRUE OR IN BASTARD FORM.
(3) Tonic and Sub Tonic	HYPOPHRYGIAN HARMONIA.
(4) Tonic and Dominant	PHRYGIAN CHARACTERISTIC BUT IN TRANSITIONAL USE.
(5) Tonic and Mediant	HYPOLYDIAN.

While it is beyond doubt that these observed types have sprung directly from the modal pivots of the several Harmoniai, it must be left to others, more competent on this ground, to decide whether changes of key in the modern sense were actually involved, i.e. whether the two pivots were invested by the early medieval musicians with the functions of two separate tonics or finals. In the case of Types 1 and 4 involving subdominant and dominant, the treatment of these as related keys may easily spring from the duplication of the first tetrachord, either conjunct on the subdominant, or disjunct on the dominant, quite irrespective of modal pivots, which in those scales of duplicated tetrachords are shelved, unless they occur in the first tetrachord, e.g. in the Hypophrygian and Hypolydian Harmoniai, an occurrence which may account for their survival during the early Middle Ages in Ecclesiastical Music, and of their gradual disappearance with the growth of polyphony and harmony. It would be interesting to discover whether, in the case of duplication in the latter two Modes, the emphasis on the pivots is likewise duplicated in the upper tetrachord, as in two of the examples given above.

THE ERA OF POLYPHONIC MUSIC HERALDS THE WANE OF THE HARMONIA IN LITURGICAL CHANTS

The dawn of the era of polyphony heralds the waning of the true feeling for the Harmonia in Ecclesiastical Music. The two pivots of the melos

[1] 'Key-Relationships in Early Mediaeval Music', *I.M.G.* Smbd. xiii, Jan. to March, 1912, pp. 250 sqq.

may certainly still be traced, wandering here and there among spread triads and 7ths, but they are halting and hesitant. In fact, the two pivots, instead of forming a common bond emphasizing modality, tend to separate and to engage in individual spheres of interest.

Dr. Frere states that the commonest key-relationship in early medieval music is that of Tonic and Sub-Tonic (i.e. the two Hypophrygian cardinal points, Tonic and keynote—K. S.). This relationship was found already in evidence when the first irruption of Folk Music into classical Plain-Song took place. The device is conspicuous in thirteenth-century compositions of non-liturgical melodies, such as those known as '*conductus*'.

The process of transformation from mere shadows of modal pivots into stereotyped figures, used in and out of closes and cadences as aids to composition, may be observed in MS. xxxi of the Vich Codex. From a casual glance at the three liturgical chants cited above, one would feel inclined to diagnose the Mode as Hypophrygian on account of the step of a major tone up and down.

In the Epistola the Mode is finally revealed as Phrygian or *D* Mode through the leaps to the dominant and the repeated use of the first tetrachord as close.

In the Paschal Trope, in the *F* Mode, the two modal pivots, at a major 3rd, are accompanied by the step of a tone thus: $\overset{\frown}{A\ F}$*—G F—G* $\overset{\frown}{A\ F}$ *F* and in the upper tetrachord, conjunct on *B*♭, there is a melodic imitation *B*♭*—C—B*♭ used many times.

In the Processional, in the *G* Mode, *A—G—G* is accompanied by *G—F—G* and as final cadence a leap to the dominant $\underbrace{G—D}$ is followed by *D—D—D—E—D—C* on 'Puer Natus'. It is obvious that nothing but a thorough knowledge of the Harmonia, and its principles, will clear the ground of all the misconceptions which have grown intertwined in the Ecclesiastical Modes of the early Greek and Roman Churches. When the essential differences which exist between Modes and Species receive practical recognition, a revision of the nomenclature of the Ecclesiastical Modes will be undertaken. At present the nomenclature allows a tetrachord of the structure S.T.T. to masquerade as Phrygian, and one of T.S.T. as Dorian, with all their implications. Moreover, the purely functional and incidental nature of Authentic and Plagal will lead to the disappearance of these terms.

* * *

Reviewing evidence of the survival of the Harmonia in theory and practice through the centuries, we may recall the set of 15 flutes with equidistant fingerholes, described by Sārañgdev in the Ratnākara, a thirteenth-century history of the Drama (see Chap. vii). One of these, No. 12, on examination of the measurements given, indicated the Hypolydian Harmonia; and as the flutes were graded, they could not fail to yield precise indications of the other Harmoniai.[1]

[1] Traces of a further survival of the Harmoniai from those early days will be found further on.

The same century in Europe provides evidence of paramount importance concerning the Harmonia, found in a document known as the Harmonic Canon of Florence, which is included in a Codex of the thirteenth century.[1]

This Canon contains a description of the division of a monochord into 28 equal segments. The value of the Canon of Florence is unique as an illustration of the intimate connexion that exists between the modal division by equal measure and the Systema Teleion Ametabolon (the P.I.S.) of Ancient Greek Music. The manuscript also provides the only known example of the practical use of the *Onomasiai Kata Thesin* and *Kata Dynamin*, first introduced in those terms by Ptolemy.[2] The division of the Canon by M.D. 28 suggests the Mixolydian Harmonia, but it is expressly stated in the opening lines that the whole series is dependent upon Proslambanomenos as 28 ; the modal octave from Hypate Meson, therefore, begins on ratio 20 and is Hypolydian. As the nomenclature—according to the P.I.S.—of each segment is expressly stated, there is no possible doubt as to the identity of the modal sequence which the scheme entails. The provenance of the ancient original manuscript of which the Canon of Florence is a thirteenth-century copy, remains obscure. In my opinion it is the work of two distinct authors ; the one dealing with a division into 28 equal segments ; the second with a division into 24 segments, both based upon Proslambanomenos. Meanwhile a third, a scribe, has amused himself by applying the ditonal scale, *tant bien que mal* to the segments, thus, of course, entailing fractions of segments with which he juggles with evident delight.

The peculiar significance of an aliquot division by a Modal Determinant is that it tells its own tale, and can afford to disregard with complacency all such well-meaning but erroneous deductions.[3]

This Canon suggests a brief review of references to the monochord which bear upon our subject. Gaudentius states that Pythagoras divided the Canon of his monochord into twelve equal parts,[4] to which we may add that this produced the Phrygian Harmonia of M.D. 12, the origin of Ptolemy's tetrachord Homalon, viz. $\frac{12}{11} \times \frac{11}{10} \times \frac{10}{9}$.[5]

[1] Cod. Graeci, Plut. lvi, § xiii in the *Bibl. Laur. Medicea*. See also Chap. v.

[2] *Harm.*, Lib. ii, Cap. xi. See also fn. 6 on the two pivots.

[3] *Re* the two modal pivots. The relative positions of the two notes in the octave εἴδη are referred by Ptolemy (*Harm.*, Lib. ii, Cap. xi) to the Onomasiai Kata Thesin and Kata Dynamin. Hypate Meson is regarded as the common Tonic or initial note of all the Species, Kata Thesin. The Mese, Kata Dynamin of each species, is given the position of the real keynote, so many degrees above the Tonic, Hyp. Mes. : thus, the Hypophrygian Mese, Kata Dynamin, is placed by Ptolemy on Parh. Mes., i.e. on the 2nd degree of the octave species (which agrees with the principles of the Harmonia according to K. S.).

These terms are sometimes considered to be confusing, but all difficulties disappear when the two terms are used in relation to the eight-stringed Kithara : Kata Thesin represents the naming of the strings, as in the Meson and Diezeugmenon tetrachords, Kata Dynamin represents the Harmonia to which the strings are tuned by ratios, and recognized by the ear. (See in this connexion the Canon of Florence, Chap. v.)

[4] *Harm. Eisagoge*, p. 14 M.

[5] *Harm.*, Lib. ii, Cap. xvi.

Thrasyllus of Rhodes is recorded by Theo of Smyrna to have used the same division of the canon (ed. Hiller, p. 47).

From Arabian sources there is a description by Safi-ed-Din of this same division [1] (thirteenth century). Still later comes the scheme of the division of a monochord into forty-eight equal parts, published by Michael Praetorius.[2] The names of the notes corresponding to the numbered segments, although not carried out *in extenso*, are correctly allocated on all the characteristic notes, a fact which testifies to the familiarity of the original author of the sketch with the genesis of the Phrygian Harmonia (not identified on the plate). One may search in vain in the text of Praetorius for further information concerning this precious evidence of the survival of the Phrygian Harmonia in the Enharmonic Genus. Praetorius is greatly embarrassed, and refers to the subject in several places, but postpones the explanation ' *To be given later if God will* '.[3]

The Phrygian tetrachord, $12/11 \times 11/10 \times 10/9 = 4/3$, the Homalon of Ptolemy, is considered at length in the treatise of George Pachymeres (thirteenth century) on the Four Mathematical Sciences (or Quadrivium) on pp. 507 sqq., and *passim*.[4]

Pachymeres states that the best manner of using the monochord is by a division into equal parts (p. 496) by means of a compass. The evident partiality exhibited in theory and practice for the Phrygian Harmonia is no doubt due to its pure consonances of 4th and 5th, and to its minor 3rd on the Tonic. Evidence has already been adduced elsewhere of the continued use as late as 1870 of the Homalon tetrachord, duplicated, in the practice of the Greek Church in Asia Minor, on the authority of Tzetzes (*op. cit.*, *passim*).

A brief reminiscent reference seems to be called for at this point to the flute schemes published in the early sixteenth-century German treatises of Sebastian Virdung, 1511 (*op. cit.*), and of Martin Agricola, 1528 (*op. cit.*) (see Chap. vii), in which flutes, bored for a Modal Scale, are brought into line with the keyboard scales of the Ecclesiastical Modes by means of cross-fingering accurately described in detail. The writers display a very competent knowledge of the technique of various flutes—fipple and side-blown. The effect of opening and closing the fingerholes corresponds accurately with the resulting change from the modality (of flutes with equidistant fingerholes) to one or other of the scales of the Ecclesiastical Modes.

CROSS-FINGERING ON THE FLUTE AS EVIDENCE OF THE SURVIVAL OF THE HARMONIA IN THE SIXTEENTH CENTURY

A flute, bored to give the Ancient Greek Dorian Harmonia, is, for example, converted by cross-fingering to one playing the Authentic Deuterus,

[1] *Le Traité des Rapports musicaux, ou l'Epitre à Scharaf ed-Din, par Safi-ed-Din*, par M. le Baron Carra de Vaux (Paris, 1891). Division of string by 12, pp. 16 sqq.

[2] *Syntagma II Teil. Von den Instrumenten* (Wolfenbüttel, 1618), *Theatrum Instrumentorum*, Taf. xxxix, ref. p. 71.

[3] This pious hope was not realized, as far as my research goes, in his works or in those of his predecessors.

[4] A. J. H. Vincent, *Notices et Extraits des Manuscrits de la Bibl. du Roi* (Paris, 1847), Tome xvi, pp. 471 sqq., pp. 506 sqq.

corresponding to the Dorian Species of the Graeco-Roman theorists. Another specimen in the Hypolydian Harmonia of M.D. 20 (the Harmoniai are diagnosed by K. S.) is changed by Agricola's skill into one that plays in what may be recognized as the Authentus Protus, the Phrygian Species of the Greek theorists. A Hypophrygian flute of M.D. 18 is converted into one playing in the Tonus Secundus and so on.[1]

Virdung [2] gives, at the end of his little treatise, full instructions in the text for cross-fingering and for half-closing fingerholes, and he prints two schemes, both of which undoubtedly relate to a Dorian Harmonia, converted into scales of tones and semitones only—the first by cross-fingering, the second by half-covering certain holes to lower the pitch.

These quotations from the earliest German Musical Treatises reveal by unmistakable implications the nature of the music of the Folk, in which the survival of the Harmonia in the first half of the sixteenth century is clearly indicated.

On the one hand, the Ecclesiastical Music in the Modes adapted to the keyboard scale of tones and semitones of organs, harpsichords, clavichords and stringed instruments with fretted necks, held sway in Church and State. On the other hand, the Folk danced and sang to the music of the Harmonia, played on the Schalmey, Shawm, Pommer and oaten pipe; on Recorders and Flutes, accompanied by stringed instruments with finger-boards innocent of frets, and on the Sackbuts, Cromornes and Cornets of Town Bands.

In the meanwhile, theorists were busy finding means of compromise which would bridge the gaps between the two systems. We may now proceed to give results of the identification of the Harmonia in the music of the Folk.

THE DORIAN HARMONIA IN FOLK MUSIC: EVIDENCE FROM THE INCAS OF PERU

The testimony of Virdung as evidence of the survival of the Dorian Harmonia among the Folk up to the beginning of the sixteenth century may be followed up by other examples of the use of this Harmonia among the Folk of various nations.

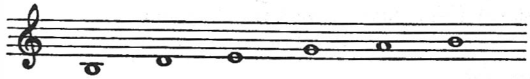
From a volume devoted to the music of the Incas of Peru,[3] we shall quote passages from a beautiful 'Hymn to the Sun' appropriately sung by Indians, descendants of the Incas, who styled themselves 'Children of the Sun'. This invocation to the Sun-God was sung in unison by a group of Indians in the mountains of Huanuco (Peru). The authors of this fine volume disclaim any responsibility for the notation of the song, which was communicated to them by a member of a French religious order in Callao who had received it from M. A. Robles.

[1] See Chap. vii and on cross-fingering, with full refs.

[2] *Musica getutscht und Auszgezogen durch Sebastianum Virdung Priesters von Amberg* (Basel, 1511). An edition of 200 copies in a Facsimile Reproduction was published by Rob. Eitner for the *Ges. f. Musikforschung*, Bd. xi (Berlin, 1882).

[3] See d'Harcourt, *op. cit.*

The scale of the notes used in the hymn is the following, together with the higher octave of the notes :



Ratios by K. S. 11/11 9 8/16 13 12 11

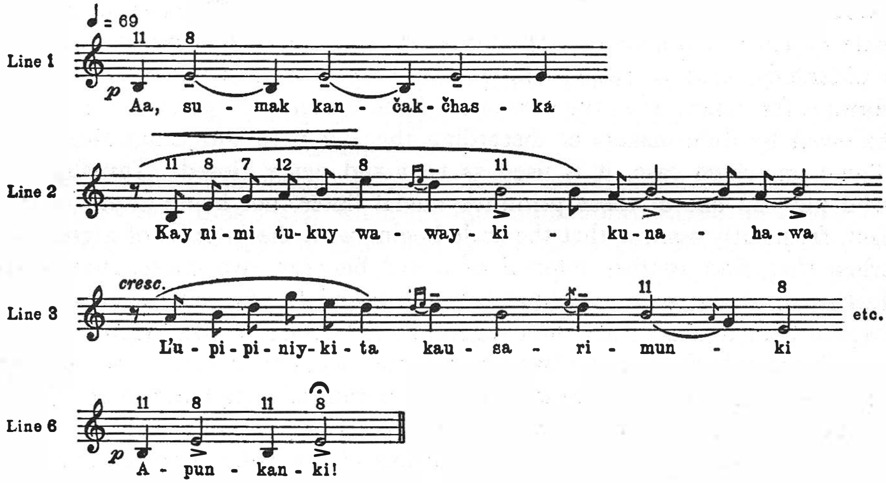
The *G* as ratio 13 would be a sharpened $\overset{\sharp}{G}$.

No indications of the shades of intonation are given : the song itself, however, bears inherent evidence of its modality, which is confirmed by the scales on the two Kena's [1]—the vertical notched flutes of the Incas—on which the Hymn is playable from Hole 1 as vent with the help of the Harmonic register ; the high *G* as ratio 13 is obtained by overblowing an octave from the exit fundamental.

FIG. 89.—'Hymn to the Sun'

'*AA, Sumak Kančakčhaska*'. *Collected by Mr. A. Robles, from Huanuco, Peru*

No. 10 (*op. cit. p. 247*)



The Hymn begins with the leap from the Tonic *B* to the raised 4th *E* as keynote (ratio 11/8, *551 cents*), characteristic of the Dorian Harmonia, sung twice impressively, and with a close on the keynote, to the words which form the title of the Hymn. The leap occurs twice again in lines 2 and 4 and as a final cadence.

A pastoral melody in the Dorian Harmonia (No. 198, pp. 523–4) selected from this fine collection, was played on the *Kena* (flute) by an Indian from the Highlands of the Peruvian province of Candarave, while herding llamas.

[1] Cf. Chap. x, Record of Flute Inca No. 12, with six fingerholes bored at equal distances, wherein the unusual features of the scale are explained.

THE SCALE OF THE DORIAN HARMONIA OF M.D. 11 ERRONEOUSLY DIAGNOSED AS PENTATONIC

FIG. 90.—Pastoral (No. 195) played by Kapakuti on the *Kena*. Hacienda de Totora, Peru

The scale of notes used in the song is the following :

Tonic *E*. Keynote *A*.		*E*	*G*	*A*	*B*	*D*	*E*
Ratios and		11/11	9	8	7	6	11/11
(by K. S.)	*Cents*		*348°*	*204°*	*231°*	*267°*	*151°*

Notes *G*, *A*, *B* and *D* would all be raised by superscript sharps.

At first sight the scale appears to belong to the Pentatonic ; it is, however, a Modal Scale. The interval between the exit fundamental and the note of Hole 1, frequently approximating to a 3rd, is but rarely, and then accidentally, true to ratio ; the reason being the intricate nature of the formula for determining the position of Hole 1, hence the practice commonly followed by flute-makers of discarding the exit note and using Hole 1 as Tonic, in which case, it is used as vent and never closed. Investigators who pick up one of these primitive modal flutes, not being aware of this fact, frequently assume that the scale begins with the interval of a 3rd, and when they find another interval of a 3rd between two consecutive holes further up in the scale—as above between *B* and *D*, of a septimal 3rd—they naturally enough consider their diagnosis of Pentatonic confirmed.

This simple flute tune, played by a llama shepherd of Peru, is an instance of a Pentatonic arising naturally from the boring of fingerholes in obedience to the progression of ratios initiated by the Modal Determinant 11.

It is a matter for regret that the authors of this volume on the music of the Incas purposely abstained from giving exact values in the intonation of songs and flute tunes, holding firmly to their belief that the Pentatonic scale of tones and semitones and a heptatonic of similar structure formed the basis of the Folk Music of the Incas. No measurements of flutes are given, but from the photographic plates it is evident that the *Kenas* have equidistant fingerholes, and could not therefore have produced the intervals attributed to them.

EVIDENCE FROM HINDOSTAN, HUNGARY AND RUMANIA

The delightful Musical Diary of Mr. A. H. Fox Strangways [1] in *The Music of Hindostan* contains many examples of Folk Tunes suggestive of the

[1] *The Music of Hindostan*, by A. H. Fox Strangways (Oxford : at the Clarendon Press, 1914). It must not be inferred that the author is in agreement with K. S.

Harmonia, e.g. No. 6, p. 21, in which on a Tonic *G*, $\overset{\sharp}{C}$ and $\overset{\sharp}{d}$, with superscript sharps, imply a raised 4th and 5th, the functions of which may be interpreted thus :

	G	*A*	*B*	$\overset{\sharp}{C}$	$\overset{\sharp}{D}$	(doubly sharpened)
Ratios K. S.	11/11	10/11	9/11	8/11	7/11	DORIAN HARMONIA OF M.D. 11
Cents	*165°*	*182°*	*204°*	*231°*		

11/7 = *782 cents*
11/8 = *551°*

In addition to the superscripts supplied by the author, the *A* and *B* would also require superscript flats for an accurate identification. It may be added that unless otherwise specified, the tunes (including No. 6) were taken down by ear.

Turning now to Béla Bartok's collections of Folk Music, a somewhat rare example of the duplication of the first tetrachord of the Dorian Harmonia on the 4th occurs among the Rumanian folk of the Maramuros (a province of Eastern Hungary separated from Galicia by the Carpathians).[1] The author categorically disclaims the practice of the repetition of the augmented 2nd in the 2nd half of the scale in Hungarian Folk Music,[2] although it is common among the Rumanians ; this is an implicit stricture against the duplication of tetrachords, *ergo* : the Hungarians favour the octave Harmonia, whereas the Rumanians might be credited with a partiality for the musical practices of the Orient.

The shades of intonation in this volume are represented by accidentals divided by two, thus : ♯/2, when the intonation approximates to a quarter-tone, this is better than nothing, but is still too vague to exclude misconceptions, where modal characteristics are not unmistakable. Our example No. 53*c*, p. 40, has the following scale and signature. (See *Volksmusik der Rumänen von Maramures*, by Béla Bartok (Munich, 1923), No. 53*c*, p. 40.)

Scale, ratios and *cents* by K. S.	11/11	10	9	8	11/11	10	9	8	11
	165°	*182°*	*204°*		*165°*	*182°*	*204°*	*98°*	
	11/8 = *551°*				11/8 = *551°*				

The 1st tetrachord of the Harmonia would require *A*♭/2 as well as *B*♭/2, otherwise the identification is perfect. (*N.B.*—The *B* is flattened to the extent of a quarter-tone only.) Should there be any doubt felt about the identification of the tune as Dorian of M.D. 11, on the strength of the raised 4th, since Hypophrygian and Hypolydian characteristics both include a

[1] Among the flutes in my collection is No. 3 from the Carpathians, presented by Mr. George Kaufmann, M.A., bored for the Dorian of M.D. 11, i.e. the scale of the song of which the closing line is quoted in Fig. 91.

[2] See Hungarian Folk Music, pp. 54–55.

raised 4th, the closes settle the point without appeal. They are neither Hypophrygian nor Hypolydian, but Dorian. The last note of line 1, the Tonic *G*, leads up by a slide to the keynote *C*♯/2, in line 2, to drop down at the end of the bar to the Tonic; this is repeated in line 3, and in variant

FIG. 91.—Rumanian Folk Song from the Maramuros, No. 53*c*
(*Collected by Béla Bartok* (*op. cit. p. 40*))

No. 5 as a final cadence. The slide down from the Tonic in this final cadence to *D* is merely to give an octave slide up to start the 2nd verse.

No. 172 (*op. cit.*, p. 145) is another example of the survival or rebirth of the Dorian Harmonia embodied in the '*fluer*', a fipple flute from 30 to 40 cm. in length with 6 equidistant fingerholes.

The fundamental of this specimen is evidently *C*; the signature is given as *B*♭/2, *F*♯/2; the harmonic register is extensively used. The final cadence does not introduce the keynote *F*♯/2, but instead the *D* of the characteristic 2nd step. The tune is playable on a Dorian flute of M.D. 11.

It is a far cry from Rumania to the South Islands of New Mecklenburg.[1] Nevertheless, we recognize in the music of those remote islands the same language of music, e.g. in No. iii, Hornbostel prints a tune from Lamassa entitled 'Magical Invocation for Rain' with a range of a 7th thus:

EVIDENCE FROM NEW MECKLENBURG, TURKEY AND THE JEWS OF THE YEMEN

NO. III FROM LAMASSA, S. ISLANDS OF NEW MECKLENBURG

Panpipe Sequence (*from the Dorian Harmonia*)

The Scale

	A	*D*	*E*	*G*	
v.f. E.M.H.	*217·5*	*300*	*348*	*405*	derived from the
v.f. K. S. and Ratios,	217·5	299	343	406	Dorian Harmonia
Dorian Harm. M.D. 11	11/11	8/11	7/11	6/11	of M.D. 11
Cents	*551°*	*231°*	*267°*		

Hornbostel obtained the scale from a phonographic record of natives playing on Panpipes. Other scales from the same source, given with the tunes, are also modal in the Hypodorian, Mixolydian, Hypophrygian (two items) and Phrygian Harmoniai. Needless to say, this scale could not originate on the Panpipe. It is a flute scale, such as might be produced on a long pipe having but three equidistant fingerholes, the first at a long distance from exit; or from a flute having five fingerholes, bored at equal

[1] See *Hornb.*, 1922, *Vergl. Mus.*, pp. 356–8.

distances, on which the scale would be played from exit, Holes 3, 4 and 5. The leap from one pivot to the other, viz. down from *D* to *A*, occurs three times in the tune and the closes are all on the keynote *D*.

In the same volume, Hornbostel [1] presents among phonographic records of Turkish Folk Music, No. 1, which is in the Dorian Harmonia (K. S.).

THE SCALE OF TURKISH TUNE NO. 1

	D	*E*	*F*	*G*	*A*	*B*	*C*
v.f. E.M.H.	*302*	*333*	*354*	*410*	*455*	*508*	*550*
v.f. K. S. Dorian Har. M.D. 22	302	332·2	351	415	456·5	507	553
Ratios K.S.	22/22	20/22	19/22 and	16/22 22/22 *D* Duplicated first Tetrachord	20/22	18/22	16/22

N.B.—The 4th from *G* to *C* (if correctly translated from the record) lies between the ratio 11/8 and the perfect 4th 4/3.

From Idelsohn's phonographic records No. 1167, p. 85,[2] we quote the following :

SCALE USED IN A PRAYER CHANT OF THE JEWS OF THE YEMEN

Range, a Sixth

Idelsohn's v.f. (octaves)	*C*	*E*	*F*	*G*	*A*	*C*
	515	620	702	762	801	(*reciting*
	257·5	*310*	*351*	*381*	*400·5*	*note 515*)
Modal v.f. and	256	312·5	352	375	401·5	
Ratios K. S.	22/22	18/22	16/22	15/22	14/22	
Cents	11/9 = *348°*	*204°*	*112°*	*119·4°*		

The four Brazilian Panpipes which play the scale of the Dorian Harmonia and the Xylophone from Burma (given in full in Hornbostel's Tables) may be recalled at this point as well as the Elgin Auloi, the Bucheum Graeco-Roman Flute, the Bali, and two Java flutes, and the Mond flute from Sicily, and my Agariche (Peru-Bolivia). These all constitute evidence of the survival or rebirth of the Dorian Harmonia among the Folk.[3]

THE LYDIAN HARMONIA IN FOLK MUSIC

The Lydian Harmonia of Ancient Greece originally had as Modal Determinant 26. During the development of the Systema Teleion Ametabolon—the Perfect Immutable System (P.I.S.)—the Determinant was lowered in pitch to 27, a change signalized by the tetrachords ascribed to Archytas, which began in all three genera with the ratio 28/27. In the

[1] E. M. Hornbostel and O. Abrahams, *op. cit.*, pp. 241 sqq.

[2] See A. Z. Idelsohn, *Gesänge der Jemenischen Juden* (Leipzig, 1914). Although the v.fs. are in agreement, it cannot be claimed that the Dorian modal canons are observed in this example.

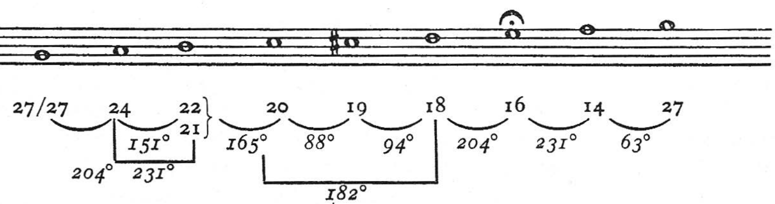
[3] See also Dr. Jaap Kunst's Tables of Results of personal investigation in Java, Indonesia, &c., and the modern shepherd flute from Nauplia, Greece, in Prof. Dayton C. Miller's collection (Chap. x, Record Nauplia).

system of the Harmonia, which I suggest was the origin of the P.I.S., the ratio 28/27 is found between Hypate and Parhypate Hypaton, so that the Lydian Species begins on ratio 27. There is evidence of the use of both Determinants 26 and 27 ; 27, however, is more especially traced to Hellenistic Asia and the Alexandrian culture, and it was followed also by the Eastern Arabs and perhaps also by the Persians (e.g. see *Zālzāl's Wosta* of ratio 27/22). The characteristic modal interval of a 6th of ratio 13/8 of the Lydian Harmonia forms a striking close (even when the Tonic is flattened a quarter-tone) of which Tune No. 49 (p. 206) of Henebry's collection is a truly remarkable and clear example, '*current everywhere in the Irish portions of Waterford*'. The song is transposed to *G*, and the author is greatly disturbed by the behaviour of the *C*, the intonation of which varies between *C* and *C*♯: with M.D. 27, the *C* should be a comma sharper than the perfect 4th $\left(\frac{27}{20} \times \frac{3}{4} = \frac{81}{80}\right)$. The two closes on  and one on *E* an octave lower as keynote, together with the initial and final *G* as Tonic, indicate the Mode as Lydian. The scale is the following :

Ratios and Cents by K. S.

HENEBRY'S COLLECTION

Modal Ratios by K. S. *Cents*



No reason can be advanced, so cogent as the modal keynote provides, for the emphatic closes on *E* in the key of *G*.

The Tunes from the Ring Promontory,[1] south-east of Dungarvan, furnish two more specimens (see Nos. 95 [vi] and 99 [x]) in which the characteristic Lydian interval of a 6th on the Tonic is indicated, not only in closes and as modal pivots, but also by the vibration frequencies of the phonograms with which, in most instances, the staff notation is entirely at variance. The Tune No. 99 is a fine example of the survival, not only of the notes and intervals of the Modal Scale, but also of the inherent feeling for the modal pivots ; the Tonic *G* is emphasized eleven times and the keynote *E* nine times : moreover, in line 1, the *G* of a vibration frequency of 410·5, is strictly in the ratio 13/8 to the keynote of 667·5 v.p.s.

THE PHRYGIAN HARMONIA IN FOLK MUSIC

This Harmonia has remained a favourite in many lands, e.g. in Java, in Asia Minor, in New Mecklenburg, in Turkey, in Mohammedan countries generally and among the Incas and Hebrews.

The modal pivots, the perfect 4th and 5th, as well as the minor 3rd on

[1] Phonographic records of these were taken, and forwarded to Dr. von Hornbostel for tonometric examination so that the v.f.s of the notes are indicated, and the values of intervals given in *cents* while Henebry is responsible for the notation.

the Tonic, are also those of our minor Mode and of the Ecclesiastical *D* Mode erroneously called Dorian even at the present day, thus perpetuating the blunder of Glareanus.

The one decisive feature in its pure octave form of Determinant 12 is the septimal 3rd between the 6th and 7th degrees—the latter as octave of the Tonic.

The same interval occurs also as a distinguishing feature in the Dorian Harmonia of M.D. 11, but between the 5th and 6th degrees. The three-quarter tone 12/11 (*151°*) on the Tonic of the Phrygian Harmonia, which divides the minor 3rd, can only be identified with certainty when vibration frequencies are available.

A scale identified as that of the Phrygian Harmonia is cited by Idelsohn as the traditional Mode of the Synagogue Melodies allotted to recitations from the Prophets, Lamentations, Esther and the Psalms; the statement is borne out by the vibration frequencies of the phonographic records of the Persian Jews from the Book of Esther, sung to the reciting notes

	e		*f*		*g*
v.f.	326		355		394
		12/11		11/10	

i.e. the opening notes of the Phrygian Harmonia (see Idelsoln, *op. cit.*, p. 99, Pl. 2127)

Josef Singer[1] (p. 11) names the Mogen Awaus

C D—E♭ F G A♭ B♭ C

(or Phrygian conjunct—K. S.) as one of the three oldest Modes used in the traditional songs of the Synagogue, and gives a melody No. 20 in 'Neginoth in the Mode Mogen Owaus' (he gives both spellings) for the Book Esther, in which the minor 3rd on the Tonic and the two modal pivots are used with great emphasis (see Tunes Nos. 15, 16, 21).

Henebry senses a definite potency in the modal unit of Tonic and Dominant, but without being able to suggest any well-established concept to explain the force of this connexion (see pp. 70–1, *op. cit.*).

When we turn to Turkish phonograms[2] we find two tunes in the Phrygian Harmonia; in both, modality is definitely and strongly emphasized. For No. ii, the scale given by the record is the following:

SCALE OF TURKISH SONG NO. II

(*Phrygian Harmonia*)

E. M. v. H.	*258·5*	*274·5*	*307*	*337*	*358*	*387*	*420*	*465*	*521*	*548*	*613*
Phrygian v.f.	256	279·2	307	341	361	384	410	472·8	512	558	614
ratios (K. S.)	24/24	22	20	18	17	16	15	13	12	11	10

[1] *Die Tonarten d. Traditionellen Synagogengesänges* (*Steiger*) (Wien, 1886).

[2] 'Phonographierte Türkische Melodien', von O. Abraham and E. von Hornbostel, first published in *Zts. f. Ethnologie*, 36, 1904, reprinted in *Abh. z. Vergl. Musikwissenschaft*, Vol. i, 1922, pp. 233–50. Twenty songs with scale of notes used and v.f.s.

The agreement between the recorded notes and the modal values was closer for the Hypophrygian Harmonia,[1] but the tune itself did not bear out the modal pivots and characteristic intervals, which are emphatically Phrygian, with closes on Tonic and sub-dominant, a strongly stressed dominant throughout, and a final cadence descending through the lower tetrachord to the Tonic.

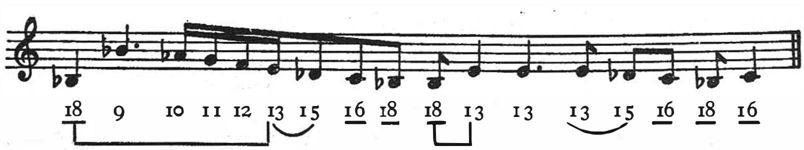
EVIDENCE FROM SYNAGOGUE CHANTS, FROM NEW MECKLENBURG, RUMANIA, THE PAWNEES, PERU, AND SUMATRA OF THE HYPOPHRYGIAN HARMONIA IN FOLK MUSIC

Turning again to Jos. Singer's Synagogue Songs, we find examples of the Hypophrygian Harmonia which display the characteristic leap to the sharpened 4th 18/13 and the distinctive 15/13 augmented 2nd, notably No. 7, which ends the following cadence, to which are added ratios by K. S.

SYNAGOGUE SONG, NO. 7, JOSEPH SINGER

(*Pismon Jisroel Noscha No. 7*)

last line



The modal pivots are *B♭* = 18, Tonic.
C = 16, keynote.
Augmented 2nd *d♭* to *e* = 15/13.
Raised 4th on Tonic 18/13 = *B♭* to *E*.

The first phrase of Song No. 12, also strongly emphasizes the Hypophrygian modal pivots, and the last line quoted ends with the leap to the sharp 4th followed by a return to the keynote through the augmented 2nd. Nos. 14 and 23 are other examples of the use of this Harmonia.

Among the Panpipe tunes and songs reproduced from phonographic records from New Mecklenburg,[2] there is a melody for the Sun festival dance in King sung by a man who used alternately chest and falsetto registers.

[1] The v.f.s and ratios of the Hypophrygian Harmonia may be compared with those of the Phrygian.

B	*c*	*d*	*e*	*f*	*f♯*	*g*	*a*	*b*	*c*	*d*
258·5	273·7	310	332·7	358	387·7	422	464·5	516	548	620
18/18	17	15	14	13	12	11	10	9	17	15

[2] E. M. von Hornbostel, 'Notiz über die Musik der Bewohner von Sud-Neu-Mecklenburg'; published in *Smb. f. Vergl. Musikwissenschaft*, ed. by Carl Stumpf and E. M. von Hornbostel, Bd. 1 (Munich, 1922), pp. 351 sqq., with illus. of Panpipes.

SUN FESTIVAL DANCE (IN KING, NEW MECKLENBURG)

The scale with vibration frequencies is the following:

chest * falsetto *

E. M. v. H.

202·5 (227·8) 240·5 {275 / 277·5} 309 367 411 (455·6) 481 555 618

* Although *B* occurs a few times in Parts I and II, no v.f. for the note of the song is given. *N.B.*—I have reduced the v.f. to the octave below in agreement with notation.

V.f.s of the Hypophrygian Harmonia on *A* by K. S.

202·5 227·8 243 280·4 304 364·8 411 455·6 486 561 623

Ratios by K. S.

18/18 16 15 13 12 or } 10 9 8 / 16 } 15 13 { 12 or 23/36

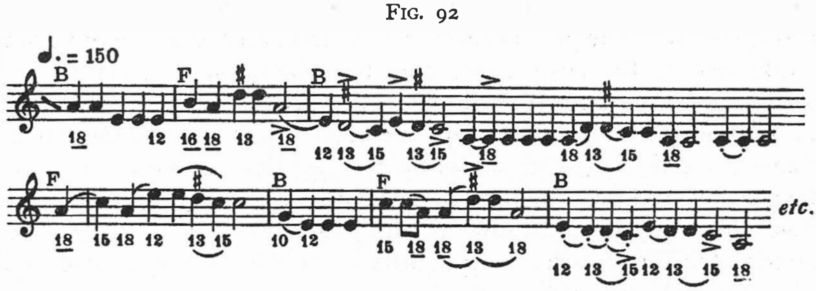
The 5th is sharpened to 23/36} in both octaves———————{23/36

202·5 × 4/3 = 270 v.p.s.

The sharpened 4th 18/13 occurs as an upward leap fourteen times in the first six lines of the song, and eight times in the four lines of the 2nd part. The augmented 2nd of ratio 15/13 innumerable times, a leap to the sharpened 5th is, with one exception, merely led up through the minor 3rd. The singer was uncertain about the intonation of the Tonic of 405 and 411 v.p.s. A few bars of the melody are given below.

MELODY FOR THE SUN FESTIVAL DANCE IN KING, NEW MECKLENBURG

FIG. 92



B = chest voice, F = falsetto
Hypophrygian Harmonia
Modal Ratios by K. S. E. M. v. H., p. 357, *op. cit.*

N.B.—*D* should bear a superscript sharp in agreement with v.f.s.

Among this collection of records from New Mecklenburg there are Panpipe melodies in the Hypodorian and songs in the Dorian and Phrygian Harmoniai. (The 5ths on the Tonic are sharp, not flat, and therefore do not lend colour to Hornbostel's exploded theory of the Cycle of Blown 5ths.)

Béla Bartok's Collection of Rumanian Folk Songs contains several in the Hypophrygian Mode. No exact values in vibration frequencies are given, but there are unmistakable indications in key signatures; and notes sharpened or flattened by less than a semitone are thus marked ♭/2, ♯/2.

Eight Hypophrygian songs examined all have the two modal pivots, Tonic and Keynote as closes on the caesurae, used either separately or in sequence; and as finals. Leaps to the 4th 18/13—the note of ratio 13 indicated by *C*♯—occur six times in No. 21*b*; in all of these examples the augmented 2nd of ratio 15/13, indicated by *b*♭ to *c*♯, is in evidence; in most of them the descending Hypophrygian lower pentachord or tetrachord, (*d* 12) *c*♯ 13 *b*♭ 15 *a* 16 *g* 18 *g* 18 is used melodically, as close, or as final cadence.

(*d*)	*c*♯	*b*♭	*a*	*g*	*g*
(12)	13	15	16	18	18

A Rumanian Folk Song, No. 21b

Bocicoiel Erina Chiesa* **(Béla Bartok,** ***op. cit.*****, p. 12) may be cited as an example.**

Hypophrygian Scale with modal ratios by K. S.

denominator constant 18/18 16 15 13 12 (11) (10)

The large group of Hora Lunga songs (Nos. 23 sqq.) are evidently based upon the Hypophrygian Harmonia (*op. cit.*, p. x, for scale and description). Béla Bartok states that they suggest an instrumental origin.

In a few examples, e.g. in No. 21*a*, the perfect 4th on *C*♮, of ratio 36/27, appears instead of the characteristic 18/13 raised 4th.

This author's collection of Hungarian Folk Music likewise contains melodies in the Hypophrygian Harmonia in which the same modal characteristics occur. Bartok,[1] however, draws attention to a distinction affecting the modality of the songs: whereas the ubiquitous augmented 2nd is frequently found in both tetrachords (i.e. indicating duplication of the first tetrachord on 4th or 5th degree) in Rumanian Folk Music of the Banat and Maramuros, and is used to excess by the Gypsies, the author claims that 'never do two different augmented 2nds appear in one [Hungarian] tune'. Bartok adds that the intonation of the augmented 2nd is usually rather uncertain, the lower note being a little too high, the upper note a little too low (*op. cit.*, p. 55).

The Hako, a song sung at a Pawnee ceremony, collected by Miss Alice Fletcher,[2] is quoted in d'Harcourt's *La Musique des Incas* (p. 213). Although noted in the Key of *F*, this is clearly an error, for the Tonic is *C* and the keynote *D*. There are three closes on *g* and three of the following, which is also the final.

[1] *Hungarian Folk Music*, trans. by M. D. Calvocoressi (Oxf. Univ. Press, London, 1931).

[2] 'The Hako', by Alice Fletcher, *Smithsonian Institution. Publ. of the Bureau of Amer. Ethnology*, Part ii, 1900–1 (Washington, 1904).

FIG. 93.—Closes from The Hako Song of the Pawnees
Hypophrygian Harmonia (K. S.)

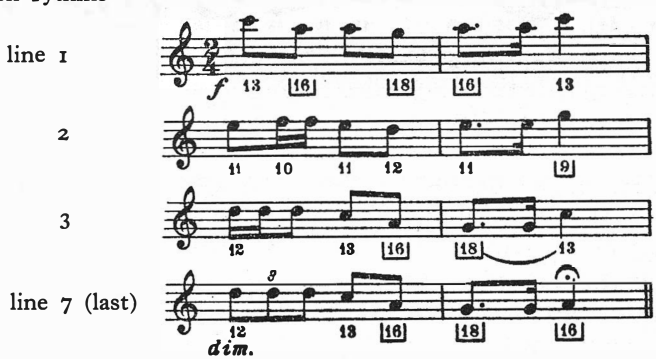
No other notes are used in the song, which consists of three lines only. This is clearly a Hypophrygian tune that exhibits the two pivots and begins with the leap up from the raised 4th to Tonic.

The music of the Incas provides several melodies in the Hypophrygian Harmonia from which we may select the lively tune No. 194 (p. 521), 'Ripusakme', '*Je m'en vais*', played upon the *Kena* (notched flute with six or seven fingerholes). The two pivots $\boxed{F \atop 18}$ and $\boxed{G \atop 16}$ occupy a central position throughout in alternate phrases, or in sequence as closes; there are leaps down from $\boxed{B \atop 13}$ to $\boxed{F \atop 18}$, but one misses the augmented 2nd 15/13. The harmonic register is greatly in demand.

No. 197 (p. 526) is a Hypophrygian pastoral melody played by an orchestra of Panpipes in the village of Totora, Peru.

FIG. 94.—Peruvian Pastoral (No. 197) in the Hypophrygian Harmonia

Orchestra of Panpipes
1 = 76 Bien rythmé



(Hacienda de Totora, Peru; from 'La Mus. des Incas', D'Harcourt, p. 526)

The scale of notes used:

g	*a*	*c*	*d*	*e*	*f*	*g*
$\boxed{18}$	$\boxed{16}$	13	12	11	10	9

18/13 = *561°*   4/3

the minor 3rd on the Tonic *B*♭/15 is absent. The characteristic interval 16/13 = *359·3 cents* of the 1st tetrachord is answered by the equally distinctive 11/9 = *347 cents* of the 2nd tetrachord.

The fact that a whole band of Syrinxes has been tuned to this Harmonia, implies that the scale forms part of the language of music of the Inca folk. The Harmonia is not born on the Panpipe as it is on the *Kena*; the instruments have to be specially tuned.[1] There is in the portfolio of fine photogravure plates, issued with the volume, one representing an orchestra of seventeen Panpipes from Peru providing an extended compass which would enable the pipes to play in a number of Harmoniai without retuning.

The Jews of the Yemen are partial to this Mode, if we may judge from the Folk song (Phonogram No. 1667, pp. 71–2). The scale is limited to the 1st tetrachord, the vibration frequencies of which are in agreement with those of a section of the Hypophrygian Harmonia.

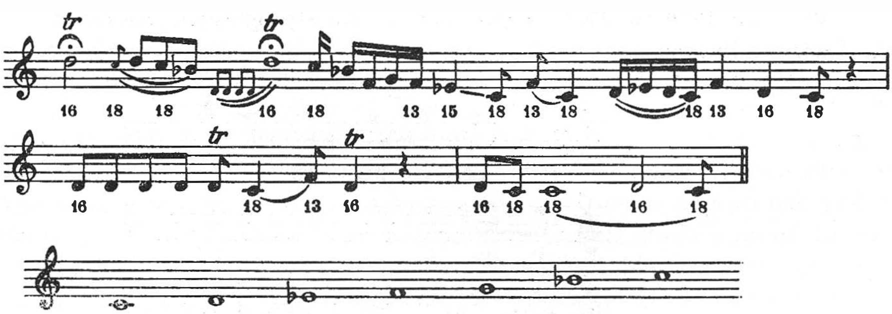
e	*f*♯	*g*	*g*♯
18	16	15	14

The vibration frequencies of the lute scale of Maqām Rāst on p. 118, No. 1601, give a very close approximation but with the curious omission of the keynote (a tone above the Tonic), probably due to error, for the 1st fret on the lute (*Sābbābā*) was at a tone above the open string. Six of the frequencies agree within 1 v.p.s. and the 7th within 2 v.p.s.

One more example in this Mode must be given in conclusion from the music of the Kubus of Central Sumatra.[2]

FIG. 95.—Tune No. 9(*a*). Music of the Kubus of Sumatra

Hypophrygian Harmonia



Modal Ratios by K. S.
The Scale of Notes used

| 18 | 16 | 15 | 13 | 12 | 10 | 9 |

$18/13 = 561°$ *The double ties indicate a slide of the voice.*

(*P.* 373, *op. cit.*)

[1] The Panpipes of Peru consist in many cases of open cylindrical reeds, having as closure a closely fitting, thin pad which can be moved up to the correct height in the tube to produce various shades of intonation, or different intervals.

N.B.—Shortening the resonant tube by 1 cm. produces in the fundamental note a proportional rise in pitch four times as great.

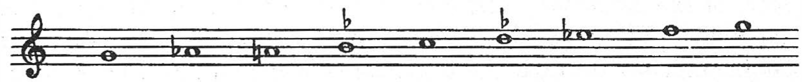
[2] 'Über die Musik der Kubu', by E. M. von Hornbostel, *Abh. zur vergl. Musikwiss.*, 1922, pp. 361–77. The scales of the tunes, pp. 375–6, need to be drastically revised; in the scale of No. 9, (*a*) for example, the note *d* is given as a quaver (unimportant); *g*, as minim with pause, and *a*, which does not occur twice, as semibreve.

The two modal pivots *C* 18 and *D* 16 are strongly emphasized throughout and form the final cadence. All the characteristic intervals of the harmonia: the minor 3rd *E*♭ on the Tonic, the *F*, which as the leap up from the Tonic, should be sharpened as 13, and even the augmented 2nd 15/13 are in evidence. The Kubus play on notched flutes with equidistant holes; therefore no surprise need be felt to find them so thoroughly at home in the Hypophrygian Harmonia. (See also Tunes Nos. 15*a*, 9*b*, 10—Tonic *D*, Keynote *E*).

The vibration frequencies of flute No. 3295 (*op. cit.*, p. 364), as recorded by Hornbostel, played the ratios of the Hypophrygian Harmonia 18/18, 16, 15, 13, 12, 11, in very close approximation. Thus it is found that this Modal Scale, in which the majority of the extant Fragments of Ancient Greek Music were sung, reverberates through the succeeding centuries among the folk of many nations.

THE HYPODORIAN HARMONIA IN FOLK MUSIC

The Modal Scale, with ratios by K. S.



Keynote and Tonic 16/16 15 14 13 12 11 10 9 8

Cents *112* *119* *128* *138* *151* *165* *182* *204* *cents*

The flattened *B* and *D* appear as flat or natural, sometimes both are used in the same tune. The 3rd on the Tonic (ratio 16/13) lies between major and minor at *359 cents.*

In the Hypodorian Harmonia, there is no 2nd pivot: Tonic and keynote are one, with ratio 16; the perfect 4th has the secondary emphasis; the augmented 2nd 15/13 is a marked characteristic, on the 2nd and 3rd degrees of the scale. A song in the Hypodorian Harmonia is found among the Hungarian Folk Songs collected by Béla Bartok (*op. cit.*, p. 73, No. 271), who gives this '*strange*' scale on p. 53 without shades of intonation.

The ratios of the Hypodorian fit the notes of the Modal Scale of this song, and the emphasis laid upon *C*, the 4th on the Tonic and secondary pivot, removes any doubt concerning the identity of the Mode, which is so closely allied to the Hypophrygian: in No. 271, the solitary appearance of *F* as passing note instead of Tonic, makes identification an easy certainty; and although the *A*♭ as 15 and the *B* as 13 are both in use, they do not occur to form an augmented 2nd 15/13.

From Jos. Singer (*op. cit.*), it is learnt that in the music of the synagogue the Mode Jichtabach is noted thus:

C—Des E—F G—AS B C (N.B.—*B* = *B*♭),

with the 1st tetrachord duplicated, which may be identified as Hypodorian (K. S.). Nos. 9 and 24 are good examples of melodies in the Hypodorian

Mode and furnish another distinctive feature, viz. the flat 5th on the Tonic $\left|\begin{smallmatrix}\flat\\ d\\ 11\end{smallmatrix}\right|$ in the scale given above. These distinctions are, of course, only important where neither vibration frequencies nor other shades of intonation are provided.

No. 94, p. 290, of Henebry's songs from the Ring Promontory, must be assigned a Hypodorian origin, in spite of its garbled notation and numerous vibration frequencies for each note ; for the song exhibits the main modal characteristics.

The most interesting Hypodorian Folk Song has been kept till the last : it hails from Wales. The emphasis in this delightful song ' Lliw Gwyn Rhosyn yr Haf ' duly lies on the Tonic, which in this Mode is also the keynote, and in the secondary pivot, the perfect 4th, the thetic Mese.

The augmented 2nd 15/13, in the true modal version (see below), is given full significance in the 2nd part of the song, and adds to its charm.

It is clear that the conventionalized form of the Mode was current in Wales when the song was taken down in staff notation ; it had its prototype, however, in the Ancient Greek musical system when, according to certain Graeco-Roman Theorists, the Tonic of the Hypodorian Harmonia led from Proslambanomenos through the tetrachords Hypaton and Meson to Mese, thus falsifying the beginning of the Harmonia with a Tone instead of a semi-tone.[1]

This Welsh Folk Song was introduced by Mr. Alfred Daniel, M.A., LL.B., D.Sc., F.R.S.E., in his address on ' Certain Vocal Traditions in Wales ' to the Cymmrodorion Society and the Welsh Folk Song Society, in London in 1909. He was bent on discovering traces of the ancient scale on which Welsh Folk Song was based. In his paper he has dealt with the ' *mixed character* ' of the intonation by means of two symbols : an acute accent for slightly sharp notes, and an obelus † to indicate a greater degree of sharpness ; the symbols appear over nearly every note of the song, and imply merely a loss of bearings quite usual when an attempt is made to translate the intervals of the Harmonia.

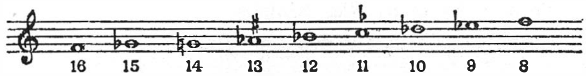
When, however, the modal version of the song (by K. S., given on p. 399), with strictly accurate modal intervals was sung to him some years later by Miss Isabel Dodds,[2] he pronounced the intonation correct and ' stated that the poignant sharp *A 13*, both ascending and descending, was also correct,

[1] Aristides Quintilianus, however (p. 18 M.), states that the Hypodorian Species starts from Mese. Ps-Euclid (p. 16 M) gives the 7th species from Mese to Nete Hyperbolaion ; or from Proslambanomenos to Mese, and states that this species is called Common Locrian and Hypodorian. Bacchius (p. 19 M) also places the Hypodorian Species from Mese to Nete Hyperbolaion, passing through the tetrachord Synemmenon, which is in agreement with the position assigned in this work to the Hypodorian Harmonia.

[2] After a study of the ancient scales of the Harmonia, Miss Isabel Dodds became known in the British Isles and the U.S.A. for her rendering, in their original ancient scales, of the songs of the Hebrides and for her recitations of the ancient Celtic romances and poems with improvised incidental accompaniment on a Celtic harp tuned to the Harmonia ; (she was duly authorized by K. S. to use the scales).

and as sung by his mother in the early 'eighties '. Needless to say, the song loses all its charm and becomes commonplace when the modal intervals are replaced by tones and semitones: e.g. in the second theme, the 15/13 augmented 2nd becomes a semitone.

FIG. 96.—' Lliw Gwyn Rhosyn Yr Haf '. Modal version by K. S. (as sung by Miss Isabel Dodds)

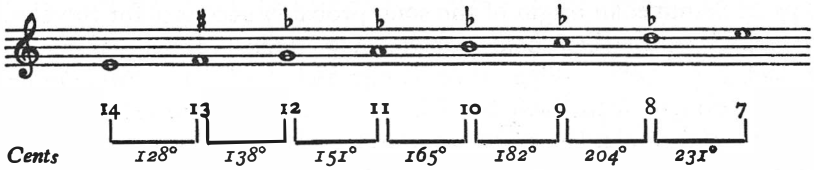


N.B. The main difference between the modal version and the one published in 1909 consists in the 15/13 modal interval instead of a semitone.

Other examples of the Hypodorian Harmonia have been given with the frequency values of the notes from the records of Dr. Kunst (see Chap. viii).

THE MIXOLYDIAN HARMONIA IN FOLK MUSIC: AS RĀG MĀLKOS IN HINDOSTAN

The Harmonia on Tonic *E*. (Ratios by K. S.)



The Mixolydian Harmonia has a very high *tessitura* with its keynote on the 7th degree of the scale; the keynote is, therefore, frequently found transferred to the lower octave as sub-Tonic. Our restricted survey has not yielded many examples of this Mode. The Rāg Mālkaus (Mālkos) of Hindostan undoubtedly derives from the Harmonia of M.D. 14; its scale is given thus by A. H. Fox Strangways (*op. cit.*, p. 150, Table No. 26).

	c	*e*♭	*f*♭	*a*♭	*b*♭	*C*
Ratios by K. S.	14/14	12	11	9	$\boxed{8}$	7
	tonic				keynote	

In his Musical Diary, Mr. Fox Strangways records a flute tune from a phonogram saying: ' assuming the *C* to be in tune, the higher notes were all a little flat, the *F* most so '. The Tune No. 29 (p. 30) has a range of a

4th C–F, evidently from Hole 1, used as vent for the Tonic; a practice which is usual with Indian flutes [1]; the A below the Tonic was produced from the exit, a note probably not in tune with the scale.

A very long piece of music in Rāg Mālkos is given by the same author: No. 380, pp. 287–99. The principal theme, built up on the characteristic notes of the Harmonia, runs thus (p. 289):

Fig. 97.—From a Song in Rāg Mālkos

(*A. H. Fox Strangways, op. cit., p. 289, lines 3 and 4*)

Ratios of Harmonia by K. S.

theme repeated with variations many times

The form of this ambitious composition is the Khyāl, a later form of the Dhrūpad; it consists of dozens of repetitions of the theme given above (which begins after the double bar) interspersed with variations upon it, and ending in virtuoso style with a long cadenza (suggestive of the European Concerto). The little introductory phrase in our quotation was given to show the use of the keynote $B\flat$ of ratio *8* in its place as 7th degree above the Tonic as well as below it. The leap from the Tonic to F, as 4th, the aṃśa of Hindu theory, and the use of Tonic and keynote as closes are all consistent characteristics of the Mixolydian Harmonia.

The Mohammedan origin of the song probably accounts for the absence of the superscripts, which belong to the Rāg Mālkos (as duly given in the table facing p. 150). When they conquered India, the Mohammedan musicians brought with them the Ditonal Scale borrowed from the Greek Theorists—and implanted it—as they did in North Africa and in Spain. The traces of the pre-Mohammedan System of the Harmonia have persisted nevertheless in Hindostan, as implied in the Ratnākara of Sārañgdev (see Chap. vii).

Another example of a simple tune in Rāg Mālkos is given as No. 407, p. 308, also with C as Tonic and $B\flat$ as keynote.

From the volume of *Thirty Indian Songs* recorded by Ratan Devī, and edited by Ananda Coomaraswamy,[2] 'The Gopi's Complaint', *Tumri* in Rāg Mālkaus: *tīn tāl* (*E* is *Sā*) may be cited as further evidence of the survival in Hindostan of the Mixolydian Harmonia. The song was sung

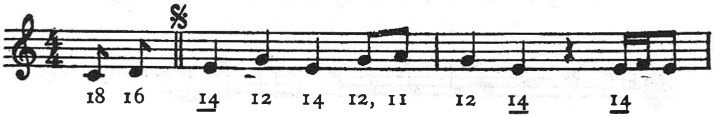
[1] See Chap. vii, the set of flutes described by Sārañgdev. As already explained, the finding of the correct position for the 1st hole is the main difficulty in boring the fingerholes: in order to avoid a false relation, the general practice is to base the scale on Hole 1 as Tonic, and used as Vent, always left uncovered.

[2] Four hundred and five copies were printed for the authors at the Old Bourne Press, 1913. Sold by Luzac and Novello, p. 9, No. 2.

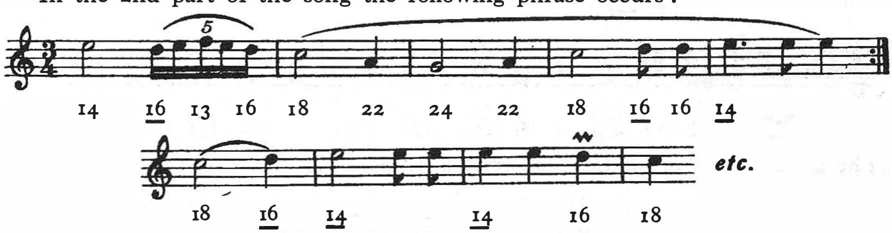
by an Ustad, Abdul Rahīm, the teacher of Ratan Devī. No superscripts are given. The Tonic is $\boxed{\frac{E}{14}}$, the keynote $\boxed{\frac{D}{8}}$, the following excerpt forms beginning and close.

FIG. 98.—'The Gopi's Complaint'
(*Recorded by Ratan Devī*)

(Mixolydian Harmonia) Ratios by K. S.



In the 2nd part of the song the following phrase occurs:



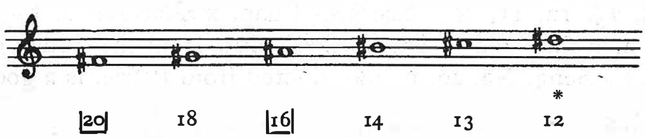
The two modal pivots, duly emphasized, and $\boxed{\frac{A}{11}}$ as aṃśa show, as in the examples in Rāg Mālkos given by Mr. Fox Strangways, that the feeling for the cardinal points of the Mode still survives in Hindostan.

From a diagnosis based upon the two modal pivots, the Mixolydian Harmonia might easily be confused with the Hypophrygian, when the keynote of the former, on the 7th degree of the scale, is transposed as sub-Tonic, since in uninflected staff notation both stand a tone apart, i.e. *D* to *E*. There is nothing, either, to indicate that in Rāg Mālkos the 4th on the Tonic is very flat (of ratio 14/11 = *417·4 cents*), whereas the 4th on the Hypophrygian Tonic is very sharp (18/13 = *561 cents*). The 15/13 augmented 2nd, moreover, is not obtainable in the Mixolydian. When the Harmoniai are actually heard, however, no confusion is possible on account of the sharply defined ethos proper to each of these two Modes.

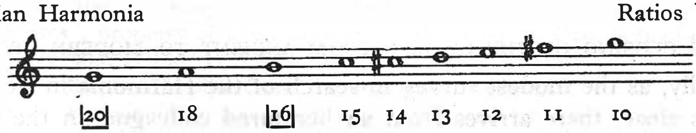
THE HYPOLYDIAN HARMONIA IN FOLK SONG

ORIGINAL SCALE, EAST GREENLAND

Ratios by K. S.



Hypolydian Harmonia Ratios by K. S.



* The D should probably be D♯.

Since the Hypolydian Harmonia (when the first tetrachord is duplicated on the Dominant) is practically identical with our modern major scale, it will not be necessary to quote more than this one song taken down by phonograph in East Greenland.[1]

The phonograph was set to *A* 435 v.p.s. Song No. 40, Group iii, p. 71, in original scale as above.

FIG. 99.—Recitative Song from East Greenland

13 $\boxed{16}$, 18, 20, 13 14 12 13 14 13

$\boxed{16}$ $\boxed{20}$ 13 $\boxed{16}$ 20 $\boxed{16}$ $\boxed{20}$ $\boxed{16}$ 13 $\boxed{16}$ $\boxed{20}$ 13

(The division symbol for *C*, ÷ signifies a flattened *C*♯ with the close

13 $\boxed{16}$ $\boxed{20}$

$\boxed{\begin{matrix}B\\14\end{matrix}}$ occurs only as grace note.

FIG. 100.—Eskimo Song (in same scale transposed)
(Collected by R. Stein. ' Eskimo Music ', p. 340 [ap. Thalbitzer, p. 35])

16 13 11 12 12 13 12 $\boxed{16}$ 20 20 20 $\boxed{20}$

Ratios by K. S.

The ratios are merely suggested, in the absence of v.f.s, by the pivots of the melody.

The two characteristic pivots of the Hypolydian Harmonia, the major 3rd on the Tonic, thus reveal the modality here also.

It may be recalled that the Burmese scale noted by Hornbostel and Kunst is (from v.f.) that of the Hypolydian Harmonia of ratios 20/20, 18, 16, 15, 13, 12, 11, 10. See also Chap. x, Records of the Bucheum, Graeco-Roman flute and of Dr. Tucker's flute from the Sudan of M.D.10.

Henebry's Song, No. 40, p. 180, quoted from Petrie, is a good example of a Hypolydian tune on Tonic, $\boxed{\begin{matrix}D\\20\end{matrix}}$; keynote, $\boxed{\begin{matrix}F\sharp\\16\end{matrix}}$.

THE CLOSING OF THE CYCLE: FROM ANCIENT TO MODERN GREECE

Finally, as the modest survey in search of the Harmonia in Folk Music nears its close, there arrives from an honoured colleague on the scientific

[1] W. Thalbitzer and Hjalmar Thuren, *op. cit.*, p. 71, and R. Stein, ex., p. 35.

side, Professor Dayton Clarence Miller, what may prove to be the coping-stone, viz. the scrupulously exact measurements of three modern Greek flutes from Olympia, Corinth and Nauplia,[1] played by shepherds.

The measurements worked out by our formulae establish the following facts :

OLYMPIA FLUTE. Phrygian M.D. 12, from Hole 1.
CORINTH FLUTE. Phrygian M.D. 12, from Hole 1.

Scale from the Fingerholes

						Half Inct.	
Fingerholes	1 used as vent	2	3	4	5	6	7
Ratios by K. S. denominator constant	12/12	11	10	9	8	7 or 15	6 or 13

Owing to the inner reactions of the air column, through what I have termed the Incremental All. No. 7, which is cumulative, the normal modal sequence is interrupted at the 7th Hole, with the result that this Hole speaks the note of half an increment lower, viz. 13 instead of 12. Hole 6 is bored midway between Holes 5 and 7 and plays ratio 13.

THE SPECIMEN FROM NAUPLIA HAS A DORIAN HARMONIA OF M.D. 11 FROM HOLE 1 with the following scale of ratios

						Half Inct.	
Hole	1	2	3	4	5	6	7
Ratios by K. S.	11/11	10	9	8	7 / 14	13	12

Owing to the operation of Incremental All. No. 7, which becomes active between Holes 5 and 6, *Conjectural* — Holes 5 6 7, Ratios 15 14 13 — would probably, therefore, speak notes corresponding in pitch to these ratios.

The above scales are conjectural, founded on proven theory, and have not yet been submitted to practical tests (for Measurements, see Chap x, Records).

These flutes from Modern Greece thus form an auspicious ending for the survey, which began with the scale of the Elgin pipes (Brit. Mus.) of *c.* 500 B.C. and the analysis of the extant fragments of Greek music as revealed by the System of Greek Musical Notation.

At the end of the journey through the centuries intervening, the Folk in Greece are still piping in the same ancient Harmonia used by their ancestors. Whether this may be claimed as a case of uninterrupted survival must be left to others to decide.

[1] *Olympia*, specimen No. 18 on photograph No. 151, and No. 27 on photo ' Exotic Flutes ', property of Prof. Dayton C. Miller of the Case School of Applied Science, Cleveland, Ohio, U.S.A.

Corinth. Shepherd's pipe from the Acropolis. Idem.

Nauplia. Bought from Shepherd Boy in Greece by Mr. Jackson of Exeter College. Flute was formerly in Taphouse Collection and later in the Brownsea Castle Collection.

After reading the foregoing chapters, it will probably be recognized that the most significant contribution made by this work to the study of the early stages in the evolution of music in general, and more specifically to the origin and development of modal systems, consists in the introduction of the inherent and infallible testimony borne by reed-blown pipes and flutes having equidistant fingerholes. It is evident, therefore, that the only hope of being able to reconstitute the scales or musical systems of a bygone age, in harmony with ascertained facts, lies in the recovery of authentic specimens of its pipes and flutes; these, at least, tell their own tale through the boring of their fingerholes; if, moreover, these are found placed at equal distances, we are at once put in possession of certain definite facts without hearing a note from the instrument itself.

It is felt that any conclusions formed on the music of the Ancients or of primitive folk of the present day, must inevitably be found related to some basic principle, capable of accounting in a reasoned, logical manner for the sequence of intervals detected. It must, moreover, be such a basic principle, for instance, as the one embodied in the Aulos Harmonia, which inevitably creates Modality. Therefore, to attempt to establish the foundations of a scale or of a musical system upon mere conjecture, or by comparison with our keyboard scale, is to court failure. As an apt application, I may cite the pair of silver pipes (dating from 2800 B.C.), discovered in the course of Sir Leonard Woolley's excavations at Ur of the Sumerians. They are illustrated and described in the sumptuous volume by Canon Galpin on 'The Music of the Sumerians, Babylonians and Assyrians'.[1] The two slender, cylindrical silver pipes found at Ur have a bore of only 4 mm.; and there are four fingerholes at equal distances. One of these pipes is said to be perfect except for about 3 mm. at the mouthpiece end (but see photograph and tests, Chap. x, Records). A facsimile of the pipe has been tested by Canon Galpin, who used at first as mouthpiece a small Zummarah single- or beating-reed, which played from exit and the four holes, the notes *c*, *d*, *e*, *f*♯, $\overset{\sharp}{g}$ (the *g* decidedly sharp), the octave[2] being completed by the use of the Harmonic register. Subsequently, with a double-reed mouthpiece of straw, fitted with a ligature,[3] the same scale was again obtained with ease. These are facts, and judging from Canon Galpin's wide experience during many years in collecting and playing upon all kinds of wind instruments,

[1] Camb. Univ. Press, 1937. The pipes are illustrated on Pl. XI, No. 3, and described on pp. 40 sqq. and p. 94. The book has come to hand just as mine is going to press, so that there is only time to add a few comments.

[2] On a pair of pipes, the octave is completed on the 2nd pipe, as I have suggested in Chap. ii.

[3] See Victor Loret, 'Sur Une Ancienne Flûte Égyptienne', *Soc. d'Anthropologie de Lyon* (Juin, 1893), Fig. 2, p. 12. Canon Galpin was unfortunately misled by Victor Loret's illustration of one of the two Ancient Egyptian wheaten double-reed mouthpieces—which appears to have a ligature—both mouthpieces were found inserted in pipes discovered by Maspero in Panopolis. Loret explains, however, in the text that there was originally no ligature, and that he himself wound a thread round the splitting straw to prevent the eventual destruction of such a precious relic. There are, however, traces of a slight natural constriction in the reed, the two blades of which are flattened towards the tip.

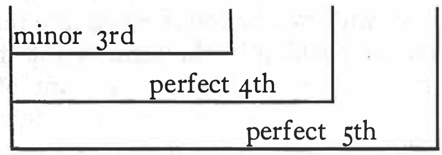
we have every confidence in the results he obtained. It is when we pass from these facts to his deductions, which he avows are a matter of conjecture, that the ground is felt to be insecure, and the serious consequence of this is that the interpretation of notation is thereby involved. With Canon Galpin's adoption of the scale of the pipe of Ur as the standard scale of the Sumerians, I am in complete disagreement. Every reed-blown pipe having equidistant fingerholes, inevitably implies modality, by virtue of the principle embodied in the pipe itself, which I have defined and shown in operation in my first chapter. Moreover, modality is a fraternity of related Harmoniai, and no single pipe with equidistant fingerholes, but lacking its mouthpiece, can stand alone; it is a specimen devoid of individuality until provided with a mouthpiece that will play it at a definite extrusion from the resonator.

No reed-blown pipe with equidistant fingerholes can have a scale of fixed intervals or pitch; its scale must always be regarded as a temporary manifestation, for it can with ease be transformed into that of another Mode, by merely pulling out or pushing in the stem of the mouthpiece, thereby lengthening or shortening the total by the amount of one increment of distance or more. For it is the mouthpiece at a definite extrusion from the resonator that decides which of the Harmoniai the pipe shall bring forth. It must not be assumed, therefore, without definite evidence that any pipe has been bored specifically with intent to produce any one scale which, by extension in both directions, might come to be generally adopted in any land as a standard scale.

Among primitives the pipe may be of any fancied length and diameter; the spread of the fingers on the pipe fixes the position of the fingerholes at an equal increment of distance; this prepares the way for the Harmonia, brought to birth as soon as the piper adjusts his mouthpiece so that it plays with ease. There may be a dozen or a hundred pipes of different dimensions in the length and breadth of the land, with increments of distance varied within limits, and mouthpieces at different extrusions, yet out of this mass of heterogeneous dimensions there does eventually emerge a Modal System. The wonder is that the Modal System, when traced and duly identified by the flutes and pipes, or by the vibration frequencies of phonographic records, inevitably does exhibit examples of the same seven Harmoniai of Modal Determinants 16, 14, 13, 12, 11, 10, 9, as, for instance, in Ancient Egypt and in present-day Java, Burma and Hindostan, &c.

Let us now return to the pipes of Ur: we left Canon Galpin playing on his facsimile pipe with two different kinds of mouthpiece, of the single- or beating-reed, and of the double-reed types, and obtaining from both the same scale with the tritone on the 4th degree. We must come down to the practical side of this weighty matter and see what it all means. The Ur pipe had a length of ·270 (like the Mond flute) and a diameter of only 4 mm.; the mouthpiece with which it was tested added another ·0385; the resonator thus now totals ·3085, the increment of distance between the holes is of ·031; therefore, in millimetres, $\frac{308{\cdot}5}{31} = 10$, as Modal Determinant, which is that of the simplest form of the Hypolydian Harmonia of ratios 10/10, 9/10, 8/10, 7/10, 6/10; i.e. a scale represented by the notes *c*, *d*, *e*, *f*♯, and *g* very sharp (or we may call it a flattened *A*, as septimal 3rd from *F*♯, also very sharp). It is not necessary to go to India, China or Java to find an analogous scale with raised 4th, for I know of three others,[1] and unless definite data were available there could be no basis of comparison. By fitting a different mouthpiece with a long shank to the silver pipe, which could be pulled out to an extrusion of about ·071, the scale would then be that of the Dorian Harmonia of M.D. 11 (·031 × 11 = ·341 − ·270 = ·071); or, again, with an extrusion of ·102, the Mode would be the Phrygian of M.D. 12 (·031 × 12 = ·372 − ·270 = ·102). The ratios of the Dorian Harmonia for the exit and four holes would be 11/11, 10/11, 9/11, 8/11, 7/11, and of the Phrygian,

12/12 11/12 10/12 9/12 8/12



The trouble is that we can have no idea which of these three was last in use on the pipes of Ur. The same fingerholes would therefore give out in turn three different series of intervals, e.g. the 2nd fingerhole would sound a major 3rd on the Tonic in the Hypolydian Mode, a 3rd 11/9 between major and minor, on the Dorian, and in the Phrygian a just minor 3rd, all in perfect tune.

The factor of differentiation in these pipes is proportion: i.e. the relation between the Increment of Distance and its multiple (represented by resonator + mouthpiece). The Increment of Distance is variable within limits, imposed by the lay of the fingers and the length of the resonator. Modality, however, is ultimately the affair of the mouthpiece, the extreme importance of which has not been sufficiently realized. Its influence is paramount, not only in the determination of the Mode, but also on Tonality—quite independently of the factor of length—through the phenomenon of resonance.

There is a remarkable resemblance between the pipes of Ur and those of the Lady Maket,[2] discovered by Sir W. Flinders Petrie, and other similar pairs shown in musical scenes on Egyptian Wall-paintings, as, for instance, on the one from a tomb in Thebes in the British Museum (see Chap. ii,

[1] These four scales with raised 4ths have been given with ratios and vibration frequencies as Fig. 85(B) earlier in this chapter: they are the Dorian with sharpened 4th of ratio 11/8; the Hypophrygian with a still sharper 4th of ratio 18/13, the Hypolydian 10/7, and the scale derived from a cycle of perfect 5ths which cannot occur on a pipe having equidistant fingerholes.

See Chap. x, Records of Auloi, for measurements and photograph No. 18 of the Ur Pipes.

[2] For further details, measurements and performance, see Chap. x, Records. For a review by K. S. of Canon Galpin's book, *Music and Letters* (April, 1938). See also July and October Correspondence.

Plate No. 3) and in other similar mural paintings preserved in continental museums. The stalks of the wheaten mouthpieces at a long extrusion are distinguishable by their straw colour, whereas the reed resonators are painted reddish brown.

It is my reasoned opinion that further research into the Music of the Sumerians and Chaldeans will reveal the existence of a Modal System based upon the Harmonia. The practice and development of the Harmonia into a Modal System, moreover, could hardly fail to be strongly influenced by the Sumerian and Babylonian astronomical and astrological sciences and beliefs, so that the characteristic Ethos of each Harmonia, by its strong appeal, might well suggest its dedication to a titular divinity or to one of the stellar spheres.[1]

Apart from these strictures I have felt obliged to make on the subject of the scale and notation, Canon Galpin's valuable investigations appear to be of absorbing interest; they will afford food for study and reflection.

I must bring this long survey to a close with the following dictum: the discovery of pipes and flutes with equidistant fingerholes must henceforth invariably imply Modality; and when the establishment of a standard scale is contemplated, pipes of this type should be rigorously excluded.

[1] See, for instance, Nicom., *op. cit.*, M., pp. 7 and 33 sqq., whose attribution to individual planets (including Sun and Moon) of the seven degrees of the Harmonia, may have been inspired by Sumerian and Chaldean traditions.

CHAPTER X

RECORDS OF MEASUREMENTS AND PERFORMANCE: (1) OF AULOI, (2) OF FLUTES, PRECEDED BY EXPLANATORY NOTES

THE RECORDS OF AULOI

Elgin Aulos 1 (straight) } Plate No. 17.
Elgin Aulos 2 (curved) }
'Lady Maket 3.'
'Lady Maket 4.'
Loret XXIII.
Cairo 'C.F.'
Cairo 'C.M.'
Cairo 'C.R.'
Cairo 'C.G.'
Primitive Oboe from N. Egypt. Plate No. 10.
Loret X.
Loret XII.
Loret XIII.
Loret XV.
Loret XVI.
Loret XVIII.
Loret XIX.
Loret XXI.
Loret XXII.
Loret XXIV.
Loret XXV.
Loret XXVI.
Loret XXVII.
Loret XXVIII.
Loret XXX.
Loret XXXI.
Loret XXXII.
Loret XXXV.
Loret XXXVI.
The Silver Pipes of Ur. Table XIV.

EXPLANATORY NOTES

N.B.—According to my invariable practice, the measurements given in records of Auloi are taken from embouchure to the centre of each fingerhole, therefore *not* as in Loret's list to the upper edge of each hole.

CONCERNING THE NAMING AND NUMBERING OF THE MOUTHPIECE

An apology must be offered for the haphazard allotment to the mouthpieces of their distinctive letters and numbers. These mouthpieces have been made and collected over a period of some twenty years, during which each was measured, tested many times and recorded, before the principles and conditions under which they operate had been discovered, and could provide a basis for classification. This is regrettable, but time is lacking for the handling of such vast material : to change the labels would involve irremediable confusion.

THE FORMULAE FOR THE DETERMINATION OF PITCH FROM LENGTH IN PRIMITIVE MOUTHPIECES

The D–R. Mouthpiece.—The total length of the wheat or oaten stalk has no bearing on pitch ; it is, however, necessary that its length within the bore of the Aulos should exceed that of extrusion, by an amount sufficient to resist the weight and pressure involved in the manipulation of the mouthpiece.

The factor which determines pitch is the distance from the open end of the straw at which the lips close upon it, and thus define the vibrating length (V.L.). The vibrating length may then be converted by formula into pitch vibration frequency, an operation for which formulae are given in Chapter iii under suitable headings for both types of mouthpiece ; see also Table of Formulae, pp. xliii–iv.

Where difficulty arises in obtaining the fundamental note of the Aulos, pressing the lower lip against the stalk brings out the fundamental note at the selected vibration length. In testing the modal sequence of an Aulos, note by note, great care must be taken to maintain unchanged this vibration length, which can be done by keeping the thumb-nail on a line at the measured distance on the mouthpiece.

INCREMENT OF DISTANCE

The Mean I.D.—This is stated with due regard to the point at which the scale of the Aulos begins : whether from exit, or from vent. The increment of distance between exit and vent is omitted from calculation as ostensibly unused, unless it is a multiple—within a few millimetres—of the increment of distance ; for as a multiple, the latent increments must, of course, be taken into account. Actually, the distances from centre to centre of fingerholes are rarely exactly equal ; it is a matter of great interest to discover the secret of the strength displayed by the proportional, propulsive impulse through the resonator of the Aulos which discriminates between these increments. A case in point may be examined in the record of Aulos ‘ Cairo R ’. The mean I.D. = ·03083 (= ·031) but the pipe decides in favour of I.D. ·032. The significance of the mouthpiece extrusion is that of ‘ adjustor ’ in the aliquot division by the Modal Determinant productive of modality and the increment of distance. To obtain the best results, the Resonator L. + Mp.-Ext. should form a multiple of the increment of distance, which appears to indicate a very simple proposition. The mouthpiece extrusion, however, is the obstinate factor : it may be irreproachable on paper, but slip the mouthpiece into the Aulos, and it may manifest its displeasure in various ways : it may play the fundamental and the note of Hole 1, but on opening Hole 2, the note may drop down to the fundamental which may unaccountably sound again ; or else the mouthpiece may refuse to sound at all. If the mouthpiece be of the D–R. variety the length of extrusion may prove insufficient to admit of the requisite vibration length (e.g. in the Maket Pipes 3 and 4). These difficulties and many others are more apparent than real, however, to any one who has experimented with the Aulos and they should not act as a deterrent. Having selected a mouthpiece that will play freely in the Aulos, the first step is to discover the lowest Modal Determinant compatible with a multiple of the increment of distance, which will allow a mouthpiece extrusion of at least ·068 or ·070 for a double-reed, or ·50 as a minimum for a beating-reed mouthpiece. Then the next M.D. in the series may be tested in theory and practice, when the altered multiple resonator length (theoretical) minus the actual R.L. gives as remainder the new mouthpiece extrusion.

The performance of the mouthpieces in these Records discloses the fact that in spite of all the theories and formulae, it is the mouthpiece—and through it the idiosyncrasy of the piper—that has the casting vote.

The preamble explains why the mean increment of distance is useful on paper, but that in practice the mouthpiece chooses the most useful of the increments, all the factors in the case being implicit in mouthpiece and resonator. The part played by the pipes is the true crux; and only after other experimentors have devoted time and patience to these problems will it become known what results, other than mine, are obtainable with these mouthpieces by another piper under like conditions.

Finally, those who would understand the nature and capabilities of the Aulos, and who wish to derive useful material from data in the Records, are invited to make a preliminary study of Chapter iii on the Aulos and more especially of pp. 91–99 (and of pp. 94–96 as of special importance); of Chapter vi on the Flute including Incremental All. No. 7 in its unsuspected cumulative aspect.

CONCERNING THE SOUND-WAVE LENGTH

In an open pipe, such as a flute, the sound-wave accomplishes its periodic journeys from embouchure to exit, and back again to embouchure. The complete course is thus approximately twice the length of the flute, or to be exact, flute length + diameter taken twice. For a closed pipe, such as the Panpipe, the Aulos, and its mouthpieces, the product must be multiplied again by two; the complete sound-wave in a closed pipe thus represents approximately four times the length of the pipe. I say approximately because the influence of diameter (Δ) as added length, must be taken into account, with its implications in individual cases.

There is one point which frequently puzzles the student, and that is the relation of the sound-wave—whether complete or fractional—to the visible and measurable length of the instrument: it is only the complete sound-wave that is productive of the vibration frequency of sound; and moreover, the length of the whole sound-wave never corresponds to the length of the bore of the instrument. To speak of a half- or quarter-sound-wave, or to quote its vibration frequency, is a pure convention for use merely on paper. The imaginary vibration frequency of a half- or quarter-sound-wave is respectively twice or four times that of the whole sound-wave. The term 'effective' added to the whole or fractional sound-wave implies the inclusion of diameter as added length.

The tongue of the beating-reed mouthpiece (hinged with its base towards exit) requires two movements for each pulsation; the first, as the piper blows through the mouthpiece, lifts the tongue free of the opening into the bore; the second movement draws the tongue back into place over the opening, which it completely closes again. The result of the two movements adds twice the length of the reed tongue + diameter + (diameter − tongue width) to the actual length of the reed tongue; and in addition four times that aggregate—since Aulos and mouthpiece both react as closed pipes—so that the length of the sound-producing wave is eight times that of the aggregate actual length, and the vibration frequency of this 1/8 sound-wave is eight times that of the whole sound-wave. (See Formulae Nos. 8 and 9.)

Finally, the effective length of a half-sound-wave is equal to the actual flute length plus diameter allowance taken once; and for the complete sound-wave, the sum of twice the length and twice the diameter is implied.

ELGIN AULOS No. 1 (Straight)

British Museum: Dept. Graeco-Roman Antiquities

Facsimile by K. S.

Owing to the delicate condition of the precious relic the measurements taken in the Graeco-Roman Department at the British Museum were made by me without actual contact with the wood of the instrument. The pipe was fixed on a flat surface of white cartridge paper; supports were placed at both ends for the rule. The measurements were taken with a millimetre rule, with the utmost care, and were checked again and again, and then compared later with those made by Mr. William Bentley, about 1915; and again in 1934. They substantially agree with these as may be seen below.

Two earlier facsimiles made during the war were discarded by K. S. as not being sufficiently exact in every detail; the present specimen dates from December 15th, 1925.

The Elgin Aulos No. 1 plays in the following Harmoniai:

(*a*) Dorian Spondaic	M.D. 11.	Extrusion of mp. ·108.	Played from Hole 1.
(*b*) Hypolydian	M.D. 10.	,, ,, ,, ·076.	,, ,, ,,
(*c*) Phrygian	M.D. 12.	,, ,, ,, ·140.	,, ,, ,,
(*d*) Lydian	M.D. 13.	,, ,, ,, ·105 from exit.	

MEASUREMENTS BY K. S.

Two separate sets of measurements were taken during the same year 1914–15 at the British Museum by W. B. (William Bentley, Hon. Librarian, Birmingham and Midland Institute) and by K. S. (author of present work).

	metres	
Length of Aulos from exit to embouchure	·311	= ·067
Length of Aulos from vent (i.e. centre of Hole 1 left open) . —	·244	
Diameter of Aulos at exit and at mp. end of bulb . ·007 or	·008 [1]	
Diameter of fingerholes	·007	
One bulb found entire with the Auloi	·0425	
Another (incomplete) measures	·033	

The distances have been measured from the embouchure to the centre of the fingerholes, since their diameters are equal:

[1] The bore of the Aulos at the bulb end or embouchure has been shaved off to admit of introduction of bulb. The diameter of the bore is approximately equal throughout the Aulos, according to test, by means of a slip of paper inserted through the fingerholes.

POSITION OF FINGERHOLES		I.D.
Centre of Hole 1 from emb. = ·244 ;	from exit to C. of Hole 1	= ·067
Centre of Hole 2 from emb. = ·204 ;	from C. of Hole 1	= ·040
Centre of Hole 3 from emb. = ·1715 ;	from C. of Hole 2	= ·0325
Centre of Hole 4 from emb. = ·1405 ;	from C. of Hole 3	= ·031
Centre of Hole 5 from emb. = ·1085 ;	from C. of Hole 4	= ·032
Centre of Hole 6 from emb. = ·0805 ;	from C. of Hole 5	= ·028
		5)·1635
	Increment of distance for 5 holes, mean	= ·0327

Mean, omitting first I.D. = ·0307

The provisional Modal Determinant (P.M.D.) is thus found to be 7 + ·020

Remainder for the Resonator of Aulos $\left(\text{i.e. } \frac{\cdot 244}{\cdot 032} = 7 + \cdot 020, \text{ \&c.}\right)$

For the Dorian Spondaic of M.D. 11 $\}$ ·032 × 11 = ·352 = length + mouthpiece

− ·244 length of resonator from vent.

extrusion of mp. = ·108

Increment of distance, ·0327 mean. ·032 adopted as better for working in practice, since the first I.D. ·040 is obviously an error : mean for 4 holes, ·0307 ; the excess in the first increment does not affect the intonation of Hole 2, which is played in tune in each Harmonia in spite of it.[1]

MEASUREMENTS BY W. B.

These were communicated in April, 1934, in the form of a diagram of the Elgin Aulos, the actual size of the original Aulos, drawn by W. B. according to his measurements taken at the British Museum. These are practically the same as those of K. S. given above. The slight differences—within the same total length ·311—are probably due to the fingerholes not being centred on the diagram and having diameters varying between ·007 and ·010.

Length from exit to embouchure	·311
,, ,, centre of Hole 1 to embouchure	·242

The distances are measured from embouchure to centre of holes.

Centre of Hole 1 from emb. = ·242 ;	from exit to C. of Hole 1	= ·069
Centre of Hole 2 from emb. = ·201 ;	from C. of Hole 1	= ·041
Centre of Hole 3 from emb. = ·168 ;	from C. of Hole 2	= ·033
Centre of Hole 4 from emb. = ·137 ;	from C. of Hole 3	= ·031
Centre of Hole 5 from emb. = ·104 ;	from C. of Hole 4	= ·033
Centre of Hole 6 from emb. = ·076 ;	from C. of Hole 5	= ·028
		·166

Increment of distance mean for 5 holes = ·033

for 4 holes mean = ·0312

The first increment is obviously an error.

Since these measurements taken by W. B. differ so slightly from those of K. S., no separate facsimile was made.

[1] Attention has already been drawn to the fact that in the Aulos played with primitive reed mouthpieces of the double-reed or of the beating-reed types, length does not influence pitch according to accepted formulae. It is a fact that excess, either positive or negative, in one increment does not affect the purity of the interval in question. An explanation of this surprising fact has been suggested in this section, see Chap. iii.

TABLE IX

Elgin Aulos I (Straight)

Record of Performance

(6 Holes, with 19 Mouthpieces on 6 fundamentals)

Length from exit to embouchure = ·311 ; Length from Hole I (Vent) = ·244
Increment of Distance = ·032 (mean). V.L. = vibration length

Mp. Type and Number	Modal Determinant	Fundamental v.p.s.	Extrusion	V.L. of Mouthpiece	
D-R. Elgin I ' BB '	11	$C = 128$	·108	·065	*from Hole I as vent*
D-R. ' D. 1 '	11	$C = 128$	·108	·060	*from Hole I as vent*
D-R. ' Cl. 6 '	11	$C = 128$	·108	·075	*from Hole I as vent*
	12	$B \frac{12}{64}$	$\left.\begin{matrix}·138\\·140\end{matrix}\right\}$	·060	*from Hole I as vent*
D-R. ' Cl. 7 '	11	$C = 128$	·108	·070	*from Hole I as vent*
D-R. ' Cl. 7 '	12	$A \frac{27}{128}$	$\left.\begin{matrix}·138\\·140\end{matrix}\right\}$	·060	
D-R. ' Cl. 18 '	11	$C = 128$	·108	·075	*from Hole I as vent*
D-R. ' N. 1 '	11	$C = 128$	·108	·060	*from Hole I as vent*
D-R. ' Elg. M. 12 '	11	$B \frac{12}{64}$	·108	·060	*from Hole I as vent*
D-R. ' N. 7 '	12	$A \frac{27}{128}$	·140	·060	*from Hole I as vent*
D-R. ' Elg. 13 ' D-R. ' Elg. 13A '	13	$A \frac{13}{64}$	·105	·067	*from exit.* *N.B.*—Elg. 13A also plays at *E* ·076 in Hypolyd. Harm. on *D* 20
D-R. ' Elg. D. 10 '	10	$D \frac{20}{128}$	·076	·070	*from Hole I as vent*
D-R. ' Cl. 1 '	10	$A \frac{27}{128}$	·076	·070	*from Hole I as vent*
D-R. ' Elg. DK. 1 '	10	$D \frac{20}{128}$	·076	·060	*from Hole I as vent* This mp. is now split and is useless
D-R. ' Elg. DK. 2 '	10	$A \frac{13}{64}$	·076	·060	*from Hole I as vent*
D-R. ' H. 6 '	10	$D \frac{20}{128}$	·076	·060	*from Hole I as vent*
B-R. ' E.E. ' River Reed	11	$G \frac{15}{64}$	·108		*from Hole I as vent*
B-R. ' Elg. B '	10	$B \frac{12}{64}$	·076		*plays from Hole I*
B-R. ' Elg. N '	11	$C = 128$	·108		*from Hole I as vent*
B-R. ' D. 7 '	10	$D \frac{20}{128}$	·076		*from Hole I as vent*

PERFORMANCE OF ELGIN AULOS I WITH VARIOUS MOUTHPIECES OF D-R. AND B-R. TYPES

A condensed statement in tabulated form gives a bird's-eye view of the result of many tests on this Aulos, and is followed by a detailed record of performance with each mouthpiece.

TEST PERFORMANCE BY INDIVIDUAL MOUTHPIECES

(1) Double-Reeds.	(2) Beating-Reeds
V.L. = Vibration length determined on D-R. by impact of lips. (T.) = Treated mp.	(U.) = Untreated mp. M.D. = Modal Determinant.

D–R. Mp. (T) ' Elgin B.B.' M.D. (11). Dorian Spondaic

L. ·133 ; $\begin{cases} \Delta \cdot 004 \text{ exit; V.L.} = \cdot 060. \text{ Ext.} \cdot 108 \\ \Delta \cdot 008, \text{ flattened.} \end{cases}$

The mouthpiece was treated by pressing the wheat stalk out gently at one end to a fan-shape, while damp after a soaking of some hours; the mouthpiece embouchure then measured ·008 across, and the V.L. was accordingly reduced to ·058. Treatment is not necessary for testing an Aulos; the natural wheat stalk speaks equally well, and may be used at once; but for a musical performance the treated mouthpiece has a finer tone as a rule. For accuracy and purity of intonation both kinds give equally good results, but for testing, the untreated mouthpiece is superior, being always instantly available and ready for use.

Mouthpiece ' Elgin B.B.' used alone (Dec. 16, 1925), after a good soaking played *F* 16/128, when held loosely in lips, and *G* 14/128 when pinched with lips; at V.L. ·035 gave *D* 20/256, &c. In the Aulos this mouthpiece (on same date) played *C* = 128 v.p.s. from Hole 1 used as vent, the whole modal sequence (given below) in perfect tune; the intonation of each note was stable time after time; whether taken in the modal sequence, or haphazard from any of the fingerholes, the correct note always came: at tests, repeated through the years, the result has never varied. As a law-giver and for purposes of research the double-reed primitive mouthpiece reigns supreme, but the tone is more hollow and muffled, less reedy and resonant, and yet mellow, than that of the beating-reed mouthpiece of primitive type. Repeated tests were made after this with the same enduring results: Sept. 27, 1930; Feb. 10, 1931; March 6, 1933, &c.

The Modal Sequence in Dorian Spondaic (11)

Holes	1		2		3		4		5		6
Ratios	$\frac{11}{11}$		$\frac{10}{11}$		$\frac{9}{11}$		$\frac{8}{11}$		$\frac{7}{11}$		$\frac{6}{11}$
Cents		*165°*		*182°*		*204°*		*231°*		*267°*	

THE ELGIN AULOS I IS A DORIAN SPONDAIC AULOS FOR THE FOLLOWING REASONS

It will be noticed that Elgin Aulos 1 has been claimed as a Dorian Aulos, and yet the Record shows that the Aulos may be played also in the Lydian, Phrygian and Hypolydian Harmoniai. Some explanation seems, therefore, to be due.

The fact is that the Dorian Spondaic Harmonia fits in best with the measurements of the Aulos resonator plus one bulb, found with the pipe, and fitting into the bore exactly. The mouthpiece extrusion which produces the requisite length for the eleven increments of distance for the Dorian Harmonia is ·108, and of this the bulb covers about ·038 or ·039 of the stem of the mouthpiece, thus leaving ·070 for a possible V.L.

This extrusion of ·108 appears from the tests (carried out by K. S.) to afford the best conditions and results. The extrusion of ·140 required for the Phrygian Harmonia, although unexceptionable on paper, when two bulbs are required (·038 × 2 = ·076) leaves ·064 for V.L. which does not give such good results in practice, owing to the length of the slender shank of the mouthpiece and probably also to less favourable conditions of resonance, set up between mouthpiece and resonator—these conditions might have been bettered by a V.L. of from ·070 to ·080 which are unobtainable here.

The most conclusive and convincing fact is, however, the modal sequence itself which can only be rendered from the six holes of the Aulos in complete melodic form in the Dorian Spondaic of M.D. 11 (see figure below). According to my opinion, this form of the Dorian Harmonia can be no other than the one attributed to Terpander, said to have contained a characteristic interval (of a septimal 3rd), explained by the Theorists as due to the omission of Trite Diezeugmenon, whereas it is clear that the interval is implicit in the Harmonia itself as part of the modal sequence proper to M.D. 11. This modal sequence is entirely satisfying when played up and down, with or without the addition of the octave on Nete Diezeugmenon; whereas the Phrygian Harmonia, breaking off on ratio 12/7 of the septimal tone, leaves one in the air and sounds meaningless as a scale. Had the Elgin Aulos been primarily intended for the Phrygian Harmonia, it would undoubtedly have started from exit and ended at Hole 6 on the octave 12/6.

The Dorian, Phrygian, Hypolydian and Lydian Harmoniai on the Elgin Aulos

Fingerholes Modal Ratios	Exit	1	2	3	4	5	6	
Dorian Spondaic.		$\frac{11}{11}$	$\frac{10}{11}$	$\frac{9}{11}$	$\frac{8}{11}$	$\frac{7}{11}$	$\frac{6}{11}$	= sept. 3rd
Phrygian Harmonia.		$\frac{12}{12}$	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{9}{12}$	$\frac{8}{12}$	$\frac{7}{12}$	
Hypolydian Harmonia.		$\frac{10}{10}$	$\frac{9}{10}$	$\frac{8}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{5}{10}$	
Lydian Harmonia.	$\frac{13}{13}$	$\frac{11}{13}$	$\frac{10}{13}$	$\frac{9}{13}$	$\frac{8}{13}$	$\frac{7}{13}$ ...	$\frac{6}{13}$	

The Lydian Harmonia is obviously the Dorian from the vent at Hole 1, merely extended downwards a flattened minor 3rd if sounded from exit; a note which was probably unused although the distance from exit to Hole 1 is equal to two increments; but the last note of Hole 6, overstepping the octave, is unconvincing in an Aulos designed for the Lydian Harmonia.

D–R. Mp. '*D. 1*'. (*T.*). M.D. (11). Dorian Spondaic

L. ·172; Δ ·003, flattened to ·006; proper note at V.L. = ·059 = $\frac{F\ 17}{128}$; V.L. at ·077 = $\frac{C\ 11}{128}$; gl. = $\frac{g\ 15}{128}$.

Tests Jan., 1926, 1930, 1931, 1933, and 8/1/34.

D–R. mp. 'D. I.' plays in Elgin Aulos 1 on *C* 11/128, at extrusion ·108, V.L. = ·060. A note in the record early in 1933 states that this mouthpiece played the whole series (as above) in perfect tune, with strong, firm notes; three days later a second note states that the performance was not so satisfactory, the notes were weaker and the note of Hole 6 difficult. There was nothing in the mouthpiece to account for this deterioration, which was found to be due to the player's physical condition which, from frequent observations, has been found responsible for unsatisfactory results.

D-R. Mp. (*U.*) *Cl. 6.* M.D. (11). Dorian Spondaic

L. ·161 ; Δ ·004 ; V.L. ·079 ; proper note $\frac{C\ 11}{256}$; gl. $\frac{g\ 15}{128}$.

Played in Aulos (as above) on C = 128 v.p.s. at Ext. ·108, V.L. ·075, the sequence in tune, all notes strong, but the mouthpiece did not speak quite freely and fluently —was needing practice. Evidently some slight detail was out of gear, probably the V.L. was at fault.

D-R. Mp. ' Cl. 6 ' also plays in Phrygian Harmonia M.D. (12) on $\frac{B\ 12}{64}$ (see scheme below for Cl. 7) at Ext. ·140, at V.L. ·060.

D-R. Mp. (*U.*) ' *Cl. 18* '. M.D. (11). Dorian Spondaic

L. ·161 ; Δ ·005 ; V.L. ·079 ; proper note C = 256 v.p.s. and at ·060 $\frac{F\ 17}{128}$

Plays in Elgin I, at Ext. ·108 on C = 128 v.p.s. V.L. ·075, the sequence of the Dorian Spondaic (11). When first tested it was found that the C and D of Holes 1 and 2 were very soft, and the C difficult to obtain, the tone strengthened, however, as pitch rose in the sequence. A test dated 4/5/34 was made with special attention to Hole 2, ratio 11/10 ; and it was discovered that the result at V.L. ·060 was excellent : the difficulty was due to the faulty relations of resonance between mouthpiece and resonator.

D-R Mp. (*U.*) ' *N. 1* '. M.D. (11). Dorian Spondaic

L. ·131 ; Δ ·004 ; proper note at V.L. ·079 = C = 256 v.p.s. plays in Elgin 1 on $\frac{C\ 11}{128}$ at Ext. ·108, as above.

D-R. Mp. (*U.*) ' *Cl. 7.*' *M.D.* (*11*). *Dorian Spondaic*

L. ·145 ; Δ ·004 ; Proper note at V.L. ·060 = $\frac{F\ 17}{256}$ gl. $\frac{A\ 13}{128}$.

Plays in Elgin Aulos 1 on $\frac{C\ 11}{128}$ but not with ease ; gives better results on $\frac{A\ 27}{128}$ for all but Hole 6 ; a change of Harmonia to Phrygian (12) on the same fundamental. $\frac{A\ 27}{128}$, proved entirely satisfactory and the sequence was obtained in perfect tune at Ext. ·132—for ·140, owing to the short stem of the mouthpiece : the negative excess of ·008 was not sufficient to upset the intonation of the Harmonia, since it only amounts to one-quarter increment of distance.

The Modal Sequence in the Phrygian Harmonia M.D. (12)

Holes	1		2		3		4		5		6
Ratios	$\frac{12}{12}$		$\frac{11}{12}$		$\frac{10}{12}$		$\frac{9}{12}$		$\frac{8}{12}$		$\frac{7}{12}$
Cents		*151°*		*165°*		*182°*		*204°*		*231°*	

D-R. Mp. (*T.*) ' *N. 7* '. M.D. (12). Phrygian Harmonia

L. ·157 ; Δ ·004 ; Proper note at V.L. ·068 with ictus $\frac{F\ 16}{128}$ = 176 v.p.s. ; of startling power $\left(\text{norm } \frac{E\ 19}{128} = 148 \text{ v.p.s.}\right)$. Tested 28/2/33.

Played in Elgin Aulos 1, at Ext. ·140 ; V.L. ·061 in the Phrygian Harmonia, at first with hesitation on the part of the mouthpiece newly treated a few hours only, but each note as it came was steady and strictly in tune ; this was more especially noticeable with the notes issuing from Holes 5 and 6, which N. 7 played easily in tune with monochord. Tested again on 1/3/33 with greatly improved result.

A set of fresh tests in a new direction revealed further resourcefulness of the reed-blown pipe.

D-R. Mp. (*U.*) '*Elg. A. 13* (*1*)'. M.D. (13) from exit. Lydian Harmonia.

L. ·138 ; Δ ·006 ; at Ext. ·105 ; at V.L. ·059 proper note $\frac{F\ 17}{128}$ norm, and

D-R. Mp. (*U.*) '*Elg. A. 13* (*2*)'. M.D. (13). Lydian Harmonia from exit.

L. ·156 ; Δ ·006 ; at V.L. ·058 = $\frac{F\ 17}{128}$ norm.

Both mouthpieces played in the Aulos in the Lydian Harmonia (13) from exit at *Ext.* ·105 all notes in tune on $\frac{A\ 13}{64}$ as below.

This mouthpiece also plays at Ext. ·076 on $\frac{D\ 20}{128}$ in the Hypolydian Harmonia from Hole 1.

The Modal Sequence in the Lydian Harmonia M.D. (13)

Holes	Exit	1	2	3	4	5	6
Ratios	$\frac{13}{13}$	$\frac{11}{13}$	$\frac{10}{13}$	$\frac{9}{13}$	$\frac{8}{13}$	$\frac{7}{13}$	$\frac{6}{13}$
Cents	*290°*	*165°*	*182°*	*204°*	*231°*	*267°*	

D-R. Mp. (*U.*) '*Elg. D. 10*'. M.D. (10). Hypolydian Harmonia

L. ·101 ; Δ ·005.

Plays in Elgin Aulos 1 on $\frac{D\ 10}{128}$ at Ext. ·076, in the Hypolydian Harmonia (10) ; all notes in tune, even the high *D* of Hole 6, of the sequence from Hole 1.

The Modal Sequence in the Hypolydian Harmonia M.D. (10)

Holes	1	2	3	4	5	6
Ratios	$\frac{10}{10}$	$\frac{9}{10}$	$\frac{8}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{5}{10}$
Cents	*182°*	*204°*	*231°*	*267°*	*316°*	

Tests on 27/4/34 and 30/4/34.

D-R. Mp. (*U*). '*Elg. Dk. 2*' (Dk. 1 now broken). M.D. (10). Hypolydian Harmonia

L. ·110 ; Δ ·006 ; Ext. ·076.

Plays in Elgin Aulos 1, on $\frac{A\ 13}{64}$ at Ext. ·076 in the Hypolydian Harmonia (10) in perfect tune with monochord ; all holes easily, in a magnificent booming tone (for a double-reed) the two mouthpieces Elg. D.K. 1, and 2 new on 27/4/34.

D-R. Mp. (*U.*) '*H. 6*'. M.D. (10). Hypolydian Harmonia

L. ·110 ; Δ emb. ·005, exit ·004.

Plays on Elgin Aulos 1 on $\frac{D\ 10}{128}$ at Ext. ·076 in the Hypolydian Harmonia from Hole 1 in tune with piano (tuned to Dorian Harmonia (22), of which Hypolydian on *D* 10 is a species.) The fundamental *D* is a little weak and difficult but all the other notes are played easily in tune and the mouthpiece plays with ease the last two holes: the 5th hole on $\frac{B\ 12}{128}$, and the 6th hole on $\frac{D}{256}$ from Hole 4, a septimal, and a minor 3rd respectively.

Tested on 14/4/34, and again on 27/4/34.

D-R. (*U.*) *Mp.* '*M. 12*' (new on 12/5/34). M.D. (11). Dorian Spondaic

L. ·121; Δ ·006; V.L. ·061; at Ext. ·108.

Plays in Elgin Aulos 1, on $\frac{B\ 12}{64}$ in the Dorian Harmonia (11) at Ext. ·108, the whole scale in tune without difficulty at first trial, when once the V.L. had been carefully defined on the mouthpiece by the pressure of the lower lip, which helps to steady the intonation, and to produce the notes of Holes 5 and 6 with ease in tune without straining.

It will be noticed from the Table that mouthpieces numbered 1, 2, 3, 4, 5, 6, all play on $\frac{C\ 11}{128}$ in the Dorian Harmonia, further that Nos. 11, 13, 15, play on $\frac{D\ 20}{128}$ in Hypolydian as species of the Dorian; that Nos. 9 and 10 play on $\frac{A\ 13}{64}$ in the Lydian Harmonia and No. 3A, plays on $\frac{B\ 12}{64}$ in the Phrygian Mode.

The difference in extrusion of the mouthpiece, corresponding in each case with the difference in the Modal Determinants, thus produces each Harmonia in consequence as a species of the Dorian M.D. 11 on *C*, i.e. these have all been tested with K. S.'s piano tuned to the Dorian Harmonia 22 on $C = 128$ v.p.s., by starting each scale from the note bearing the proper M.D. of the Harmonia in question. These results indicate that the Aulos played by D-R. mouthpiece will —when due attention is paid to the laws of resonance operating between Aulos, resonator, and mouthpiece—strictly conform to the proportional basic law of the Harmonia to which the Aulos is subject.

ELGIN AULOS PERFORMANCE BY BEATING-REEDS

T.L. = tongue length of mp.
T.W = tongue width

B-R. Mp. '*E.E.*', of seasoned river reed, cut and fashioned 27/11/33 M.D. (11) in the Dorian Spondaic Harmonia

L. ·124; T.L. ·046; T.W. ·0025; Δ ·004; norm $\frac{A\ 27}{64}$

By formula:

$$\begin{aligned} \text{T.L.} \times 2 &= \cdot 092 \\ \Delta \times 2 &= \cdot 008 \\ \Delta - \text{T.W.} \times 2 &= \underline{\cdot 003} \\ & \cdot 103 \end{aligned} \left\{ \begin{array}{l} \cdot 008 - \cdot 005 = \cdot 003 \text{ length of} \\ \text{pulse of the vibrating tongue} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Length of sound-wave in} \\ \text{pipe} = \cdot 103 \times 4 = \cdot 412 \\ \text{as closed pipe} \end{array} \right. \quad \frac{340}{\cdot 412}\text{ m.} = \frac{825 \cdot 2}{8} = 103 \text{ v.p.s.} \quad \frac{A\ 27}{64} = 104 \text{ v.p.s.}$$

The normal proper note of B-R. mp. 'E.E.' is $\frac{A\ 27}{64}$.

This is a beating-reed mouthpiece of very fine tone, rich and resonant. By dint of much scraping the tongue has been rendered resilient; it is a remarkable instance of what the beating-reed mouthpiece is capable of doing. This river-reed mouthpiece may be accepted as representing the type described by Theophrastus which had, he stated, come into favour with Antigenidas. B-R. mp. 'E.E.' lends itself admirably to the device of shortening the vibrating tongue by a half, a quarter, or third, with the result that the pitch of the whole Aulos, fundamental and finger-holes, is raised an octave, a 4th, or a 5th.

The Elgin I on $\frac{A\ 27}{64}$ —the norm of the mouthpiece's proper note—in the Dorian Spondaic Harmonia (11). B-R. mp. 'E.E.' plays from Hole 1 at Ext. ·108 the modal sequence in tune, and produces very readily, in beautiful tone, all the notes of the scale. As stated above, moreover, on shortening the tongue of the mouthpiece by one-third, the Aulos fundamental rises to $\frac{E\ 18}{128}$, and continues the sequence in tune to the end in the tonality of the dominant. This, as already pointed out, is a feat which lies beyond the scope of the double-reed mouthpiece. This device may be accounted, partly or wholly, responsible for the form of our major scale, with the first tetrachord of the Hypolydian Harmonia, from which our major scale is undoubtedly derived, repeated on the dominant.

B-R. '*Elg. B*' *wheat straw.* Ext. at ·076 ; M.D. (10) from Hole 1 in the Hypolydian Harmonia. 28/4/34

L. ·127 ; Δ ·004 ; T.L. ·048 ; T.W. ·003.

Plays in Elgin Aulos 1 on $\frac{B\ 12}{64}$ at Ext. ·076 in the Hypolydian Harmonia (10) ; all notes strong and in tune (as given above under D-R. mouthpieces).

B-R. '*D. 7*' at Ext. ·076 ; M.D. (10) from Hole 1 in the Hypolydian Harmonia

L. ·163 ; Δ ·0035 ; T.L. ·049 ; T.W. ·0025 to ·003 irregular in shape.

Plays in Elgin 1 on $\frac{D\ 20}{128}$ in the Hypolydian Harmonia (10) at Ext. ·076 ; all holes of the sequence in tune, even Hole 6, the octave of the fundamental.

B-R. '*Elg. N.*' at Ext. ·108 ; M.D. (11), from Hole 1 in the Dorian Spondaic.

L. ·183 ; Δ ·003 ; T.L. ·034 ; T.W. ·002.

Plays in Elgin 1 at once on C = 128 v.p.s., all notes in good tune easily with the exception of Hole 6 which needs high glottis action.

Elgin Aulos 1, being our most important document as evidence of the use of the Harmonia in Ancient Greece (*c.* 500 B.C.), has been subjected to drastic and numerous tests ; some of which are here recorded. The relative brevity of the records of other Auloi is merely dictated by the necessity for economy of space.

ELGIN AULOS No. 2 (Curved)

British Museum : Dept. Graeco-Roman Antiquities

Facsimile by K. S. (See Elgin Aulos 1)

MEASUREMENTS		DISTANCES FROM EMBOUCHURE TO CENTRE OF HOLES	
Length from exit to embouchure ; Resonator	·343	Hole 1 from emb. = ·299 ; from exit to C. of Hole 1	= ·044
Length from centre of Hole 1 to emb.	·299	Hole 2 from emb. = ·246 ; from C. of Hole 1 (two I.D.)	= ·053
Diameter of bore (Δ)	·008	Hole 3 from emb. = ·212 ; from C. of Hole 2	= ·034
Diameter of fingerholes	·008	Hole 4 from emb. = ·180 ; from C. of Hole 3	= ·032
		Hole 5 from emb. = ·149 ; from C. of Hole 4	= ·031
		Hole 6 from emb. = ·112 ; from C. of Hole 5	= ·037
		For all I.D. Total ·231 ; 7 | ·033	4 | ·134 ; ·033

Mean I.D. for the four regular I.D. = ·033

Mean I.D. for the 7 = ·033

I.D. × M.D. = multiple length

·033 × 14 = ·462 − ·343 Resonator = ·119 Extrusion of mp. for the Mixolydian Harmonia of M.D. 14.

ELGIN AULOS 2. PERFORMANCE (FROM EXIT)

With *D-R. Mp. ' Curved ' ' G.14. K. S.' at Mp. Ext. ·119 Δ ·007 at V.L. ·060*
Plays the whole Sequence from exit in tune as below

The Mixolydian Harmonia on G $\frac{\mathbf{14}}{\mathbf{64}}$

Holes	Exit	Hole 1	2	3	4	5	6
Ratios	14/14	13	12	11	10	9	8
Cents		*128*	*138·5*	*151*	*165*	*182*	*204*

With *D-R. Mp. ' Elgin 2 K. S. ' on* $\frac{A\ 13}{64}$. L. ·138 ; Δ ·006 at V.L. ·060 Mp. Ext. ·085. Plays in Elgin 2 in tune ; tone strong and resonant, in the Lydian Harmonia of M.D. 13. Tested April 18, 26, 34.

Lydian Modal Sequence

Holes	Exit	Hole 1	2	3	4	5	6
Ratios	13/13	12	11	10	9	8	7
Cents		*138·5*	*151*	*165*	*182*	*204*	*231*

D-R. Mp. ' K. S. Elgin 3 ' plays on $\frac{A\ 13}{64}$, *the same sequence in the Lydian Harmonia at Mp. Ext. ·085.* Length of mp. ·156 ; Δ ·007 ; V.L. ·060.

PERFORMANCE WITH BEATING-REED MP. ' ELGIN 2, 13 '

Length of straw ·103, Δ· 006, T.L. ·043, T.W. ·0025. Proper note of Mp. $\frac{G\ 14}{64}$ to $\frac{B\ 12}{64}$.

BY FORMULA NO. 8

$$\begin{aligned} \text{T.L.} \times 2 &= \cdot 086 \\ \Delta \times 2 &= \cdot 012 \\ (\Delta - \text{T.W.}) \times 2 &= \underline{\cdot 007} \\ & \ \cdot 105 \times 4 = \cdot 420 \end{aligned} \qquad \frac{340}{\cdot 420} = 101 \text{ v.p.s.} = \frac{G\ 14}{64}$$

The Aulos plays with Mp. Ext. ·085 on $\frac{A\ 13}{64}$ in the Lydian Harmonia M.D. 13, with a fine resonant tone, all notes of the Sequence in tune easily except the 6th note, which was a little difficult to keep steady.

LADY MAKET PIPE 3

Berlin, in the Musikhist. Museum, Charlottenburg. (*3 Fingerholes*)

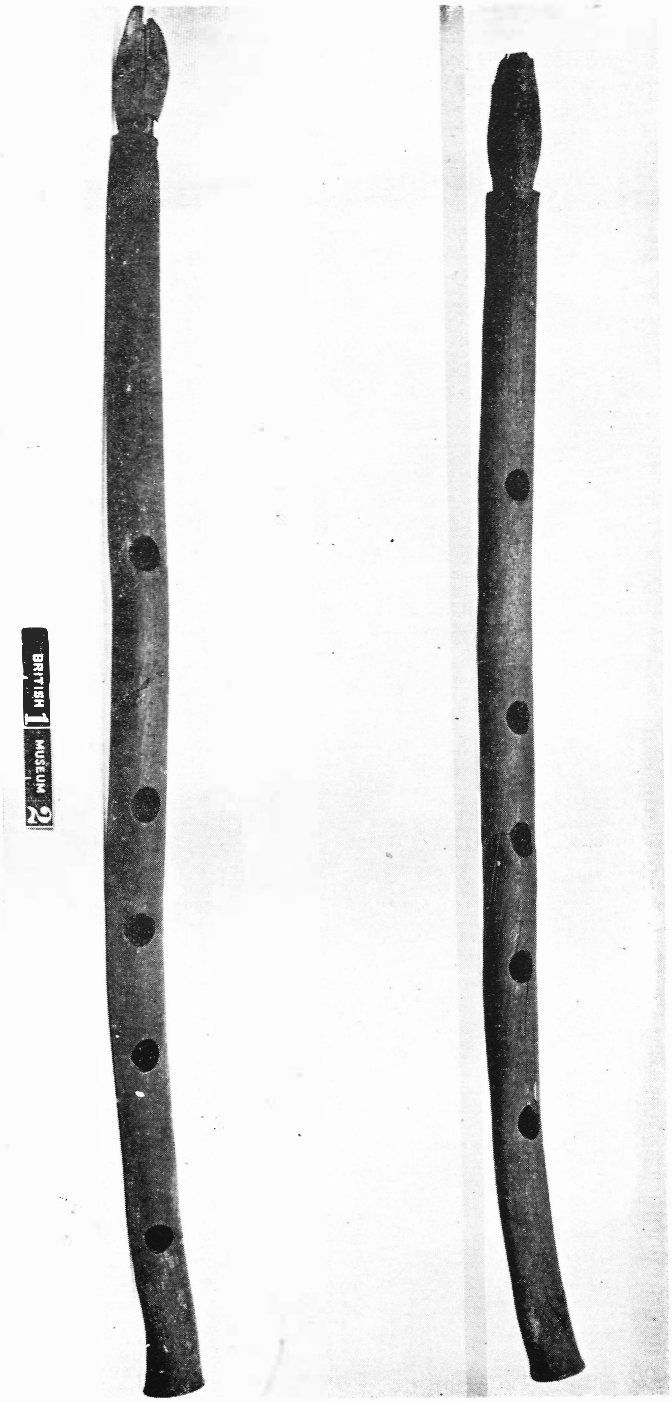
Discovered in 1890 by Sir W. Flinders Petrie in Kahun in the Fayum.[1] Facsimile in brass by Besson & Co., made after the originals in 1890. Facsimile by K. S. in 1925 in river reed (English) bound with waxed linen thread. The measurements of this facsimile agree with those of Mr. Blaikley—

From a letter dated August 28, 1917, addressed to me by Mr. D. J. Blaikley, I quote the following interesting facts :

I enclose a copy of my results as given to the Musical Association in the discussion

[1] For a discussion on the date of the pipes see W. v. Bissing, ' Die Datierung des Maketgrabes ', *Zts. f. Aegypt. Sprache* (Leipzig, 1897), Tom. xxxv, pp. 94–7. *Mus. Times*, Dec., 1890.

PLATE 17



THE ELGIN AULOI WITH BULBS, 5TH CENTURY B.C.

British Museum. By courtesy of the Director

following upon Dr. Southgate's paper, and published in the *Proceedings* for the Session 1890–1. The reeds I used were cut from wheat-straws, and although the absolute pitch of the notes obtained would vary slightly with the stiffness, &c., of the reed, the relative pitches or intervals between the notes would not, and the related pitch is the all-important point in looking for scale systems. My observations were made by comparisons with a carefully tuned harmonium and tuning-forks, and I do not think they can be more than one or two vibrations in error. . . . The main point to my mind is that the notes of the pipes agreeing with the notes 9, 10, 11, 12, on the Harmonic Scale ; or *D*, *E*, *F*, or *F*♯ and *g* are in accordance with the bagpipe scale, and certain other scales, Arabic or Eastern. In any examination of these, *we must accept the fact that notes in the ratios 10, 11, 12 are used* and not attempt to force No. 11 (the trumpet *F* or *F*♯) into modern notation. In conclusion, I would say that though I am certain about the pitches of the notes observed, I think it is still open to question whether the reeds or the fingerings agreed with those for which the flutes were made, and therefore there is room for uncertainty in the matter.

(*Signed*) D. J. Blaikley.

It is important to note that Mr. Blaikley, a shrewd and learned authority on wind instruments and on Pitch in general, preserved an open mind on the intervals used in Eastern scales. In his experiments with the Maket pipes, he used both Double-reed and Beating-reed mp.s ; but the former were of the Oboe type, and the latter were cut with the hinge of the tongue close up to the knot. The tongue was, moreover, wide and was stiff, producing harsh, high-pitched proper notes. Mr. Blaikley did not take into account the influence of the length of mouthpiece extrusion ; he did not realize the implications of fingerholes bored at equal distances, nor the necessity for increasing the compression of breath, interval by interval, as the sequence rises in pitch. Finally, he had not experimented with the B–R, with hinge in direction of the holes, nor with reed tongues cut to a width of 1 or 2 mm. He was amazed to hear the strong, sonorous and beautiful reedy tone of a tiny mp. about 4 inches in length, of a tongue-length of some ·030 and of a width of only ·0015, which boomed out on an 8 ft. *C* ! (in 1917). His results are therefore those of the Harmonic Series in its upward progression from 9, 10, 11, 12, and they may be compared with mine.

This Aulos plays from exit the 1st tetrachord of the Mixolydian Harmonia with D-R. mp.s and with many B-R. mp.s in a number of tonalities ; it also plays the 1st tetrachord of the Hypodorian Harmonia M.D. 16 on $\frac{F\ 1\ 6}{128}$

MEASUREMENTS

R.L. from exit to emb. J.D. Blaikley and K. S. ; (Besson : ·448) ·452
Δ exit and emb. ·005

THE FINGERHOLES — I.D.

C. of Hole 1 from emb. = ·413 ; from exit = ·039
C. of Hole 2 from emb. = ·377 ; from C. of Hole 1 = ·036
C. of Hole 3 from emb. = ·343 ; from C. of Hole 2 = ·034

3)·109
·0363

Increment of Distance (mean) ·0363

in practice ·036 and ·037.

·036 × 14 = ·504 − ·452 = ·052 Mp. Ext.

·037 × 14 = ·518 − ·452 = ·066 Mp. Ext. (in practice up to ·082).

The Mp. Ext. is often varied in practice to suit the idiosyncrasies of mouthpieces ; this may be done without prejudice to intonation, so long as the variation does not amount to a half I.D. The minimum Mp. Ext. of practical use is ·047 and for B-R. mp. only.

THE MOUTHPIECES AND THEIR PERFORMANCE

D-R. Mp. '*E. 18*' (treated). L. ·112 ; pressed out to fan-shape after soaking, to flat Δ ·007 ; proper note at V.L. ·060 = $\frac{F\ 17}{128}$; plays the Mixolydian tetrachord in perfect tune by monochord tuned to $\frac{E\ 18}{64}$. Tested 26/9/30.

The Mixolydian Tetrachord M.D. 14

The Fingerholes	Exit	Hole 1	2	3
Ratios and Notes	14/14	13	12	11
Cents		*128*	*138·5*	*151*

D-R. Mp. '*C.21*' (treated). L. ·133 ; flattened at tip to ·0065 ; proper note $\frac{F\ 16}{256}$; plays the Mixolydian Tetrachord as above on $\frac{D\ 20}{128}$, easily in tune 14/5/26.

D-R. Mp. '*O·8*' *at Mp. Ext. ·112* (untreated). L. ·179 ; Δ ·004 (round). Proper Note at V.L. ·060, $\frac{F\ 17}{256}$, a strong note ; gl. note, $\frac{A\ 13}{256}$ equally strong, the resonance of the mp. is in octave relation with the glottis note ; plays in Aulos in Hypodorian Harmonia M.D. 16 on $\frac{E\ 18}{128}$ and on $\frac{F\ 16}{128}$ at V.L. ·055. Tested 15/1/34.

PERFORMANCE WITH BEATING-REED MP.S

B-R. Mp. '*44*'. L. ·099 ; Δ ·004 ; T. L. ·029 ; T.W. ·002 ; one of the best mp.s ; plays in the Aulos in the Mixolydian Harmonia on $\frac{G\ 14}{64}$ with a fine tone, the whole tetrachord. Tested 6/6/29 and 29/6/30.

B-R. Mp. '*K.3*'. L. ·152 ; Δ ·003 ; T.L. ·035 ; T.W. ·00225 ; Proper Note C = 128 v.p.s. This mp. had at first a T.L. of ·032 and a proper note $\frac{E\ 18}{128}$, but was not a success in the Aulos ; the tongue on being cut to ·035, then played with proper note C in pipe at Mp. Ext. ·047 on $\frac{G\ 14}{64}$ correctly. A further trial at Mp. Ext. ·062 still gave the series, but not with the same ease, and at Mp. Ext. ·072 the series was distorted. This trial belongs to an early series (1925) before the exact significance of the Mp. Ext. had been ascertained. Similar tests were carried out by B-R. mp. 'D.5' on November 7, 1925.

The Harmonic Register with B-R. Mp. of oaten straw, '*Harmonics*' (the first with which a harmonic register was obtained on May 14, 1926).

This B-R. mp. plays in Maket 3 on $\frac{E\ 18}{128}$ easily, all holes closed, then quite easily by pinching the tip with compressed lips and breath the pipe gave out $\frac{B\ 12}{256}$, a 12th Harmonic, and rose with the opening of each hole, with reedy quality lessened to $\frac{C\ 22}{512}$, C 21 and D 20. This test and also others with B-R. mp. 'Lyd. 20' confirms my opinion that the harmonic compass can be obtained by the mp. alone without use of a speaker-hole.

This same mp. also plays the series on the fundamental register on G 14/64.

B-R. '*Lyd. 20*'. L. ·068 ; T.L. ·034 ; T.W. ·0025 ; Δ ·004 ; proper note $\frac{B\ 12}{128}$; in the Aulos at Mp. Ext. ·047 the mp. plays on $\frac{G\ 14}{128}$ (gl. note) the full harmonic compass on the 12th with ease, hole by hole. Tested 1/7/34 and 11/7/34.

N.B.—With this record, K. S.'s comments on A. A. Howard's study, *The Aulos*, should be read (Chap. ii ; see index).

LADY MAKET 4

For details of provenance, &c., see Lady Maket 3. (*With 4 Fingerholes.*)

This Aulos plays from Exit in the Mixolydian Harmonia M.D. 14 and from Hole 1 in the Phrygian M.D. 12; at Mp. Ext. ·047.

MEASUREMENTS

Proportions of original pipe, by D. J. Blaikley [*Mus. Times*, Dec. 1, 1890].

R. L. ·448, with natural knot at exit (pierced), which flattened the pitch slightly, making it equal in value to Maket 3, which measures ·452. Δ ·005 at exit and emb.

δ: the fingerholes are elliptical, ranging in length from ·006 to ·003, decreasing in the direction of emb. *N.B.*—A decrease in δ implies a lengthening of the air column, a fact which has but little significance on reed-blown pipes.

FINGERHOLES

FINGERHOLES	I.D.
C. of Hole 1 from emb. = ·372; from exit	·076*
C. of Hole 2 from emb. = ·340; from C. of Hole 1	= ·032
C. of Hole 3 from emb. = ·305; from C. of Hole 2	= ·035
C. of Hole 4 from emb. = ·270; from C. of Hole 3	= ·035
	5)·178
	·0356

* The distance from Exit to Hole 1 evidently covers *two* Increments of Distance.

Increment of Distance (mean) ·0356

in practice ·035.

I.D. ·035 × 14 = ·490 − ·448 = ·042; the minimum distance in practice for Mp. Ext. of B-R.s is ·047. It has been found that for this Aulos ·050 and ·052 give best results.

THE MOUTHPIECES AND THEIR PERFORMANCE

B-R. Mp. '*K*' (*2*). L. ·151; Δ ·0035; T.L. ·036; T.W. ·0025; plays in Aulos at Mp. Ext. ·047 on $\frac{G\ 14}{64}$, in tune with modal piano, with fine resonant tone. Tested 30/10/30.

B-R. Mp. '*xxviii E.9*'. L. ·105; Δ ·003; T.L. ·029; T.W. ·002; plays at Mp. Ext. ·050 on $\frac{G\ 14}{64}$, the whole series in tune.

The Mixolydian Sequence with the Phrygian from Vent as Species

Fingerholes	Exit	1	2	3	4
Ratios and notes	14/14 (13)	12	11	10	9
Cents	*267*		*151*	*165*	*182*
Phrygian species		12/12	11	10	9
Cents			*151*	*165*	*182*

B-R. Mp. '*G.1*'. L. ·140; Δ ·0035; T.L. ·035; T.W. ·0025; gave the same results as above; with full, reedy and sonorous tone.

LORET XXIII

(*Leyden Museum 1, 475* (*cf. Loret xvii, note*). (*4 Holes*)

3 Facsimiles by K. S. A remarkable Record

Plays, on same fundamental note from Hole 1 in the Mixolydian M.D. 14; the Lydian M.D. 13. The Phrygian M.D. 12 and from exit in the Hypodorian Harmonia. All these Harmoniai are obtained with D-R. Mp. xxiii, 'N. 32' and 'N. 33' and in addition the Dorian Harmonia of M.D. 11 with B-R. mp.s.

MEASUREMENTS			FINGERHOLES	I.D.
L.R.	exit to emb.	·354	(The distances are from emb. to Centre of Holes.)	
L.	emb. to C. of Hole 1	·265	C. of Hole 1 from emb. = ·265 ; from exit	= ·089
Δ	at emb. ·005 ; at exit	·0035	C. of Hole 2 from emb. = ·237 ; from C. of Hole 1	= ·028
			C. of Hole 3 from emb. = ·210 ; from C. of Hole 2	= ·027
			C. of Hole 4 from emb. = ·180 ; from C. of Hole 3	= ·030
				3)·085
				·0283

Increment of distance mean from Hole 1, ·0283
(useful) ·028
,, ,, ,, ,, from exit ·029

It is my practice where the increment from exit to Hole 1 is greatly in excess of the mean, without being, as here, a multiple of it, to conclude that the note was unused. With Aulos xxiii it is probable that the note from exit was used as lower octave of the Lydian Mese, as explained further on.

THE MOUTHPIECES

D-R. Mp. '*N.32*'. L. ·160 ; Δ ·004 ; V.L. ·065.

D-R. Mp. '*N. 33*'. Golden oaten straw, L. ·139 ; Δ ·0035 at exit and ·004 at emb.

D-R. Mp. '*xxiii.*' L. ·222 ; Δ ·004 ; fine wheat, silica, satiny, excellent tone, strong.

B-R. Mp. '*xviii C*'. L. ·150 ; Δ ·0035 ; T.L. ·035 ; T.W. ·0025.

PERFORMANCE

With D-R. Mp. '*N. 32*'. Aulos xxiii played at first at Mp. Ext. ·112 and V.L. ·065 ; the tone was weak and hesitating. When the extrusion was reduced to ·099 and the V.L. to ·060, the sequence was played in a firm tone, in tune with piano on $\frac{A\ 13}{128}$. The reason for this is plain: Lydian M.D. = 13 I.D. ·028 × 13 = ·364 ; ·364 − ·265 [L. emb. to C. Hole 1] = ·099 Mp. Ext. This experiment is recorded to show the effect of an exact amount of Mp. extrusion ; it is, indeed, possible in some cases to play the sequence in tune with an approximate extrusion, but here the excess amounted to almost half an increment which disturbed the proportional impulse. Tests 25–28 April, 1933, both characterized as very good.

A further important test, made with this Aulos, demonstrated that a change of extrusion, by the addition or subtraction of an I.D.—the visible graphic sign of which is the bulb added or taken off (see Plates 9 and 13, Chap. ii)—may signify a change of species, or more important still, a change of Harmonia when the fundamental remains unchanged. How this can be is a mystery, for which there is perhaps a possible explanation.

With D-R. Mp. '*N. 32*' *at Mp. Ext.* ·*099*. The performance of the Aulos in the Lydian Harmonia of M.D. 13 on $\frac{A\ 13}{128}$ was repeated many times.

The Modal Sequence

Fingerholes	1	2	3	4
Ratios and Notes	13/13	12	11	10
Cents	*138·5*	*151*	*165*	

This was followed immediately by another, on pulling out the mp. to Ext. ·127, while keeping the V.L. at ·060. This new sequence was played without change of fundamental on $\frac{A\ 13}{128}$, in spite of the additional length of one increment through the extrusion prolonged to ·127. The Harmonia now became Mixolydian.

The Modal Sequence at Mp. Ex. ·127

Fingerholes	1	2	3	4
Ratios and Notes	14/14	13	12	11
Cents		*128*	*138·5*	*151*

A phenomenon may now be observed on this Aulos, viz. from the same fundamental and through the same fingerholes, a change in the ratios of the intervals has occurred : although the addition of an increment of length produced a response from the open fingerholes in accordance with proportional length, this additional lengthening in resonator + mp. did not affect the pitch of the fundamental $\frac{A\ 13}{128}$, added length could not break through the paramount influence of the mouthpiece on the pitch of the fundamental.

It has already been suggested that the fundamental resulting from the alliance of Mp. + Aulos resonator is due to the law of resonance; in this case investigation has proved the correctness of the suggestion.

The proper note of D-R. Mp. 'N. 32' at V.L. ·060 and Δ ·004 may be computed thus :

$$4(\cdot060 + \cdot004) = \cdot256 \text{ and } \frac{340 \text{ m./s.}}{\cdot256} = 332 \text{ v.p.s.}$$

i.e. $\frac{F\ 17}{256}$; the fundamental is $\frac{A\ 13}{128}$, approximately a 5th below

$$\left[\frac{26}{17} \times \frac{2}{3} = \frac{52}{51} = \text{a difference of } + \textit{34 cents}\right].$$

Before proceeding to mention other remarkable manifestations connected with this Aulos, we must note that a new D-R. Mp. 'N. 33' gives exactly the same results as 'N. 32', viz. Lydian of M.D. 13.

D-R. Mp. 'N. 33'. A golden oaten straw ; L. ·139 ; Δ ·0035 at exit, and ·004 at emb., played at Extrusion ·099 in Aulos on $\frac{A\ 13}{128}$, on a strong fundamental, the sequence as above, in the Lydian Harmonia. When pulled out to ·127 (i.e. one additional increment) Mp. 'N. 33' still plays on *A*13/128 faultlessly in the Mixolydian Harmonia with good tone and in tune, thus duplicating the performance of 'N. 32'. It is, therefore, clear that the additional increment *has the power to impose a new division on the proportional impulse of the air column, but leaves the resonance factor unchanged.*

On investigation of the relation of length to pitch in Aulos xxiii when played by D.R. Mp.s 'N. 32' and 'N. 33', the following results are obtained : I.D. × 14 = ·392 and $\frac{340 \text{ m./s.}}{\cdot392}$ = 217 v.p.s., viz., $\frac{A\ 13}{128}$, i.e. the pitch of the fundamental.

Thus accidentally, it is found that the length formula and the actual pitch elicited in practical tests of Aulos + Mp. correspond exactly, which is a rare experience.

When worked out according to the same formula, for the Lydian Harmonia, the correspondence, of course, no longer exists, for I.D. × 13 = ·364 and $\frac{340 \text{ m./s.}}{\cdot364}$ = *B* 12 = 117 v.p.s.

The working out of the formula for each note of the modal sequence is shown below for the Mixolydian of M.D. 14 in Table A, and for the Lydian 13 in Table B (Nov. 6, 1933). Tested again a few days later (Nov. 25, 1933) with the same results.

With *B-R. Mp. 'xviii C'*.—A new mp. ; L. ·150 ; Δ ·0035 ; T.L. ·035 ; T.W. ·0025 ; Aulos xxiii with mp. at Ext. ·047, M.D. 11 played in the Dorian Harmonia on *C* 128 v.p.s. in perfect tune, and fine tone. Pulling out the same mp. to ·071, the Aulos played on the same fundamental *C* = 128 v.p.s. in the Phrygian Harmonia of M.D. 12, likewise in perfect tune and with a musical, resonant tone.

Thus this remarkable Aulos plays in four Harmoniai : Dorian and Phrygian with a B-R. Mp. and Lydian and Mixolydian with D-R. Mp.s. The short extrusion demanded for the Dorian Harmonia cannot, according to my experience, for obvious reasons (viz. insufficient V.L.), be obtained with D.R. mp.s of primitive type. The D-R. mp.s likewise refused to play in the Phrygian Harmonia, although the extrusion was not then at fault.

Tests of the correspondence of length with pitch by formula in Auloi reveal the fact that this correspondence, when present, can only be accidental. This Aulos, with a fundamental $A 13/64 = 108{\cdot}5$ v.p.s. has a correspondence when used in the Mixolydian Harmonia, but not when used in the Lydian Harmonia. This Test means that a pipe having a fundamental sounding A 13 is only *en rapport* with the fundamental when the length of resonator + mp. totals ·392. With the Lydian Harmonia, the total is ·364 = 116·7 v.p.s., i.e. $\frac{B\ 12}{64}$ instead of $\frac{A\ 13}{64}$.

TABLE A

Showing Correspondences Hole by Hole

Hole 1 at ·265 + Mp. Ext. ·127 = ·392 from Hole 1 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 392} = 217 \text{ v.p.s.} = A\ 13/128 \text{ (by Formula No. 1)}$$

14/14 = 217 v.p.s. in practice.

Hole 2 at ·237 + Mp. Ext. ·127 = ·364 Hole 2 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 364} = 233{\cdot}4 \text{ v.p.s.} = B\ 12/128 \text{ very slightly flattened (exact} = 234{\cdot}6 \text{ v.p.s. by Formula).}$$

14/13 Hole 2 = B 233·7 v.p.s. in practice.

Hole 3 at ·209 + ·127 = ·336 = Hole 3 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 336} = 252{\cdot}8 \text{ v.p.s. (253) (theor. by formula).}$$

14/12 Hole 3 = 253 v.p.s.
217 v.p.s. × 7/6 = 253·16 v.p.s. in practice.

Hole 4 at ·182 + ·127 = ·309 Hole 4 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 309} = 275 \text{ v.p.s. (by formula)}$$

14/11 Hole 4. 217 × 14/11 = 276 v.p.s. in practice.

TABLE B

The same test applied to Aulos xxiii in the Lydian Harmonia and played by the same D-R. Mps. Nos. ' N. 32 ' and ' N. 33 ' at extrusion ·099, on fundamental A 13—the tonic of the Lydian species in the Dorian Harmonia—does not give resonance results in accordance with the formula.

I have only found a half-dozen of these correspondences among all the pipes ; they are therefore the exceptions, not the rule. That it cannot be the rule is proved by this Aulos, for the change of extrusion destroys the correspondence.

Hole 1 at ·265 + Mp. Ext. ·099 = ·364 from Hole 1 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 364} = 233{\cdot}5 \text{ v.p.s.} = \frac{B\ 12}{128} \text{ by formula (no correspondence between theory and practice)}$$

13/13 Hole 1 = 217 v.p.s. $\frac{A\ 13}{128}$ in practice.

Hole 2 at ·237 + ·099 = ·336 from Hole 2 to tip of mp.

$$\frac{340 \text{ m./s.}}{\cdot 336} = 252{\cdot}8 \text{ v.p.s. } (C\ 11 = 256 \text{ v.p.s.}) \text{ (theoretical by formula)}$$

217 × 13/12 = 235·08 v.p.s. in practice.

Hole 3 at ·209 + ·099 = ·308 from Hole 3 to tip of mp.

$$\frac{340\text{ m./s.}}{\cdot 308} = 276\text{ v.p.s. (by formula)}$$

217 × 13/11 = 256·4 v.p.s. in practice.

Hole 4 at ·182 + ·099 = ·281 from Hole 4 to tip of mp.

$$\frac{340\text{ m./s.}}{\cdot 281} = 301\text{ v.p.s. (by formula)}$$

217 × 13/10 = 282·1 v.p.s. in practice.

It is thus obvious that in case ' B ' the practical results of resonance tests are not in agreement with the theoretical obtained by formula.

Two more facsimiles of Aulos xxiii were made in Malacca, in order to test still further the persistence of the fundamental with mouthpieces at different lengths of extrusion. These are the results :

Facsimile xxiii No. 2 (*Malacca*). Tested with D-R. Mp. 23 ; L. ·222 ; Δ ·004 ; fine wheat silica, satiny, excellent tone, strong.

PERFORMANCE

In Aulos xxiii (2) at Mp. Ext. ·099, played in the Lydian Harmonia on $\frac{A\ 13}{128}$, easily in tune with modally tuned piano. The mp. was then pulled out to ·127, and still played on fundamental *A* 13 in the Mixolydian Harmonia with a strong tone. Tested April 10, 1934.

The same mp. and Aulos were then played with Mp. Ext. at ·071 on the same fundamental *A* 13, this time in the Phrygian Harmonia, in tune, and with a strong tone. Tested April 10, 1934. (I.D. ·028 × 12 = ·336 − ·265 = ·071 Mp. Ext.)

In the 3rd facsimile xxiii (3), *D-R. Mp.' xxiii 3c '*, played in the Mixolydian Harmonia M.D. 14 at Mp. Ext. ·127 on *C* = 128 v.p.s. in tune (April 10, 1934). Tested again, June 6, 1934, when the Aulos played as above, easily and in tune on the new fundamental.

CAIRO MUSEUM No. 28 : 10 and 23·5

Facsimile by K. S. February, 1934. ' C. F.' (5 Holes)

Measured by M. F. Grant in Cairo : ' Wooden pipe, polished but worn ; there is a slight nick in emb. 2 × 2 mm., not opposite fingerholes, seemingly due to accident.'—M. F. G.

Plays from exit : Phrygian Harmonia M.D. 24
Lydian Harmonia M.D. 26
Mixolydian Harmonia M.D. 28
Dorian Harmonia M.D. 22.

MEASUREMENTS		FINGERHOLES	I.D.
L.R. from emb. to exit	·2255 ;	(The distances are from emb. to centre of holes)	
Δ	·0055 ;	C. of Hole 1 from emb.	=·1995
δ	·005, constant ;	C. of Hole 1 from emb. = ·1995 ; from exit	= ·0265
		C. of Hole 2 from emb. = ·1745 ; from C. of Hole 1	= ·025
		C. of Hole 3 from emb. = ·1475 ; from C. of Hole 2	= ·027
		C. of Hole 4 from emb. = ·1335 ; from C. of Hole 3 (½ I.D.)	= ·014
		C. of Hole 5 from emb. = ·1095 ; from C. of Hole 4	= ·024
			.1165
			9)·2330
			·0259

H. 3–4. I.D. for half-increment = ·013 (mean). ·1165 × 2/9 = 026.
(i.e. nine half-increments)

THE MOUTHPIECES

D-R. Mp. ' *C.F. 13/2* '. L. ·074 ; Δ ·004 ; Mp. Ext. ·060 ; R.L. ·226 + ·060 = ·286 [·026 × 11 = ·286].

D-R. Mp. ' *C.F. 13* '. L. ·109 ; Δ ·004 ; Mp. Ext. ·086. R.L. ·226 + ·086 = ·312 [·026 × 12 = ·312].

D-R. Mp. ' *C.F. 13/3* '. L. ·134 ; Δ ·004 ; Mp. Ext. ·112. R.L. ·226 + ·112 = ·338 [·026 × 13 = ·338].

D-R. Mp. ' *C.F. 12* '. L. ·129 ; Δ ·004 ; Mp. Ext. ·138. R.L. ·226 + ·138 = ·364 [·026 × 14 = ·364].

D-R. Mp. ' *C.F. 14 (1)* '. L. ·158 ; Δ ·0035. R.L. ·226 + ·138 = ·364 [·026 × 14 = ·364].

D-R. Mp. ' *C.F. 14(2)* '. L. ·151 ; Δ ·004. R.L. ·226 + ·138 = ·364 [026 × 14 = ·364].

PERFORMANCE

D-R. Mp. ' *C.F. 13(2)* '. The Aulos plays on *C* = 256 v.p.s. at Mp. Ext. ·060 the Dorian Harmonia M.D. 22 in tune.

The Sequence

Fingerholes	Exit	1	2	3	4	5
Ratios and Notes	22/22	20	18	16	1/5	13
	11/11	10	9	8	15/22	13
Cents		*165*	*182*	*204*	*112*	*247*

D-R. Mp. ' *C.F. 13* ' plays on $\frac{B\ 12}{128}$ in tune in the Phrygian Harmonia.

Fingerholes	Exit	1	2	3	4	5
Ratios and Notes	24/24	22	20	18	17	15
	12/12					
Cents		*151*	*165*	*182*	*99*	*216·4*

D-R. Mp. ' *C.F. 13/3* ' plays on *A* 13 in tune (at once) in the Lydian Harmonia M.D. 26 or 13.

Fingerholes	Exit	1	2	3	4	5
Ratios and Notes	26/26	24	22	20	19	17
	13/13					
Cents		*138·5*	*151*	*165*	*89*	*192*

D-R. Mp. ' *C.F. 12* ' played on *B* 12 in tune in the Phrygian Harmonia (as above for ' C.F. 13 ').

D-R. Mp. ' *C.F. 14(1)* '. Played on *G* 14 at Mp. Ext. ·138 in the Mixolydian Harmonia from exit.

Fingerholes	Exit	1	2	3	4	5
Ratios and Notes	28/28	26	24	22	21	19
	14/14	13	12	11	21/28	19/28
Cents		*128*	*138·5*	*151*	*80·5*	*173·4*

D-R. Mp. ' *C.F. 14(2)* ' as above. All these Mp.s tested in the Aulos, March 7, 1934.

It will be noticed that in this Aulos, the change in the extrusion by one I.D.

produces the species in order from Dorian to Mixolydian all in perfect tune on their proper tonic.

Cf. with Loret xxiii for a contrast.

CAIRO MUSEUM Nos. 28 : 10 and 23 : 13

Measured in Cairo Museum by M. F. Grant. (*4 Fingerholes*)
Facsimile by K. S. February, 1934. '*C.M. 4*'

'Bone pipe with natural grooves back and front ; flat at mp. end ; underlip about 1 mm. longer than top-lip.'—M. F. G.

Dorian Harmonia M.D. 11 from Exit on $\frac{G\ 15}{128}$ = 187·7 v.p.s.

MEASUREMENTS

L.R. Exit to emb. =·161
Δ exit ·005 × ·006, depth ·002 constant ;
Δ emb. ·004 × ·007 ;

FINGERHOLES I.D.

(The distances are from emb. to centre of holes)
C. of Hole 1 from emb. = ·1375 ; from exit = ·0235
C. of Hole 2 from emb. = ·115 ; from Hole 1 = ·023
C. of Hole 3 from emb. = ·092 ; from Hole 2 = ·023
C. of Hole 4 from emb. = ·0695 ; from Hole 3 = ·023
·0925

The Increment of Distance = ·023

THE FIVE D-R. MP.S

D-R. Mp. '*M. 11**': L. ·145 ; Δ ·004 ; V.L. ·061.
D-R. Mp. '*M. 11/2*': L. ·128 ; Δ ·004 ; V.L. ·061.
D-R. Mp. '*M. 11*/A': L. ·131 ; Δ ·004 ; V.L. ·061.
D-R. Mp. '*H.5*': L. ·141 ; Δ ·006 (flattened) ; V.L. ·061.
D-R. Mp. '*ARE.*': L. ·128 ; Δ ·007 (oval).
In Aulos at Mp. Ext. ·092. I.D. ·023 × 11 = ·253.
R.L. + ·161

Total length ·253

PERFORMANCE

With D-R. Mp. '*M. 11**' *at Mp. Ext. ·092, V.L. ·061.* Played in Aulos from exit on $\frac{G\ 15}{128}$ in tune ; notes fairly strong. Hole 4, at ·092 from exit, is past the middle of the resonator and demands greater compression of breath, and the note 15/22 sounds instead of 7/11. Tested March 5, 1934.

With D-R. Mp. 11/2 at Mp. Ext. ·092. Plays in Aulos as above on $\frac{G\ 15}{128}$; tone not quite so strong but in tune with monochord ; and plays 7/11 ; the mp. overcomes the slight difficulty due to the position of the Hole near emb. and plays Hole 4 without straining. When the *tessitura* is not too high, a little glottis action will enable a flexible wheat straw of light texture to reach a note beyond the boundary (of the half resonator) in tune, whereas with a stiffer reed, the half segment note will prove the limit.

The Modal Sequence

Fingerholes	Exit	H.1	2	3	4
Ratios and Notes	11/11	10	9	$\boxed{8}$ *231°*	7
Cents	*165*	*182*	*204* Mese		15/22
					112

D-R. Mp. 'M. 11/A' at Mp. Ext. ·092, V.L. ·061 on G 15. The result here is the same Dorian Sequence from exit. Ratio 7 is attacked in tune with only a very slight glottis action.

D-R. Mp. 'H.5' plays the sequence on $\frac{C\ 11}{128}$ *but at Mp. Ext. ·094.* With this mp. the Aulos plays with precision ratio 15/22 instead of 7/11.

D-R. Mp. 'ARE' plays at Mp. Ext. ·098 on G 15. The Aulos with this Mp. plays the whole sequence with ratio 7 from Hole 4 in tune with ease and with a good Tone.

'C.R.' CAIRO MUSEUM Nos. 27 : 10 and 23 : 4

Facsimile made by K. S.

Measured in Cairo by M. F. Grant. (*7 Fingerholes*), *played from Vent*[1]

'Brass pipe; brass band at about 14 mm. from exit ornamented with a row of holes for a quarter of the way round. Brass (or gold) band near thumb-hole.'—M. F. G.

MEASUREMENTS	FINGERHOLES I.D.
	(The distances are from emb. to centre of Hole 1)
R.L. from exit to emb. ·388 ;	C. of Hole 1 from exit (from emb. ·251) =·137
L. from exit to C. of Hole 1 ·251 ;	C. of Hole 2 from emb. = ·218 ; from C. of Hole 1 = ·033
Δ ·0065 exit and emb.	C. of Hole 3 from emb. = ·184 ; from C. of Hole 2 = ·034
δ = ·006 and 6 = ·005 at Hole 6	C. of Hole 4 from emb. = ·155 ; from C. of Hole 3 = ·029
	C. of Hole 5 from emb. = ·124 ; from C. of Hole 4 = ·031
	C. of Hole 6 from emb. = ·0985 ; from C. of Hole 5 = ·0255
	C. of Hole 7 from emb. = ·066 ; from C. of Hole 6 = ·0325
	6)·1850
	·03083

Increment of distance (mean) ·03083
Useful I.D. ·031 (or better ·032 in practice)
R.L. to C. of. H. 1 = ·251 + Mp. Ext. ·101 = *·352* ; I.D. ·032 × 11 = *·352*.

THE MOUTHPIECES

D-R. Mp. 'C.R. 11/1'. Fine stiff satiny wheat. L. ·123 ; Δ ·006 ; Mp. Ext. ·090 or better ·101.

If I.D. = ·031 × 11 = ·341, L. vent + Mp. Ext. ·090 = ·341, and whereas with I.D. = ·032 × 11 = ·352. L. Vent + Mp. Ext. = ·101 = ·352.

D-R. Mp. 'C.R. 11/2'. L. ·159 ; Δ ·006. Thin wheat, lined, elastic, in every way a better mp. than the above. Tested March 7, 1934.

PERFORMANCE

With D-R. Mp. 'C.R. 11/1', played in Aulos on *C* = 128 v.p.s. at Mp. Ext. ·101 ; Holes 6 and 7 difficult.

Modal Sequence

Fingerholes	Hole 1	2	3	4	5	6	7
Ratios and Notes	11/11	10	9	8	7	6	5½
Cents		*165*	*182*	*204*	*231*	*267*	*151*

D-R. Mp. 'C.R. 11/2'. Played the sequence as above on *C* = 128 v.p.s., and gives the notes of Holes 6 and 7 very clearly and all in tune. Tested March 7, 1934.

[1] In the original pipe, there are two holes in the back, awkwardly placed for my fingers, which I transferred to the front at the same distance from emb. In addition, I transferred one fingerhole from front to back in a convenient position, while keeping the distance from emb. correct. These changes entail no modification of modal increments of distance.—K. S.

'C.G.' CAIRO MUSEUM Nos. 28 : 10 and 23 : 13

K. S. facsimile February, 1934. 'C.G.' (5 Holes)

Measured in Cairo by M. F. G.

'Wooden pipe similar to "*F*", polish slightly worn. Has been bound round with thread; knot at emb.'—M. F. G.

Plays the Phrygian Harmonia from Exit.

MEASUREMENTS		FINGERHOLES I.D.
		(The distances are emb. from centre of holes)
L. Resonator	·215 ;	C. of Hole 1 from emb. = ·1895 ; dist. from exit = ·0255
Δ at exit	·005 ;	C. of Hole 2 from emb. = ·1670 ; dist. from Hole 1 = ·0225
Δ at emb. Knot	·003–·005 ;	C. of Hole 3 from emb. = ·1445 ; dist. from Hole 2 = ·0235
δ of Hole 1, 2, 3	·005 ;	C. of Hole 4 from emb. = ·1210 ; dist. from Hole 3 = ·0235
δ of Hole 4, 5	·004 ;	C. of Hole 5 from emb. = ·0975 ; dist. from Hole 4 = ·0235
		5)·11850
		·0237

Increment of Distance ·0237 (mean)
·023 useful mean

D-R. Mp. 'Y. 2'. Wheat straw untreated; L. ·142; Δ at emb. ·005; Δ at exit ·0045.

$$\begin{array}{lr} \text{In Aulos R.L.} & \cdot 215 \\ \text{Mp. Ext.} & +\ \cdot 061 \\ \hline \text{Total length} & \cdot 276 \end{array} \qquad \frac{\cdot 276}{\cdot 023} = 12. \quad \text{M.D. 12.} \quad \text{Phrygian.}$$

D-R. Mp. No. 144. L. ·144; Δ ·0035; V.L. ·059; Mp. Ext. ·061 plays on $\frac{B\ 12}{128}$ in the Aulos.

PERFORMANCE

With D-R. Mp. 'Y. 2', from Exit; Mp. Ext. ·061. Played on $\frac{A\ 27}{64}$ in the Phrygian Harmonia, all notes in tune.

The Resonance note of the Resonator alone (blown across the top), was a *D* of 1152 v.p.s. Therefore the adjustment by resonance between Aulos and mp. was in the proportion of 4 to 3, i.e. *A* to *D*.

With D-R. Mp. No. 144 at Mp. Ext. ·061 and V.L. ·059. The Aulos played on $\frac{B\ 12}{128}$ all the notes of the Phrygian Harmonia in tune.

The Modal Sequence of the Phrygian Harmonia of M.D. 12

Fingerholes	Exit	Hole 1	2	3	4	5
Ratios and Notes	12/12	11	10	9	8	7
Cents		*151*	*165*	*182*	*204*	*231*

With this mp. the Aulos also plays from Hole 1 as vent, the Dorian Spondaic, as species of the Phrygian Harmonia on $\frac{C\ 11}{256}$ of ratios $\frac{11}{11}$, 10, 9, 8, 7.

By forcing a glottis note from Hole 5, the septimal 3rd could be obtained from the same hole as 7.

B-R. Mp. 'C.G. 12/5'. A fine stiff wheat straw; L. ·134; Δ ·003; T.L. ·003; T.W. ·002. When tested on April 4, 1933, this mp., which has a very fine elastic tongue, played alone the harmonics *D* 20 and *F* 17. The mp.'s proper normal note is $\frac{B\ 12}{64}$; with glottis action $\frac{G\ 15}{64}$.

B-R. Mp. '*C.G. 12/2*'. Wheat straw lined; L. ·121; Δ = ·004; T.L. = ·004; T.W. ·0025 (at base, ·003). Plays as proper normal note $\frac{C\ 11}{128}$. Mp. cut for 'C.G.' Tested March 5, 1934.

PERFORMANCE OF B-R. MP.S IN 'CAIRO G'

B-R. Mp. '*C.G. 12/5*'. Played at Mp. Ext. ·061 in Phrygian Harmonia on $\frac{B\ 12}{64}$ (the mp.'s own proper normal note), from exit in perfect tune with rich resonant tone, and from Hole 1 on $C = 128$, the Dorian Species, giving the septimal 3rd 7–6 from Hole 5 by forcing with glottis action.

B-R. '*C.G. 12/2*' plays on $\frac{B\ 12}{64}$ from Hole 1, the Dorian Harmonia, but with this mp. the septimal 3rd of the Dorian Spondaic is a mere staccato note.

N.B.—This mp. plays from exit on $\frac{B\ 12}{64}$ and from Hole 1 also on $\frac{B\ 12}{64}$: difference in length = ·0225.

PRIMITIVE OBOE FROM N. EGYPT

Presented by H. A. Burgess

Original Specimen used in Tests. (*See Photograph of the Instrument in Chap. ii, Plate* 10)

This Oboe, or Zamr, plays in the Dorian Harmonia from Vent, and also in the Phrygian Harmonia; the Hypolydian is obtainable as species of the Dorian from Holes 2 to 8.

MEASUREMENTS

L. from emb. to rim of bell, ·381

L. from emb. to vent, i.e. 1st hole of sequence of 8 at equal distances of ·226

Δ appears from exterior to be of conical bore, but when checked, hole by hole, gives the following results:

Δ at Holes 1 and 2	·020;	δ of fingerholes is constant at ·007
Δ at Holes 3 to 7	·019;	
Δ at Holes 8	·014;	

Hole 8 is just below the block that forms the support or holder for the mp.

THE BELL

There are 7 holes in the bell itself; 3 of these centre at ·053, 1 at ·08, 3 at ·105 from rim. These holes do not affect the pitch of the instrument when played from vent, the beginning of the series of 8 fingerholes—including No. 7, as thumbhole in the back.

The Double-reed tested at Mp. Ext. ·080 gave $\frac{E\ 19}{128}$ from vent, both before and after closing the seven holes in the bell; the closing produced an increased fullness of tone.

The emb. has a wooden rim pierced with a bore of Δ ·01; at a distance below this rim of ·049, the emb. narrows to ·007 when passing through the leather band, through which the thumbhole is bored. Below this the Δ = ·019.

This oboe or Zamr from N. Egypt is of a type with cylindrical bore as far as the fingerholes extend, but widening out beyond into a bell, for the purpose of amplifying the sound. The bell has no effect on pitch, since the 1st hole is always left uncovered as vent.

FINGERHOLES		I.D.	
C. of Hole 1 from emb. = ·226 ;	to rim of bell	= ·155	
C. of Hole 2 from emb. = ·197 ;	from C. of Hole 1	= ·029	
C. of Hole 3 from emb. = ·168 ;	from C. of Hole 2	= ·029	
C. of Hole 4 from emb. = ·139 ;	from C. of Hole 3	= ·029	
C. of Hole 5 from emb. = ·111 ;	from C. of Hole 4	= ·028	
C. of Hole 6 from emb. = ·083 ;	from C. of Hole 5 [1]	= ·028	
C. of Hole 7 from emb. = ·070 ;	from C. of Hole 6	= ·013	·028
C. of Hole 8 from emb. = ·055 ;	from C. of Hole 7	= ·015	
		6)·171	
		·0285	

Increment of Distance (mean) ·0285
Useful increment ·029

I.D. ·029 × 11 = ·319 − ·226 = Mp. Ext. ·093 (Dorian)
I.D. ·029 × 12 = ·348 − ·226 = Mp. Ext. ·122 (Phrygian)

THE MOUTHPIECES

D-R. Mp. '*Mor. 1*'. Wheat. L. ·165, later cut to ·143 ; Δ exit ·005 ; emb. ·007 ; V.L. ·058 ; proper note *F* 17.

D-R. Mp. '*Mor. 2*'. Wheat ; L. ·165, later cut to ·143 ; Δ exit ·0045 ; emb. ·005.

D-R. Mp. '*Mor. 3*'. L. ·182, later cut to ·143 ; Δ exit ·005 ; Δ emb. ·007.

PERFORMANCE

With D-R. '*Mor. 1*', the Aulos plays at Mp. Ext. ·093 on *C* = 128 v.p.s. in the Dorian Harmonia, every note in tune with the piano.

Hole 7 gives the octave of Hole 1. It was noticed that the stem of the mp. at ·165 was too long, and extended past the centre of Hole 8 ; it was therefore cut down to ·143, when it was found that the pitch of the Aulos fundamental had not risen in pitch in consequence.

The Dorian Sequence played by the Oboe

Fingerholes	Hole 1	2	3	4	5	6	7	8
Ratios and Notes	11/11	10	9	8	7	6	5½	5
Cents		*165*	*182*	*204*	*231*	*267*	*151*	*165*

It is obvious that the 1st species of the Dorian Harmonia, viz. the Hypolydian of M.D. 10, finds its octave through Hole 8 of ratio 5/11.

With D-R. Mp. '*Mor. 2*', it was found as a contrast to the experience with 'Mor. 1' that the mp. at length ·165 played in the Aulos a fundamental of $\frac{B\ 12}{64}$ from vent, but when cut down to ·143, the fundamental rose to $\frac{C\ 11}{128}$, and the mp. played in excellent tune and tone all the notes with ease, giving a more satisfactory performance than 'Mor. 1', which experienced some difficulty in negotiating Holes 7 and 8.

With D-R. '*Mor. 3*' *pulled out to Mp. Ext. ·122*, the Aulos played the sequence of the Phrygian Harmonia on *B* 12/64.

Phrygian Sequence

Fingerholes	1	2	3	4	5	6	7	8
Ratios and Notes	12/12	11	10	9	8	7	13/24	6
Cents		*151*	*165*	*182*	*204*	*231*	*128*	*138·5*

Mp. '*Mor. 3*' experienced some difficulty in playing holes 7 and 8.

[1] The distance between Holes 6 to 8 measures one I.D. = ·028 and is divided into ·013 + ·015.

LORET X

Turin Museum, No. 6. 3 Fingerholes

Second Facsimile, by K. S. 5/11/25

This Aulos plays in the Mixolydian Harmonia of M.D. 14 from Exit.

MEASUREMENTS	FINGERHOLES	I.D.
R.L. from exit to emb. ·518 ;	C. of Hole 1 from emb. = ·472 ; from exit to C. of Hole 1	= ·046
L. from C. of Hole 1 to emb. ·472 ;	C. of Hole 2 from emb. = ·432 ; from C. of Hole 1	= ·040
Δ at emb. ·007 ; at exit (crushed) ·0045 ;	C. of Hole 3 from emb. = ·392 ; from C. of Hole 2	= ·040
		3)·126
		·042

Increment of Distance (mean) = ·042
or practical = ·040

THE MOUTHPIECES

D-R. Mp. '*X. 5*'. L. 144 ; Δ ·004 ; V.L. ·050 ; Mp. Ext. ·056.

B-R. Mp. '*X.*'. L. ·167 ; D. ·0025 ; T.L. ·038 ; T.W. ·002. Proper note norm. $\frac{A\ 13}{64}$ gl. $\frac{D\ 20}{64}$.

B-R. Mp. '*X. 4*'. L. ·133 ; Δ ·0035 ; T.L. ·04 ; T.W. ·0015.

B-R. Mp. '*X. 7*'. L. ·134 ; Δ ·0035 ; T.W. ·002.

D-R. Mp. '*X. D*'. L. ·118 ; Δ ·006 emb. ; exit ·004 ; V.L. ·060 ; at Mp. Ext. ·070 (i.e. I.D. mean = ·042 × 14 = ·588 − ·518 = ·070).

PERFORMANCE

With *D-R. Mp.* '*X. 5*' the Aulos plays in the Mixolydian Harmonia M.D. 14 at Mp. Ext. ·056 on $\frac{G\ 15}{128}$,

With *B-R. Mp.* '*X.*' the Aulos plays on $\frac{G\ 14}{64}$ with fine resonant tone in tune.

The Mixolydian Harmonia. 1st Tetrachord

Fingerholes	Exit	H. 1	2	3
Ratios and Notes	14/14	13	12	11
Cents		*128*	*138·5*	*151*

With *B-R. Mp.* '*X. 4*', the Aulos plays the sequence as above on $\frac{E\ 19}{64}$ with a full rich tone.

With *B-R.* '*X. 7*', the Aulos plays at Mp. Ext. ·065 (with mean I.D. ·040), all fine rich notes.

With *D-R.* '*X. D*', the Aulos plays the Mixolydian tetrachord on $\frac{D\ 20}{128}$ at Mp. Ext. ·070.

FINAL REMARKS

The great length of the Aulos and the wide I.D., together with the narrow bore of the Aulos, contribute to determine and limit the modality at 14 increments (M.D. 14) Mixolydian. The I.D. could not be less with a Mp. Ext. of ·070. To try 16 as M.D. would render the length unmanageable and the resulting sequence, 16, 15, 14, 13, meaningless. Thus Loret X may be pronounced a Mixolydian Aulos.

It will not be forgotten that with a Beating-reed mp., the compass could be extended to an octave, by repeating the tetrachord, 14, 13, 12, 11 on the dominant, through shortening the vibrating tongue by a 3rd; this device produces a scale which destroys the modality of the Harmonia by the repetition of the 1st tetrachord instead of proceeding with the 2nd 10, 9, 8, 7, that would, of course, require four more fingerholes. The Arabian practice at the present day still regards such scales as pure Modes, and those of the Harmonia as mixed, a point of view with far-reaching implications, which are a revelation of the course followed by their musical evolution.

LORET XII

Turin Museum No. 8. 4 Fingerholes

2 Facsimiles made by K. S. xii and 12 (made during the War while materials were scarce)

This Aulos plays in the Hypodorian Harmonia at Mp. Ext. ·059 and also at ·078.

MEASUREMENTS	FINGERHOLES		I.D.
R.L. from exit to emb. ·450;	C. of Hole 1 from emb. = ·403; from exit	= ·047	
R.L. from C. of Hole 1 to emb. ·403;	C. of Hole 2 from emb. = ·367; from C. of Hole 1	= ·036	·036
Δ at emb. ·005; at exit ·004	C. of Hole 3 from emb. = ·337; from C. of Hole 2	= ·030	·030
δ constant at ·004	C. of Hole 4 from emb. = ·304; from C. of Hole 3	= ·033	·033
		4)·146	3)·099
		·036	·033

Increment of Distance (mean) from exit ·036 (A).
Hole 1 ·033 (B).

N.B.—When playing the Aulos from Hole 1, the 1st I.D. from exit is omitted, and the mean (B), derived from the Holes only, is used in computation.

THE MOUTHPIECES

D-R. Mp. '*12 F.*'. L. ·134; Δ at exit ·0035; emb. ·004.
D-R. Mp. '*12 F. 16*'. L. ·149; Δ ·004.
B-R. Mp. X. L. ·108; Δ ·0035; T.L. ·035; T.W. ·002.
B-R. Mp. E 3. L. ·166; Δ ·0035; T.L. ·034; T.W. ·002.

PERFORMANCE

With *D-R. Mp.* '*12 F.*' *from exit*, the Aulos plays at Mp. Ext. ·078 on $\frac{F\ 16}{128}$ in the Hypodorian Harmonia.

The Hypodorian Modal Sequence

Fingerholes	Exit	H 1	2	3	4
Ratios and Notes	16/16	14	13	12	11
Cents		*231*	*128*	*138·5*	*151*

The same D-R. Mp. '12 F.' from C. of Hole 1 as Vent plays the Aulos in good tune and readily, at Mp. Ext. ·059 in the Dorian Harmonia 1st tetrachord of ratios 11/11, 10, 9, 8, on $\frac{F\ 16}{128}$.

D-R. Mp. '*12 F. 16*' repeats the performance of '12 F.'.

B-R. Mp. X. plays in the Aulos at Mp. Ext. ·078 the Hypodorian tetrachord from exit on $\frac{A\ 13}{64}$.

B-R. Mp. '*E 3*' also plays in the Aulos the Hypodorion Sequence as above with a fine musical tone.

LORET XIII

British Museum No. 6385. 4 Fingerholes
Wooden flute 1917
Facsimile made by K. S. (mottled bamboo)

This Aulos plays in the Mixolydian Harmonia from Exit.

MEASUREMENTS	FINGERHOLES	I.D.
R.L. from exit to emb. ·440 ;	C. of Hole 1 from emb. = ·400 ;	from exit = ·040
Δ exit ·017 ; emb. ·014 ;	C. of Hole 2 from emb. = ·365 ;	from C. of Hole 1 = ·035
δ constant at ·006	C. of Hole 3 from emb. = ·332 ;	from C. of Hole 2 = ·033
	C. of Hole 4 from emb. = ·2975 ;	from C. of Hole 3 = ·0345
		4)·1425
		·0356

Increment of Distance (mean) ·0356
useful ·036

·036 × 14 = ·504 R.L. ·440 + Mp. Ext. ·070 = ·510; ·067 = ·507

THE MOUTHPIECES

D-R. Mp. '*xiii D.*' wheat straw, untreated ; L. ·131 ; Δ ·005 ; V.L. ·059 ; at Mp. Ext. ·076.

D-R. Mp. '*xiii G. 14*'. Untreated ; L. ·091 ; Δ ·005 ; at Mp. Ext. ·067.

D-R. Mp. '*xiii B.*'. Untreated ; L. ·162 ; Δ ·007 ; V.L. ·057 ; at Mp. Ext. ·067 ; proper note norm. $\frac{F\ 17}{128}$; glottis note $\frac{C\ 11}{128}$.

PERFORMANCE

With *D-R. Mp.* '*xiii D.*', the Aulos plays on $\frac{D\ 20}{128}$ in the Mixolydian Harmonia in tune but in a husky, unmusical tone, owing to the wide diameter of resonator in relation to that of mp. which necessitated the addition of a socket to fit the resonator. The socket consists of a short length of bamboo = ·040, fitting tightly into the embouchure, and extruding by ·007 ; the mp. shank passes through this socket and is firmly held. (Cf. Chirimia of the Indians of Mexico, presented by Miss Marian Storm [Chap. ii, Plate 10].)

The Mixolydian Sequence

Fingerholes	Exit	1	2	3	4
Ratios and Notes	14/14	13	12	11	10
Cents		*128*	*138·5*	*151*	*165*

With *B-R. Mp.* '*xiii G. 14*' (untreated), the Aulos, at Mp. Ext. ·067, played on $\frac{G\ 14}{64}$ in tune as above. The tone less husky, but still rough and unmusical.

With *D-R. Mp.* '*xiii B.*', the Aulos played at Mp. Ext. ·067 on $\frac{B\ 12}{64}$ in tune the sequence as above.

LORET XV

Turin No. 10, 8 Fingerholes

Facsimile made by K. S., Feb., 1934

Plays in the Hypophrygian Harmonia M.D. 18 from Exit.

MEASUREMENTS		FINGERHOLES (The distances are from emb. to centre of Hole)		I.D.
R.L. exit to emb.	·435 ;	C. of Hole 1	from emb. = ·369 ;	from exit = ·066
L. Hole 1 to emb.	·369 ;	C. of Hole 2	from emb. = ·342 ;	from C. of Hole 1 = ·027
Δ ·008 at exit and emb.		C. of Hole 3	from emb. = ·316 ;	from C. of Hole 2 = ·026
δ ·004 constant		C. of Hole 4 thumb back	from emb. = ·284 ;	from C. of Hole 3 = ·032
		C. of Hole 5	from emb. = ·257 ;	from C. of Hole 4 = ·027
		C. of Hole 6	from emb. = ·232 ;	from C. of Hole 5 = ·025
		C. of Hole 7	from emb. = ·206 ;	from C. of Hole 6 = ·026
		C. of Hole 8	from emb. = ·174 ;	from C. of Hole 7 = ·032
		from Exit to Hole 1 = two increments		9)·261
				·029

Increment of Distance = ·029 (**mean**).

Ext. of Mp. = ·087
R.L. + ·435
·522 and ·029 × 18 = ·522

M.D. 18 ; Hypophrygian

MOUTHPIECES

D-R. Mp. '*xv* (*c*)'. L. ·181 ; Δ ·006 emb. ; ·007 exit ; V.L. ·077 ; normal proper note C = 256 v.p.s. ; glottis note $\frac{E\ 18}{256}$ and D 20.

D-R. Mp. '*xv* (*d*)'. L. ·188 ; Δ exit ·0055 ; emb. ·005 ; proper normal note at V.L. ·059 = $\frac{F\ 17}{128}$ at V.L. ·070 = $\frac{D\ 20}{128}$.

D-R. Mp. '*xv* $\left(\frac{a}{27}\right)$'. *L.* ·156 ; Δ exit and emb. ·006 (round) ; at V.L. ·058 = $\frac{F\ 17}{128}$ = normal proper note of mp.

B-R. Mp. '*E. 18*'. River reed, blue seal ; Mp. Ext. ·087 ; L. ·154 ; T.L. ·039 ; T.W. ·003 ; Δ ·004.

PERFORMANCE

With *D-R.* '*xv* (*c*)' at Mp. Ext. ·087 plays on C = 128 v.p.s. Played in Aulos at M.D. 18 in the Hypophrygian Harmonia on $\frac{C\ 11}{128}$ in tune with Monochord. The 1st hole is at about two I.D.s from exit and Hole 8 at ·261 from emb., is exactly at half the length of resonator + Mp. Ext. which falls to ratio 9.

The Modal Sequence

Fingerholes	Exit	H 1	2	3	4	5	6	7	8
Ratios and Notes	18/18	16	15	14	13	12	11	10	9
Cents	*204*	*112*	*119·4*	*128*	*138·5*	*151*	*165*	*182*	

The octave is actually played on Ratio 9 at Hole 8 (regardless of any influence due to diameter) because there are 9 increments. It will be seen that the Resonator has a length of ·435 + Mp. Ext. ·087 = ·522, half of which = ·261 − ·087 = ·174, which marks the actual distance of the C. of Hole 8 from emb.

With *D-R. Mp.* '*xv* (*d*)'. At V.L. ·059 the proper normal note is *F* 17, but at V.L. ·070 the mp. plays in the Aulos on $\frac{D\ 20}{128}$, in the Hypophrygian Harmonia at Mp. Ext. ·087, the sequence given above in tune with monochord, and the octave $\frac{D\ 20}{256}$ is played clearly and easily from Hole 8.

With *D-R. Mp.* '*xv* $\left(\frac{a}{27}\right)$' at Mp. Ext. ·087, the Aulos plays the whole sequence in tune and sounds the octave clearly.

With *B-R. Mp.* '*E. 18*', cut from a river reed, the Aulos played staccato, in tune, the Modal Sequence as far as Hole 5, but could not be coaxed to continue further in tune. The interval of the 4th is the useful limit for the B-R. Mp., although sometimes Hole 6 would sound ratio 11.

LORET XVI

Paris, Louvre, E. 5404. Reed Pipe. 4 Fingerholes

Facsimile made by K. S.

This Aulos plays from Vent (H. I.) in the Dorian Harmonia at Mp. Ext. ·060; and in the Phrygian at Mp. Ext. ·087.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·400;	C. of Hole 1 from emb. = ·309;	from exit	= ·091
L. from C. of Hole 1	·309;	C. of Hole 2 from emb. = ·272;	from C. of Hole 1	= ·037
Δ emb. ·005; exit	·004;	C. of Hole 3 from emb. = ·2415;	from C. of Hole 2	= ·0305
δ^1 and δ^2 ·006; δ^3 and δ^4	·005;	C. of Hole 4 from emb. = ·2105;	from C. of Hole 3	= ·031
				3)·0985
				·0328

Increment of Distance (mean) ·0328
Useful I.D. ·033

THE MOUTHPIECES

D-R. Mp. '*R. R*'. Wheat stalk, fine silica, untreated (ʊ). L. ·141; Δ ·003; Mp. Ext. ·087; V.L. ·080; proper normal note *C* = 128.

D-R. Mp. '*Z. 7*'. L. ·137; Δ ·003; V.L. ·061; normal proper note $\frac{F\ 17}{256}$; gl. $\frac{A\ 13}{128}$; Mp. Ext. ·087.

D-R. Mp. '*H. 8*'. L. ·145; Δ ·003; V.L. ·061; Mp. Ext. ·087; norm. $\frac{F\ 17}{256}$.

D-R. Mp. '*R. 8*'. L. ·159; Δ ·003.
D-R. Mp. '*V. 3 ii*'. L. ·188; Δ ·0035.
D-R. Mp. '*D. 1*' (see Aulos xxi).
B-R. Mp. No. xvi. L. ·098; Δ exit ·003; T.L. ·033; T.W. ·0025; proper note *B* 12/64; fine tone.

PERFORMANCE

With *D-R. Mp.* '*R. R*'. Plays from Vent in Aulos at Mp. Ext. ·087 on $\frac{B\ 12}{128}$ at V.L. ·073; in the Phrygian Harmonia in tune, but subdued tone (due to narrow diameter of Mp.).

The Phrygian Modal Sequence

Fingerholes	H. 1	2	3	4
Ratios and Notes	12/12	11	10	9
Cents		151	165	182

With *D-R. Mp.* '*Z. 7*'. The Aulos plays, as above, on $\frac{B\ 12}{128}$ at Mp. Ext. ·087 in tune.

With *D-R. Mp.* '*H. 8*'. The Aulos plays, as above, on $\frac{B\ 12}{128}$ at Mp. Ext. ·087. Tone a little stronger.

With *D-R. Mp.* '*R. 8*'. The Aulos plays, as above, in the Phrygian Harmonia at Mp. Ext. ·087, on $\frac{B\ 12}{128}$, and at Mp. Ext. ·060 on $\frac{C\ 11}{256}$ in the Dorian Harmonia.

With *D-R. Mp.* '*V. 3*'. The Aulos plays at Mp. Ext. ·087 on $\frac{B\ 12}{128}$ in the Phrygian Harmonia from Hole 1 as above; and at Mp. Ext. ·060 in the Dorian Harmonia on $\frac{G\ 15}{128}$, both with a fine, musical tone.

With *D-R. Mp.* '*D. 1*' (treated reed), the Aulos plays on $\frac{B\ 12}{64}$ at Mp. Ext. ·087, the sequence as above.

With *B-R. Mp. No. xvi*, the Aulos plays in the Phrygian Harmonia on $\frac{G\ 15}{128}$, the sequence as above. The same Mp. plays at Mp. Ext. ·052 in the Dorian Harmonia on $\frac{F\ 16}{64}$ with a fine resonant tone, as below.

The Dorian Sequence (from Vent) on $\frac{F\ 16}{64}$

Fingerholes	H. 1	2	3	4
Ratios and Notes	11/11	10	9	8 (Mese)
Cents		165	182	204

LORET XVIII

Leyden I No. 479. (4 Fingerholes)
Facsimile by K. S. in 1925

This Aulos plays best from Vent in the Lydian Harmonia M.D. 13 on $\frac{A\ 13}{64}$; it will also play in the Phrygian Harmonia M.D. 12 on $\frac{B\ 12}{64}$, likewise from Hole 1.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·397 ;	C. of Hole 1 from emb. = ·305 ;	from exit	= ·092
L. from C. of Hole 1	·305 ;	C. of Hole 2 from emb. = ·272 ;	from C. of Hole 1	= ·033
Δ at emb.	·005 ;	C. of Hole 3 from emb. = ·243 ;	from C. of Hole 2	= ·029
Δ at exit ·0035 δ ·003 to ·0045;		C. of Hole 4 from emb. = ·215 ;	from C. of Hole 3	= ·028
				3)·090
				·030

Increment of Distance (mean) ·030
and practical ·030

I.D. × 13 = ·390 ; ·305 + Mp. Ext. ·085 = ·390.
I.D. × 12 = ·360 ; ·305 + Mp. Ext. ·055 = ·360.

THE MOUTHPIECES AND THEIR PERFORMANCE

With *D-R. Mp.* '*Cl. 5*'. L. ·130 reduced to ·120 ; Δ emb. ·004 ; Δ exit ·005 ; V.L. ·060 ; proper note $\frac{F\ 17}{256}$; gl. note $\frac{A\ 13}{128}$; plays in the Aulos in the Lydian Harmonia, M.D. 13 from Vent at Mp. Ext. ·085 and V.L. ·060, on $\frac{A\ 13}{128}$ (the glottis note of the mp.) in perfect tune with modal piano. Tested Jan. 4, 1934.

With *D-R.* '*N. 8*'. L. ·155 ; Δ emb. round ·0035 ; proper note at V.L. ·060 = $\frac{F\ 17}{256}$; gl. note $\frac{A\ 13}{64}$, the Aulos played at once in tune. Tested Jan. 8, 1934.

With *D-R. Mp.* '*D. 1*'. L. ·172 ; Δ ·003 (flattened by treatment to ·006) at V.L. ·045 ; proper note $\frac{B\ 12}{128}$; at V.L. ·060–·062 = $\frac{G\ 15}{128}$; plays in the Lydian Harmonia on $\frac{E\ 18}{128}$ at Mp. Ext. ·085, and at V.L. ·080, in perfect tune. Tested Jan. 8, 1932.

B-R. Mp. xviii made to supersede xviii A and B in October, 1930. L. ·150 ; Δ ·0035 ; T.L. ·035 ; T.W. ·0025 ; proper note $\frac{G\ 14}{256}$, plays in the Aulos at Mp. Ext. ·085 on $\frac{A\ 13}{128}$ in tune with modal piano. Tested Oct., 1930. The other two B-R. mp.s bearing the same number (Roman) were not satisfactory in this Aulos. 'xviii A' could not be pushed into the resonator farther than ·060 ; and xviii B did not speak freely until later when I lengthened the Tongue to ·032, clearing it carefully on the underside, then the mp. improved greatly and spoke freely on $\frac{B\ 12}{64}$.

The Lydian Harmonia, 1st Tetrachord

Fingerholes	1 Vent	2	3	4
Ratios and Notes	13/13	12	11	10
Cents		*138·5*	*151*	*165*

LORET XIX

British Museum No. 6388. 3 Fingerholes

Facsimile by K. S., Feb., 1934. One experimental hole added at ·031 from exit

The Aulos plays at Mp. Ext. ·104 in the Hypodorian Harmonia on C = 128 v.p.s. and on $\frac{B\ 12}{32}$ from exit.

MEASUREMENTS FINGERHOLES I.D.

(The distances are from emb. to centre of Holes)

R.L. from exit to emb. ·392 ;	C. of Hole 1 from emb.	=·3005 ; from exit	=·0915
Δ at exit and emb. ·005 ;	C. of Hole 1 (added by K. S.)	=·361 ; from exit	=·061
δ constant at ·003 ;	C. of Hole 2 from emb.	=·2675 ; from C. of Hole 1	=·033
	C. of Hole 3 from emb.	=·2385 ; from Hole 2	=·029

The addition of the Hole I.A. gives the I.D. as follows :

·0305 ; $\left\{\begin{matrix}\cdot061\\ \text{two I.D.}\end{matrix}\right\}$; ·033 ; ·029 Total 5)·1535 = ·0307

Average Increment of Distance ·0307

useful I.D. ·031

THE MOUTHPIECES

D-R. Mp. '*xix C. 16*' (untreated) at Mp. Ext. ·104; L. ·206; Δ ·006; V.L. ·058; proper normal note $= \frac{F\ 17}{128}$ with glottis action $\frac{A\ 13}{64}$.

B-R. Mp. '*Z. 2*' (oat straw). L. ·179; T.L. ·038; T.W. ·003; Δ ·005; norm. $\frac{B\ 12}{32}$.

Scheme of Aulos division I.D. 16

		H. 1*a*	H. 1	H. 2	H. 3
		·061			
	Exit	O	O	O	O
	L. ·392	(·0305)	at ·0915	·033	·029
Ratios	16/16	15	13	12	11

PERFORMANCE

With *D-R. Mp.* '*xix C. 16*', at Mp. Ext. ·104, the Aulos played on C = 128 v.p.s. in the Hypodorian Harmonia of M.D. 16, the 1st tetrachord and the dominant or tone of disjunction. The extra hole added at ·0305 from Exit produced the 2nd step in the Harmonia, omitted in the original specimen; which may have been used as a Lydian pipe of M.D. 13; but this seems pointless, since the sequence from vent would consist only of the three notes 13/13, 12, 11.

The Modal Sequence (with added 2nd Step)

Fingerholes	Exit	H. 1*a*	1	2	3
Ratios and Notes	16/16	15	13	12	11
			two I.D.		
Cents		*112*	*247*	*138·5*	*151*

The Sequence is played in tune with monochord; first tested March 13, 1934.

With *B-R. Mp.* '*Z. 2*', the Aulos played the sequence in tune as above on $\frac{B\ 12}{32}$; the tone is powerful, reedy and rich at Mp. Ext. ·104.

The extra fingerhole added to the facsimile forms an illustration of scale-building on the Aulos in order to produce a specified Harmonia. The addition of a finger-hole at one I.D. above exit is also a proof that no allowance for diameter in the determination of the position of Hole 1 is needed on the Aulos—although this allowance is an absolute necessity on the Flute. (See Chap. vi and Flute Records further on.)

LORET XXI

Leyden I No. 477. Reed. 4 Fingerholes

Facsimile made by K. S. in 1916

The Aulos plays in the PHRYGIAN Harmonia M.D. 12 with D-R. Mp.s from Vent, and with B-R. Mp.s in the DORIAN Harmonia M.D. 11. From exit the Aulos plays in the HYPODORIAN Harmonia M.D. 16.

MEASUREMENTS		FINGERHOLES	I.D.
R.L. from exit to emb.	·357;	C. of Hole 1 from emb. = ·271;	from exit, 3 (I.D.) = ·086
L. from emb. to C. of Hole 1 as Vent	·271;	C. of Hole 2 from emb. = ·240;	from C. of Hole 1 = ·031
Δ emb. ·005; exit	·004;	C. of Hole 3 from emb. = ·210;	from C. of Hole 2 = ·030
		C. of Hole 4 from emb. = ·183;	from C. of Hole 3 = ·027
			6)·174
			·029

Increment of Distance (mean) **·029.**

I.D. ·029 × 11 = ·319	L. from Vent	·271
„ ·029 × 12 = ·348	Mp. Ext.	·077
„ ·029 × 16 = ·464		·348

R.L. ·357
Mp. Ext. ·107
·464

The Dorian Harmonia is not suitable for use with D.R. Mp. for the Mp. Ext. is too short to allow the necessary V.L. The B-R. mp.s are the best for this Harmonia (see Performance).

THE MOUTHPIECES

D-R. Mp. ' *D. 1* ', treated wheat stalk. L. ·172 ; Δ ·003 ; flattened at emb. to ·006 plays in a number of Auloi. (See Table viii, Chap. iii.)

D-R. Mp. ' *xxvii* (*a*) ', flattened and treated. L. ·140 ; Δ ·0045 flattened to ·01 at emb. ; very responsive after soaking for a couple of hours.

D-R. ' *Cl. 4* ', wheat stalk. L. ·154 ; Δ ·004 ; V.L. ·060. Proper normal note $\frac{F\ 17}{256}$; glottis notes $\frac{D\ 20}{128}$ and $\frac{A\ 13}{128}$.

D-R. Mp. ' *O. 1* '. Mp. Ext. ·080. L. ·161 ; Δ ·0035 ; V.L. ·080 ; proper note *C* = 256 v.p.s. ; at V.L. ·060. *F* 17/128.

D-R. Mp. ' *xxi B* ', at Mp. Ext. ·107 ; V.L. ·60 ; L. ·125 ; Δ ·0035 emb., ·004 exit. Tested March 30, 1934.

B-R. Mp. ' *No. 12* ', wheat. L. ·210 ; Δ ·004 ; T.L. ·037 ; T.W. ·0025 ; proper note norm. *C* = 128 v.p.s., fine tone ; the T.L. was extended from ·034 to ·037.

B-R. Mp. ' *K. 12* ', wheat. L. ·185 ; T.L. ·032 ; T.W. ·002 ; Δ ·003. Tested March 30, 1934.

B-R. ' K. 12 ' also plays in the Dorian Harmonia at Mp. Ext. ·050, but not quite so successfully as No. 12 ; the Mp. is at its best in the Phrygian Harmonia. It will just play the Hypodorian.

PERFORMANCE

With *D-R. Mp.* ' *D. 1* ', at Mp. Ext. ·077–·080 and at V.L. ·060, the Aulos plays from Vent on $\frac{D\ 20}{128}$, in the Phrygian Harmonia at Mp. Ext. ·080 in tune with monochord.

The Phrygian Modal Sequence

Fingerholes	Vent 1	2	3	4	
Ratios and Notes	12/12	11	10	9	
Cents		*151*	*165*	*182*	

With *Mp.* ' xxvii (*a*) ' at ·077 Mp. Ext., and at V.L. ·060, the Aulos plays from Vent on $\frac{B\ 12}{64}$, the Modal Tonic, in the Phrygian Harmonia, the Sequence as above.

With *D-R. Mp.* ' *Cl. 4* ' at Mp. Ext. ·080, V.L. ·060, the Aulos plays from Vent on $\frac{B\ 12}{64}$ in the Phrygian Harmonia, as above, in tune with modal piano.

With *D-R. Mp.* ' *O. 1* ', at Mp. Ext. ·080 and at V.L. ·060, the Aulos played from Vent on $\frac{B\ 12}{128}$ in the Phrygian Harmonia as above ; in tune with modal piano.

With *D-R. Mp.* ' *xxi. B* ', at Mp. Ext. ·107 and at V.L. ·060, the Aulos plays from Exit in the Hypodorian Harmonia M.D. 16, but in the 1st tetrachord, only the two fixed notes, Tonic and 4th of ratios 16 and 12. I.D. ·029 × 16 = ·464 – ·357 (L. from Exit) = ·107 Mp. Ext. The 1st Hole being at ·086 from Exit, includes three I.D.

The Hypodorian Modal Sequence

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	16/16	13	12	11	10
	(15, 14)	128			
Cents		*359·2*	*138·5*	*151*	*165*

B-R. Mp. 12, wheat. Tested Nov. 8, 1925. Plays in Aulos on $\frac{F\ 16}{64}$ freely and correctly, with sonorous, musical tone, in the Dorian Harmonia from vent, at Mp. Ext. ·050, tested April 22, 1933, and March 29, 1934; the same mp., at extrusion ·077, plays also from Vent in the Phrygian Harmonia.

B-R. Mp. '*K. 12*' plays from Vent in the Aulos at Mp. Ext. ·077 on $\frac{B\ 12}{64}$ in the Phrygian Harmonia, freely with a strong, sonorous tone and with less success in the Hypodorian from exit.

[I.D. ·029 × 12 = ·348 − ·271 = ·077 Mp. Ext.]

Thus Aulos xxi (4 Holes) plays in three Harmoniai: from Vent (Hole 1) in the Dorian Harmonia with a B-R. mp. only, owing to the short Mp. Ext. The Aulos plays in the Phrygian Harmonia with four D-R. mp.s at ·077 Mp. Ext., and with one D-R. Mp. (xxi B) in the Hypodorian Harmonia at Mp. Ext. ·107.

LORET XXII

British Museum No. 12742. Bronze pipe (or flute). 4 Fingerholes
Facsimile by K. S.

Loret states that engraved on the flute is an inscription in Demotic of a single line, spoilt by the oxidation of the metal.

The Aulos plays in the Mixolydian M.D. 14 at Mp. Ext. ·105 on *G* 14 and in the Lydian Harmonia M.D. 13 at Mp. Ext. ·072 on $\frac{C\ 11}{128}$.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·357;	C. of Hole 1 from emb. = ·318;	from exit	= ·039
Δ	·013;	C. of Hole 2 from emb. = ·291;	from C. of Hole 1	= ·027
δ ·006 constant		C. of Hole 3 from emb. = ·260;	from C. of Hole 2	= ·031
		C. of Hole 4 from emb. = ·224;	from C. of Hole 3	= ·034
			4)	·131
				·03275

Increment of Distance (mean) ·03275.
useful ·033.

R.L. ·357 + ·105 = ·462; $\frac{\cdot 462}{\cdot 033}$ = 14 M.D.

·033 × 14 = ·462.

THE MOUTHPIECES

D-R. Mp. '*22*; *L.C. 128*'. Wheat stalk; at Mp. Ext. ·105; L. ·143; Δ ·005 at V.L. ·077; proper note = *C* 128 v.p.s.

D-R. Mp. '*xxii. G. 14*'. Wheat stalk, bound with thread; L. ·164; Δ ·005.

D-R. Mp. '*xxii. C. 13*'. Wheat stalk; Mp. Ext. ·072; L. ·142; Δ ·006.

PERFORMANCE

With *D-R.* '*22 L.C.*', at Mp. Ext. ·105, the Aulos plays on $C = 128$ v.p.s. in the Mixolydian Harmonia M.D. 14 in tune with monochord.

The Mixolydian Sequence

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	14/14	13	12	11	10
Cents		*128*	*138·5*	*151*	*165*

With *D-R.* '*xxii G. 14*', at Mp. Ext. ·105, the Aulos played on $\frac{G\ 14}{128}$ at V.L. ·070 in the Mixolydian Harmonia. The tone was soft, the mp. needs more practice.

With *D-R. Mp.* '*xxii C. 13*', at Mp. Ext. ·072, the Aulos played on $\frac{C\ 11}{128}$ in the Lydian Harmonia in good tune, and gave better results than in the Mixolydian Sequence.

The Lydian Modal Sequence

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	13/13	12	11	10	9
Cents		*138·5*	*151*	*165*	*182*

LORET XXIV

Leyden I No. 478. 4 Fingerholes

Facsimile by K. S., 1925

This Aulos plays from Vent in the Lydian Harmonia, M.D. 13, at Mp. Ext. ·062 with mouthpieces of both types. On $\frac{D\ 20}{128}$ and on $\frac{A\ 13}{128}$ and at Mp. Ext. ·090 in the Mixolydian Harmonia, M.D. 14, on $\frac{B\ 12}{64}$, on $\frac{A\ 27}{64}$ and on $\frac{G\ 14}{128}$.

MEASUREMENTS

R.L. from exit to emb. ·353 ;
R.L. from C. of Hole 1 to emb. ·302 ;
Δ emb. ·005 ;
Δ exit ·0045 ;

FINGERHOLES — I.D.

C. of Hole 1 from emb. = ·302 ;	from exit	= ·051
C. of Hole 2 from emb. = ·275 ;	from C. of Hole 1	= ·027
C. of Hole 3 from emb. = ·248 ;	from C. of Hole 2	= ·027
C. of Hole 4 from emb. = ·219 ;	from C. of Hole 3	= ·029
		3)·083
		·0276

Increment of Distance (mean) **·0276**
practical ·028

·028 × 13 = ·364 − ·302 = Mp. Ext. ·062.
·028 × 14 = ·392 − ·302 = Mp. Ext. ·090.

THE MOUTHPIECES

D-R. Mp. '*D. 1*' (treated). L. ·172 ; Δ ·003 flattened to ·006 ; V.L. ·060.

D-R. Mp. '*xxvii (a)*' (treated). L. ·140 ; Δ ·0045 flattened to ·01 ; proper note at V.L. ·060 = $\frac{F\ 17}{128}$.

D-R. Mp. '*Cl. 6*' (untreated). L. ·200 cut to ·161 ; Δ ·004 exit ; Δ ·005 emb. ; proper note at V.L. ·079 = $\frac{C\ 11}{256}$; gl. $\frac{G\ 14}{128}$.

D-R. Mp. ‘ *R. 3* ’ (untreated). L. ·119 ; Δ emb. ·004 ; exit ·005. Proper note at ·060 = $\frac{F\ 17}{256}$; gl. $\frac{A\ 13}{128}$; fine tone.

PERFORMANCE

With *B-R. Mp. No. xxiv*, the Aulos plays on $\frac{D\ 20}{128}$ in the Lydian Harmonia, from Vent at Mp. Ext. ·062 with a good firm tone in perfect tune and without hesitation, with monochord. Tested Nov. 9, 1925, and Nov. 6, 1930.

With *B-R. Mp.* ‘ *R. 3* ’, the Aulos plays the Lydian Harmonia M.D. 13, on $\frac{A\ 13}{128}$ with a singularly fine powerful tone, due probably to the fine elastic texture of the straw, and to the fact that the mp. is in the octave relation of resonance with the fundamental of the resonator on $\frac{A\ 13}{128}$.

With *D-R. Mp.* ‘ *Cl. 6* ’ (untreated), the Aulos plays at Mp. Ext. ·090 in the Mixolydian Harmonia M.D. 14, 1st tetrachord, on $\frac{G\ 14}{128}$ (at V.L ·079. Proper note $\frac{C\ 11}{256}$ and glottis note $\frac{G\ 14}{128}$) in perfect tune with the modal piano. Tested Jan. 6, 1934.

This same mp. ‘ Cl. 6 ’ also plays at Mp. Ext. ·065 in the Lydian Harmonia M.D. 13 on *A* 13, V.L. ·075. Tested Jan. 6, 1934, in tune with piano (for modal sequence, see Loret xviii).

Since this mp. played in Aulos xxiv in both Harmoniai on their own Tonics as species of the Dorian Harmonia and therefore in unison with modal piano, a special test was carried out, giving one breath to each note, while keeping the thumb nail at the correct V.L. in each case, for the whole sequence. When the M.D. is the next in arithmetical succession, the difference in the intervals is slight and a rigid test becomes necessary.

D-R. Mp. xxvii (*a*) gave a similar result in the Mixolydian Harmonia on $\frac{A\ 27}{64}$ at Mp. Ext. ·090 and V.L. ·060.

LORET XXV

Paris, Louvre No. E 5404. Reed pipe. 3 Fingerholes
Facsimile by K. S., 1934

The Aulos plays in the Mixolydian Harmonia M.D. 14 from Exit.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·351 ;	C. of Hole 1 from emb. = ·291 ;	from exit	= ·060
Δ emb. ·006 ; exit	·005 ;	C. of Hole 2 from emb. = ·253 ;	from C. of Hole 1	= ·038
δ ·006 ;	·0055 ;	C. of Hole 3 from emb. = ·222 ;	from C. of Hole 2	= ·031
				4)·129
				·0322

Increment of Distance (mean) ·0322.
Useful I.D. ·032

THE MOUTHPIECES

D-R. ‘ *xxv G. 14* ’. Wheat ; L. ·140 ; Δ exit ·006 ; emb. ·008 ; proper normal note at V.L. ·058 = $\frac{F\ 17}{128}$.

D-R. xxv. Wheat straw ; L. ·172 ; Δ emb. ·006 ; exit ·005 ; proper normal note at V.L. ·058 = $\frac{F\ 17}{128}$.

R.L. = ·351 + Mp. Ext. ·097 = ·448 ; $\frac{.448}{.032}$ = 14 M.D. ; ·032 × 14 = ·448.

PERFORMANCE

With *D-R. xxv* '*G. 14*', at Mp. Ext. ·097 on $\frac{G\ 14}{128}$, the Aulos played in the Mixolydian Harmonia M.D. 14 in tune with modal piano and monochord. The centre of Hole 1 is at about two I.D.s (within a few mm.) from exit and the Aulos has no difficulty in playing the septimal 3rd 14/12 and the short sequence as follows :

The Modal Sequence

Fingerholes	Exit	H. 1	2	3
Ratios and Notes	14/14	12	11	10
Cents		*267*	*151*	*165*

With *D-R. Mp. xxv* at Mp. Ext. ·098, the Aulos played on $\frac{D\ 20}{128}$ the sequence as above in tune and with sonorous tone.

B-R MOUTHPIECES

B-R. '*25 Z.*' (*14*). L. ·131 ; Δ ·006 ; T.L. ·045 ; T.W. ·0035 ; Mp. Ext. ·097 ; proper note $\frac{D\ 20}{128}$; plays in Aulos on $\frac{A\ 27}{64}$. Of great power.

B-R. '*Z. 2*'. L. ·169 ; T.L. ·039 ; T.W. ·003 ; Mp. Ext. ·097 plays in Aulos on $\frac{C\ 11}{64}$.

B-R. '*E*'. *Reed.* Plays in Aulos on *G* 14 ; L. ·139 ; T.L. ·038 ; T.W. ·003 ; Δ ·004 ; norm. $\frac{A\ 27}{64}$.

B-R. blue seal mp. of river reed. Plays in Aulos on $\frac{G\ 14}{64}$; L. ·154 ; T.L. ·039 ; T.W. ·003 ; Δ ·006.

PERFORMANCE

The performance with these 4 B-R. mp.s in the Aulos was noticeably more powerful in tone ; but the intonation was not so certain as with the D-R. mp.s. Knowing the sequence one could get the note exactly in tune on opening the finger-hole but the B-R. mp. is the artist rather than law-giver.

LORET XXVI

Leyden I No. 480. 3 Fingerholes

Facsimile by K. S. ; a crack about $1\frac{1}{4}$ inches appeared at emb. and has been repaired with brown bands and blue linen thread and is now airtight.

This Aulos plays from Exit in the Mixolydian Harmonia on $\frac{G\ 14}{64}$ and also on $\frac{D\ 20}{64}$ and $\frac{A\ 27}{64}$.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·322 ;	C. of Hole 1 from emb. = ·292 ;	from exit	= ·030
Δ emb.	·005 ;	C. of Hole 2 from emb. = ·264 ;	from C. of Hole 1	= ·028
Δ exit	·0045 ;	C. of Hole 3 from emb. = ·236 ;	from C. of Hole 2	= ·028
				3)·086
				·0286

Increment of Distance (mean) ·0286
practical ·028

I.D. ·028 × 14 = ·392. ·392 − ·322 = ·070 Mp. Ext.

THE MOUTHPIECES

D-R. Mp. xxvii (treated). L. ·140 ; Δ ·0045 ; flattened ·01 ; proper note at V.L. ·050 $\frac{F\ 17}{128}$.

D-R. Mp. '*Cl. 9*' (untreated). Proper note $\frac{F\ 17}{128}$; and glottis $\frac{C\ 21}{256}$ and $\frac{A\ 27}{128}$; L. ·200 reduced to ·161 ; Δ ·003 ; V.L. ·070.

B-R. '*xxvi No. 4*'. L. ·154 ; Δ ·0035 ; T.L. ·032 ; T.W. ·0025 ; proper note *F* 16 = 176 v.p.s. (now slightly cracked).

B-R. Mp. xxvi. L. ·098 ; T.L. ·034 ; T.W. ·002 (exit slightly crushed ; no importance).

D-R. Mp. '*V. 2*' (untreated). L. ·206 ; Δ ·0035 wide oval ; V.L. ·060 ; proper note $\frac{F\ 17}{256}$ also at same V.L. the octave below $\frac{F\ 17}{128}$ and with glottis action a strong *C* = 256 v.p.s.

D-R. Mp. '*Y. 1*' (untreated). L. ·181 ; Δ ·0045 emb. ; ·004 exit ; V.L. ·079 ; proper note *C* = 256 v.p.s., strong tone ; also at V.L. ·060 $\frac{F\ 17}{256}$.

PERFORMANCE

With *D-R.* '*Cl. 9*', the Aulos plays from exit at V.L. ·070, and with Mp. Ext. ·070, the Mixolydian tetrachord on $\frac{G\ 14}{128}$ the four notes in tune ; needs a little coaxing at first, otherwise good. Tested April 28, 1933, and May 23, 1933. The mp. is now fixed in Aulos, airtight. Tested March 8, 9, 11, 1933.

With *B-R. Mp.* '*xxvi, No. 4*', the Aulos played the Mixolydian tetrachord in correct intonation on *E* 19, tested Nov. 5, 1925 ; and the next day on *D* 20, with Mp. Ext. ·070, in tune with modal piano.

With *B-R. Mp. xxvi*, the Aulos plays on its own Tonic $\frac{G\ 14}{64}$ with a full, rich tone in tune with modal piano.

With *D-R. Mp. xxvii*, the Aulos plays at Mp. Ext. ·080 and V.L. ·050 the Mixolydian Tetrachord on $\frac{A\ 27}{64}$. All in perfect tune. Changing the V.L. to ·069, proved adverse ; the ratio $\frac{13}{14}$ of Hole 1 was raised to $\frac{25}{28}$ and Hole 3 from $\frac{11}{14}$ to $\frac{20}{28}$; at Hole 2 there was wobbling.

The Mixolydian Tetrachord

Fingerholes	Exit	H. 1	2	3
Ratios and Notes	14/14	13	12	11
Cents		*128*	*138·5*	*151*

LORET XXVII

Turin No. 12. 6 Fingerholes

Facsimile by K. S. xxvii in 1925

The Aulos plays best in the Mixolydian Harmonia, M.D. 14, from Vent, but also from exit in the Hypodorian Harmonia M.D. 16.

MEASUREMENTS	FINGERHOLES (Distances from emb. to Centre of Hole.)	I.D.
R.L. from exit to emb. ·320 ;	C. of Hole 1 from emb. = ·278 ;	from exit = ·042
R.L. from C. of Hole 1 to emb. ·278 ;	C. of Hole 2 from emb. = ·253 ;	from C. of Hole 1 = ·025
Δ ·006	C. of Hole 3 from emb. = ·229 ;	from C. of Hole 2 = ·024
δ ·004 constant	C. of Hole 4 from emb. = ·206 ;	from C. of Hole 3 = ·023
	C. of Hole 5 from emb. = ·182 ;	from C. of Hole 4 = ·024
	C. of Hole 6 from emb. = ·155 ;	from C. of Hole 5 = ·027

Increment of Distance (mean) ·0246 from Vent
(mean) ·0275 from Exit.

Practical I.D. Vent ·025 × 14 ·350
− ·278

I.D. ·072 Mp. Ext.

THE MOUTHPIECES

D-R. Mp. '*xxvii* (*A*) (treated). V.L. ·060–·65 ; L. ·140 ; Δ ·0045 flattened at emb. to ·01 ; proper note norm. $\frac{F\ 17}{128}$.

D-R. Mp. '*R. 6*'. L. ·158 ; fine satiny wheat stalk ; Δ ·004 round ; at V.L. ·060–·64 ; normal note *F* 17 ; at V.L. ·075 ; norm. *C* = 256 v.p.s.

D-R. Mp. '*Z. 1*'. L. ·142 ; Δ ·0035 ; V.L. at ·079 ; proper note $\frac{C\ 11}{256}$; the results are only passable in the Aulos.

D-R. Mp. '*D. 1*' (treated). L. ·172 ; Δ ·003 flattened to ·006 ; V.L. ·062 (for performance of this mp. in various Auloi see Chap. iii, Table viii).

D-R. Mp. '*H. 7*'. Firm wheat, silica striped. L. ·142 ; Δ ·0045 round ; V.L. ·065 ; proper note $\frac{F\ 17}{256}$; gl. note $\frac{A\ 13}{128}$.

D-R. Mp. '*R. 9*'. L. ·158 ; Δ ·004 round ; proper note at ·060 $\frac{F\ 17}{256}$; gl. $\frac{A\ 13}{128}$ and also $\frac{F\ 17}{128}$; at V.L. ·075 ; proper note *C* = 256 v.p.s. Tested June 10, 1933.

D-R. Mp. '*H. 6*'. L. ·111 ; Δ ·0045 round ; proper note at ·060 $\frac{F\ 17}{256}$; $\frac{D\ 20}{256}$ gl. ; and $\frac{A\ 13}{128}$. Fine strong tone.

D-R. Mp. '*O. 6*'. L. ·135 ; Δ ·004 ; V.L. ·060 ; proper note $\frac{F\ 17}{256}$; gl. $\frac{D\ 20}{256}$ and $\frac{A\ 13}{128}$; the best note, mellow and easy.

PERFORMANCE

D-R. Mp. '*R. 6*', at Mp. Ext. ·115 and V.L. ·068, plays in the Aulos on $\frac{G\ 14}{128}$;

Hypodorian Harmonia M.D. 16 ; all notes in good tune with Monochord ; this mp. will also play in Aulos xxvii on $\frac{B\ 12}{128}$ at V.L. ·074.

B-R. Mp. '*S*', of hard thick reed, with sealing-wax on base. Proper note $\frac{G\ 14}{256}$ plays in Aulos the whole sequence at normal breath on $\frac{G\ 14}{128}$ in the Mixolydian Harmonia M.D. 14 at Mp. Ext. ·063 ; all notes in tune with modal piano. D-R. '47' plays in Aulos at Mp. Ext. ·068 on $\frac{B\ 12}{64}$ in the Mixolydian Harmonia M.D. 14. Tested June 10, 1933, and again June 30, 1933 for pitch, found correct at $\frac{B\ 12}{64}$. With D-R. Mp. D. 1 at V.L. ·062 Mp. Ex. ·090 played in Mixolydian on $\frac{G\ 15}{128}$ only 4 holes ; and at Mp. Ext. ·065, V.L. ·045 the Aulos played on *D* 20 the whole series strong, steady and in tune.

Mixolydian Sequence M.D. 14

Fingerholes	H. 1	2	3	4	5	6
Ratios and Notes	14/14	13	12	11	10	9
Cents		*128*	*138·5*	*151*	*165*	*182*

The majority of these mouthpieces play in Aulos xxvii the modal sequence as above at Mp. Ext. ·068.

Hypodorian Sequence M.D. 16

Fingerholes	Exit	H. 1	2	3	4	5	6
Ratios and Notes	16/16	15 } 14 }	13	12	11	10	9

Only D-R. Mp. 'R. 6' plays the full sequence on $\frac{B\ 12}{128}$ or on $\frac{G\ 14}{128}$.

LORET XXVIII

Paris, Louvre (unnumbered). 6 Fingerholes

Facsimile by K. S.

This Aulos plays from Exit in the Mixolydian Harmonia with a Mp. Ext. of ·078.

MEASUREMENTS		FINGERHOLES (The distances all from emb. to centre of Hole.)	I.D.
R.L. from exit to emb.	·300 ;	C. of Hole 1 from emb. = ·2735 ;	from exit = ·0265
Δ at emb.	·006 ;	C. of Hole 2 from emb. = ·2475 ;	from C. of Hole 1 = ·026
Δ at exit	·007 ;	C. of Hole 3 from emb. = ·2185 ;	from C. of Hole 2 = ·029
Mp. Ext.	·078 ;	C. of Hole 4 from emb. = ·1905 ;	from C. of Hole 3 = ·028
		C. of Hole 5 from emb. = ·1665 ;	from C. of Hole 4 = ·024
		C. of Hole 6 from emb. = ·1415 ;	from C. of Hole 5 = ·025
			6)·1585
			·0264

Increment of Distance (mean) **·0264**
practical ·027.

I.D. ·027 × 14 = ·378 ; ·378 − ·300 = Mp. Ext. ·078.

THE MOUTHPIECES

D-R. Mp. xxvii (wheat, treated). L. ·140 ; Δ emb. ·0045 flattened ·01 ; proper note at V.L. ·055 = $\frac{F\ 17}{128}$.

D-R. Mp. '*Cl. 3*' (untreated). L. ·158 ; Δ ·004 ; V.L. ·065 ; proper note $\frac{F\ 17}{128}$.

B-R. Mp. '*E. 5*'. L. ·168 ; Δ ·0035 exit ; Δ ·0025 emb. ; T.L. ·030 ; T.W. ·0025.

B-R. Mp. '*D. 7*'. Oaten stalk. L. ·124 ; Δ exit ·0035 ; emb. knot ; T.L. ·042 ; T.W. ·0025.

PERFORMANCE

With *Mp. D-R. xxvii*, plays from exit in the Mixolydian Harmonia at Mp. Ext. ·078 (and also at ·067) on $\frac{A\ 27}{64}$ with a good, strong tone, every note in perfect tune with monochord. Tested Feb. 4, 1933.

With *D-R. Mp.* '*Cl. 3*' (untreated), plays from exit in the Aulos the Mixolydian Harmonia on $\frac{G\ 14}{128}$ in tune with modal piano. *N.B.*—*G* 14 is the Tonic of the Mixolydian Species of the Dorian Harmonia to which the piano is tuned. Tested March 8, 1933. A 2nd test, March 9, 1933, revealed the tone weak at first, but with perseverance the Aulos again played on *G* 14 ; Hole 6 was at first difficult but the note came true and clear with the lips close up to emb., i.e. with a longer V.L. Tested April 28, 1933.

With *B-R.* '*E. 5*', the Aulos plays the whole sequence on *G* 14 = 100 v.p.s. quite correctly with modal piano. Tested Nov. 5 and 6, 1925.

With *B-R.* '*D. 7*' (also used with Elgin A), the Aulos gave similar results up and down many times in tune with piano also.

Mixolydian Sequence from Exit

Fingerholes	Exit		H. 1		2		3		4		5		6
Ratios and Notes	14/14		13		12		11		10		9		8 Mese
Cents		*128*		*138·5*		*151*		*165*		*182*		*204*	

LORET XXX

Louvre. No number. Brownish black reed. 5 Holes

Facsimile by K. S., Feb. 28, 1934

Harmonia Mixolydian M.D. 14 from exit at Mp. Ext. ·111.
Harmonia Lydian M.D. 13 from exit at Mp. Ext. ·075.

MEASUREMENTS	FINGERHOLES I.D. (The distances are from emb. to centre of Holes)
Length of Aulos R. exit to emb. ·263 ;	C. of Hole 1 L. from emb. = ·2285 ; L. from exit = ·0345
Δ of bore emb. ·004 ;	C. of Hole 2 L. from emb. = ·198 ; dist. from Hole 1 = ·0305
at exit ·0035 ;	C. of Hole 3 L. from emb. = ·173 ; dist. from Hole 2 = ·025
	C. of Hole 4 L. from emb. = ·148 ; dist. from Hole 3 = ·025
	C. of Hole 5 L. from emb. = ·126 ; dist. from Hole 4 = ·022
	5)·137
	·0274

Increment of Distance (mean) ·027.

D-R. Mp. xxx A. Wheat straw, untreated. L. = ·149 ; Δ = ·004 ; V.L. at ·060 ; norm. $\frac{F\ 17}{256}$.

In Aulos at Mp. Ext.	·111	·374 = 14, Mixolydian Modal Determinant
L. of resonator	+ ·263	·027 (I.D.)
	·374	

In Aulos at Mp. Ext. ·088 ·351 = 13 Lydian Modal Determinant
L. of resonator + ·263 ·027 (I.D.)
·351

PERFORMANCE

(1) With *D-R. Mp.* '*xxx A*', from exit. At Mp. Ex. ·111, the Aulos had as Modal Determinant 14, and played the Mixolydian Harmonia on $\frac{G\ 14}{128}$ in the tonality proper to the Mixolydian species of the Dorian Harmonia on *C* (as on my modal piano).

Modal Sequence

Fingerholes	Exit	H. 1	2	3	4	5
Ratios and Notes	14/14	13	12	11	10	9
Cents		*128*	*138·5*	*151*	*165*	*182*

Played in tune with modal piano; last note needs practice.

(2) With *D-R. Mp.* '*xxx A*', again from exit, was tried at Mp. Ext. ·075 and rejected for ·088. The M.D. of the Aulos was now 13 and the Harmonia Lydian.

The Aulos now played from exit on $\frac{A\ 13}{128}$, the Lydian Tonic in the Dorian Harmonia.

Modal Sequence

Fingerholes	Exit	H. 1	2	3	4	5	
Ratios and Notes	13/13	12	11	10	9	8	Mese
Cents		*138·5*	*151*	*165*	*182*	*204*	

TESTS MARCH, 1934

(1) I.D. = ·027 × 14 M.D. = ·378. R.L. = ·263 + Mp. Ext. = ·111 = ·374.
(2) I.D. = ·027 × 13 M.D. = ·351. R.L. = ·263 + Mp. Ext. ·088 = ·351.

LORET XXXI

Louvre No. 1714; C. 22, No. 62. 6 Fingerholes. Reddish polished wood
Facsimile made by K. S., Feb., 1934

The Aulos plays from Exit in the Dorian Harmonia M.D. 22 (half-segments) with an Enh.-Chr. Pyknon with Mp. Ext. ·075, and in the Phrygian Harmonia with Mp. Ex. ·102.

MEASUREMENTS		FINGERHOLES		I.D.
L. from emb. to exit	·258;	C. of Hole 1 from emb. = ·245	; from exit	= ·013
Δ at exit and emb.	·010;	C. of Hole 2 from emb. = ·219	; from C. of Hole 1	= ·026
δ ·004 except for Hole 3	·005;	C. of Hole 3 from emb. = ·1905	; from C. of Hole 2	= ·0295
		C. of Hole 4 from emb. = ·157	; from C. of Hole 3	= ·0335
		C. of Hole 5 from emb. = ·126	; from C. of Hole 4	= ·031
		C. of Hole 6 from emb. = ·096	; from C. of Hole 5	= ·030
				11)·1630
				·0148

Half Increment of Distance (mean) ·0148
useful ·015

The 1st distance from exit is obviously of a half-segment.

THE MOUTHPIECES

D-R. Mp. xxxi '*C. 1*'. Wheat straw, untreated, golden; L. ·128; Δ ·004; Mp. Ext. ·075; V.L. ·060; proper normal note *F* 17.

D-R. Mp. xxxi '*C. 2*'. Wheat straw, untreated (dark); L. ·126; Δ ·004; Mp. Ext. ·075; V.L. ·060; proper note *F* 17.

D-R. Mp. xxxi '*F.*'. L. ·118; Δ ·005; Mp. Ext. ·075; V.L. ·059.

D-R. Mp. xxxi '*B. 12*' (untreated). L. ·117; Δ ·0045; Mp. Ext. ·102.

PERFORMANCE

With *D-R. Mp. xxxi* '*C. 1*'. Played in Aulos in the Dorian Harmonia on *C*. = 256 v.p.s. at Mp. Ext. ·075 in tune (but without enthusiasm); last note difficult.

Modal Sequence

Fingerholes	Exit	H. 1	2	3	4	5	6
Ratios and Notes	22/22	21	20	18	16	15	14
Cents		*80·5*	*85*	*182*	*204*	*112*	*119·4*

N.B.—The note of ratio 21, played from Hole 1, is the half-segment note required for the Enh-Chr. Pyknon.; from that hole to Hole 4, the ratios are double those of M.D. 11; Ratio 16 heralds a new octave of genesis, and the arithmetical succession is by *one*. This is a basic principle in these modal sequences, viz. **a note interpolated from a half-segment demands the doubling of the M.D. as far as the Mese, which always introduces a new octave of genesis.**

With *D-R. Mp.* '*xxxi C. 2*'. The Aulos played in the Dorian Harmonia on *C* = 256 v.p.s. at Mp. Ext. ·075 and V.L. ·060 as above, in tune more easily than with Mp. C. 1. With a little forcing and glottis tension, the interval from Holes 5–6 could be obtained as 15/13 (*247 cents*), instead of 15/14.

With D-R. Mp. '*xxxi F.*'. The Aulos played the sequence in the Dorian Harmonia in tune with monochord, but on $\frac{F\,16}{128}$ the last hole a little difficult at first with ratio 14.

With *D-R. Mp.* '*xxxi B.* 12' at Mp. Ext. ·102. The Aulos played on $\frac{B\,12}{128}$ in the Phrygian Harmonia easily in tune, the last note of ratio 15 a little difficult.

The Phrygian Modal Sequence

Fingerholes	Exit	1	2	3	4	5	6
Ratios and Notes	24/24	23	22	20	18	16	15
Cents		*74*	*77*	*165*	*182*	*204*	*112*

It must not be assumed that the addition of an I.D. to the Mp. Ext. invariably has the result of lowering the fundamental (e.g. cf. Loret xxiii). The tonality of that note will be found to be the affair of the mp. in its proportional relations of resonance with the Resonator of the Aulos.

LORET XXXII

Louvre, No. 1714; C. 22, No. 63. 6 Fingerholes. Polished wood

Facsimile made by K. S. Feb., 1934

N.B.—Auloi xxxi and xxxii are companions

This Aulos plays from exit the 1st tetrachord of the Dorian Harmonia, M.D. 22, with the Chrom-Enh. Pyknon. and the Phrygian Modal Sequence M.D. 24 at Mp. Ext. ·100.

MEASUREMENTS		FINGERHOLES		I.D.
R.L. from exit to emb.	·260 ;	C. of Hole 1 from emb. = ·242 ;	from exit	= ·018
R.L. from C. of Hole 1 to emb.	·242 ;	C. of Hole 2 from emb. = ·215 ;	from C. of Hole 1	= ·027
Δ	·010 ;	C. of Hole 3 from emb. = ·185 ;	from C. of Hole 2	= ·030
δ	·004 constant ;	C. of Hole 4 from emb. = ·152 ;	from C. of Hole 3	= ·033
		C. of Hole 5 from emb. = ·122 ;	from C. of Hole 4	= ·030
		C. of Hole 6 from emb. = ·098 ;	from C. of Hole 5	= ·024
				11)·162 × 2
				= ·0294

The 1st I.D. is of a half-segment: $\cdot162 \times 2 = \frac{\cdot324}{11} = \cdot0294$.

Increment of Distance (mean) ·0294, useful ·029

Since the distance from exit to Hole 1 is obviously a half-segment, it is necessary to double the I.D., now 22.

$$\cdot260 + \cdot075 = \cdot335 \text{ and } \cdot029 \times 11 = \cdot319 \qquad \begin{array}{r} \cdot335 \\ -\ \cdot319 \\ \hline \cdot016 \end{array}$$

A discrepancy of over a half-increment

The Aulos nevertheless plays in tune. A different mp. at extrusion ·060 would conform with theory, and would probably yield still better results.

THE MOUTHPIECES

D-R. Mp. ‘*xxxii D.*’ (untreated). L. ·133 ; Δ ·006 ; split for a few mm. on one side of emb., which proves to be of no importance. V.L. ·060 ; Mp. Ext. ·075.

D-R. Mp. ‘*xxxii E. 19*’ (untreated). L. ·140 ; Δ ·006 ; V.L. = ·060. Mp. Ext. ·075 ; normal proper note *F* 17.

D-R. Mp. ‘*xxxii C. 11*’ (untreated). L. ·132 ; Δ ·006 ; at V.L. ·060 ; Mp. Ext. ·075 ; normal proper note *F* 17.

PERFORMANCE

With *D-R. Mp.* ‘*xxxii D.*’. At Mp. Ext. ·075 the Aulos played on $\frac{D\ 20}{128}$ in the Dorian Harmonia with Enh.-Chrom. Pyknon. in tune ; the notes from Holes 5 and 6 good and clear.

The Modal Sequence

Fingerholes	Exit	1	2	3	4	5	6
Ratios and Notes	22/22	21	20	18	16	15	14
Cents		*80·5*	*85*	*182*	*204*	*112*	*119·4*

With *D-R. Mp.* ‘*xxxii E. 19*’, the Aulos played on $\frac{E\ 19}{128}$, the Dorian Sequence as above ; all notes clear and in tune.

With *D-R. Mp.* ‘*xxxii C. 11*’, the Aulos played on $\frac{C\ 11}{128}$ at ·075 Mp. Ext., the Dorian Sequence as above in tune, and the top notes good and clear.

LORET XXXV

British Museum. 4 Fingerholes

Facsimile by K. S.

This Aulos plays from exit in the Dorian Harmonia M.D. 11 ; in the Phrygian M.D. 12 and in the Hypolydian M.D. 10.

MEASUREMENTS	FINGERHOLES	I.D.
L. from exit to emb. ·222 ;	C. of Hole 1 from emb. = ·1795 ; from exit	= ·0425
Δ at emb. ·003 ; at exit ·005 ;	C. of Hole 2 from emb. = ·1485 ; from C. of Hole 1	= ·031
δ ·003 constant ;	C. of Hole 3 from emb. = ·1195 ; from C. of Hole 2	= ·029
	C. of Hole 4 from emb. = ·0935 ; from C. of Hole 3	= ·026
		4)·1285
		·0321

Increment of distance (mean) ·0321
practical ·029 for 3, or ·028

I.D. ·028 × 11 = ·308 − ·222 = Mp. Ext. ·086 for Dorian.
I.D. ·028 × 12 = ·336 − ·222 = 114 Mp. Ext. for Phrygian
I.D. ·029 × 11 = ·319 − ·222 = Mp. Ext. ·097 for Dorian.
I.D. ·029 × 12 = ·348 − ·222 = Mp. Ext. = ·126 for Phrygian.
I.D. ·029 × 10 = ·290 − ·222 = Mp. Ext. = ·068 for Hypolydian.

The tiny bore of only ·003 in this Aulos creates difficulties for D-R. mp.s.

THE MOUTHPIECES

D-R. mp.s have been tested at the requisite extrusion for means ·029 and ·028 ; it has been found that individual mp.s respond with good results at either one or other of the Mean Increments of Distance and Mp. Ext.

D-R. ‘*xxxv 10*’. L. = ·083 ; Δ exit ·003, emb. ·0035 ; V.L. ·055 ; Mp. Ext. ·068.

D-R. Mp. ‘*xxxv A.*’. L. ·122 ; Δ ·003 emb. and exit.

D-R. Mp. ‘*xxxv A.A.*’. New ; L. ·130 ; Δ ·003.

D-R. Mp. ‘*xxxv A. 12*’. L. ·130 ; Δ ·003.

D-R. Mp. ‘*xxxv 11*’. L. ·105 ; Δ ·003.

B-R. Mp. ‘*xxxv B.*’. L. ·154 ; T.L. ·038 ; T.W. ·001 ; Δ = ·003.

B-R. Mp. ‘*xxxv A.*’. L. ·134 ; T.L. ·033 ; T.W. ·0015 ; Δ ·0025. Proper note norm. $\frac{A\ 27}{64}$.

B-R. ‘*xxxv F.*’. L. ·128 ; Δ ·003 ; T.L. ·031 ; T.W. ·001 ; proper note norm. $\frac{E\ 18}{64}$.

PERFORMANCE

With *D-R. Mp.* ‘*xxxv 10*’. The Aulos plays at Mp. Ext. ·068 in the Hypolydian Harmonia M.D. 10 on C = 256 v.p.s., the sequence from exit.

Hypolydian Harmonia

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	10/10	9	8	7	6
Cents		*182*	*204*	*231*	*267*

D-R. Mp. ‘*xxxv A.*’ plays in a 2nd facsimile ‘xxxv B.’ from exit at Mp. Ext. ·114 in the Phrygian Harmonia on $\frac{B\ 12}{128}$.

With *D-R. Mp.* ‘*xxxv A.A.*’ The Aulos ‘xxxv A.’ plays from exit on $\frac{B\ 12}{128}$ in the Phrygian Harmonia the sequence :

Phrygian Sequence

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	12/12	11	10	9	8
Cents		*151*	*165*	*182*	*204*

D-R. Mp. ‘*xxxv A. 12*’ plays in facsimile B. with Mp. Ext. ·114 in the Phrygian Harmonia on G 15 : the practical I.D. is here ·028.

With *D-R. Mp.* '*xxxv* II' the Aulos plays in the Dorian Spondaic at Mp. Ext. ·097 on $\frac{C\ 11}{128}$.

PERFORMANCE WITH B.R. MP.S

B-R. Mp. '*xxxv B.*', plays in the Aulos in the Dorian Spondaic M.D. 11 from exit on $\frac{C\ 11}{128}$ at Mp. Ext. ·086 (I.D. = ·028) the full modal sequence.

B-R. Mp. '*xxxv A.*' plays in Aulos B in the Dorian Spondaic on $\frac{F\ 16}{64}$ at Mp. Ext. ·086 in full, rich tone. No difficulty arises from the positive excess in the 1st I.D. (i.e. of ·042. See remark above under Record Elgin Aulos, 1).

B-R. Mp. '*xxxv F.*' plays in Aulos B, the Dorian Spondaic M.D. 11 at Mp. Ext. ·086 on $\frac{F\ 16}{64}$ with a fine, rich tone of great sonority, all notes in tune.

The Dorian Spondaic, M.D. 11

Fingerholes	Exit	H. 1	2	3	4	4
Ratios and Notes	11/11	10	9	8	7	6 with high glottis action from Hole 4
Cents		*165*	*182*	*204*	*231*	*267*

Tested again June 29, 1934.

LORET XXXVI[1]

Berlin Kgl. Mus. No. 6823. 4 Holes. Date ascribed to reign Rameses II
Facsimile by K. S. Feb., 1934

Harmonia Dorian M.D. 11 on $\frac{C\ 11}{128}$ from Exit.

MEASUREMENTS.

Length of Aulos resonator exit to emb.	·214 ;
Δ of bore at emb.	·005 ;
Δ of bore at exit	·0045 ;

FINGERHOLES I.D.
(The distances are from emb. to centre of Holes)

C. of Hole 1 from emb. = ·1695 ;	from exit	= ·0445
C. of Hole 2 from emb. = ·1425 ;	from Hole 1	= ·027
C. of Hole 3 from emb. = ·1115 ;	from Hole 2	= ·031
C. of Hole 4 from emb. = ·0835 ;	from Hole 3	= ·028
		3)·086
		·0286

Increment of distance (mean) = ·0286
Useful I.D. ·029

D-R. Mp. xxxvi. Wheat straw untreated ; L. ·214 ; Δ at emb. ·005 ; at exit ·004 ; V.L. at ·060 ; Mp. Ext. at ·105.

In Aulos Resonator	·214	·319	= 11 M.D.
Mp. Ext.	+ ·105	·029	I.D.
Total Length	·319		

Proper note of Mp. at V.L. ·060 = *F* 17 proportional resonance with Resonator + Mp. in the ratios of 4 : 3.

[1] This Aulos was discovered at Thebes in a sarcophagus lying at the side of the mummy, by M. Passalacqua ; it bore the number 565 in his collection. See *Catalogue raisonné et hist. des Antiquités découvertes en Egypte* (Paris, 1826), pp. 30 and 157, acquired later by Berlin. See *Kgl. Museen zu Berlin*, Ausführliches Verzeichniss d. Aegyptischen Altertümer, 2nd ed. (Berlin, 1899), pp. 190 and 219, No. 6823.

PERFORMANCE

With *D-R. Mp. xxxvi* on $\frac{C\,11}{128}$ v.p.s. in Aulos at Mp. Ext. ·105 played in the Dorian Harmonia M.D. 11 from exit on $\frac{C\,11}{128}$ in tune with modal piano. Tests Feb. 28, March 1 and 2, 1934.

The Modal Sequence

Fingerholes	Exit	H. 1	2	3	4
Ratios and Notes	11/11	10	9	$\boxed{8}$ Mese	7
Cents		*165*	*182*	*204*	*231*

LIST OF FLUTES (in order)

Classification of Modal Flutes.
Balance between Proportional and Effective Allowance at Hole 1.
Notes concerning the Factor of Length in Pipes, Flutes and Mouthpieces.
'Sensa A'. Hypophrygian Harmonia M.D. 18.
'Sensa B'. Hypophrygian Harmonia M.D. 18.
'Sensa C'. Hypophrygian Harmonia M.D. 18.
Flute No. 1. Graeco Roman (Bucheum). Dorian Harmonia M.D. 11.
The Mond Sicilian Flute No. 2. Phrygian Harmonia (Exit), Dorian Harmonia (Vent).
Carpathian Flute No. 3. Dorian Harmonia M.D. 11.
Java i 'Soeling' No. 5. Dorian Harmonia (Exit).
Java ii 'Soeling' No. 6. Dorian Harmonia (Vent).
Flute No. 10 (India). Hypodorian Harmonia (Vent) M.D. 16.
Inca Flute No. 12. Dorian Harmonia (Vent).
Japanese Flute No. 13. Lydian Harmonia M.D. 13 (Vent).
Bali Flute No. 20. Dorian Harmonia (Exit).
Greek Flute No. 26 from Olympia (Modern). Phrygian Harmonia (Vent).
Greek Flute No. 27 from Nauplia (Modern). Dorian Harmonia M.D. 11 from Vent.
Dr. A. N. Tucker's 8 Flutes from the Sudan (Acholi Tribe, with measurements and performance tabulated).
Eleven Flutes from N. Egypt, presented by Sir Robert Mond, with measurements and performance tabulated.

PRELIMINARY NOTES

THE CLASSIFICATION OF MODAL FLUTES : ABSTRACT FROM CHAP. VI

CLASS Ia and CLASS Ib. Flutes in which the length from exit or vent is a multiple (within a few mm.) of the Increment of Distance, a distinguishes flutes having a normal, uninterrupted sequence, b those in which the modal sequence—interrupted by the operation of Inc. All. No. 7—plays an interpolated note due at a half-increment lower.

CLASS IIa and b. Flutes in which the actual length from embouchure to exit or vent, with the addition of the allowance in respect of diameter due at exit (All. No. 5) or at centre of Hole 1 (No. 3), is a multiple of the I.D. × the M.D. Flutes of this class are free from interruptions in the Modal Sequence.

CLASS IIIa and b. Flutes in which the actual length of the Flute less Allowance No. 3 or 5 is equal to the I.D. × M.D. ; or conversely, in which the I.D.

× M.D. + Allowance is equal to the length of the flute at exit or vent. Examples of this class are rare (see Nos. 1 and 10).

THE SUB-DIVISIONS A AND B IN EACH CLASS are determined by the Inc. All. No. 7.

(A) This cumulative allowance—actually on the flute between exit and c. of Hole 1 when it has not reached a nodal point allows the sequence to proceed uninterrupted.

(B) When the accumulated allowance No. 7 has reached a nodal point, the note issuing from the hole is that due at a half-increment lower, as a consequence of the allowance having accrued as additional length to the effective half-wave.

THE BALANCE BETWEEN PROPORTIONAL ALLOWANCE BY RATIO FROM THE STANDARD ALLOWANCE AT HOLE 1, AND THE ALLOWANCE DEDUCED FROM THE V.F. OF THE NOTE IS EFFECTED AS FOLLOWS:

(1) In flutes belonging to Class IIA, in which the I.D. bears its share of Allowance, the excess of Actual I.D. over Prop. I.D. must be added per I.D. and per hole to balance the Proportional Allowance (e.g. in ‘ Sensa A ’ and in Japanese Flute ‘ No. 13 ’).

(2) The Inc. All. No. 7 should be carried forward; increment by increment and watched: *latent* indicates that the accumulated amount has not reached a nodal point; the sequence is therefore unbroken.

(3) The Floating Allowance = the effective half-wave I.D. less the actual I.D.; it represents the amount of allowance available on the half-wave length, at the pitch in question *per increment*. In flutes of class IIA, in which the actual I.D. bears an allowance, the excess of the effective ½ w. I.D. over the actual—i.e. *the Floating Allowance*—forms a reserve.

(4) The effective half-wave I.D. No. 9 represents a length additional to the actual, which must be effectively borne whenever a sound is elicited; it must be taken into account as a constituent of the sound-wave.

NOTES CONCERNING THE FACTOR OF LENGTH IN PIPES, FLUTES AND MOUTHPIECES

Length as a factor in wind instruments, more especially in flutes and pipes, may be considered from two aspects: absolute and proportional.

Length, as an attribute resulting from pitch, is fixed in the flute in respect of any given v.f. and has here been termed *effective* in contradistinction to *actual* visible and measurable length.

In the I.D., length is considered in both aspects. The influence of length in the concrete, e.g. in the tube, or in the diameter of the flute, takes effect in lowering the pitch. In the I.D. it is the reverse: the larger the I.D. between fingerholes, the nearer to embouchure is any given hole, and therefore the shorter the wave length. But on the other hand, since all the increments are of the same length, and yet bear different ratios; further, since ratio determines pitch, and pitch results in wave length, it is patent that the casting vote is with *proportion*. Hence the curious anomaly that although at every fingerhole, there is an actual position or length on the surface of the flute, measured from exit or embouchure, which should, according to formula, produce a note of definite pitch, yet it frequently happens that the modal sequence is interrupted at that point by the intrusion of a note belonging by position to a half or whole increment lower. (See, for instance, flute ‘ Sensa B ’, at Hole 4.) What is the cause of this abnormal lengthening of the sound-wave at what I have called the *virtual* position of the fingerhole? The length here seems to be determined, not so much by millimetres, as by the ratio between two pitch notes: the measure of the increment counts for nothing. Thus the curious fact emerges that on two flutes of the same length and diameter of bore, embouchure and fingerholes, playing in the same Harmonia, at the same fundamental pitch, but differentiated by their I.D., it will be found that notes bearing the same ratio number in that Harmonia, consequently of the same v.f. and therefore of the same wave length, are issuing from fingerholes centred at points on the surface of the flute, which may differ by as much as 20 or 30 millimetres. An example of this is exhibited by the three 'Sensa' flutes A, B and C. The modal sequence in 'B' and 'C' proceeds for a few steps without perceptible change in intonation; then, suddenly, in 'B', at the 4th hole, and in 'C' at the 6th, comes a surprise: a note of different intonation sounds, not out of tune, but of a definite intermediate ratio, due at one half increment lower, between two consecutive fingerholes (e.g. in 'B' the note between ratio numbers 14 and 13, viz. of ratio 27). The reason for this interruption in the modal sequence seems to be connected with the correct placing of Hole 1, at a distance from exit which includes two increments of distance representing the first step in the Harmonia (i.e. 18 17 16), and besides this the whole amount of the diameter allowances at exit in concrete values. Thus, when this has been correctly computed (Formula No. 3), and carried out on the flute, and then only, may the position of the rest of the fingerholes be correctly bored to produce the modal sequence in integrity. Hole 1 has thus been made to bear the burden of the whole Allowance at exit—a burden which has a lengthening influence—and this has been paradoxically effected by driving the centre of Hole 1 nearer to the embouchure by the amount of this allowance. The paradox is only apparent since the physical laws, based upon the nature of diameter, decree a proportional elongation of the wave length. Compensation must be provided by decreasing the distance of the centre of Hole 1 from embouchure by the same proportional length in order to maintain the exact ratio between the notes of exit and vent. It is evident that not only Hole 1, but also the other fingerholes in succession, all bear their share, increment by increment, in this elongating and compensating process. The wave length, determined by Formula No. 2 being fixed and unalterable for each v.f., and the actual distance of the centre of each fingerhole from embouchure measurable, it follows that the allowance for diameter thus displayed on the flute itself is likewise an indisputable fact. It seems worth while in the interests of modality on the flute, to endeavour to trace the proportional allotment of the allowance per increment. The v.f., together with the implications of length mentioned above, progresses by superparticular ratios. A proportional amount—according to the same ratios—based upon the allowance accruing at Hole 1, should therefore correspond with the allowance comprised in the effective wave length, ascertained by Formula No. 2. In comparing these two amounts, it is found that the Proportional Allowance in flutes belonging to Class II, in which Hole 1 is correctly placed, shows a discrepancy that is cumulative by increment, but which is covered by what I have termed 'Incremental Allowance No. 7', added for each increment of distance at the fingerhole in question. This allotment may be followed in operation in records of flutes, hole by hole, and its results traced as (*a*) normal or uninterrupted, or (*b*) a broken modal sequence.

MODAL FLUTE RECORDS. No. 18

SENSA A

Class IIA

Made by Kathleen Schlesinger in 1918 (red vulcanite)

Hypophrygian Harmonia

Modal Determinant 18

$$on\ \frac{E\,18}{256} = 312{\cdot}8\ v.p.s.$$

Modal Sequence

Holes	Exit	1	2	3	4	5	6	7
Ratios	$\frac{18}{18}$	$\frac{16}{18}$	$\frac{15}{18}$	$\frac{14}{18}$	$\frac{13}{18}$	$\frac{12}{18}$	$\frac{11}{18}$	$\frac{10}{18}$
Cents.		*204*	*112*	*119·4*	*128*	*138·5*	*151*	*165*

All in perfect tune.

N.B.

In flutes belonging to Class IIA, in which the I.D. is not an exact aliquot of the actual length, but of actual length + allowance for diameter, the following adjustment has to be made in balancing the allowance per fingerhole (derived from the pitch-equivalent length of half-sound-wave) with the proportional allowance computed by ratio of the note from the standard (= Std.) allowance at Hole 1.

The actual I.D. between the fingerholes on the flute, in Class IIA, is in excess of the proportional I.D., which is an exact aliquot of the length of the flute; the actual I.D. thus carries its proportion of the allowance for diameter, whereas the proportional I.D. bears no allowance.

* * *

To the proportional allowance must be added the difference between the actual I.D. and the proportional I.D., viz., in Sensa A ·028 − ·0234 = ·0046, cumulatively per fingerhole.

MEASUREMENTS		FINGERHOLES AND I.D.	
L.C. of emb. to exit	·465 ;	H. 1 fr. emb. = ·375 ;	H. 1 to exit = ·090
L.C. ,, ,, to c. H. 1	·375 ;	H. 2 ,, ,, = ·347 ;	H. 1 to H. 2 = ·0275
L.C. H. 1 to exit	·090 ;	H. 3 ,, ,, = ·319 ;	H. 2 to H. 3 = ·028
Δ of bore	·023 ;	H. 4 ,, ,, = ·291 ;	H. 3 to H. 4 = ·028
d = Δ of emb.	·010 ;	H. 5 ,, ,, = ·263 ;	H. 4 to H. 5 = ·028
δ = Δ of fingerhole	·009 ;	H. 6 ,, ,, = ·235 ;	H. 5 to H. 6 = ·028
de = depth of walls	·003 ;	H. 7 ,, ,, = ·207 ;	H. 6 to H. 7 = ·028
I.D. =	·028 ;		

Increment of Distance ·028.

Modal Determinant, 18 ; Hypophrygian Harmonia.

Note of Exit (all holes closed) $\frac{E\ 18}{256} = 312{\cdot}8$ v.p.s.

THE THREE I.D.[1] IN SENSA A, NO. 18

No. 1(A) = ·028 actual I.D. between fingerholes

No. 2 = ·0234 i.e. $\left(\frac{{\cdot}375}{16} = {\cdot}0234\right)$ Prop. I.D.

No. 3 = ·0301 i.e. $\left(\frac{{\cdot}543}{18} = {\cdot}0301\right)$ Eff. ½ w.l. I.D.

Incremental All. No. 7 $= \frac{\text{Exit All.}}{18} = \frac{{\cdot}039}{18} = {\cdot}002166$

[1] THE THREE KINDS OF I.D. (GENERAL NOTES) TO BE FOUND IN MODAL FLUTES

No. 1. The actual I.D. on the flute may be 'A' intermediate in length between No. 2 the Prop. I.D., and No. 3, the Eff. ½ w.l. I.D.

No. 1(A) carries its own allowance. No. 1(B) is less than No. 2, and No. 1(C) is equal to No. 2.

No. 2. The Proportional I.D. $= \frac{\text{Exit L.}}{\text{M.D.}}$

or

When Hole 1 has been correctly placed $\frac{\text{Vent L.}}{\text{M.D.}}$

No. 3. The Eff. ½ w.l. I.D. derived from the Eff. ½ w.l. of the fundamental note of exit

$$\frac{\text{Eff. } \tfrac{1}{2} \text{ w.l.}}{\text{M.D.}}$$

Incremental Allow. No. 7 (cumulative)

$$= \frac{\text{Exit. All.}}{\text{M.D.}} = \text{Inc. All. No. 7.}$$

The '*floating allowance*' is the difference between the Eff. ½ w.l. I.D. and the Prop. I.D. or the actual I.D. (whichever is the lesser)

$$\text{i.e. } \begin{array}{r} \cdot 0301 \\ -\ \cdot 0234 \\ \hline \cdot 0067 \end{array} \textit{ Floating Allowance}$$

The floating allow. = (Fl. All.) multiplied by the Ratio number of Hole 1 should be equal (within a mm. or two) to the standard allowance at Hole 1, i.e.

$$\cdot 0067 \times 16 = \cdot 1072$$

$$\text{Standard All.} = \cdot 1079 \text{ difference } \frac{7}{10} \text{ of a mm.}$$

The Fl. All. thus represents the difference between the Eff. ½ w. I.D. and the actual I.D., i.e. ·0301 − ·028 = ·0021 + the difference between the actual I.D. and the Prop. I.D., i.e. ·028 − ·0234 = ·0046 thus: ·0021 + ·0046 = ·0067 Fl. All.

EXIT

L. = ·465 Note $\frac{E\,18}{256}$ = 312·8 v.p.s.

Modal Ratio 18/18

BY PITCH FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{312\cdot 8 \text{ v.p.s.} \times 2} = \frac{\cdot 340}{625\cdot 6 \text{ v.p.s.}} \begin{array}{rl} = \cdot 5434 & \text{Eff. } \tfrac{1}{2} \text{ w.l. of } E\,18 \\ -\ \cdot 465 & \text{L. of flute} \\ \hline \cdot 0784 & \text{All. for } \Delta \end{array}$$

ANALYSIS OF DIAMETER ALLOWANCE AT EXIT—FORMULA NO. 5

$$\begin{array}{rl} \cdot 046 & = 2\Delta \\ \cdot 026 & = 2(\Delta - d) \\ \cdot 006 & = 2\,de \\ \hline \cdot 078 & \text{theoretical Eff. All. for } \Delta. \end{array}$$

Thus the All. derived from pitch equivalent half-wave-length is in agreement with the theoretical All. based on the actual Δ of the flute.

Effective I.D. derived from ½ w.l.

$$\frac{\cdot 543}{18} = \cdot 0301 = \text{Eff. I.D.}$$

POSITION OF HOLE 1 BY FORMULA NO. 3

$$\begin{array}{ll} \frac{\Delta}{2} = & \cdot 0115 \\ \Delta - d & \cdot 013 \\ \Delta - \delta & \cdot 014 \\ \text{I.D.} \times 2 & \cdot 056 \\ \hline & \cdot 0945 \text{ Position of H. 1.} \end{array}$$

Hole 1 is actually at ·090 in this flute; since the I.D. carries its All., it is clear that the Incr. All. No. 7 (q.v.) for the two I.D. included in the computation must be deducted: therefore Exit All. $\frac{\cdot 039}{18}$ = .00216 × 2 = ·0043, and

$$\begin{array}{rl} \cdot 0945 & \text{Pos. of H. 1} \\ -\ \cdot 0043 & \\ \hline \cdot 0902 & \text{correct actual Pos of H. 1} \end{array} \qquad \begin{array}{lrl} \text{L. of Flute} & = \cdot 465 & \\ \text{H. 1 at} & -\ \cdot 090 & \text{fr. exit} \\ \hline \text{H. 1 at} & \cdot 375 & \text{fr. emb.} \end{array}$$

Hole 1 at ·375, Ratio $\frac{18}{16}$ (9/8), Note $\frac{F\,16}{256}$ = 352 v.p.s.

$$\frac{340 \text{ m./s.}}{352 \text{ v.p.s.} \times 2} = \frac{340}{704 \text{ v.p.s.}} \begin{array}{rl} = \cdot 4829 & \text{L. Eff. } \tfrac{1}{2} \text{ w.} \\ -\ \cdot 375 & \text{L. at H. 1} \\ \hline \cdot 1079 & (= \cdot 108) \end{array}$$

Analysis of All. fr. Δ by Formula No. 4

2Δ	$= \cdot 046$	(or with depth, $+ \cdot 003 = \cdot 049$
$2(\Delta - d) + de$	$\cdot 029$	Stand. All. $= \cdot 109$)
$2(\Delta - \delta) + de$	$\cdot 031$	
	$\cdot 106$	

Stand. All. fr. pitch $= \cdot 108 - \cdot 106 =$ diff. $\cdot 002$

Inc. All. No. 7 = ·00216 latent

The standard allowance from pitch equivalent L. is used for testing the proportional allowance, according to the ratio of the increment of distance for each successive hole : should this proportional allowance not agree with the allowance derived from pitch, adjustment will be indicated to balance the two.

Hole 2 at ·347, Ratio $\frac{16}{15}$; Note $\frac{G\ 15}{256} = 375{\cdot}5$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{375{\cdot}5 \text{ v.p.s.} \times 2} = \frac{340}{751 \text{ v.p.s.}} = \cdot 4527 \text{ Eff. } \tfrac{1}{2} \text{ w.l.}$$
$$- \cdot 347 \text{ Act. L. H. } 2$$
$$\cdot 1057 \text{ Stand. Vt. All.}$$

Prop. All. tested from Stand. All. at Hole 1, to which must be added the difference between the Actual I.D. ·028 and the Prop. I.D. ·0234 = ·0046.

Prop. All.

$$\frac{\cdot 108 \times 15}{16} = \cdot 1012$$
$$+ \cdot 0046 \text{ Excess of Act. over Prop. I.D.}$$
$$\cdot 1058 \text{ Agreement within one-tenth mm.}$$

Inc. All. No. 7 = ·00216 latent

Hole 3 at ·319, Ratio $\frac{16}{14}$ (8/7) ; Note $\frac{G\ 14}{256} = 402{\cdot}2$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 402{\cdot}2 \times 2} = \frac{340}{804{\cdot}4 \text{ v.p.s.}} = \cdot 4226 \text{ Eff. } \tfrac{1}{2} \text{ w.l.}$$
$$- \cdot 319 \text{ Act. L. to H. } 3$$
$$\cdot 1036 \text{ Std. All. Vt.}$$

Prop. All.

$$\frac{\cdot 108 \times 7}{8} = \cdot 0945$$
$$+ \cdot 0092 \text{ Excess of Act. over Pr. I.D.} \times 2$$
$$\cdot 1037 \text{ Agreement within one tenth mm.}$$

Inc. All. No. 7 = ·00216 × 2 = ·00432 latent

Hole 4 at ·291, Ratio $\frac{16}{13}$; Note $\frac{A\ 13}{256} = 433{\cdot}2$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 433{\cdot}2 \times 2} = \frac{340}{866{\cdot}4 \text{ v.p.s.}} = \cdot 3924 \text{ Eff. } \tfrac{1}{2} \text{ w.l.}$$
$$- \cdot 291 \text{ Act. L. at H. } 4$$
$$\cdot 1014 \text{ Std. Vt. All. at H. } 4$$

Prop. All.

$$\frac{\cdot 108 \times 13}{16} = \begin{array}{rl} \cdot 0877 & \text{Prop. All.} \\ + \underline{\cdot 0138} & \text{Excess of Act. over Pr. I.D.} \times 3 \\ \underline{\underline{\cdot 1015}} & \text{Agreement within one-tenth mm.} \end{array}$$

Inc. All. No. 7 = ·00216 × 3 = ·00648 latent

Hole 5 at ·263, Ratio $\frac{16}{12}$; Note $\frac{B\ 12}{256}$ = 469·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 469\cdot 2 \times 2} = \frac{340}{938\cdot 4 \text{ v.p.s.}} \begin{array}{rl} = \cdot 3623 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 263} & \text{Act. L. at H. 5} \\ \underline{\underline{\cdot 0993}} & \text{Std. All. at H. 5} \end{array}$$

Prop. All.

$$\frac{\cdot 108 \times 3}{4} = \begin{array}{rl} \cdot 081 & \text{Prop. All.} \\ + \underline{\cdot 0184} & \text{Excess of Act. over Pr. I.D.} \times 4 \\ \underline{\underline{\cdot 0994}} & \text{Agreement within one tenth mm.} \end{array}$$

Inc. All. No. 7 = ·00216 × 4 = ·00864 latent

Hole 6 at ·235, Ratio $\frac{16}{11}$; Note C 11 = 512 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 512 \times 2} = \frac{340}{1024} = \begin{array}{rl} \cdot 332 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 235} & \text{Act. L. at H. 6} \\ \underline{\underline{\cdot 097}} & \text{Std. All. at H. 6} \end{array}$$

Prop. All.

$$\frac{108 \times 11}{16} = \begin{array}{rl} \cdot 074 & \text{Prop. All.} \\ + \underline{\cdot 023} & \text{Excess of Act. over Pr. I.D.} \times 5 \\ \underline{\underline{\cdot 097}} & \text{Agreement} \end{array}$$

Inc. All. No. 7 = ·00216 × 5 = ·01080 latent

Hole 7 at ·207, Ratio $\frac{16}{10}$; Note $\frac{D\ 10}{512}$ = 563·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 563\cdot 2 \times 2} = \frac{340}{1126\cdot 4 \text{ v.p.s.}} \begin{array}{rl} = \cdot 3018 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 207} & \text{Act. L. at H. 7} \\ \underline{\underline{\cdot 0948}} & \text{Std. All. at H. 7} \end{array}$$

Prop. All.

$$\frac{108 \times 5}{8} = \begin{array}{rl} \cdot 0675 & \text{Prop. All.} \\ + \underline{\cdot 0276} & \text{Excess of Act. over Pr. I.D.} \times 6 \\ \underline{\underline{\cdot 0951}} & \text{Agreement to three tenths of a mm.} \end{array}$$

Inc. All. No. 7 = ·00216 × 6 = ·01296 latent

N.B.—The Modal Sequence is uninterrupted, for the Inc. All. No. 7 has not cumulatively reached a Nodal point; it would do so at the next I.D. $= \frac{\cdot 028}{2} = \cdot 014$.

THE EFFECTIVE I.D. DERIVED FROM THE EFFECTIVE HALF-WAVE LENGTH, DIVIDED BY THE MODAL DETERMINANT

Fingerhole	Eff. ½ w.l.	Eff. I.D.
Exit	·543	·0301
Hole 1	·4829	·0601 (I.D. × 2)
Hole 2	·4527	·0302
Hole 3	·4226	·0301
Hole 4	·3924	·0302
Hole 5	·3623	·0301
Hole 6	·332	·0303
Hole 7	·3018	·0302

N.B.—The half-wave-length is obtained for each hole from the operation of Formula 2 so that the Eff. I.D. per hole forms independent evidence of accuracy in theory and practice.

MODAL FLUTE RECORDS.

SENSA B

9 Fingerholes Experimental Flute made by K. S.

Cf. with Sensa A and Sensa C

Class IIIB

Modal Determinant 18

Hypophrygian Harmonia from Exit. Hypodorian from Vent

Modal Sequence

Exit		Hole 1	2	3	4	5	6	7	8	9
Ratios	18	16	15	14	27	26 } 13	12	11 } 22	21	20

Of the three Sensa flutes, ' Sensa A ' is the most perfect specimen : B and C were designed in order to test the significance of the Increment of Distance (I.D.) between the fingerholes, since the I.D. is the only factor which differentiates the 3 flutes : Total length ; position of Hole 1, diameter of bore, of embouchure and of fingerholes and depth of walls ; notes of fundamental and vent, and Modal Determinants from exit and vent ; all are identical in Sensa A, B and C.

Their differentia is the effect of a different I.D. on the modal sequence.

In the Sensa A with I.D. ·028 the sequence is uninterrupted.

In Sensa B (with an I.D. of ·023 equal to the diameter) at Hole 4, a note due at half an increment lower, viz. of ratio 27, instead of 13 or 26, is interpolated in the modal sequence ; the note proper to Hole 4 is now played from Hole 5 ; after which, the sequence proceeds in order from that new basis.

In Sensa C (the actual I.D. and the Prop. I.D. are identical, viz. ·0235) the break in the modal sequence occurs at Hole 6, where ratio 11 is replaced by ratio 23, due half-way between Holes 5 and 6.

My explanation of the cause of the broken sequence—the progression of the cumulative Incremental Allowance No. 7 from hole to hole—is given in detail in Chapter vii and *passim* in these records.

MEASUREMENTS		FINGERHOLES AND I.D.			
L. fr. emb. to exit	·465 ;	H. 1 fr. emb.	·375 ;	H. 1 fr. exit	·090
L. ,, ,, to c. Hole 1	·375 ;	H. 2 ,, ,,	·352 ;	fr. c. H. 1	·023
Δ of bore	·023 ;	H. 3 ,, ,,	·329 ;	fr. c. H. 2	·023
d = Δ of emb.	·010 ;	H. 4 ,, ,,	·306 ;	fr. c. H. 3	·023
δ = Δ of fingerholes	·009 ;	H. 5 ,, ,,	·283 ;	fr. c. H. 4	·023
de = depth of walls	·0025 ;	H. 6 ,, ,,	·260 ;	fr. c. H. 5	·023
	·003 ;	H. 7 ,, ,,	·237 ;	fr. c. H. 6	·023
Exit to c. Hole 1	·090 ;	H. 8 ,, ½ I.D.	·2255 ;	fr. c. H. 7	·0115
I.D. = ·023 (·0005 less than the Prop. I.D.)		H. 9 ,, ½ I.D.	·214 ;	fr. c. H. 8	·0115

N.B.—Holes 8 and 9 are at a half-increment of distance.
I.D. = ·023 × M.D. 18 = ·414.

THE THREE INCREMENTS

1. The Actual I.D. = ·023
2. The Proportional I.D. = ·0235
3. The Effective I.D. = $\frac{\cdot543}{18}$ = ·03017 (= ·0302)

The Actual Allowance is a minus quantity = ·0005
The Floating Allowance = ·0067
  and ·0067 × 16 = ·1072
  Vent Standard All. = ·108
The reserve allowance is practically the same as the Fl. All. viz. = ·0072
Inc. All. No. 7 = ·00216

Exit Note $\frac{E\ 18}{256}$ = 312·8 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.p.s. } 312{\cdot}8 \times 2} = \frac{340}{625{\cdot}6} =$$

·543	Eff. ½ w.l.
− ·465	Act. length
·078	Eff. All. at exit

Exit Allowance by Formula No. 5

Δ = ·023
Δ − d = ·013
de = ·003
·039 Exit All.; in agreement with Effective Allowance at exit by Formula No. 2, viz. $\frac{.078}{2}$ = ·039.

Position of Hole 1 by Formula No. 3

$\frac{\Delta}{2}$	= ·0115
Δ − d	= ·0130
Δ − δ	·0140
Two I.D.	= ·046
	·0845
	+ ·00532
(= ·090)	·08982

For this flute having an I.D. ·0005 below the Prop. I.D., and therefore carrying no allowance, an addition must be made for the two I.D. latent between exit and H.I., viz.:

·00432 = Inc. All. No. 7 × 2
·001 negative excess of Act. below Prop. I.D. × 2
·00532

Standard Allowance at Vent by Formula No. 4

$$
\begin{aligned}
2(\Delta) \quad + de &= \cdot046 + \cdot0025 = \cdot0485 \\
2(\Delta - d) + de &= \cdot026 + \cdot0025 = \cdot0285 \\
2(\Delta - \delta) + de &= \cdot028 + \cdot0025 = \underline{\cdot0305} \\
& \qquad\qquad\qquad\quad \cdot1075 \text{ Vt. Std. All.}
\end{aligned}
$$

Hole 1 at ·375 fr. emb., Ratio $\frac{16}{16}$; Note, $\frac{F\,15}{256} = 352$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 352 \times 2} = \frac{340}{704} = \begin{array}{rl} \cdot4829 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot375} & \text{Act. L. at H. 1} \\ \cdot1079 & \text{Eff. All. at H. 1} \end{array}$$

(= ·108)

Hole 2 at ·352 fr. emb., Ratio $\frac{16}{15}$; Note $\frac{G\,15}{256} = 375{\cdot}4$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 375{\cdot}4 \times 2} = \frac{340}{750{\cdot}8} = \begin{array}{rl} \cdot4527 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot352} & \text{Act. L. at H. 2} \\ \underline{\underline{\cdot1007}} & \text{Eff. All. at H. 2} \end{array}$$

Prop. All.
fr. Stand. All.

$$\begin{array}{rl} \cdot1007 & \text{Eff. All. at H. 2} \\ + \underline{\cdot0005} & \text{excess[1] to balance.} \\ \underline{\underline{1012}} & \end{array}$$

$$\frac{\cdot108 \times 15}{16} = \cdot1012$$

Inc. All. No. 7 = ·00216 × 3 = ·00648 latent

Hole 3 at ·329 fr. emb., Ratio $\frac{16}{14}$ (8/7); Note $\frac{G\,14}{256} = 402{\cdot}2$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 402{\cdot}2 \times 2} = \frac{340}{804{\cdot}4} = \begin{array}{rl} \cdot4227 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot329} & \text{Act. L. at H. 3} \\ \cdot0937 & \text{Eff. All. at H. 3} \\ + \underline{\cdot001} & \text{Neg. excess} \times 2 \\ \underline{\underline{\cdot0947}} & \end{array}$$

Prop. All.
fr. Std. All.

$$\frac{\cdot108 \times 7}{8} = \cdot0945$$

Inc. All. No. 7 = ·00216 × 4 = ·00864 latent

Hole 4 at ·306 from emb.; Ratio $\frac{16}{13}$; Note $\frac{A\,27}{256} = 417{\cdot}1$ v.p.s.

Actual Ratio of note $\frac{32}{27}$ instead of $\frac{A\,13}{256}$.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 417{\cdot}1 \times 2} = \frac{340}{834{\cdot}2 \text{ v.p.s.}} = \begin{array}{rl} \cdot4075 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot306} & \text{Act. L. at H. 4} \\ \underline{\underline{\cdot1015}} & \text{Eff. All. at H. 4} \end{array}$$

[1] The nature of the excess has been explained at H. 1.

This allowance is, of course, not regular; the modal sequence has been interrupted by the interpolation of ratio 27, due at a Half I.D. lower. If the virtual position at ·306 + ·0115 $\left(\frac{\text{I.D.}}{2}\right)$ = ·3175 be substituted, the result is satisfactory.

Prop. All. fr. Std. All. and

$$\frac{\cdot108 \times 27}{32} = \underline{\underline{\cdot0911}} \longleftrightarrow \begin{array}{rl} \cdot4075 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot3175 & \text{Virtual L.} \\ \hline \cdot0900 & \text{All. at H. 4} \\ + \cdot0015 & \text{Neg. excess} \times 3 \\ \hline \cdot0915 & \end{array}$$

Inc. All. No. 7 = ·00216 × 5 = ·01080 active

N.B.—Hole 4 sounds the note due at half an increment lower, thus necessitating an adjustment in order to bring the allowance into agreement with the Proportional Allowance, for the reason that the I.D. on 'Sensa B', not only carries no allowance, but also falls short of the Prop. I.D. by ·0005 cumulative per I.D.

Thus the Inc. All. No. 7, cumulative for 5 I.D. (including the two between Exit and Hole 1, + what I have termed a negative excess), viz.:

$$\begin{array}{rrl} & \cdot0108 & \text{Inc. All. No. } 7 \times 5 \\ & + \cdot0015 & \text{neg. exc.} \times 3 \\ \hline \text{Aggregate Cumulative All.} = & \cdot0123 & \text{at Hole 4,} \end{array}$$

having reached the nodal point (·0115) at the half-increment, provides the requisite stimulus for the production of the note of ratio 27/32, due at the half-increment lower. It may be recalled that accumulated length by increment means receding from the embouchure, with the consequent lowering of pitch involved. The Proportional Allowance is reckoned from the Standard Allowance of the Vent (Formula No. 4 at Hole 1), whereas Inc. All. No. 7, is cumulative from exit and at Hole 4, therefore the I.D. is taken 3 times for the Proportional Allowance and 5 times for the Inc. All. No. 7.

Hole 5 at ·283 actual L. + ·023 virtual L. = ·306 ; Ratio $\frac{16}{13}$; Note $\frac{A\ 13}{256}$ = 433·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 433\cdot2 \times 2} = \frac{340}{866\cdot4 \text{ v.p.s.}} = \begin{array}{rl} \cdot3925 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot306 & \text{virtual L. at H. 5} \\ \hline \cdot0865 & \text{Eff. All. at H. 5 (due at H. 4)} \\ + \cdot0015 & \\ \hline \cdot088 & \text{Eff. All.} \end{array}$$

Prop. All.

$$\frac{\cdot108 \times 13}{16} = \underline{\underline{\cdot0875}} \ (= \cdot088)$$

Inc. All. No. 7 = ·00216 × 6 = ·01296 active

N.B.—As *A* 13 is the normal note at Hole 4, that distance (·306) must be taken as Virtual L. + the negative excess ·0005 × 3 (number of I.D. at Hole 4); this brings the two sources of allowance into agreement.

Hole 6 at ·260 (Virtual L. = ·283), Ratio $\frac{16}{12}\left(\frac{4}{3}\right)$; Note $\frac{B\ 12}{256}$ = 469·2 v.p.s.
(one I.D. lower)

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4} = \begin{array}{rl} {\cdot}3623 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}283 & \text{Virtual L. at Hole 6} \\ \hline {\cdot}0793 & \text{Eff. All. at H. 6} \\ +\ {\cdot}002 & \text{Neg. excess} \times 4. \\ \hline {\cdot}0813 & \end{array}$$

Prop. All.

$$\frac{{\cdot}108 \times 3}{4} = {\cdot}081$$

Inc. All. No. 7 = ·00216 × 7 = ·01512 active

Hole 7 at ·237 Act. L. (·260 Virtual L.), Ratio $\frac{16}{11}$; Note $C\ 11 = 512$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \times 2} = \frac{340}{1024 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}332 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}260 & \text{Virtual L. at H. 7} \\ \hline {\cdot}072 & \\ +\ {\cdot}0025 & \text{Neg. excess} \times 5 \\ \hline {\cdot}0745 & \end{array}$$

Prop. All.

$$\frac{{\cdot}108 \times 11}{16} = {\cdot}0742$$

Inc. All. No. 7 = ·00216 × 8 = ·01728 active

Hole 8 at ·2255 (Virtual L. ·2485) Ratio $\frac{32}{21}$; Note $\frac{C\ 21}{512} = 536{\cdot}4$ v.p.s.
(half I.D.)

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 536{\cdot}4 \times 2} = \frac{340}{1072 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3160 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}2485 & \text{Virtual L. at H. 8} \\ \hline {\cdot}0675 & \text{Eff. All. at H. 8} \\ +\ {\cdot}003 & \text{Neg. Excess} \times 6 \\ \hline {\cdot}0705 & \end{array}$$

Prop. All.

$$\frac{{\cdot}108 \times 21}{32} = {\cdot}0709$$

Inc. All. No. 7 = ·00216 × 9 = ·1944 active

Hole 9 at ·214 (at ½ I.D.), Act. L.; ·237 Virtual L.; Ratio $\frac{16}{10}$; Note $\frac{D\ 10}{512}$ = 563·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 563{\cdot}2} = \frac{340}{1126{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3018 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}237 & \text{Virtual L. H. 9} \\ \hline {\cdot}0648 & \text{All. at H. 9} \\ +\ {\cdot}0035 & \text{Neg. excess} \times 7 \\ \hline {\cdot}0683 & \text{Eff. All. at H. 9} \end{array}$$

Prop. All.

$$\frac{{\cdot}108 \times 10}{16}(5/8) = {\cdot}0675$$

Inc. All. No. 7 = ·00216 × 10 = ·0216 active

EFFECTIVE HALF-WAVE-LENGTH I.D. AT EACH FINGERHOLE

Half w.-l.		Eff. I.D. per fingerhole
Exit	·543	
H. 1	·4829	·0601 (2 I.D.)
H. 2	·4527	·0302
H. 3	·4227	·030
H. 4	·4075	·0152 (½ I.D.)
H. 5	·3925	·0150 (½ I.D.)
H. 6	·3623	·0302
H. 7	·3320	·0302
H. 8	·3160	·016 (½ I.D.)
H. 9	·3018	·0142 (½ I.D.)

COMPARISON OF LENGTHS ON THE TWO SENSA FLUTES A AND B

Sensa A			Note and Ratio	Sensa B
Exit	*E* 18	·465	*E* 18	*E* 18 = ·465
Hole 1	*F* 16	·375	16	*F* 16 = ·375
Hole 2	*G* 15	·348	15	*G* 15 = ·352
Hole 3	*G* 14	·320	14	*G* 14 = ·329
Hole 4	*A* 13	·291	13	*A* 27 = ·306
Hole 5	*B* 12	·264	12	*A* 13 = ·283
Hole 6	*C* 11	·236	11	*B* 12 = ·261
Hole 7	*D* 10	·208	10	*C* 11 = ·237
			21/32	*C* 21 = ·2255
			20	*D* 20 = ·214

Like Ratios indicate identical pitch ; it will be noticed that this occurs at different lengths for several of the notes.

MODAL FLUTE RECORDS : No. 19
SENSA C

9 Holes. Experimental

Facsimile of Sensa A except for the I.D. ; in Sensa C the I.D. made proportional, therefore carries no allowance

Flute made by Kathleen Schlesinger, September, 1934. Includes explanatory note on the virtual position of Fingerholes

Class IB
Modal Determinant 18

Hypophrygian Harmonia on $\frac{E\ 18}{256} = 312{\cdot}8$ v.p.s.

Modal Sequence

Holes	Exit	1	2	3	4	5	6	7	8	9
Ratios	$\frac{18}{18}$	$\frac{16}{18}$	$\frac{15}{18}$	$\frac{14}{18}$	$\frac{13}{18}$	$\frac{12}{18}$	$\frac{23}{36}$	$\frac{22}{36}$	$\frac{21}{36}$	$\frac{20}{36}$
Cents		*204*	*112*	*119·38*	*128*	*138·5*	*73·6*	*77*	*80·5*	*84·4*
Notes	*E*	$\overset{\sharp}{F}$	*G*	$\overset{\sharp}{G}\sharp$	$\overset{\sharp}{a}$	*b*	$\overset{\sharp}{b}$	*c*	*c*♯	*d*

MEASUREMENTS

L. from emb. to exit	·466 ;
L. „ „ to c. H. 1	·376 ;
Δ of bore	·023 ;
d = Δ of emb.	·010 ;
δ = Δ of fingerholes	·009 ;
de = depth of walls	·0025 ; ·003 ;
Exit to c. H. 1	·090 ;
I.D. (proportional)	·0235 ;
(i.e. $\frac{376}{16}$ vt.	= ·0235)

FINGERHOLES AND I.D.

H. 1 fr. emb.	·376 ;	from exit	·090
H. 2 „ „	·3525 ;	c. H. 1 to H. 2	·0235
H. 3 „ „	·329 ;	c. H. 2 to c. H. 3	·0235
H. 4 „ „	·3055 ;	c. H. 3 to c. H. 4	·0235
H. 5 „ „	·282 ;	c. H. 4 to c. H. 5	·0235
H. 6 „ „	·2585 ;	c. H. 5 to c. H. 6	·0235
H. 7 „ „	·235 ;	c. H. 6 to c. H. 7	·0235
H. 8 „ „	·2115 ;	c. H. 7 to c. H. 8	·0235
H. 9 „ „	·188 ;	c. H. 8 to c. H. 9	·0235

Increment of Distance Constant at ·0235
L. vent = ·376 = I.D. ·0235 × 16 = ·0376

Modal Determinant at Exit 18
at Vent 16

Hypophrygian Harmonia as Species of Dorian on *C* = 256.
Note of Exit *E* 18 = 312·8 v.p.s.
Note of Vent *F* 16 = 352 v.p.s.

THE THREE I.D. IN FLUTE NO. 19

No. 1.	Actual I.D. between fingerholes	·0235
No. 2.	Proportional I.D. aliquot of length from emb. to vent	·0235
No. 3.	Effective half-wave I.D.	·0302

Floating allowance = No. 3 – No. 2 always present per I.D. — ·0302 Eff. I.D.
·0235 Prop. I.D.
·0067 Fl. All.

The Floating Allowance × M.D. at Vent
= ·0067 × 16 = ·1072
is equal to Allow. No. 4, termed '*standard*'
vent allowance = ·107 (·1069)
exact within $\frac{3}{10}$ of one mm.

Incremental Allowance No. 7 (Cumulative) = ·00216 $\left(\frac{\cdot 039}{18}\right)$

This allowance, No. 7, remains latent in Sensa C, as far as Hole 5 $\left(\text{ratio } \frac{12}{16}\right)$, after which it becomes active, the cumulative allowance having reached a nodal point at Hole 6. The effect of this stimulation of a nodal point at a half-increment calls forth the note due at the half-increment lower, and thus the sequence is interrupted by the interpolation of notes belonging to half-increments.

EXIT

Exit at ·466 from emb., Ratio $\frac{18}{18}$; Note $\frac{E\,18}{256}$ = ·312·8 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 312{\cdot}8 \times 2} = \frac{340}{625 \text{ v.p.s.}} = \begin{array}{rl} \cdot 5434 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 466} & \text{Act. L.} \\ \cdot 0774 & \text{Eff. All. at Exit.} \end{array}$$

ALLOWANCE FOR DIAMETER AT EXIT

Δ	= ·023
$\Delta - d$	= ·013
de	·003
	·039 Act. diameter All. at Exit

POSITION OF HOLE 1

Determination by Formula No. 3

2 (I.D.)	·047	
$\frac{\Delta}{2}$	·0115	All. = ·039
$\Delta - d$	·013	
$\Delta - \delta$	·0145	
	·086	
	+ ·00432	[1] All. for two latent I.D.
	·09032	

Hole 1 at ·376 fr. emb., Ratio $\frac{16}{16}$; Note $\frac{F\,16}{256}$ = 352 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 352 \times 2} = \frac{340}{704 \text{ v.p.s.}} = \begin{array}{rl} \cdot 4829 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 376} & \text{Actual L.} \\ \underline{\underline{\cdot 1069\ (107)}} & \text{Eff. All. of H. 1.} \end{array}$$

Standard Allowance No. 4

2Δ	·046
$2(\Delta - d)$	·026
de	·003
$2(\Delta - \delta)$	·029
de	·003
	·107 in agreement with the All. from pitch.

Floating All. ·0067 × 16 (M.D. at H. 1) = ·1072, also in agreement.

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[1] *N.B.*—·00432 represents the missing allowance for the two latent I.D., which being proportional increments, carry no allowance. Inc. All. No. 7 = ·00216 taken twice and added to the computation for the determination of the position of Hole 1, provides the necessary adjustment.

Hole 2 at ·3525 fr. emb., Ratio $\frac{16}{15}$; Note $\frac{G\,15}{256}$ = 375·4 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 375{\cdot}4 \times 2} = \frac{340}{750{\cdot}8 \text{ v.p.s.}} = \begin{array}{rl} & {\cdot}4527 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{{\cdot}3525} \text{ Act. L.} \\ & \underline{\underline{{\cdot}1002}} \text{ All. at H. 2} \end{array}$$

Prop. All.

$\frac{{\cdot}107 \times 15}{16} = \underline{\underline{{\cdot}1003}}$ Pr. All. in agreement.

Inc. All. No. 7 = ·00216 latent

Floating All. ·0067 × 15 = ·1005 at Hole 2.

Hole 3 at ·329, Ratio $\frac{16}{14}$; Note $\frac{G\,14}{256}$ = 402·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 402{\cdot}2 \times 2} = \frac{340}{804{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} & {\cdot}4227 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{{\cdot}329} \text{ Act. l.} \\ & \underline{\underline{{\cdot}0937}} \text{ All. at H. 3} \end{array}$$

Prop. All.

$\frac{{\cdot}107 \times 7}{8} = \underline{\underline{{\cdot}0936}}$ Prop. All. in agreement at H. 3

Floating All. ·0067 × 14 = $\underline{\underline{.0938}}$ at Hole 3.

Inc. All. No. 7 = ·00216 × 2 = ·00432 latent

Hole 4 at ·3055, Ratio $\frac{16}{13}$; Note $\frac{A\,13}{256}$ = 433·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 433{\cdot}2 \times 2} = \frac{340}{866{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} & {\cdot}3925 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{{\cdot}3055} \text{ Act. L.} \\ & \underline{\underline{{\cdot}0870}} \text{ All. at H. 4} \end{array}$$

Prop. All.

$\frac{{\cdot}107 \times 13}{16} = \underline{\underline{{\cdot}087}}$ Prop. All. in agreement

Floating All. ·0067 × 13 = $\underline{\underline{{\cdot}0871}}$ at H. 4.

Inc. All. No. 7 = ·00216 × 3 ·00648 latent

Hole 5 at ·282, Ratio $\frac{16}{12}$; Note $\frac{B\,12}{256}$ = 469·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} & {\cdot}3623 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{{\cdot}282} \text{ Act. L.} \\ & \underline{\underline{{\cdot}0803}} \text{ All. at H. 5} \end{array}$$

Prop. All.

$\frac{{\cdot}107 \times 3}{4} = \underline{\underline{{\cdot}08025}}$ in agreement.

Floating All. = ·0067 × 12 = $\underline{\underline{{\cdot}0804}}$.

Inc. All. No. 7 = ·00216 × 4 = ·00864 latent

Hole 6 at ·2585 Act. position, Ratio $\frac{32}{23}$; Note $\frac{B\ 23}{256}$ = 489·7 v.p.s.

+ ·0117 = I.D./2

·2702 virtual position

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 489{\cdot}7 \times 2} = \frac{340}{979{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3471 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}2702 & \text{Virtual L.}^{1} \\ \hline {\cdot}0769 & \text{All. at H.6} \end{array}$$

Prop. All.

$$\frac{{\cdot}107 \times 23}{32} = {\cdot}0769$$

Floating All. = ·0067 × 11 = ·0737 (Ratio due to H. 6)

= ·0737

+ ·00335 Floating All./2

·07705

Inc. All. No. 7 = ·00216 × 5 = ·01080 (= ·011) active

Nodal point at ·01175 $\left(\text{I.D.} = \frac{{\cdot}0235}{2}\right)$.

EXPLANATION OF THE SIGNIFICANCE OF THE 'VIRTUAL POSITION' OF A HOLE

The virtual lengthening of the position of Hole 6 from ·2585 to ·2702 is a direct consequence of the actual lengthening of the effective half-sound-wave at that Hole. The cause of this lengthening is the sudden activity of the Incremental Allowance No. 7 = ·00216; being cumulative, this increment, taken 5 times for the five I.D.s at Hole 6, has now reached the total of ·0108 (= ·011), i.e. practically a half-increment of distance.

With an I.D. which is directly proportional to the length from emb. to vent, i.e. ·0235, the proportional impulse, primarily aided, perhaps, by the hollows in the interior of the flute at the fingerholes (as already suggested with regard to the Aulos), finds itself further greatly stimulated by the Incremental Allowance No. 7. As soon as this accumulated allowance reaches an amount equal to a nodal point, at half an increment (viz. ·0117), the note is heard that belongs to that half-increment, i.e. between Holes 6 and 5, although there is no fingerhole at that point.

$\frac{C\ 11}{512}$ is normally due at Hole 6 which has been usurped by $\frac{B\ 23}{32}$. This curious and unexpected phenomenon (which is frequently observed in flutes having an actual I.D. that measures less than the Proportional I.D.), is due apparently to the Incremental Allowance No. 7 becoming active at a nodal point. This Incremental Allowance No. 7 may be regarded as a kind of reflection of the exit allowance on the actual flute, and a symbol of the allowance due to Hole 1 as Vent. But it must not be forgotten that this allowance is vested in Exit or Vent in trust for all the fingerholes, and that its significance is therefore cumulative.

On flute No. 19, after the interpolation of the *B* 23 at Hole 6, the sequence continues to proceed by half-increments as to ratios, but from holes at whole increments. This constitutes a further evidence of the reality of the allowance for diameter at exit and embouchure in every flute, whether represented or not in the correct placing of Hole 1.

A glance at the table of effective half-wave lengths of the notes (at the end of the Record) shows that the first five of these Effective half-wave-length I.D.s are whole, and that from Hole 6 onwards these increments are halved, thus confirming the break in the modal sequence beginning with *B* 23 at Hole 6, which should normally produce

[1] For an explanation of the significance of this virtual position see further on.

$\frac{C\,11}{512}$, as indicated by its position on the flute at ·258 from emb., i.e. ·0235 × 11 = ·258.

It is clear, therefore, that since the I.D. of the half-wave-length includes diameter allowance, and that at Hole 6 the half-wave-length has only advanced by ·0152, i.e. $\frac{\text{I.D.}}{2}$, an explanation is due for this half-increment length, which is provided by the functional activity of Incremental Allowance No. 7 at a Nodal point.

Hole 7 at ·235 actual position, ·2585 virtual position } Ratio $\frac{16}{11}$; Note C 11 = 512 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \text{ v.p.s.} \times 2} = \frac{340}{1024 \text{ v.p.s.}} = \begin{array}{rl} \cdot 3320 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 2585 & \text{Virtual L.} \\ \hline \cdot 0735 & \text{All. at H. 7} \end{array}$$

Prop. All.

$$\frac{\cdot 107 \times 11}{16} = \cdot 0735 \text{ Pr. All. in agreement}$$

Floating All. = ·0067 × 11 = ·037.

Inc. All. No. 7 = ·00216 × 6 = ·01296 active

Thus Hole 7, actually at ·235 from emb., plays C 11, although normally it should produce D 10 from this hole, while C 11 should issue from Hole 6 at ·2585.

Hole 8 at ·2115 actual position, Ratio $\frac{32}{21}$; Note $\frac{B\,21}{512}$ = 536·4 v.p.s.

$$\begin{array}{rl} +\ \cdot 0235 & \text{I.D.} \\ +\ \cdot 0117 & \text{I.D./2} \end{array} \Big\} = 1\tfrac{1}{2} \text{ I.D.}$$
$$\cdot 2467 \text{ virtual position}$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 536{\cdot}4 \times 2} = \frac{340}{1072 \text{ v.p.s.}} = \begin{array}{rl} \cdot 3169 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 2467 & \text{virtual l.} \\ \hline \cdot 0702 & \text{All. due at H. 8} \end{array}$$

Prop. All.

$$\frac{\cdot 107 \times 21}{32} = \cdot 0702 \text{ Prop. All. in agreement}$$

Floating All. = $\frac{\cdot 0067}{2}$ × 21 = ·0703.

Inc. All. No. 7 = ·00216 × 7 = ·01512 active

Hole 9 at ·188 actual position, + ·047 I.D. × 2, ·235 virtual position } Ratio $\frac{16}{10}$; Note $\frac{D\,10}{512}$ = 563·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 563{\cdot}2 \times 2} = \frac{340}{1126{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} \cdot 3018 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 235 & \text{Virtual L.} \\ \hline \cdot 0668 & \text{All. at H. 9} \end{array}$$

Prop. All.

$$\frac{\cdot 107 \times 10}{16} = \cdot 0668$$

Floating All. ·0067 × 10 = ·067.

Inc. All. No. 7 = ·00216 × 8 = ·01728 active

PROGRESSION OF THE EFFECTIVE HALF-WAVE LENGTHS AND INCREMENTS OF DISTANCE BY FINGERHOLES

Eff. ½ w.l. I.D. = half-wave-length ÷ M.D.

Fingerhole	*Eff. ½ w.l.*	*Eff. I.D.*
Exit	·5434	·0302
1	·4829	·0605 2 I.D.
2	·4527	·0302
3	·4227	·0300
4	·3925	·0302
5	·3623	·0302
6	·3471	·0152 ½ I.D.
7	·3320	·0151 ½ I.D.
8	·3169	·0151 ½ I.D.
9	·3018	·0151 ½ I.D.

Thus the effective half-wave lengths proceed from Hole 6 by half-increments, whereas the fingerholes proceed by whole increments.

The I.D. on Sensa C, being proportional, carries no allowance, so that the corresponding amount is lacking on the flute itself; and as the fingerholes progress by increments throughout the sequence, the lack accumulates likewise, hence the aspect of allowance termed No. 7 which, in its accumulation, accounts for the break in the sequence and affords compensation.

MODAL FLUTE RECORDS: No. 1

Græco-Roman (Bucheum) Vertical or End-Blown Flute

Class IIIA Exit, Class IA Vent

Facsimile of Original made by Mr. J. D. Coates

Modal Determinant 11

on $\frac{B12}{256}$ = *469·2 v.p.s.*

Modal Sequence

Holes	Exit	1	2	3	4
Ratios	$\frac{11}{11}$	$\frac{10}{11}$	$\frac{9}{11}$	$\frac{8}{11}$	$\frac{7}{11}$
Cents		*165*	*182*	*204*	*231*
Notes	$\frac{B\,12}{256}$	*C*	*D*♭	*E*	*F*♯

Ratio $\frac{6}{11}$ may be obtained as 3rd Harmonic of Hole 2.

Harmonic (*A*)

MEASUREMENTS	
L. from emb. to exit	·335 ;
L. ,, ,, to c. H. 1	·277 ;
L. ,, c. H. 1 to exit	·058 ;
Δ of bore at emb. and exit	·014 ;
δ = Δ of fingerholes	·0055 ;
de = depth of walls	·0015 ;
I.D. = increment of distance	·0276 ;
I.D. × M.D., i.e. ·0276 × 11 =	·3036

FINGERHOLES			
C. of H. 1 fr. emb. =	·277 ;	H. 1 to exit	·058
C. ,, H. 2 ,, ,,	·248 ;	H. 1 to H. 2	·029
C. ,, H. 3 ,, ,,	·220 ;	H. 2 to H. 3	·028
C. ,, H. 4 ,, ,,	·194 ;	H. 3 to H. 4	·026
			·083/3

Mean I.D. = ·0276.

Modal Determinant, 11 ; Harmonia : Dorian Spondaic from Exit.

Note of exit $\frac{B\,12}{256}$ = 469·2 v.p.s.

THE THREE I.D. IN FLUTE NO. I AND THEIR SIGNIFICANCE

No. 1. Actual I.D. between fingerholes $= \cdot 0276$

No. 2. Proportional I.D., aliquot of L. at exit $= \frac{\cdot 335}{11} = \cdot 0305$

No. 3. Effective I.D. (exit) $\frac{\cdot 3623}{11} = \cdot 0329$

Aliquot of the half-wave-length at exit.

N.B.—The Actual I.D. on this flute is less than the Prop. I.D.; therefore it carries no allowance; and in comparing the proportional allowance per hole with the allowance derived from pitch-equivalent-length, it will be necessary to add the missing allowance, which has been termed in this work '*floating allowance*'; it consists in the difference between the effective $\frac{1}{2}$ w. I.D. and the Actual I.D., i.e., here:

$$\begin{array}{r} \cdot 0329 \\ - \underline{\cdot 0276} \\ \cdot 0053 \end{array}$$

The Incremental Allowance No. 7, taken cumulatively, i.e.

$\frac{\cdot 031}{11}$ Allowance at exit $= \cdot 0028$, (M.D.)

at each hole, remains latent until the aggregate amounts to the length at a Nodal point, i.e. at an $\frac{\text{I.D.}}{2}$ (which does not occur on this flute), for the three increments of distance at Hole 4 $= \cdot 0028 \times 3 = \cdot 0084 : \frac{\text{I.D.}}{2} = \cdot 0138$.

The floating allowance may be used to check the standard allowance for the fingerholes (given for Hole 1, q.v.); thus: Fl. All. $\times$ by the ratio number of Hole 1 (in this flute 10) should be equal to the standard allowance, within a millimetre or two, e.g.

Fl. All. $\cdot 0053 \times 10 = \cdot 053$
Standard All. $= \cdot 0523$
Difference, $\frac{7}{10}$ of one mm.

and similarly for each hole.

CLASSIFICATION: FLUTE NO. I BELONGS TO CLASS IIIA

This flute belongs to Class III*A*, for the length of the flute, minus the exit allowance—present between exit and Hole 1—is the multiple of I.D. and M.D., viz.

$$\begin{array}{rl} \cdot 335 & = \text{L. of flute} \\ - \underline{\cdot 031} & \text{All. at exit} \\ \cdot 304 & = \text{I.D.} \times \text{M.D. viz. } \cdot 0276 \times 11 = \cdot 3036 = \cdot 304. \end{array}$$

The exit allowance, therefore, from which is derived Incr. All. No. 7, becomes, in this class of flute, immediately active, increment by increment, instead of remaining latent and cumulative until a nodal point has been reached—a contingency which arises only when the actual I.D. carries an allowance for diameter.

The reason for this becomes apparent on examinaton of the three increments of distance (*v. supra*). The actual I.D. is the shortest: it is clear, therefore, that the actual I.D. not only carries no allowance at all, but also falls short of the Proportional I.D. by $\cdot 0029$, viz.

$$\begin{array}{rl} \cdot 0305 & \text{Prop. I.D.} \\ - \underline{\cdot 0276} & \text{Actual I.D.} \\ \cdot 0029 & \text{difference} \end{array}$$

Since the Eff. $\frac{1}{2}$ w.l. includes the allowance for diameter, which, as we have seen, functions as floating allowance, i.e. covering the allowance per increment (i.e. $\cdot 0053$), this should approximately correspond within a millimetre or so with Allowance No. 7 doubled (i.e. $\cdot 0056$) as relating to wave length and not to actual flute length only.

It will be seen in the analysis of pitch values for each hole how accurately this works out, when the Incr. All. No. 7 is added to the Prop. All. in order to balance the allowance derived from pitch-equivalent-length values.

EXIT

Exit at ·335 fr. emb., Ratio $\frac{11}{11}$; Note $\frac{B\ 12}{256} = 469{\cdot}2$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3623 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{{\cdot}335} & \text{Act. L.} \\ {\cdot}0273/2 & \text{Eff. All. at exit} \\ = \underline{\underline{{\cdot}01365}} & \end{array}$$

Analysis of Allowance at Exit by Formula No. 5

$$\begin{array}{rl} {\cdot}014 & \Delta \\ +\ \underline{{\cdot}0015} & \text{depth (difference 2 mm., or without ‘ depth ’ none)} \\ {\cdot}0155 & \text{All. at exit} \end{array}$$

In comparing this allowance with that resulting from pitch given above, it must be doubled.

N.B.—The two allowances should be in agreement : the slight difference implies one of three causes : (*a*) the pitch inaccurately estimated ; (*b*) the diameter of the bore ; (*c*) that depth has no significance. The state in which the precious relic was found, unsoldered down the length of the flute and the edges bent inwards, made it impossible to estimate the diameter with greater certainty.

Both the fundamental $\frac{B\ 12}{256}$ and the overblown octave are in perfect tune.

POSITION OF HOLE I

By Formula No. 3

$$\left.\begin{array}{ll} \frac{\Delta}{2} + de & {\cdot}008 \\ \Delta \text{ emb.} & {\cdot}014 \\ \Delta - \delta & {\cdot}0085 \end{array}\right\} = {\cdot}0305 = \left\{ \begin{array}{l} ^{1} \text{Exit allowance actually transferred} \\ \text{to the flute between Exit and H.I.} \end{array}\right.$$

$$\begin{array}{ll} \text{I.D.} & \underline{{\cdot}0276} \\ & {\cdot}0581 \text{ position of H.1. from exit} \end{array}$$

Hole 1 at ·277, Ratio $\frac{11}{10}$; Note $C = \frac{469{\cdot}2}{10} \times 11 = 516{\cdot}1$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 516{\cdot}1 \times 2} = \frac{340}{1032{\cdot}2} = \begin{array}{rl} {\cdot}3293 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{{\cdot}277} & \text{Act. L.} \\ \underline{\underline{{\cdot}0523}} & \text{Std. All. at H. 1} \end{array}$$

Analysis of Standard All. by Formula No. 4

$$\begin{array}{ll} 2(\Delta + de) & = {\cdot}031 \\ 2(\Delta - \delta) & = {\cdot}017 \\ 2(\text{ depth}) & \ \ \underline{{\cdot}003} \\ & \ \ \underline{\underline{{\cdot}051}} \end{array}$$

The Floating All. × Ratio 10 at Hole 1, i.e. ·0053 × 10 = $\underline{\underline{{\cdot}053}}$ in agreement with Std. All. at Hole 1 within 7/10 of one mm.

---

[1] This Exit All. as ·031 forms the basis of the Inc. All. No. 7 cumulative by increment, i.e. $\frac{.031}{11} = {\cdot}0028$. Hole 1 on this Graeco-Roman Bucheum flute is actually at ·058 from exit, and therefore in exact agreement with theory.

Hole 2 at ·248 (Act. I.D. = ·029), Ratio $\frac{11}{9}$; Note *D*

$$= \frac{469{\cdot}2 \text{ v.p.s.} \times 11}{9} = 573{\cdot}4 \text{ v.p.s.}$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 573{\cdot}4 \times 2} = \frac{340}{1146{\cdot}8 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}2965 & \text{Eff. } \tfrac{1}{2} \text{ w.l. at H. 2} \\ -\ {\cdot}248 & \text{Act. L.} \\ \hline {\cdot}0485 & \text{All. at H. 2} \end{array}$$

Prop. All. (from Std. All.)

$$\frac{{\cdot}0523 \times 9}{11} = \begin{array}{rl} {\cdot}0428 & \text{Prop. All.} \\ +\ {\cdot}0053 & \text{Eff. I.D.} - \text{Act. I.D.} \\ \hline {\cdot}0481 & \text{Adjusted All. at H. 2} \end{array}$$

Inc. All. No. 7 = ·0028 latent

Hole 3 at ·220 (I.D. = ·028), Ratio $\frac{11}{8}$; Note *E* = 645·15 v.p.s.

$$\left(\text{i.e. } \frac{469{\cdot}2 \times 11}{8}\right)$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 645{\cdot}15 \times 2} = \frac{340}{1290{\cdot}3 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}2635 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}220 & \text{Act. L. at H. 3} \\ \hline {\cdot}0435 & \text{All. at H. 3} \end{array}$$

Prop. All.

$$\text{Std. All. } \frac{{\cdot}0523 \times 8}{11} = \begin{array}{rl} {\cdot}038 & \text{Prop. All.} \\ +\ {\cdot}0053 & \\ \hline {\cdot}0433 & \text{Adjusted All. at H. 3} \end{array}$$

Inc. All. No. 7 = ·0028 × 2 = ·0056 latent

Hole 4 at ·194 (I.D. = ·026), Ratio $\frac{11}{7}$; Note *F* = 737·3 v.p.s.

$$\left(\text{i.e. } \frac{469{\cdot}2 \text{ v.p.s.} \times 11}{7}\right)$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 737{\cdot}3 \times 2} = \frac{340}{1474{\cdot}6 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}23057 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}194 & \text{Act. L.} \\ \hline {\cdot}0366 & \text{All. at H. 4} \end{array}$$

Prop. All.

$$\frac{{\cdot}0523 \times 7}{11} = \begin{array}{rl} {\cdot}0332 & \text{Prop. All.} \\ +\ {\cdot}0037 & \text{Eff. I.D.} - \text{Act. I.D. } ({\cdot}026) \\ \hline {\cdot}0369 & \text{Adjusted All.} \end{array}$$

N.B.—As the I.D. at H. 4 is only ·026, i.e. ·0016 further from emb., the adjustment due to additional length from emb. may be stated thus:

$$\begin{array}{rl} {\cdot}194 & \\ +\ {\cdot}0016 & \\ \hline {\cdot}1956 & \text{Act. L.} \end{array} \qquad \begin{array}{rl} {\cdot}2306 & \text{Eff. } \tfrac{1}{2} \text{ w.l. at H. 4} \\ -\ {\cdot}1956 & \text{Adjusted Act. L.} \\ \hline {\cdot}0350 & \text{All. at H. 4} \end{array}$$

and for Prop. All. thus:

$$\begin{array}{r} {\cdot}0369 \\ -\ {\cdot}0016 \\ \hline {\cdot}0353 \end{array}$$

Inc. All. No. 7 = ·0028 × 3 = ·0084 latent

The next note in the modal sequence, after the one given out through Hole 4, is of ratio $\frac{11}{6}$, a fifth $\left(\frac{3}{2}\right)$ above the note D of Hole 2, through which the note A may be obtained as 3rd harmonic overtone. The complete sequence may thus be played in the harmonic register by overblowing the fundamentals from exit and Holes 1 to 4, an octave, and Hole 2, a 12th.

Through Hole 2 at ·248 Ratio $\frac{11}{6}$ as 3rd harmonic of D.

$D = 573{\cdot}4$ v.p.s. $\times\ 3 = A$ of $1720{\cdot}4$ v.p.s.

$$A = \frac{469{\cdot}2 \text{ v.p.s.} \times 11}{6} = 860{\cdot}2 \text{ v.p.s.} \times 2 = 1720{\cdot}4 \text{ when overblown an octave.}$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{860{\cdot}2 \text{ v.p.s.} \times 2} = \frac{340}{1720{\cdot}4} = \underline{\underline{{\cdot}1976}} \text{ Eff. } \tfrac{1}{2} \text{ w.l.}$$

The effective ½ w.l. of the D. of Hole 2 = ·2965

and
$$\frac{{\cdot}2965 \times 2}{3} = \underline{\underline{{\cdot}1976.}}$$

THE EFFECTIVE I.D. DERIVED FROM THE EFF. ½ W.L. DIVIDED BY THE MODAL DETERMINANT FOR THE FINGERHOLES

Fingerhole	*Eff. ½ w.l.*	*Eff. I.D.*
Exit	·3623	·0329
Hole 1	·3293	·0330
Hole 2	·2965	·0328
Hole 3	·2635	·0330
Hole 4	·2306	·0329
Harm. Hole 2	·1976	·0330

No break in the modal sequence is indicated by the Eff. I.D., or by the Incremental Allowance No. 7, which remains latent throughout.

MODAL FLUTE RECORDS: No. 2
MOND SICILIAN FLUTE

Presented by the late Mrs. Ludwig Mond (*made by a peasant*)

Class IIB

Phrygian Harmonia from Exit. M.D. 12

Dorian Harmonia from Vent. M.D. 11

Modal Determinants 12 and 11. Phrygian on $\frac{B\ 12}{256}$ *and on Dorian* $\frac{C\ 11}{512}$

Modal Sequences

Exit	Hole 1	2	3	4	5	6	back 7
12/12	11	10	9	8	(7) 15/24	13/24	12
Cents	*151*	*165*	*182*	*204*	*231*	*128*	*138·5*

Increment of Distance ·025.

SCHEME OF FINGERING

Exit		
● back	●	o = C = 1024 v.p.s.
●	o = $B\frac{6}{12}$	●
●	●	●
●	●	●
●	●	●
●	●	●
o Harmonic C = 1024 v.p.s.	o	●

For explanatory matter on cross-fingering see Chap. vi (Theoretical) and vii (Historical).

N.B.

It will be noticed that in the computation of extra allowances for the adjustment of Proportional Allowance, hole by hole, the number of increments has been increased by one; e.g. at Hole 4 adjustment is made by 4 times the Fl. All., whereas, from Holes 1 to 4 there are but 3 I.D. The reason for this proceeding lies in the curiously disproportionate diameter of the Mond Flute in relation to its length, and will be found more fully stated further on.

MEASUREMENTS

L. emb. to exit	·270 ;
L. emb. to c. of Hole 1	·214 ;
L. exit to c. of Hole 1	·056 ;
Δ of bore	·020 ;
d of emb.	·009 ;
δ of fingerholes	·008 (constant);
I.D.	·025 ;
depth	·0025

FINGERHOLES AND I.D.

C. of H. 1 from emb. ·214 ; from Exit	·056
C. of H. 2 from emb. ·190 ; from c. H. 1	·024
C. of H. 3 from emb. ·165 ; from c. H. 2	·025
C. of H. 4 from emb. ·140 ; from c. H. 3	·025
C. of H. 5 from emb. ·115 ; from c. H. 4	·025
C. of H. 6 from emb. ·090 ; from c. H. 5	·025
C. of H. 7 (back) from emb. ·080 ; from c. H. 6	·010 ½ I.D.
	·134×2/11 = ·02436

Increment of Distance (mean) = **·02436.**
practical ·025.

I.D. ·025 × 12 M.D. = ·300.

Modal Determinant at Exit = 12.
Mode Phrygian.
Modal Determinant at Vent = 11.
Mode Dorian.

Note of Exit (all holes closed) $= \dfrac{B\ 12}{256}$.

THE THREE INCREMENTS OF DISTANCE

No. 1.	Actual, between fingerholes, centre to centre	= ·025
No. 2.	Prop. I.D. from c. of vent. $\dfrac{\cdot 270}{12}$	= ·0225
No. 3.	Eff. ½ w. I.D. $\dfrac{\cdot 3623}{12} = \cdot 03019$	= ·0302

Fl. All. The floating allowance is based upon

$$\left(\frac{\text{the Eff. } \frac{1}{2} \text{ w.l.}}{\text{M.D.}}\right) - \text{I.D.} = \cdot 0052 \text{ Fl. All. per I.D.}$$

viz.

$$\left(\text{Eff. } \tfrac{1}{2} \text{ w.l. } \frac{\cdot 3623}{12} = \cdot 03019 - \cdot 025 = \cdot 0052 \text{ eff. or } \cdot 0026 \text{ actual}\right).$$

N.B.—The I.D. at ·025 thus consists of the Prop. I.D. = ·0225 + Fl. All. of ·0025 = ·025.

Inc. All. No. 7 = All. from Exit $\dfrac{\cdot 034}{12}$ = ·0028

The Floating Allowance on the Mond Flute is thus equal to the difference between the Actual I.D. and the Proportional I.D. =

·025	Act. I.D.
− ·0225	Prop. I.D.
·0025	actually, on the Flute, or ·005 effective.

Thus the actual I.D. in this flute carries its requisite allowance in actual millimetres.

The Inc. All. No. 7 is actually present likewise on the flute between exit and Hole 1. It remains latent, however, from increment to increment (being included in the position of the hole centres, as length from emb.) until by accumulation it has reached the extent of a nodal point at a half-increment, when it becomes operative as an addition at half-increment of length and produces from the fingerhole the note due half an increment further from emb. and therefore lower in pitch.

THE TRIPLE DIAMETER ALLOWANCE

The Mond flute has a wide diameter in relation to its length at exit, i.e. $\frac{\cdot 270}{\cdot 020} = 13\frac{1}{2}$ times, and at vent $\frac{\cdot 214}{\cdot 020} = 10\frac{1}{2}$ times. From the exact pitch of the exit note and from its eff. ½ w.l., it becomes evident that the usual formula No. 2 does not fit the case; the allowance is ·0923 instead of ·0667, a difference of ·0256 more than one diameter, or equal to one I.D. The diameter × 3 instead of × 2 is only a matter of allowance on effective length, however, and does not affect actual length. It will be seen from the record of each fingerhole that the Prop. All. works out accurately on this basis.

THE DIAMETER ALLOWANCE FROM EXIT

L. from exit to emb. = ·270.

BY FORMULA NO. 5

$$\begin{array}{ll} \Delta & = \cdot 020 \\ \Delta - d & = \cdot 011 \\ \quad de & \quad \underline{\cdot 003} \\ & \quad \cdot 034 \text{ Allowance at Exit} \end{array}$$

I.D. ·025 × 12 = ·300 ; L. from exit ·270
Allowance + ·034
·304

NOTE OF EXIT FUNDAMENTAL

$$\frac{B\ 12}{256} = 469{\cdot}2 \text{ v.p.s.}$$

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4} = \cdot 3623 \text{ eff. L. of } \tfrac{1}{2} \text{ sd.-wave}$$

·3623	Eff. L.
− ·270	Act. L.
·0923	Allowance based upon pitch
− ·068	All. at exit × 2
·0243	i.e. an excess of more than one diameter.

N.B.—The tripling of the diameter for effective length is merely brought forward as a suggestion, which has not yet been thoroughly tested upon other specimens having similar dimensions.

POSITION OF HOLE 1 AND STANDARD ALLOWANCE FROM VENT

Hole 1 *Position of centre of Hole by Formula No. 3*

$$\begin{array}{ll} \frac{\Delta}{2} & = \cdot 010 \\ \Delta - d & = \cdot 011 \\ \Delta - \delta & = \cdot 012 \\ 1 \text{ (I.D.)} & = \underline{\cdot 025} \\ & \quad \cdot 058 \text{ Theoretical position of Hole 1 centred at } \cdot 058 \end{array}$$

Act. position of H. 1 = ·056, difference 2 mm.

Standard Allowance for Diameter by Formula No. 4 (Mond)

$$\begin{array}{lll} 3(\Delta) & = & \cdot 060 \\ 2(\Delta - d) + de & = & \cdot 025 \\ 2(\Delta - \delta) + de & = & \cdot 027 \\ \hline & & \cdot 112 = \text{Effective Standard All. for all holes} \end{array}$$

Floating allowance ·0052 × 11 (ratio of H. 1) = ·057 actual, and ·114 eff. is therefore ample.

PITCH OF NOTE OF HOLE 1

Hole 1 at ·214 from emb. ratio 11/11.
,, ,, ·056 from exit.
Note C = 512 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \times 2} = \frac{340}{1024} = \begin{array}{rl} \cdot 332 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot 214 & \text{Actual L.} \\ \hline \cdot 118 & \text{Eff. Allowance} \end{array}$$

The excess of the theoretical over the standard of ·006 is doubtless due to the abnormally wide diameter of the Mond Flute in relation to its length.

Hole 2 at ·190 (I.D. = ·024), Ratio $\frac{11}{10}$, Note of Hole 2 = $\frac{D\ 10}{512}$ = 563·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340}{563\cdot 2 \times 2} = \frac{340}{1126\cdot 4 \text{ v.p.s.}} = \begin{array}{rl} \cdot 3018 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot 190 & \text{Act. L.} \\ \hline \cdot 1118 & \text{Allowance} \end{array}$$

Prop. All.

$$\text{St. All. } \frac{\cdot 112 \times 10}{11} = \frac{1\cdot 120}{11} = \begin{array}{rl} \cdot 1018 & \\ + \cdot 0052 & \text{Fl. All.} \times 2 \\ + \cdot 005 & \text{Excess} = \cdot 0025 \times 2 \text{ for } 2 \text{ (I.D.)} \\ \hline \cdot 1120 & \text{Adjustment of Prop. All.} \end{array}$$

Inc. All. No. 7 = ·0028 × 2 = ·0056 latent

Hole 3 at ·165 from emb., Ratio $\frac{11}{9}$, Note of H. 3 $\frac{E\ 18}{512}$ = 625·6 v.p.s.

BY FORMULA NO. 2

$$\frac{340}{\text{v.f. } 625\cdot 6 \times 2} = \frac{340}{1251\cdot 2 \text{ v.f.}} = \begin{array}{rl} \cdot 2717 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot 165 & \text{Act. L.} \\ \hline \cdot 1067 & \text{Allowance} \end{array}$$

Prop. All.

$$\text{St. All. } \cdot 112 \times 9/11 = \frac{\cdot 1008}{11} = \cdot 0916 \text{ St. All. for H. 3.}$$

$$\begin{array}{rl} \cdot 0916 & \\ + \cdot 0078 & = \text{Fl. All.} \times 3 \\ + \cdot 0075 & \text{Excess} \times 3 \\ \hline \cdot 1069 & \text{Adjustment of Prop. All.} \end{array}$$

Inc. All. No. 7 = ·0028 × 3 = ·0084 latent

Hole 4 at ·140, Ratio $\frac{11}{8}$; Note $\frac{F\,16}{512} = 704$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 704 \times 2} = \frac{340}{1408} = \begin{array}{rl} \cdot 2414 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 1400 & \text{Act. L.} \\ \hline \cdot 1014 & \text{All. at H. } 4 \end{array}$$

Prop. All.

$$\frac{\cdot 112 \times 8}{11} = \begin{array}{rl} \cdot 0814 & \text{Prop. All. at H. } 4 \\ +\ \cdot 0104 & \text{Fl. All.} \times 4 \\ +\ \cdot 0100 & \text{Excess} \times 4 \\ \hline \cdot 1018 & \text{Adj. of Prop. All.} \end{array}$$

Inc. All. No. 7 = ·0028 × 4 = ·0112 latent

Hole 5 at ·115, Ratio $\frac{22}{15}$; Note $G\,15 = 750{\cdot}8$ v.p.s.

$$\begin{array}{rl} +\ \cdot 0125 & = \text{I.D./2} \\ \hline \cdot 1275 & \text{virtual position} \end{array}$$

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 750{\cdot}8 \times 2} = \frac{340}{1501{\cdot}6 \text{ v.p.s.}} = \begin{array}{rl} \cdot 2264 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 1275 & \text{Virtual L. at H. } 5 \\ \hline \cdot 0989 & \text{All. at H. } 5 \end{array}$$

N.B.—The increments now number $4\frac{1}{2}$.

Prop. All.

$$\frac{\cdot 112 \times 15}{22} = \begin{array}{rl} \cdot 07636 & \text{Prop. All. at H. } 5 \\ +\ \cdot 0117 & \text{Fl. All.} \times 4\tfrac{1}{2} \\ +\ \cdot 01125 & \text{Excess} \times 4\tfrac{1}{2} \\ \hline \cdot 09931 & \text{Adjustment of Prop. All.} \end{array}$$

Inc. All. No. 7 = ·0028 × 5 = ·0140 active, having reached and passed the nodal point $\left(\frac{\text{I.D.}}{2} = \cdot 0125\right)$ and the corresponding note of the half-increment, $\frac{15}{22}$ now sounds instead of $\frac{7}{11}$. It is well to remember that in playing a modal flute there must be an increase of compression in the blowing as pitch rises.

Hole 6 at $\cdot 090 + \frac{\text{I.D.}}{2} = \cdot 090 + \cdot 0125 = \cdot 1025$, Actual Ratio $\frac{11}{6}$; Virtual Ratio $\frac{22}{13}$.

Note of H. 6 $= \frac{A\,13}{512} = 866{\cdot}4$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 866{\cdot}4 \times 2} = \frac{340}{1732{\cdot}8 \text{ v.p.s.}} = \begin{array}{rl} \cdot 1962 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 1025 & \text{Virtual L.} \\ \hline \cdot 0937 & \text{All. at H. } 6 \end{array}$$

Prop. All.

$$\text{St. All. } \cdot 112 \times \frac{13}{22} = \begin{array}{rl} \cdot 0661 & \text{Prop. All. at Hole } 6 \\ +\ \cdot 0143 & \text{Fl. All.} \times 5\tfrac{1}{2} \\ +\ \cdot 01375 & \text{Exc.} \times 5\tfrac{1}{2} \\ \hline \cdot 09415 & \text{Adjustment of Prop. All.} \end{array}$$

Inc. All. No. 7 = ·0028 × 6 = ·0168 active

The incremental allowance is still active but has not reached a further nodal point. Thus, although Hole 6 has been bored at one whole increment about Hole 5, the effect in sound is that of one half-increment, so that the hole is virtually at ·090 + ·0125 = ·1025 from embouchure. This curious position causes what has been noticed also in other flutes (e.g. Sensa C), i.e. an interrupted sequence through the interpolation of a note due at a half-increment lower. This arises, as already explained, through the cumulative effect of Inc. All. No. 7, here stimulated into activity by the accumulated length which has overtaken the nodal point, but not sufficiently to cause a second interpolation. The lowering effect has merely been passed on from Hole 5.

Hole 7 at ·080 Actual L. from emb.
+ ·0125 half-incr. lower.
·0925 Virtual L. from emb.
Ratio 12/6 from exit
„ 11/6 from vent.

Note of Hole 7 $\dfrac{B\ 12}{512} = 938{\cdot}4$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{938{\cdot}4 \times 2} = \frac{340}{1876{\cdot}8} = \begin{array}{r} \cdot 18115 \\ -\ \cdot 09250 \\ \hline \cdot 08865 \end{array}$$

Prop. All.

St. All. $\dfrac{\cdot 112 \times 1}{2} = \begin{array}{r} \cdot 056 \\ +\ \cdot 0156 \\ +\ \cdot 0150 \\ \hline \cdot 0866 \end{array}$ | Prop. All. / Fl. All. × 6 / Exc. × 6

Adjustment of Prop. All.

Inc. All. No. 7 = ·0028 × 7, still active as at Hole 5 = ·0196

It will be noticed that Hole 7 is at ·010 above Hole 6 instead of at a full half-increment = ·0125. The note played is undoubtedly *B* 12 in tune. The flute-maker has planned his boring scheme so efficiently so far—probably by empirical experiments aided by familiarity with the pure modal sequence—that it cannot be that he has suddenly become careless. This may be regarded as the triumph of the proportional impulse over the factor of length, which allows the *B* 12 to sound in tune.

THE EFFECTIVE HALF-WAVE LENGTH AT EACH FINGERHOLE, PER INCREMENT

Holes	*½ w.l.*	*per Inct.*	*Ratio Nos.*
Exit	·3623	·0302	12
Hole 1	·332	·0303	11
Hole 2	·3018	·0302	10
Hole 3	·2717	·0301	9
Hole 4	·2414	·0303	8
Hole 5	·2113	·0301 (·0151)	7 (15)
Hole 6	·1962	·0151 $\frac{\text{I.D.}}{2}$	13
Hole 7	·1811	·0151 $\frac{\text{I.D.}}{2}$	12

Between Holes 5 and 7 the ratio is $\frac{7}{6}$, i.e. of one I.D., as shown by the effective half-wave length, but the actual holes on the flute are placed at one and a half increment.

MODAL FLUTE RECORDS: No. 3

From the Carpathians, presented by Mr. George Kaufmann, M.A. (7/9/33)

Golden brown bamboo with 6 Fingerholes

Class IIA from Vent

Dorian Harmonia of M.D. 11 (Spondaic)

Modal Sequence

Fingerholes	1	2	3	4	5	6
Modal Ratios	11/11	10	9	8	7	6
Cents		*165*	*182*	*204*	*231*	*267*

Plays from Vent on $\frac{G\ 15}{256} = 375{\cdot}4$ v.p.s.

This flute is mentioned in Chap. ix in connexion with the Rumanian Folk Song (from the Maramuros district bordering on the Carpathians), which is also in the Dorian Harmonia. Tested on July 26, 1934, on $\frac{G\ 15}{256}$

N.B.

Since the ratios of the intervals given by the fingerholes (and therefore by their v.f.s) are known and fixed, through the Modal Determinant (here 11), the theoretical distance of the holes from the embouchure may be ascertained by reckoning the I.D. as equal according to the average; this being a test of the value and correctness of the formula.

The distance from Exit to Hole 1 = ·086, i.e. two I.D. + ·014 for allowance at exit which should be ·024.

The total length = ·460
Allowance for Δ = ·024
·484 All. at Exit (theoretical).

The total length is not an exact multiple of the I.D. (mean ·036) × 13 = ·468 or × 14 = ·504,

therefore

the flute's scale may be taken to commence from the centre of Hole 1 as vent, the v.f. of which note is $375{\cdot}4 = \frac{G\ 15}{256}$ and the ratio $\frac{11}{11}$.

MEASUREMENTS	
Total length from C. of emb. to C. of vent	·460;
Δ of bore	·014;
Δ of emb. = d	·008;
Δ + d	·006;
δ of fingerholes (irregular circle) average	·006;
Depth of walls	·004

FINGERHOLES	
Total length ·460 from emb.; C. of H. 1 to exit; C. of H. 1 at ·374 from emb.	·086
C. of H. 2 at ·341; from C. of Hole 1	·033
C. of H. 3 at ·306; „ „ of H. 2	·035
C. of H. 4 at ·261; „ „ of H. 3	·045
C. of H. 5 at ·227; „ „ of H. 4	·034
C. of H. 6 at ·191; „ „ of H. 5	·036
	·183
	5)·0366

I.D. mean ·0366, useful mean ·036.

I.D. from vent $= \frac{{\cdot}374}{11} = {\cdot}034$.

·036 × 11 = ·396 − actual position of Hole 1 at ·374
= ·022 approximates to All. for diameter.

Since the length from embouchure to vent + allowance is equal (within 2 mm.) to the I.D. multiplied by the Modal Determinant, the flute No. 3 therefore belongs to Class IIA.

The mean I.D. at ·036 includes ·002 as allowance for Δ over the Prop. I.D. at vent of ·034.

THE THREE INCREMENTS OF DISTANCE

Actual I.D. useful mean $= \cdot 036$

Proportional I.D., i.e. $\frac{\cdot 374}{11} = \cdot 034$

Effective I.D., i.e. $\frac{\cdot 4528}{11} = \cdot 0412$
$- \cdot 034$ Prop. I.D.
$\cdot 0072$

Floating Allowance ·0072

Inc. All. No. 7 $\frac{\cdot 024}{11} = \cdot 00218$

The actual I.D. therefore bears its allowance of ·002 with a reserve of ·005.

POSITION OF HOLE I

By Formula No. 3

$\frac{\Delta}{2} =$	·007	
$\Delta - d$ (·014 − ·008)	·006	
$\Delta - \delta$ (·014 − ·006)	·008	
	·021	actual position at ·086 from exit
2 (I.D.)	·072	
Theoretical position	·093	

Hole 1 is, therefore, ·007 too low, according to theory.

EXIT ALLOWANCE IN RESPECT OF DIAMETER

Δ	= ·014	
$\Delta - d$	= ·006	
de	= ·004	
	·024	diameter allowance at exit

Hole 1 at ·374, Ratio 11/11 ; Note $\frac{G\,15}{256} = 375{\cdot}4$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 375{\cdot}4 \times 2} = \frac{340}{750{\cdot}8 \text{ v.p.s.}} = \cdot 4528 \text{ Eff. } \tfrac{1}{2} \text{ w.l.}$$

$- \cdot 374$ Act. L.

$\cdot 0788$ All. at H. 1

Standard All. No. 4. By Formula No. 4

$2(\Delta) + de =$	·032	
$2(\Delta - d) + de$	·016	
$2(\Delta - \delta) + de$	·020	
	·068	Standard Allowance at H. 1

It will be seen that the Standard Allowance (theoretical) is ·010 less than the Effective Allowance at Hole 1, which is computed from the v.f. of the note. The difference may be accounted for thus : the position of Hole 1 was found to be actually ·007 too low. The I.D. between Hole 1 and Hole 2 = ·033 is less than the mean pointing to irregularity in the boring of the flute.

Hole 2 at ·341 from emb., Ratio 11/10; Note *A* = 375·4 v.p.s. × 11/10 = 412·9 v.p.s.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 412{\cdot}9 \times 2} = \frac{340}{825{\cdot}8\ \text{v.p.s.}} = \begin{array}{rl} {\cdot}4117 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}341 & \text{Act. L.} \\ \hline {\cdot}0707 & \text{All. at H. 2} \end{array}$$

Prop. All. (from St. All. No. 4)

$$\frac{{\cdot}0788 \times 10}{11} = \frac{{\cdot}780}{11} = \underline{\underline{{\cdot}0709.}}$$

Inc. All. No. 7 = ·00218 × 1 (I.D.) = ·00218 latent

Hole 3 at ·306 from emb., Ratio 11/9; Note *B* = 375·4 × 11/9 = 458·8 v.p.s.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 458{\cdot}8 \times 2} = \frac{340}{917{\cdot}6\ \text{v.p.s.}} = \begin{array}{rl} {\cdot}3705 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}306 & \text{Act. L.} \\ \hline {\cdot}0645 & \text{All. at H. 3} \end{array}$$

Prop. All. (from St. All.)

$$\frac{{\cdot}0788 \times 9}{11} = {\cdot}0644.$$

Inc. All. No. 7 = ·00218 × 2 = ·00436 latent

Hole 4 at ·261 from emb., Ratio 11/8; Note *C* = 375·4 v.p.s. × 11/8 = 516·17 v.p.s.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 516{\cdot}17 \times 2} = \frac{340}{1032{\cdot}34} = \begin{array}{rl} {\cdot}3294 & \text{Eff. } \tfrac{1}{2} \text{ w.l. at H. 4} \\ -\ {\cdot}261 & \text{Act. L. at H. 4} \\ \hline {\cdot}0684 & \text{All. at H. 4} \end{array}$$

Prop. All. (by Formula No. 4)

$$\frac{{\cdot}0788 \times 8}{11} = {\cdot}0573 + {\cdot}009 = {\cdot}0663 \text{ Adjusted Prop. All.}$$

N.B.—The I.D. is here ·045 instead of ·036, therefore Hole 4 is placed ·009 higher than it should be, which accounts for the allowance being irregular.

Inc. All. No. 7 = ·00218 × 3 (I.D.) = ·00654 latent

Hole 5 at ·227, Ratio 11/7; Note *D* = 3754 v.p.s. × 11/7 = 589·9 (590) v.p.s.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 590 \times 2} = \frac{340}{1180\ \text{v.p.s.}} = \begin{array}{rl} {\cdot}2881 & \text{Eff. } \tfrac{1}{2} \text{ w.l. at H. 5} \\ -\ {\cdot}227 & \text{Act. L.} \\ \hline {\cdot}0611 & \text{All. at H. 5} \end{array}$$

Prop. All. by Formula No. 4

·0788 × 7/11 = ·05014 Prop. All.
The difference of 10 mm. is due to the irregular position of Hole 5.

Inc. All. No. 7 = ·00218 × 4 = ·00872 latent

Hole 6 at ·191, Ratio 11/6; Note $F = 375{\cdot}4$ v.p.s. $\times$ 11/6 = 688·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 688{\cdot}2 \times 2} = \frac{340}{1376{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}247 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ {\cdot}191 & \text{Act. L. at H. 6} \\ \hline {\cdot}056 & \text{All. at H. 6} \end{array}$$

Prop. All. by Formula No. 4

$$\cdot0788 \times 6/11 = \cdot04298 = \begin{array}{rl} {\cdot}043 & \\ +\ {\cdot}011 & \text{diff. as above} \\ \hline {\cdot}054 & \end{array}$$

Inc. All. No. 7 = ·00218 × 5 = ·1090 latent

N.B.—The nodal point is at ·018.

THE EFFECTIVE HALF-WAVE LENGTHS AND I.D.

Hole 1	·4528	·041
Hole 2	·4117	·0411
Hole 3	·3714	·0403
Hole 4	·3293	·0521 [1]
Hole 5	·2881	·0412
Hole 6	·247	·0411

MODAL FLUTE RECORDS: No. 5

JAVA I

6 Fingerholes. Side-blown Flute (Soeling)

Facsimile from original from Batavia belonging to Mrs. Elizabeth Ayres Kidd, of Winnetka, U.S.A.

Class IA from Vent
Class IIIA from Exit

Modal Determinant 11 from Exit and 10 from Vent

Dorian Harmonia

Modal Sequence

	Exit	Hole 1	2	3	4	5	6
	11/11	10	9	8	7	6	5½ (11)
Cents		*165*	*182*	*204*	*231*	*267*	*151*

The fundamental note of exit = $\overset{\flat}{C}$ 494 v.p.s.

" " " " vent $= \dfrac{D\ 20}{512} = 563{\cdot}2$ v.p.s.

MEASUREMENTS

L. emb. to exit	·310;
L. emb. to C. Hole 1 (Vent)	·258;
L. (internal) from exit to stopper	·3165;
L. stopper from centre of emb.	·0065;
Δ of bore	·0115;
d of emb.	·008;
δ of fingerholes (constant)	·006;
de depth of walls	·002;
Exit to C. of Hole 1	·052;
I.D. (constant)	·026

FINGERHOLES AND I.D.

L. C. Hole 1 from emb.	·258; from ext.	·052
C. of H. 2 " "	·232; from C. H. 1	·026
C. of H. 3 " "	·206; from C. H. 2	·026
C. of H. 4 " "	·180; from C. H. 3	·026
C. of H. 5 " "	·154; from C. H. 4	·026
C. of H. 6 " "	·128; from C. H. 5	·026

[1] This eff. I.D. in excess of all others draws attention to the actual I.D. at Hole 4 which is of ·045 − mean ·036 = ·009.

I.D. Constant = ·026.

Modal Determinant from exit 11.

Interval from Exit to H. 1 slightly sharpened.

Note of exit fundamental $= C =$ ·494 v.p.s. $\dfrac{\overset{\flat}{C}\ 11}{512}$

Note of Vent $\dfrac{D\ 10}{512}$ = 563·2 v.p.s. (exact)

I.D. = ·026 × 10 = ·260 (actual ·258 L. emb. to C. Hole 1)

I.D. = ·026 × 11 = ·286 L. exit to emb. = ·310

(according to ratio the fundamental note is flattened).

THE THREE INCREMENTS OF DISTANCE

(1) The Increment of Distance (I.D.) between the fingerholes = ·026

(2) The Proportional I.D., aliquot of the length from emb. to vent $\dfrac{258}{10}$ = ·0258

(3) I.D. of the Effective half-wave length $\dfrac{3018}{10}$ = ·0302

The Floating Allowance = No. 3 − No. 2 = ·0302 − ·0258 = ·0044

Inc. All. No. 7 = Vent All. $\dfrac{\cdot 019}{10}$ ·0019 × 5 = ·0095 = ·0019

(This allowance does not approach a nodal point (·013) and therefore causes no interruption in the Modal Sequence.)

EXIT

BY FORMULA NO. 5

L.	= ·310 from exit to emb.
Δ	= ·0115
Δ − d	= ·0035
de	= ·002
	·017 All. at exit (·017 × 2 = ·034, Eff. All. at exit)

L. from exit to emb. ·310

+ ·034 Eff. All. at exit

·344 Eff. ½ w.l.

BY FORMULA NO. 1

L. $\dfrac{340 \text{ m./s.}}{\cdot 344 \times 2} = \dfrac{340}{\cdot 688}$ = 494·1 v.p.s. fundamental at exit $\left(\text{a flattened } \dfrac{C\ 11}{512}\right)$

POSITION OF HOLE 1

By Formula No. 3.

$\dfrac{\Delta}{2}$	= ·00575
Δ − d	= ·0035
Δ − δ	= ·0055
2 (*de*)	·004
	·01875 (= ·019)
1 (I.D.)	+ ·026
	·04475 (= ·045)

Actual distance from exit of C. of Hole 1 = ·052, which is 7 mm. higher than the theoretical. The flute-maker has obviously allowed two I.D. between exit and Hole 1, which provides a surplus of ·007 (or of ·014 for pitch values or effective length).

The implication is that there can be no commensurate ratio between exit and vent. The table giving the Eff. ½ w. I.D. confirms the implication.

Hole 1 Standard Allowance No. 4.

BY FORMULA NO. 4

2Δ	$= \cdot 023$
$2(\Delta - d)$	$= \cdot 007$
$2(\Delta - \delta)$	$= \cdot 011$
$2\ (de)$	$= \cdot 004$
	$\cdot 045$ Standard allowance (cf. with above)

N.B.—The Actual I.D. is practically equal to the Vent proportional I.D., and therefore carries no allowance. The Floating Allowance therefore becomes operative at each Hole.

Hole 2 at ·232 from emb., Ratio 10/9; Note $\frac{E\ 18}{512} = 625\cdot 6$ v.p.s.

BY FORMULA NO. 2

$$\frac{340\text{ m./s.}}{\text{v.f. } 625\cdot 6 \times 2} = \frac{340}{1251\cdot 2} = \begin{array}{rl} & \cdot 2718 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \cdot 232 \text{ Act. L.} \\ \hline & \cdot 0398 \text{ All. at H. 2 (eff.)} \end{array}$$

Prop. All.

$$\frac{\cdot 0438 \times 9}{10} = \frac{\cdot 3942}{10} = \cdot 0394 \text{ Prop. All.}$$

Inc. All. No. 7 = ·0019 × 2 = ·0038 latent

Hole 3 at ·206 from emb., Ratio 10/8 = 5/4; Note $\frac{F\ 16}{512} = 704$ v.p.s.

BY FORMULA NO. 2

$$\frac{340\text{ m./s.}}{\text{v.f. } 704 \times 2} = \frac{340}{1408\text{ v.p.s.}} = \begin{array}{rl} & \cdot 24147 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \cdot 206 \quad \text{Act. L.} \\ \hline & \cdot 0355 \text{ Eff. All. at H. 3} \end{array}$$

Prop. All.

$$\frac{\cdot 0438 \times 4/5}{5} = \cdot 03504$$

Inc. All. No. 7 = ·0019 × 3 = ·0057 latent

Hole 4 at ·180 from emb., Ratio 10/7; Note *G* 7 = 804·4 v.p.s.

BY FORMULA NO. 2

$$\frac{340\text{ m./s.}}{\text{v.f. } 804\cdot 4 \times 2} = \frac{340}{1608\cdot 8\text{ v.p.s.}} = \begin{array}{rl} & \cdot 2113 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \cdot 180 \text{ Act. L.} \\ \hline & \cdot 0313 \text{ All. at H. 4} \end{array}$$

Prop. All.

$$\frac{\cdot 0438 \times 7/10}{10} = \cdot 03066 = \cdot 031$$

Inc. All. No. 7 = ·0019 × 4 = ·0076 latent

Hole 5 at ·154 from emb., Ratio 10/6; Note $\frac{B\ 6}{512}$ = 938·4 v.p.s.

BY FORMULA NO. 2

$$\frac{340\text{ m./s.}}{\text{v.f. } 938{\cdot}4 \times 2} = \frac{340}{1876{\cdot}8\text{ v.p.s.}} = \begin{array}{rl} & {\cdot}1811 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{{\cdot}154} \text{ Act. L.} \\ & {\cdot}0271 \text{ All. at H. } 5 \end{array}$$

Prop. All.

$$\frac{{\cdot}0438 \times 6/10\ (= 3/5)}{5} = {\cdot}0263$$

Inc. All. No. 7 = ·0019 × 5 = ·0095 latent

Hole 6 at ·128 from emb., Ratio $\frac{20}{11} = \frac{5\frac{1}{2}}{10}$; Note $\frac{C\ 11}{1024}$ v.p.s.

$$\begin{array}{rl} + & \underline{{\cdot}013}\ \tfrac{1}{2}\text{ (I.D.)} \\ & {\cdot}141 \text{ virtual L. from emb.} \end{array}$$

BY FORMULA NO. 2

$$\frac{340\text{ m./s.}}{\text{v.f. } 1024 \times 2} = \frac{340}{2048\text{ v.p.s.}} = \begin{array}{rl} & {\cdot}166 \text{ Eff. w.l.} \\ - & \underline{{\cdot}141} \text{ Virtual L.} = (+\ {\cdot}013) \\ & {\cdot}025 \end{array}$$

Prop. All.

$$\frac{{\cdot}0438 \times 11}{20} = {\cdot}0241 = \text{Prop. All.}$$

Inc. All. No. 7 = ·0019 × 6 = ·0114 becomes active
Nodal point = ·013

SCHEME OF FLUTE FOR NOTE OF HOLE 6

○ = *C* 11 of 1024 v.p.s. in tune	○ = $\frac{D\ 10}{1024}$ v.p.s.
●	●
●	●
●	●
●	●
●	○ = $\frac{D\ 10}{512}$
exit = *C* of 494·1 v.p.s.	

N.B.—Thus although Hole 6 is at a whole I.D., the note is that of a half I.D. Hole 5 = *B* 6 or 12 and Hole 6 should have ratio number 5 or 10. But instead, a pure *C* 11 is played, although the Inc. All. No. 7 had by accumulation = ·0114 not reached the nodal point which occurs at ·013. This unusual activity of Inc. All. No. 7 may have been stimulated by the position of Hole 6 at a whole I.D. past the middle length of the flute. This influence of the upper half of the flute (at the same diameter as the lower half) has been found operative in halving the ratio in other flutes, e.g. in Sensa C (q.v.).

The fact that in both the flutes in question the actual and the proportional I.D. are equal, and that therefore no allowance is included in the position of the holes may, on the other hand, prove to be the predominant factor.

EFFECTIVE HALF-WAVE LENGTH I.D.

Holes	*Eff. L.*	*Eff. L. I.D.*
Exit	·344	·0312
Hole 1	·3018	·0422 (actually 2 I.D.)
Hole 2	·2718	·0300
Hole 3	·2415	·0303
Hole 4	·2113	·0302
Hole 5	·1811	·0302
Hole 6	·166	·0151 (note of one-half I.D. although the hole is actually at one whole I.D. on the flute)

MODAL FLUTE RECORDS: No. 6

JAVA II

Side-blown Flute (Soeling). With 6 Fingerholes

Facsimile by K. S. from Mrs. Elizabeth Ayres Kidd's original from Batavia

Class IIA from Vent

Modal Determinant 11

Dorian Spondaic on $C = 512$ from Vent

Modal Sequence

Holes	Exit	1	2	3	4	5	6
Ratios	No true ratio to vent	$\frac{11}{11}$	$\frac{10}{11}$	$\frac{9}{11}$	$\frac{8}{11}$	$\frac{7}{11}$	$\frac{6}{11}$
Cents		165	182	204	231	267	
Notes		*c*	*d*♭	*e*	*f*♯	*g*♯	*b*

MEASUREMENTS	
L. from emb. to exit	·335;
L. from emb. to C. of Hole 1	·2805;
L. stopper from exit (internal)	·342;
L. stopper from C. of emb.	·007;
Δ of bore	·013;
d of emb.	·0085;
δ of fingerholes (constant)	·0065;
de depth of walls	·0015;
Exit to C. of Hole 1	·0545;
I.D. (mean) ·028	·0278

FINGERHOLES

C. H. 1 from emb. ·2805; from exit = ·0545
C. H. 2 ,, ,, ·253; H. 1 to H. 2 = ·0275
C. H. 3 ,, ,, ·2245; H. 2 to H. 3 = ·0285
C. H. 4 ,, ,, ·197; H. 3 to H. 4 = ·0275
C. H. 5 ,, ,, ·169; H. 4 to H. 5 = ·028
C. H. 6 ,, ,, ·1415; H. 5 to H. 6 = ·0275
I.D. mean ·0278; useful I.D. ·028 ·139/5
I.D. × 11 = ·028 = ·0278 (·028)

L. Vent ·2805 + allowance ·0255 = ·306.
Modal Determinant 11.
Harmonia Dorian Spondaic from Vent.
Note of Vent $c = 512$ v.p.s.

THE THREE I.D. IN FLUTE NO. 6

No. 1.	Actual I.D. between fingerholes	·028
No. 2.	Proportional I.D., aliquot of length from Vent	·0255
No. 3.	Effective ½ w.l. I.D. (Vent)	·03018
	(,, ,, I.D. (Exit ·031))	

Floating All. = No. 3 − No. 2 = ·0302
Prop. All. always present per I.D. − ·0255
·0047 Fl. All.

The Floating Allowance $\times$ M.D. = ·0047 $\times$ 11 = $\underline{\underline{·0517}}$, should equal the allowance No. 4, termed Standard Vent Allowance = ·0515 q.v. exact in this flute within $\frac{1}{5}$ of one mm.

Incremental All. No. 7 = ·0016, i.e. $\frac{·0175}{11}$ = ·0016 cumulative.

This allowance remains latent in this flute, not approaching the nodal point even at Hole 6: viz. ·0016 $\times$ 5 = ·008.

Nodal point ·014. Therefore the Modal Sequence follows a normal course without interruption.

EXIT

Exit at ·335.

The position of Hole 1 indicates one I.D. only from exit, the note of which should thus bear the ratio $\frac{B\ 12}{256}$ = 469·2 v.p.s. But the fundamental of Java II is a flattened *B* of 455·7 v.p.s., owing to Hole 1 having been placed ·009 higher than theory demands (a paradox! see further on). To be in tune with the modal sequence from Hole 1 as Vent, the flute should have measured from emb. to exit ·3243; it would then have given a fundamental $\frac{B12}{256}$ of 469·2 v.p.s. The distance is thus too long by ·009.

As the flute stands, its correct pitch at *B* = 455·7 v.p.s. may be verified by formula thus:

Allowance for diameter at exit by Formula No. 5

$$\begin{array}{ll} \Delta & = ·013 \\ \Delta - d & = ·0045\ (·013 - ·0085 = ·0045) \\ de. & \underline{·0015} \end{array}$$

$$\left.\begin{array}{r}\text{Allow. inalterable for a flute} \\ \text{of this diameter}\end{array}\right\}\ ·0190 \text{ allow. } \underline{\times\ 2 = ·038}$$

$$\begin{array}{rl} \text{L.} = & ·335 \text{ at exit} \\ + & \underline{·038} \text{ Eff. Allow.} \\ & ·373 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \end{array}$$

BY FORMULA NO. 1

$\frac{340\text{m./s.}}{·373 \times 2} = \frac{340}{·746}$ = 455·7 v.p.s. which is the v.f. of the flute at exit.

If used for a modal sequence of determinant 12, the exit note, as flattened *B* 12 of 455·7 v.p.s. instead of 469·2, would distort the Harmonia of the Dorian Spondaic, which proceeds from the Vent at Hole 1, on $\frac{C\ 11}{512}$ in perfect tune.

To give the $\frac{B\ 12}{256}$ in correct intonation, the flute would have to be at least ·009 shorter, as shown by Formula No. 2, viz. ·3243

$$\frac{340}{469·2 \text{ v.p.s.} \times 2} = \frac{340}{938·4 \text{ v.p.s.}} = \begin{array}{rl} & ·3623 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{·0380} \text{ Eff. All.} \\ & ·3243 \text{ Act. L.} \end{array}$$

The length, however, could not be altered after the fingerholes had been bored without causing distortion of the Harmonia.

therefore

the pitch of exit note, as determined by theory, must be, and is, of a frequency of 455·7, of an effective half-wave length of ·373 with an allowance—fixed by formula from the diameter of the bore—at ·038 as shown above.

It is clear that since the I.D. taken 12 times (as determinant of modal genesis)

= ·336, a flute having a length of ·335 could carry no allowance. Moreover, the Vent, as Modal Tonic at ·2805 = I.D. × 10 while playing the sequence of determinant 11, carries an allowance equal to one I.D. It is our task now to discover how nearly this allowance approximates to the amount required by theory.

I.D. = ·028 × 11	= ·308	
length emb. to H. 1	− ·2805	
	·0275	allowance actually available theoretically at Vent

Act. Position of Hole 1 from exit	·0545	
I.D.	− ·028	
	·0265	difference 1 mm.

It is seen, however, that this position of Hole 1 does not conform with the one demanded by formula.

POSITION OF HOLE 1 AT ·2805 FROM EMBOUCHURE

By Formula No. 3

$\frac{\Delta}{2}$	= ·0065
$\Delta - d$ (·013 − ·0085)	= ·0045
$\Delta - \delta$ (·013 − ·0065)	= ·0065
Allowance	·0175
One I.D.	+ ·028
Distance exit to H. 1	= ·0455

by theory actual distance = ·0545 (difference ·009 as shown below).

PITCH, EFFECTIVE HALF-WAVE LENGTH AND STANDARD ALLOWANCE AT HOLE 1 AS VENT

Hole 1 at ·2805 from emb., Ratio $\frac{11}{11}$; Note *C* 11 = 512 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \times 2} = \frac{340}{1024 \text{ v.p.s.}} = \begin{array}{rl} \cdot332 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot2805 & \\ \hline \cdot0515 & \text{Standard All.} \end{array}$$

Standard Allowance at Vent by Formula No. 4

2Δ	= ·026	
$2(\Delta - d)$	= ·009	Fl. All. = ·0047 × 11 = ·0517
de	·0015	
$2(\Delta - \delta)$	= ·013	
de	·0015	
	·0510 correct	

Inc. All. No. 7 = ·0016 latent

The position of Hole 1, as determined by theory (and tested in practice repeatedly by the present writer), is at ·0455, whereas in actuality it is nearer the emb., viz. at

·0545	Act. distance from exit
− ·0455	Theoretical distance
·009	excess length of flute according to the requirements of the Vent.

The Floating Allowance in relation to the Standard Allowance at Vent

Floating Allowance = ·0302 Eff. I.D.
− ·0255 Prop. I.D.
·0047 Floating All.

Fl. All. = ·0047 × 11 = ·0517 Allow. at Hole 1.

Hole 2 at ·253 from emb., Ratio $\frac{11}{10}$; Note $\frac{D\ 10}{512}$ = 563·2 v.p.s.

BY FORMULA NO. 2

$\frac{340 \text{ metres}}{\text{v.f. } 563{\cdot}2 \times 2} = \frac{340}{1126{\cdot}4 \text{ v.p.s.}} =$ ·3018 Eff. ½ w.l.
− ·253 Act. L.
− ·0488 (·049)

Prop. All.

$\frac{{\cdot}0515 \times 10}{11} =$ ·0468 Prop. All.
\+ ·0025 Actual—Prop. I.D.
·0493 Difference = half of one mm.

Inc. All. No. 7 = ·0016 latent

Hole 3 at ·2245 from emb., Ratio $\frac{11}{9}$; Note $\frac{E\ 18}{512}$ = 625·6 v.p.s.

BY FORMULA NO. 2

$\frac{340 \text{ metres}}{\text{v.f. } 625{\cdot}6 \times 2} = \frac{340}{1251{\cdot}2 \text{ v.p.s.}} =$ ·2718 Eff. ½ w.l.
− ·2245 Act. L.
·0473 All. at H. 3

Prop. All.

$\frac{{\cdot}0515 \times 9}{11} =$ ·0421 Prop. All.
\+ ·0050 Act. − Prop. I.D. × 2
·0471 Adjusted All.

Fl. All. ·0047 × 9 M.D. at H. 3 = ·0423 Prop. All. at H. 3

Inc. All. No. 7 ·0016 × 2 = ·0032 latent

Hole 4 at ·197 from emb., Ratio $\frac{11}{8}$; Note $\frac{F\ 16}{512}$ = ·704 v.p.s.

BY FORMULA NO. 2

$\frac{340 \text{ metres}}{\text{v.f. } 704 \times 2} = \frac{340}{1408} =$ ·2414 Eff. ½ w.l.
− ·197 Act. L.
·0444
\+ ·0005 (I.D. only ·0275)
·0449 All. at H. 4

Prop. All.

$\frac{{\cdot}0515 \times 8}{11} =$ ·0374 Prop. All.
\+ ·0075 Act. − Prop. I.D. = ·0025 × 3
·0449 exact

Inc. All. No. 7 = ·0016 × 3 = ·0048 latent

Hole 5 at ·169 from emb., Ratio $\frac{11}{7}$; Note $\frac{G\,7}{512}$ = 804·4 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ metres}}{\text{v.f. } 804{\cdot}4 \times 2} = \frac{340}{1608{\cdot}8 \text{ v.p.s.}} = \begin{array}{rl} \cdot 2113 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 169} & \text{Act. L. at H. 5} \\ \underline{\underline{\cdot 0423}} & \text{All. at H. 5} \end{array}$$

Prop. All.

$$\frac{\cdot 0515 \times 7}{11} \begin{array}{rl} = \cdot 0327 & \text{Prop. All. at H. 5} \\ + \underline{\cdot 0100} & \text{Act. — Prop. I.D. } = \cdot 0025 \times 4 \\ \underline{\underline{\cdot 0427}} & \text{Prop. All. adjusted} \end{array}$$

Fl. All. = ·0047 × 7, M.D. at H. 5 = ·0329 = Prop. All. at H. 5.

Inc. All. No. 7 = ·0016 × 4 = ·0064 latent

Hole 6 at ·1415 from emb., Ratio $\frac{11}{6}$; Note $\frac{B\,12}{512}$ = 938·4 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ metres}}{\text{v.f. } 938{\cdot}4 \times 2} = \frac{340}{1876{\cdot}8 \text{ v.p.s.}} = \begin{array}{rl} \cdot 1812 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{\cdot 1415} & \text{Act. L. at H. 6} \\ \underline{\underline{\cdot 0397}} & \text{All. at H. 6} = \cdot 040 \end{array}$$

Prop. All.

$$\frac{\cdot 0515 \times 6}{11} = \begin{array}{rl} \cdot 0281 & \text{Prop. All. at H. 6} \\ + \underline{\cdot 0125} & \text{Act. — Prop. I.D. } = \cdot 0025 \times 5 \\ \underline{\underline{\cdot 0406}} & \text{Prop. All. adjusted} \end{array}$$

Fl. All. = ·0047 × 6 M.D. at H. 6 = ·0282 = Prop. All. at H. 6.

Inc. All. No. 7 = ·0016 × 5 = ·008 [1] latent

THE EFFECTIVE I.D., DERIVED FROM THE EFFECTIVE HALF-WAVE LENGTH, DIVIDED BY THE MODAL DETERMINANT

Fingerhole	*Eff. ½ w.l.*	*Eff. I.D.* $\left(\frac{\cdot 373}{12} = \cdot 031\right)$
Exit	·373	·031
Hole 1	·332	·041
Hole 2	·3018	·0302
Hole 3	·2718	·0300
Hole 4	·2414	·0304
Hole 5	·2113	·0301
Hole 6	·1811	·0302

N.B.—The effective ½ w.l. for *B* 12 = ·3623, the Eff. I.D. should be $\frac{\cdot 3623}{12}$ = ·0302, and the Eff. I.D. from Vent = $\frac{\cdot 332}{11}$ = ·03018.

The discrepancies are only apparent; they are caused by the fact that Java II is a few mm. longer than it should be, so that the M.D. 12 assumed is not exact for the exit note.

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[1] *N.B.*—The Modal Sequence is thus allowed to run its course without interruption; for the Incremental Allowance No. 7 has not approached the nodal point at ·014.

MODAL FLUTE RECORDS: No. 10

Long White Vertical Flute (scratched zig-zag lines with red dots).

6 Fingerholes

Class IIIA

Modal Determinant 16

Hypodorian Harmonia on F 16 from Vent

Modal Sequence (Chromatic Genus)

Hole	1	2	3	4	5	6	
Ratios	16/16	15	14	12	11	10	(8)
Cents		112	119	267	151	165	

MEASUREMENTS		FINGERHOLES AND I.D.			
L. exit to emb.	·569 ;	C. H. 1 from emb.	·464 ;	from exit	·105
L. emb. to C. of Hole 1	·464 ;	C. H. 2 „ „	·437 ;	from C. H. 1	·027
L. exit to C. of Hole 1	·105 ;	C. H. 3 „ „	·410 ;	from C. H. 2	·027
Δ	·013 ;	C. H. 4 „ „	·339 ;	from C. H. 3	·071 [1]
δ	·008 ;	C. H. 5 „ „	·312 ;	from C. H. 4	·027
de	·002	C. H. 6 „ „	·2855 ;	from C. H. 5	·0265

Increment of Distance ·027 mean.

The Modal Determinant from Vent is 16.

The Harmonia in chromatic genus is Hypodorian.

On F = 176 difficult to obtain except as overblown Octave.

THE THREE INCREMENTS OF DISTANCE

No. 1.	The Actual (Act. I.D.)	·027
No. 2.	The Proportional from Vent $= \frac{·464}{16}$	·029
No. 3.	The Effective ½ w.l. I.D. from Vent $= \frac{·4829}{16}$	·0302

Inc. All. No. 7 $= \frac{·028}{16}$ = ·00175 Inc. All. No. 7 (cumulative) per increment.

N.B.—The Actual I.D. (·027) not only carries no allowance, but is less by ·002 than the Prop. I.D. (·029). This difference of ·002 is cumulative per I.D. and must be deducted for each fingerhole in order to balance the Prop. Allowance.

The Inc. All. No. 7 = ·00175 remains latent: for ·00175 × 7 increments (= 2 between Holes 3 and 4) = ·01225 ; I.D. $= \frac{·027}{2}$ = ·0135.

There is, therefore, no break in the modal sequence.

Exit Allowance, by Formula No. 5

Δ exit	·013	
Δ emb.	·013	
de	·0025	
	·0285	exit allowance
3 I.D. (latent)	+ ·081	
	·1095	Theor. dist. between exit and C. H. 1
	− ·105	Actual dist. from exit to H. 1
	·0045	

[1] The I.D. ·071 between Holes 3 and 4 contains two I.D. latent = ·054 + allowance of ·017.

If the exit note were to be used as fundamental, the 3 I.D. would produce a M.D. 19, unused as a Harmonia.

The modal sequence is based on the Vent, Hole 1, which is correctly placed according to theory, as shown below.

POSITION OF HOLE 1

By Formula No. 3

$\frac{\Delta}{2}$	$=\cdot 0065$
Δ (– d) no emb.	$=\cdot 013$
$\Delta - \delta$	$\cdot 005$
3 I.D. from exit	$\cdot 081$
	$\cdot 1055$ actual dist. of C. of H. 1 from Exit.

Standard Allowance

2Δ	$\cdot 026$
$2(\Delta - \delta)$	$\cdot 010$
$2de$	$\cdot 005$
St. Allowance	$\cdot 041$

Hole 1 at ·464, Note *F* 16 = 352 v.p.s.; Ratio 16/16

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 352 \times 2} = \frac{340}{704 \text{ v.p.s.}} = \begin{array}{rl} \cdot 4829 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 464 & \text{Actual L. at Vent} \\ \cdot 0189 & \text{All. at Vent} \end{array}$$

Hole 2 at ·437, Note *G* 15 = 375·4 v.p.s.; Ratio 16/15.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 375{\cdot}4 \times 2} = \frac{340}{750{\cdot}8} = \begin{array}{rl} \cdot 4528 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 437 & \text{Act. L.} \\ \cdot 0158 & \text{All. at H. 2} \end{array}$$

Prop. All.

$$\cdot 041 \times 15/16 = \left\{ \begin{array}{l} \frac{\cdot 0384}{2} \\ = \\ \cdot 0192 \end{array} \right.$$

	·0192	Prop. All.
–	·004	Diff. between Prop. I.D. and Actual I.D. × 2 for two Holes
	·0152	Adjusted Prop. All.

Hole 3 at ·410, Note *G* 14 = 402·2 v.p.s.; Ratio 16/14 (8/7).

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 402{\cdot}2 \times 2} = \frac{340}{804{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} \cdot 4226 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \cdot 410 & \text{Act. L.} \\ \cdot 0126 & \text{All. at H. 3} \end{array}$$

Prop. All.

$$\cdot 041 \times 7/8 = \left\{ \begin{array}{l} \frac{\cdot 036}{2} \\ = \\ \cdot 018 \end{array} \right.$$

	·018	Prop. All.
–	·006	Diff. Pr. I.D. – act. I.D. × 3 for 3 holes
	·012	Adjusted Prop. All.

Hole 4 at ·339, Ratio 16/12; Note *B* 12 = 469·2 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3623 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}339} & \text{Act. L.} \\ {\cdot}0233 & \text{All. at H. } 4 \end{array}$$

Prop. All.

$$\frac{{\cdot}041 \times 3}{4} = \begin{array}{rl} {\cdot}03075 & \text{Prop. All.} \\ - \underline{{\cdot}010} & \text{Diff. Pr. I.D.} - \text{Act. I.D.} \times 5 \\ {\cdot}02075 & \\ + \underline{{\cdot}002} & \text{Extra All. on flute between H. 3 and 4} \\ {\cdot}02275 & \text{Adjusted Prop. All.} \end{array}$$

Hole 5 at ·312, Ratio $\frac{16}{11}$; Note *C* 11 = 512 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \times 2} = \frac{340}{1024} = \begin{array}{rl} {\cdot}332 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}312} & \text{Act. L.} \\ {\cdot}020 & \text{All. at H. } 5 \end{array}$$

Prop. All.

$$\frac{{\cdot}041 \times 11}{16} = \begin{array}{rl} {\cdot}028 & \text{Prop. All.} \\ - \underline{{\cdot}012} & \text{Diff. Prop. I.D.} - \text{Actual I.D.} \times 6 \\ {\cdot}016 & \\ + \underline{{\cdot}004} & \text{Extra All. on Flute for two I.D. at H. 3–4} \\ {\cdot}020 & \text{Adjusted Prop. All.} \end{array}$$

Hole 6 at ·2855, Note *D* 10 = 563·2 v.p.s.; Ratio 16/10.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 563{\cdot}2 \times 2} = \frac{340}{1126{\cdot}4} = \begin{array}{rl} {\cdot}3018 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}2855} & \text{Act. L.} \\ {\cdot}0163 & \text{All. at H. } 6 \end{array}$$

Prop. All.

$$\cdot041 \times 10/16\ (= 5/8) = \begin{array}{rl} {\cdot}025 & \text{Prop. All.} \\ - \underline{{\cdot}014} & \text{Diff. Pr. I.D.} - \text{Act. I.D.} \times 7 \\ {\cdot}011 & \\ + {\cdot}006 & \text{Extra All. on flute between H. 3 and 4 fo} \\ & \text{3 I.D.} \\ \hline {\cdot}017 & \end{array}$$

EFFECTIVE I.D. OF HALF-SOUND WAVE

	Eff. ½ *w.l.*	*I.D.*	
Hole 1	·5178	·0324	
Hole 2	·4854	·0324	
Hole 3	·4529	·0325	
Hole 4	·3968	·0561	} = $\frac{{\cdot}0969}{3}$ = ·0323
Hole 5	·356	·0408	
Hole 6	·3236	·0324	

Between Holes 3 and 5 there is an Eff. L. of ·0969 for 3 I.D. = ·0323 per I.D

The actual distance on the flute between Holes 3 and 4 is ·071, an excess over 2 I.D. (= ·054) of ·017. The two I.D. (with the hole due for ratio 13 missing) carry an excess allowance of $\frac{\cdot 017}{6}$ *actually on the flute*, a sufficient additional provision for the 3 I.D. between exit and Hole 1 and the 6 I.D. between Hole 1 and 6 (inclusive of two between Holes 3 and 4). Hence the exact correspondences between theory and practice in this flute.

MODAL FLUTE RECORDS: No. 13

JAPANESE

Loaned[1] *by Miss V. C. C. Collum. 7 Fingerholes*

Class IIA

Modal Determinant (Vent) 13

Lydian Harmonia

Modal Sequence

Vent Hole	1		2		3		4		5		6		7
Ratios	13/13		12		11		10		9		8		7
Cents		*138·5*		*151*		*165*		*182*		*204*		*231*	

Plays from Vent on C 11 = 256 v.p.s.

MEASUREMENTS	
Total length, exit to emb.	·373 ;
L. C. of Hole 1 to emb.	·294 ;
L. exit to C. of Hole 1	·079 ;
Δ	·011 ;
δ mean (·0085 × ·0065)	·008 ;
d	·011 ;
de negligible	·0005

FINGERHOLES AND I.D.				
C. H. 1	from emb.	·294 ;	from exit	= ·079
C. H. 2	,, ,,	·269 ;	from C. H. 1	= ·025
C. H. 3	,, ,,	·245 ;	from C. H. 2	= ·024
C. H. 4	,, ,,	·220 ;	from C. H. 3	= ·025
C. H. 5	,, ,,	·196 ;	from C. H. 4	= ·024
C. H. 6	,, ,,	·171 ;	from C. H. 5	= ·025
C. H. 7	,, ,,	·146 ;	from C. H. 6	= ·025
				6)·148
				·02466

Increment of Distance (mean) ·02466.
(practical) ·025.

·025 (I.D.) × M.D. 13 = ·325.

THE THREE INCREMENTS OF DISTANCE

Actual I.D.	·025
Proportional I.D. = (Vent) = $\frac{\cdot 294}{13}$	·0226
Effective I.D. on ½ w.l. = $\frac{\cdot 332}{13}$	·0255
Floating All.	·0005
Inc. All. No. 7 $\frac{\cdot 0226}{13}$	·00173

remains latent throughout.

In this flute the Actual I.D. ·025 carries its own allowance of ·0024, which must be added per I.D. to the Prop. All. as an adjustment, in comparing it with the Allowance computed by Formula 2.

[1] Tested as on this record 22/9/33.

POSITION OF HOLE 1

$$\begin{array}{lrl}
\dfrac{\Delta}{2} & & \cdot0055 \\
\Delta - d & & \cdot011 \\
\Delta - \delta & & \underline{\cdot0025} \\
 & & \cdot019 \\
2\ (\text{I.D.}) & + & \underline{\cdot050} \\
\text{Theoretical position} & & \cdot069
\end{array}$$

(actual position from exit at ·079 incommensurable)

STANDARD ALLOWANCE AT VENT

$$\begin{array}{ll}
2(\Delta) & = \cdot022 \\
\Delta - d\ (\text{emb.} = \Delta) & \quad \cdot011 \\
2\Delta - \delta & \quad \cdot005 \\
(de \text{ negligible}) & \\
 & \quad \overline{\cdot038} \text{ Standard All. at Vent}
\end{array}$$

Hole 1 at ·294, Note *C* = 512 v.p.s.; Ratio 13/13.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 512 \times 2} = \frac{340}{1024 \text{ v.p.s.}} = \begin{array}{rl} & \cdot332 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{\cdot294} \text{ Act. L.} \\ & \cdot038 \text{ All. at H. 1 and Prop. All.} \end{array}$$

Hole 2 at ·269, Note *D* = 554·6; Ratio 13/12.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 554{\cdot}6 \times 2} = \frac{340}{1109{\cdot}2} = \begin{array}{rl} & \cdot3065 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{\cdot269} \text{ Act. L.} \\ & \cdot0375 \text{ All. at H. 2} \end{array}$$

Prop. All.

$$\begin{array}{lrll}
\cdot038 \times 12/13 = & & \cdot035 & \text{Prop. All.} \\
 & + & \underline{\cdot0024} & \text{All. borne by I.D.} \times 1 \\
 & & \cdot0374 & \text{Adjusted Prop. All.}
\end{array}$$

Inc. All. latent throughout, i.e. ·00173 × 6 I.D. = ·01038

$$\mathbf{\frac{I.D.}{2} = \cdot0125}$$

Hole 3 at ·245, Note *E* (= 512 v.p.s. × 13/11) = 605·1 v.p.s.; Ratio 13/11.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 605{\cdot}1 \times 2} = \frac{340}{1210{\cdot}2 \text{ v.p.s.}} = \begin{array}{rl} & \cdot2809 \text{ Eff. } \tfrac{1}{2} \text{ w.l.} \\ - & \underline{\cdot245} \text{ Act. L.} \\ & \cdot0359 \text{ All. at H. 3} \end{array}$$

Prop. All.

$$\begin{array}{lrll}
\cdot038 \times 11/13 = & & \cdot0321 & \text{Prop. All. at H. 3} \\
 & + & \underline{\cdot0048} & \text{All. borne by 2 (I.D.)} \\
 & & \cdot0369 & \text{Adjusted Prop. All.}
\end{array}$$

Hole 4 at ·220, Note *F* = 665·6 v.p.s. ; Ratio 13/10.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 665{\cdot}6 \times 2} = \frac{340}{1331{\cdot}2} = \begin{array}{rl} \cdot2554 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{\cdot220} & \text{Act. L.} \\ \text{[1]}\ \cdot0354 & \text{All. at H. 4} \end{array}$$

Prop. All.

$$\frac{\cdot038 \times 10}{13} = \begin{array}{rl} \cdot0292 & \text{Prop. All.} \\ +\ \underline{\cdot0072} & \text{All. borne by 3 (I.D.)} \\ \cdot0364 & \text{Adjusted Prop. All.} \end{array}$$

Hole 5 at ·196, Note *G* = 737·2 v.p.s. ; Ratio 13/9.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 737{\cdot}2 \times 2} = \frac{340}{1474{\cdot}4} = \begin{array}{rl} \cdot2306 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{\cdot196} & \text{Act. L.} \\ \cdot0346 & \text{All. at H. 5} \end{array}$$

Prop. All.

$$\cdot038 \times 9/13 = \begin{array}{rl} \cdot0263 & \text{Prop. All.} \\ +\ \underline{\cdot0096} & \text{All. borne by 4 (I.D.)} \\ \cdot0359 & \text{Adjusted All.} \end{array}$$

Hole 6 at ·171, Note *A* 13 = 832 v.p.s. ; Ratio 13/8.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 832 \times 2} = \frac{340}{1664\ \text{v.p.s.}} = \begin{array}{rl} \cdot2043 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{\cdot171} & \text{Act. L.} \\ \cdot0333 & \text{All. at H. 6} \end{array}$$

Prop. All.

$$\cdot038 \times 8/13 = \begin{array}{rl} \cdot0234 & \text{Prop. All.} \\ +\ \underline{\cdot0120} & \text{All. borne by 5 (I.D.)} \\ \cdot0354 & \text{Adjusted Prop. All.} \end{array}$$

A difference of 2 mm. due to the two I.D. at ·024 instead of ·025 assumed by the Prop. All.

Hole 7 at ·146, Note *B* = 950·8 v.p.s. ; Ratio 13/7.

BY FORMULA NO. 2

$$\frac{340\ \text{m./s.}}{\text{v.f. } 950{\cdot}8 \times 2} = \frac{340}{1901{\cdot}6\ \text{v.p.s.}} = \begin{array}{rl} \cdot178 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\ \underline{\cdot146} & \text{Act. L.} \\ \cdot032 & \text{All. at H. 7} \end{array}$$

Prop. All.

$$\cdot038 \times 7/13 = \begin{array}{rl} \cdot0204 & \text{Prop. All.} \\ +\ \underline{\cdot0144} & \text{All. borne by 6 (I.D.)} \\ \cdot0348 & \text{Adjusted Prop. All.} \end{array}$$

[1] It will be noticed that at Hole 4 the I.D. is only ·024, 1 mm. less than mean, while the Prop. All. assumes equal I.D.

EFFECTIVE I.D. OF HALF-SOUND WAVE

	½ *w.l.*	*I.D.*
Exit	·4287	
Hole 1	·332	·0255 M.D. 13 at Vent
Hole 2	·3065	·0255
Hole 3	·2809	·0255
Hole 4	·2554	·0255
Hole 5	·2306	·0248
Hole 6	·2043	·0263
Hole 7	·178	·026

MODAL FLUTE RECORDS: No. 20

BALI No. 20

Notched Flute. 7 Fingerholes

Presented by M. Soekawati

Class IB from Exit

Modal Determinant 11 exit

Dorian Harmonia

Modal Sequence

Exit	Hole 1	2	3	4	5	6	7
Ratios 11/11	10	9	8	15/22	13	12	11
Cents	*165*	*182*	*204*	*112*	*247*	*138·5*	*151*

Class IB

The I.D. (= ·033) is contained eleven times in the total length of the flute + ·007 (·033 × 11 = ·363). The I.D. thus carries no allowance. The Series is interrupted at Hole 4, where ratio 15/22 is produced instead of 7/11.

MEASUREMENTS

L. from C. of fipple (or notch) to exit	·370 ;
L. from C. of fipple to C. of Hole 1	·300 ;
C. of Hole 1 to exit	·070 ;
Δ of bore	·023 ;
Centre of fipple	·0035 ;
d ,,	·005 ;
δ of 6 front fingerholes	·007 ;
δ of thumb hole (back)	·006 ;
I.D. (useful mean) ·033	·033 ;
M.D. from exit	11

FINGERHOLES AND I.D.

From fipple to exit	·370
C. H. 1 from C. fipple ·300 ; from exit	·070
C. H. 2 from C. fipple ·267 ; from C. Hole 1	·033
C. H. 3 from C. fipple ·235 ; from C. H. 2	·032
C. H. 4 from C. fipple ·202 ; from C. H. 3	·033
C. H. 5 from C. fipple ·168 ; from C. H. 4	·034
C. H. 6 from C. fipple ·149 ; from C. H. 5	·019 } I.D.
C. H. 7 from C. fipple ·136 ; from C. H. 6	·013 } I.D.
	7·)234
	·0334

Increment of Distance, mean = ·0334.
useful mean ·033.

I.D. = ·033 × 11 = ·363.

The length from exit to C. Hole 1, although amounting to two I.D., only represents one I.D., the remainder being allowance at exit. Holes 6 and 7 divide one I.D. unevenly between them ·019 + ·013 = ·032.

THE THREE INCREMENTS OF DISTANCE

Actual I.D. between the Fingerholes (mean)	·033
Proportional I.D. at exit $= \frac{·370}{11}$	·0336
Difference between Actual and Prop. I.D.	·0006
Effective ($\frac{1}{2}$ w.l.) I.D. = ·457/11 eff. $\frac{1}{2}$ w.l. at exit	·0415
Floating Allowance ·0415 Eff. I.D. − ·0336 Prop. I.D.	
Per I.D. not cumulative ·0079	·0079

Inc. All. No. 7, $\frac{·0435}{11}$, Exit All. ·00395 = ·004.

N.B.

The actual I.D. on Bali I = ·033 is the least of the three increments; the Proportional I.D. carries no allowance, and No. 1 (actual) not only carries none, but falls below the Proportional I.D. by ·0006, which should therefore be added cumulatively hole by hole to balance the Proportional Allowance, since it means displacement of each hole by ·0006. The displacement of Holes—due to Hole 1 being placed at ·0065 too near exit, therefore too low, must be taken into account in the analysis. Standard Allowance at ·118 is based on the actual position of Hole 1; ·120 on the theoretical.

EXIT

L. from C. of fipple to exit = ·370

Note $\frac{G\ 15}{22}$ (flattened by one v.p.s.) = 372 v.p.s.

BY FORMULA NO. 2

$\frac{340 \text{ m./s.}}{\text{v.f. } 372 \times 2} = \frac{340}{744} =$ ·457 Eff. $\frac{1}{2}$ w.l.
− ·370 Act. L.
·087 Eff. All. (= ·0435 actual)

BY FORMULA NO. 5

Allowance at Exit

Δ of bore	= ·023
Δ − d	= ·018
de	·002
	·043 All. at exit in agreement with the All. by Formula No. 2

POSITION OF HOLE 1 FROM EXIT

by Formula No. 3

$\frac{\Delta}{2}$	·0115
Δ − d	·018
Δ − δ	·016
	·0455 All. at H. 1
	+ ·033 One I.D.
	·0785 Theoretical position of H. 1
	− ·070 Actual position of H. 1
	·0085 too near exit

Hole 1 at ·300 from fipple, Ratio 11/10; Note v.f. $\frac{375{\cdot}4 \times 11}{10} = 412{\cdot}9$ v.p. (= 413).

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 413 \times 2} = \frac{340}{826 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}4116 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}300} & \text{Act. L. from fipple} \\ {\cdot}1116 & \text{All. at H. 1} \end{array}$$

and ·1116 All. at H. 1
+ ·0076 Floating All. per Hole
·1192 Effective All. at H. 1 (cf. Standard All. ·118)
·070 Act. position of H. 1 from exit
·0085 too near exit, therefore length from fipple to exit should be

$$\begin{array}{rl} {\cdot}300 & \\ - \underline{{\cdot}0085} & \\ {\cdot}2915 & \text{virtual position of H. 1 from fipple} \end{array}$$

BY FORMULA NO. 4

Standard Allowance at Hole 1

$$\begin{array}{ll} 2(\Delta) & = {\cdot}046 \\ 2(\Delta - d) & = {\cdot}036 \\ 2(\Delta - \delta) & = {\cdot}032 \\ 2\,de & \quad \underline{{\cdot}004} \\ & \quad {\cdot}118 \text{ Standard All. No. 4} \end{array}$$

N.B.—The results of analysis based upon the actual position of Hole 1 may be compared with those derived from theory. The Standard Allowance No. 4 is based upon the Allowances in respect of diameter—with all its implications—at Hole 1. It forms the basis of the Proportional Allowance, computed hole by hole, as allocated by the ratio of each hole to the fundamental of the sequence. To balance the effective allowance for each hole, derived by Formula No. 2 from pitch, the Floating Allowance must, for the Bali flute, be added at each Hole—not cumulatively.

Hole 2 at ·267, Note = v.f. 375·4 × 11/9 = 458·8 v.p.s.; Ratio $\frac{11}{9}$.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 458{\cdot}8 \times 2} = \frac{340}{917{\cdot}6} = \begin{array}{rl} {\cdot}3705 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}267} & \text{Act. L.} \\ \underline{{\cdot}1035} & \text{Eff. All. at H. 2} \end{array}$$

Prop. All.

$$\text{St. All. } {\cdot}118 \times 9/11 = \begin{array}{rl} {\cdot}0965 & \text{Prop. All. at H. 2} \\ + \underline{{\cdot}0076} & \text{Floating All.} \\ \underline{\underline{{\cdot}1041}} & \text{Adjusted Prop. All. at H. 2} \end{array}$$

Inc. All. No. 7 = ·004 × 2 = ·008 latent

Hole 3 at ·235, Ratio $\frac{11}{8}$; Note C = 516·17 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 516{\cdot}17 \times 2} = \frac{340}{1032{\cdot}34 \text{ v.p.s.}} = \begin{array}{rl} {\cdot}3293 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \underline{{\cdot}235} & \text{Act. L.} \\ {\cdot}0943 & \text{Eff. All. at H. 3} \end{array}$$

Prop. All.

Std. All. $\frac{\cdot 118 \times 8}{11} =$ $\cdot 0858$ Prop. All.
$+ \cdot 0076$ Fl. All.
$+ \cdot 0012$ Diff. Pr. I.D. − Act. I.D.
$\cdot 0946$ Adjusted Prop. All.

Inc. All. No. 7 = ·004 × 3 = ·012 latent

Hole 4 at ·202, Ratio $\frac{22}{15}$, Note *C*♯ = 550·66 v.p.s.

$\cdot 202$ Act. position of H. 4
$+ \cdot 0165 = \frac{\text{I.D.}}{2}$ Inc. All. No. 7 active at nodal point
$\cdot 2185$ Virtual position of H. 4

BY FORMULA NO. 2

$\frac{340 \text{ m./s.}}{\text{v.f. } 550{\cdot}66 \times 2} = \frac{340}{1101{\cdot}32} =$ $\cdot 3087$ Eff. ½ w.l.
$- \cdot 2185$ Virtual L. at H. 4
$\cdot 0902$ Eff. All.

Prop. All.

$\frac{\cdot 118 \times 15}{22} =$ $\cdot 0804$ Prop. All.
$+ \cdot 0076$ Fl. All.
$+ \cdot 0018$ Prop. I.D. − Act. I.D. = ·0006 × 3 increments
$\cdot 0898$ Adjusted Prop. All.

Inc. All. No. 7 = ·004 × 4 = ·016 active

Nodal point $\frac{\cdot 033}{2} = \cdot 0165$.

Hole 5 at ·168
$+ \cdot 017 \left(\frac{\text{I.D.}}{2}\right)$ at Hole 5, Ratio 22/13 = Note *E* = 635·3 v.p.s.
$\cdot 185$ Virt. position

BY FORMULA NO. 2

$\frac{340 \text{ m./s.}}{\text{v.f. } 635{\cdot}3 \times 2} = \frac{340}{1270{\cdot}6} =$ $\cdot 2648$ Eff. ½ w.l.
$- \cdot 185$ Virt. L. at H. 5
$\cdot 0798$ Eff. All. at H. 5

Prop. All.

St. All. ·118 × 13/22 = $\cdot 0695$ Prop. All.
$+ \cdot 0076$ Fl. All.
$+ \cdot 0024 \begin{cases}\text{Prop. I.D.} - \text{Act. I.D.} \\ = \cdot 0006 \times 4 \text{ increments}\end{cases}$
$\cdot 0795$

Inc. All. No. 7 = ·004 × 5 = ·020 active

Hole 6 at ·149 Half I.D. at Hole 6, Note of Hole 6, v.f. 375·4 × 11/6 = 688·2 v.p.s.
$+ \cdot 019$
$\cdot 168$ Virt. position

BY FORMULA NO. 2

$\frac{340 \text{ m./s.}}{688{\cdot}2 \times 2} = \frac{340}{1376{\cdot}4} =$ $\cdot 2472$ Eff. ½ w.l.
$- \cdot 168$ Virt. L. at H. 6
$\cdot 0792$ Eff. All. at H. 6

Prop. All.

St. All. ·118 × 6/11 = ·0643 Prop. All.
+ ·0076 Fl. All.
+ ·0075 Surplus[1] from Inc. All. No. 7
·0794

Inc. All. No. 7 = ·004 × 6 = ·024
Nodal Point at — ·0165
·0075 active surplus

Hole 7 at ·136, Note 375·4 × 2 = 750·8 v.p.s.
+ ·013 This hole gives the octave of the fundamental when all
·149 holes are open below it.

EFFECTIVE I.D. OF HALF-SOUND-WAVE M.D. 11

	Eff. L.	*I.D.*
Exit	·457	·0415
Hole 1	·4116	·0454
Hole 2	·3705	·0411
Hole 3	·3293	·0412
Hole 4	·3087	·0206 ½ I.D.
Hole 5	·2648	·0439
Hole 6	·2472	·0176 ½ I.D.
Hole 7	·2264	·0208 ½ I.D.

MODAL FLUTE RECORDS: No. 12

INCA FLUTE (KENA)

Presented by Miss Anita Berry

The Flute was copied in Bolivia from an ancient Inca Specimen

Class 1B Vent

Modal Determinant 11

Dorian Harmonia

Modal Sequence

Exit	Vent	H. 2	3	4	5	6
	11/11	10	9	8	15 (7)	13 (6)

Sequence interrupted at Hole 5, by the active operation of Inc. All. No. 7 at a nodal point.

Vent Note = $\frac{B\ 12}{256}$ as 11/11.

MEASUREMENTS

Total length, extremity at exit to emb.	·370 ;
L. emb. (fipple) to C. of Hole 1 (vent)	·327 ;
Δ of bore	·015 ;
L. of fipple or notch	·007 ;
Width of fipple	·007 ;
8 fingerholes	·007 ;
de	·0025

FINGERHOLES AND I.D.

C. H. 1 from emb.	·327 ;	from exit	·043
C. H. 2 ,, ,,	·297 ;	from C. H. 1	·030
C. H. 3 ,, ,,	·267 ;	from C. H. 2	·030
C. H. 4 ,, ,,	·237 ;	from C. H. 3	·030
C. H. 5 ,, ,,	·208 ;	from C. H. 4	·029
C. H. 6 ,, ,,	·177 ;	from C. H. 5	·031

[1] The surplus from Inc. All. No. 7 must be added to the Floating All. to balance the Prop. All.

Increment of Distance (**Mean**) **·030.**

The diameter of the bore ·015 has been constricted at exit to ·004, which exercises a lengthening effect of ·011 (·015 − ·004 = ·011) and that must be added to allowance at exit.

THE THREE INCREMENTS OF DISTANCE

Actual I.D.	·030
Proportional I.D. at Vent $= \frac{327}{11}$	·0297 (= ·030)
Effective ½ w.l. I.D. Vent $\frac{\cdot 3623}{11}$	·0329
Floating All. ·0329 Eff. ½ w.l. I.D. − ·0297 Prop. I.D. = ·0032 Floating All.	
Inc. All. No. 7 $\frac{\cdot 026}{13}$ (= Exit All.)	·002

N.B.—The actual I.D. is equal to the Prop. I.D. and therefore carries no allowance in respect of diameter: the Floating All. $\frac{\cdot 0032}{2}$ = ·0016 must be added to the Inc. All. No. 7 at each hole in order to estimate the activity of the sound-wave at the Nodal point which occurs at ·015.

Thus: Inc. All. No. 7 = ·002 × 5 I.D. = ·010
Fl. All. = ·0016 × 5 = ·008
·018

EXIT

Exit at ·370; Note *G* = 402·15 v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 402{\cdot}15 \times 2} = \frac{340}{804{\cdot}3} = \begin{array}{rl} \cdot 4227 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ -\cdot 370 & \text{Act. L.} \\ \hline \cdot 0527/2 = \cdot 026 & \text{Eff. All. at exit} \end{array}$$

Exit Allowance by Formula No. 5

Δ	·015	
Δ − ·004 at exit	·011	the Δ is constricted at exit to ·004; a lengthening of sound-wave (viz. ·015 − ·004 = ·011) therefore results
	·026	

POSITION OF HOLE 1

$\frac{\Delta}{2}$	·0075
Δ − δ	·008
Δ − d (width of fipple)	·008
All. at Vent	·0235
1 (I.D.)	+ ·030
Theoretical position at	·0535

Standard Allowance

$2(\Delta)$	$= \cdot 030$
$2(\Delta - \cdot 004 = \cdot 011)$	$= \cdot 022$
$2\Delta - \delta$	$\cdot 016$
$2\Delta -$ d (fipple width)	$\cdot 016$
2 *de*	$\cdot 004$
	$\cdot 088$ Eff.
	$\cdot 044$ Act.

Hole 1 at ·327, Ratio $\frac{11}{11}$; Note $B = 469{\cdot}2$ v.p.s.

BY FORMULA NO. 2

$$\frac{340 \text{ m./s.}}{\text{v.f. } 469{\cdot}2 \times 2} = \frac{340}{938{\cdot}4} = \begin{array}{rl} \cdot 3623 & \text{Eff. } \tfrac{1}{2} \text{ w.l.} \\ - \cdot 327 & \text{Act. L.} \\ \hline \cdot 0353 & \text{Eff. All. at H. 1} \\ + \cdot 011 & \text{for constriction} \\ \hline \cdot 0463 & \end{array}$$

Standard All. = ·044.

The other fingerholes may easily be worked out from the indications given in the preceding records.

THE V.F. OF THE FINGERHOLE NOTES

Hole 2	516·1 v.p.s.	Ratio	11/10
Hole 3	573·4 ,,	,,	11/9
Hole 4	645·1 ,,	,,	11/8
Hole 5	704 ,,	,,	22/15
Hole 6	804·3 ,,	,,	22/13

The exit note stands in no commensurable ratio with the Modal Sequence from Hole 1, and is therefore of no practical use.

RECORDS OF MODAL FLUTES: VERTICAL FLUTE (SYRINX MONOCALAMUS), No. 26

Greek Shepherd's Flute (Modern) from Olympia, in the Collection of Professor Dayton C. Miller, Case School of Applied Science, Cleveland, Ohio, U.S.A.

7 Fingerholes

*Class II*B *from Vent*

Modal Determinant 12

Phrygian Harmonia

Modal Sequence from Vent

Vent	1	2	3	4	5	6	7
Ratios	12/12	11	10	9	8	15 (7)	13 (6)
Cents		*151*	*165*	*182*	*204*	*112*	*247*

Note of Vent F 16 = 696 v.f.

N.B.—The Modal Sequence is interrupted at Hole 6 by the active operation of cumulative Inc. All. No. 7 at a nodal point, when instead of ratios 7 and 6 at Holes 6 and 7, ratios 15 and 13, respectively, due at a half increment lower, sound from those holes.

FLUTES NOS. 26 AND 27 FROM GREECE

The measurements of these two shepherd flutes from modern Greece unfortunately reached me too late to allow of facsimiles being made. The records are therefore incomplete and lack results of tests.

MEASUREMENTS FINGERHOLES AND I.D.

Supplied by courtesy of Professor Dayton C. Miller

Total L. exit to emb.	·2461 ;	C. of H. 1 from emb.	·2189 ;	from exit	·0272
Total L. C. of Hole 1 to emb.	·2189 ;	C. of H. 2 ,, ,,	·1975 ;	from C. H. 1	·0214
$\Delta + de$	·0155 ;	C. of H. 3 ,, ,,	·1771 ;	from C. H. 2	·0204
Δ of bore at emb.	·010 ;	C. of H. 4 ,, ,,	·1581 ;	from C. H. 3	·019
Δ of bore at exit	·011 ;	C. of H. 5 ,, ,,	·1392 ;	from C. H. 4	·0189
Knot near exit only partially bored through, opening at exit = ·005, ∴ Δ ·011 − ·005 = ·006, which must be added to form virtual length of flute		C. of H. 6 ,, ,,	·1298 ;	from C. H. 5	·0094
		C. of H. 7 ,, ,,	·1188 ;	from C. H. 6	·011
					5)·1001
					·020
δ (mean)	·0055				

Increment of Distance (mean) = ·020.
Modal Determinant 12.

THE THREE I.D.

(1) The Actual I.D.		·020
(2) The Prop. I.D. $\left(\frac{\cdot 2189}{12} = \cdot 01824\right)$		·01824
(3) The Eff. ½ w. I.D. $\left(\frac{\cdot 2443}{12} = \cdot 02036\right)$		·02036
(4) Inc. All. No. 7 $\left(\frac{\cdot 0254}{12}\right.$ All. at H. 1 $\left.= \cdot 0021\right)$		·0021
(5) No. 1 − No. 2 = ·02000 − ·01824	=	·00176
(6) No. 3 − No. 2 = ·02036 − ·01824	=	·00212 Floating All.
Nos. 5 + 6 = ·00176 + ·00036	=	·00212 = No. 4

The 6 factors are used to balance results of comparison theoretical and practical. See Records in full.

POSITION OF HOLE 1

By Formula No. 3

$\frac{\Delta}{2}$	=	·0055	
$\Delta(-d)$	=	·011	
$\Delta - \delta$	=	·0055	
		·0220	
1 (I.D.)	+	·0200	
		·0420	Theoretical position of H. 1
	−	·0272	Actual ,, ,, ,,
The Ratio between exit and Hole 1 is incommensurate		·0148	excess distance from emb. of Hole 1, and loss of Act. All.

RECORDS OF MODAL FLUTES: VERTICAL FLUTE (SYRINX MONOCALAMUS), No. 27

Greek Shepherd's Flute (Modern) from Nauplia, in the collection of Professor Dayton C. Miller, Case School of Applied Science, Cleveland, Ohio, U.S.A., by whose courtesy the measurements have been supplied

7 Fingerholes

Class 1A

Modal Determinant 11 (original) 22 (converted)

Dorian Harmonia (Spondaic)

Modal Sequence from Vent

Hole	1	2	3	4	5	6 thumb at ½ I.D.	7
Ratios	11/11	10	9	8	7	13/22	6
Cents		*165*	*182*	*204*	*231*	*128*	*138·5*

N.B.—The interpolation into the Modal Spondaic of ratio 13 at Hole 6, bored half-way (approximately) between Holes 5 and 7, is not to be regarded as interference due to Inc. All. No. 7 (as was the case with Flute No. 26). This is a deliberate extension of compass obtained by actually halving a segment between Holes 5 and 7 and introducing a thumb-hole. The fact that Inc. All. No. 7 reaches a nodal point at Hole 6 merely facilitates the playing of the intermediate ratio 13.

MEASUREMENTS

Total L. exit to emb.	·365 ;
Total L. C. of Hole 1 to emb.	·3226 ;
L. C. of Hole 1 to exit	·0424 ;
Δ bore at emb.	·0124 ;
Δ bore at exit	·0085 ;
Δ (bore + *de*) emb.	·019 ;
Δ (bore + *de*) exit	·018 ;

δ of fingerholes varies from ·0071 to ·0081 at H. 1 mean ·0075

In this flute the exit broadens out bell-shaped from an inner bore of ·0085 to ·018. The difference in the diameter of the bore between emb. and at bell exit does not affect the Modal Sequence.

FINGERHOLES AND I.D.

C. of H. 1 from emb.	·3226 ;	from exit	·0424
C. of H. 2 ,, ,,	·2936 ;	from C. H. 1	·029
C. of H. 3 ,, ,,	·2618 ;	from C. H. 2	·0318
C. of H. 4 ,, ,,	·227 ;	from C. H. 3	·0348
C. of H. 5 ,, ,,	·196 ;	from C. H. 4	·031
C. of H. 6 ,, ,,	·1787 ;	from C. H. 5 }	·0173
C. of H. 7 ,, ,,	·165 ;	from C. H. 6 }	·0137
			5)·1576
			·0315

Increment of Distance (mean) = ·0315.
(practical) = ·031.

THE THREE INCREMENTS OF DISTANCE IN FLUTE NO. 27

(1) Actual I.D. (mean useful)	·031
(2) Proportional I.D.	·0293
(3) Eff. ½ w. I.D. at Vent	·0319
Inc. All. No. 7 $\frac{\cdot 0288}{11}$	·0026
No. 3 − No. 2 = Floating All.	·0026

I.D. ·031 × M.D. 11 = ·341 − ·3226 L. at Vent = ·0184

M.D. from Eff. ½ w.l. $\frac{\cdot 3514}{\cdot 031} = 11.$

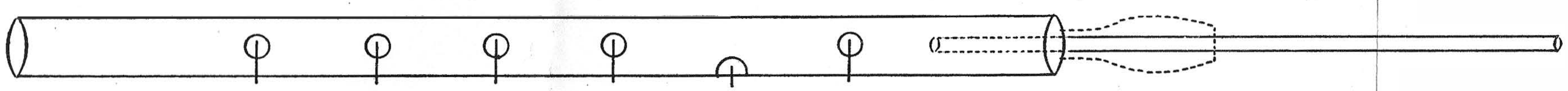
therefore

Flute No. 27 belongs to Class 1A.

TABLE X. SCALES OBTAINED FROM THE ELGIN AULOS (THE STRAIGHT ONE) AT THE BRITISH MUSEUM *by* KATHLEEN SCHLESINGER

(Type of Mp. used Double-reed)

Fingerholes 1 2 3 4 5 6 Reed or straw mp. *N.B.* 'Mp.' = Mouthpiece



Bulb cir. ·0425 in length

V.f. *A* 108·3 mp.s D.I. Cl. 6 Cl. 7 Cl. 18 N 1 — *C* 128, *D* 140·8, *E* 156·4, *F* 176, *G* 201, *B* 234·6 v.f.

$\left(\frac{13}{11}\right)$ $\frac{11}{11}$ $\frac{10}{11}$ $\frac{9}{11}$ $\frac{8}{11}$ $\frac{7}{11}$ $\frac{6}{11}$

Pulled out to extrusion ·108 to ·110 one bulb — *165 cents* *182* *204* *231* *267* *cents*

The 11 Mode on *C*. Through Hole 1 used as vent } left always open. Dorian

A 108·3 v.f. from exit $\frac{13}{13}$ — *C* 128 $\frac{11}{13}$, *D* 140·8 $\frac{10}{13}$, *E* 156·4 $\frac{9}{13}$, *F* 176 $\frac{8}{13}$, *G* 201 $\frac{7}{13}$, *B* 234·6 v.f. $\frac{6}{13}$

Mp. extrusion ·105 one bulb — *290 cents* *165 cents* *182* *204* *231* *267* *cents*

The 13 Mode on *A* from the exit. Lydian

Aulos + Mp. 'Elgin D. 10' and mp. H. 6 } fundamental at extrusion ·078 — v.f. *D* 140·8 $\frac{10}{10}$, *E* 156·4 $\frac{9}{10}$, *F* 176 $\frac{8}{10}$, *G* 201 $\frac{7}{10}$, *B* 234·6 $\frac{6}{10}$, *D* 281·6 v.f. $\frac{5}{10}$*

182 cents *204* *231* *267* *316* *cents*

The 10 Mode on *D* through Hole 1 used as vent

* If the mp. does not play Hole 6 easily, it will play the lower octave *D* from the 6th hole.

Aulos + Mp. Cl. 1 plays the same series on *A* 208·6 v.p.s. at extrusion ·078 no bulb also B.R. X at ·078 on $\frac{B\ 12}{64}$ whole sequence — v.f. *A* 208·6 $\frac{10}{10}$, *B* 231·7 $\frac{9}{10}$, *C* 260·7 $\frac{8}{10}$, *D* 298 $\frac{7}{10}$, *F* 347·6 $\frac{6}{10}$, *A* 417 v.f. $\frac{5}{10}$

182 cents *204* *231* *267* *316* *cents*

The 10 Mode on *A* Hypolydian

Aulos + Mp. Cl. 7 fundamental *A* = 208·6 v.p.s. pulled out to extrusion ·110 one bulb — v.f. *A* 208·6 $\frac{11}{11}$, *B* 229·5 $\frac{10}{11}$, *C* 255 $\frac{9}{11}$, *D* 287 $\frac{8}{11}$, *E* 327·8 $\frac{7}{11}$, *G* 382·4 v.f. $\frac{6}{11}$

165 cents *182* *204* *231* *267* *cents*

The 11 Mode on *A* through Hole 1 used as vent

Aulos + Mp. Cl. 7 fundamental *A* = 208·6 v.p.s. extrusion of Mp. ·138 two bulbs — v.f. *A* 208·6 $\frac{12}{12}$, *B* 227·5 $\frac{11}{12}$, *C* 250·3 $\frac{10}{12}$, *D* 278 $\frac{9}{12}$, *E* 312·9 $\frac{8}{12}$, *F* 357·6 v.f. $\frac{7}{12}$

151 cents *165* *182* *204* *231* *cents*

The 12 Mode on *A* through Hole 1 used as vent Phrygian

Aulos + Mp. Cl. 6 fundamental *B* = 117·3 v.p.s. at extrusion ·138 two bulbs — v.f. *B* 117·3 $\frac{12}{12}$, *C* 128 $\frac{11}{12}$, *D* 140·7 $\frac{10}{12}$, *E* 156·4 $\frac{9}{12}$, *F* 176 $\frac{8}{12}$, *G* 201 v.f. $\frac{7}{12}$

151 cents *165* *182* *204* *231* *cents*

The 12 Mode on *B* through Hole 1 used as vent

N.B.—The names of the Notes are approximate only: the v.f. gives the exact intonation.

Notes and Comments

The results of my tests, researches and experiments since August, 1914, prove conclusively that in the resonator of the Aulos, the factor of length is no criterion whatever of the pitch of the fundamental of the Aulos or of the notes obtained by opening the fingerholes, when the pipe is played by means of a double-reed, or of a beating-reed mouthpiece. The proper note of the mouthpiece, by accommodation through the law of resonance with the resonator or pipe of the Aulos, provides the fundamental produced through the exit or through the first hole used as vent and left open. It will be noticed from the Table that each pair of fingerholes gives out an interval of different ratio in each of the Modes, occasionally on the same fundamental (as with Cl. 7; or on a different fundamental, as with mouthpiece Cl. 6).

Bulbs

The complete bulb found with the Elgin Auloi measures *c.* ·0425. From the vase paintings, it is evident that these bulbs were used to hide the portion of the straw mouthpiece not retained in the piper's mouth. Auletes are shown putting on an extra bulb, or removing one according to the length of extrusion of the mouthpiece from the resonator demanded by the Mode.

POSITION OF HOLE 1

By Formula No. 3

$\frac{\Delta}{2}$ =	·0062	
Δ emb.	·012	
$\Delta - \delta$	·004	
	·0222	All. at H. 1 actually on the flute
+	·031	one I.D.
	·0532	Theoretical position of H. 1
—	·0424	Actual
	·0108	too near exit according to formula
Act. L. to H. 1 =	·3226	
2Δ at emb. and exit	·0248	
$\Delta - \delta$	·004	
Eff. ½ w.l. at H. 1	·3514	and by Formula No. 1 = 241·4 v.p.s.

TABLE XI

11 Flutes from N. Egypt

Selection from a Collection of 50 presented by Sir Robert Mond (Condensed Record)

No.	Total Length Diameter Allowance and Eff. ½ w.l.	Δ Exit Δ Emb. and Mean	Eff. L. of Half-Sound-Wave	Exit to Centre of Hole 1	Hole 1 as Vent	Increments of Distance between Fingerholes	Modal Determinant	Modal Sequence and Ratios (non-existent holes within brackets merely indicate the number of I.D. included between Exit and Hole 1)
2	·477 ·014 = Δ ·004 = *de* ·495	·013 ·015 mean Δ ·014	·495	·084	·392	·027 + ·026 + ·041 + ·026 useful mean ·026	18 Exit Hypophrygian $\frac{\cdot 477}{\cdot 026} = 18$	Ex. H. 1 (o o o) ● ● ● ● ● 18/18 15 14 13 23 22 $\underbrace{13\ 23}_{=9/8}$
3	·492 ·015 = Δ ·004 = *de* ·511	·014 ·017 mean ·015	·511	·086	·406	·031 + ·029 + ·058 + ·031 useful mean ·030	16 Hypodorian $\frac{\cdot 492}{\cdot 030} = 16$	Ex. H. 1 (o o o) ● ● ● ● ● 16/16 13 12 11 9 8
7	·541 ·023 = Δ ·004 = *de* ·568	·022 ·025 mean ·023	·568	·117	·424	·035 + ·031 + ·037 + ·031 useful mean ·033	16 Hypodorian $\frac{\cdot 541}{\cdot 033} = 16$	Ex. H. 1 (o o o) ● ● ● ● ● 16/16 13 12 11 10 9
11	·517 ·020 = Δ ·004 = *de* ·541	·02 ·02	·541	·095	·422	·030 + ·030 + ·048 + ·029 useful mean ·030	(17 Exit) 14 Vent Mixolydian	Ex. (o o o) ● ● ● ● ● 17 14/14 13 12 21 19 $\underbrace{12\ 21}_{=8/7}$

14	·478 ·015 = Δ ·004 = *de* ·497	·014	·015	·497	·092	·386	·029 + ·029 + ·052 + ·028 useful mean ·028	(17 Exit) 14 Vent Mixolydian	Ex. (O O O) ● ● ● ● ● 17 14/14 13 12 10 9
19	·458 ·016 = Δ ·004 = *de* ·478	·016	·017	·478	·084	·374	·030 + ·030 + ·055 + ·030 useful mean ·030	(15 Exit) 13 Vent Lydian	Ex. H. 1 (O O) ● ● ● ● ● 15 13/13 12 11 9 8
26	·494 ·017 = Δ ·004 = *de* ·515	·017	·017	·515	·090	·404	·030 + ·030 + ·061 + ·030 regular	16 Exit Hypodorian 13 Vent Lydian	Ex. H. 1 (O O O) ● ● ● ● ● 16/16 13 12 11 9 8
27	·564 ·018 = Δ ·004 = *de* ·586	·019	·018	·586	·116	·448	·035 + ·033 + ·059 + ·033 useful mean ·032	18 Exit Hypophrygian 14 Vent Mixolydian	Ex. H. 1 (O O O) ● ● ● ● ● 18/18 (16) (15) 14 13 12 10 9
37	·582 ·021 = Δ ·004 = *de* ·607	·021	·021	·607	·116	·416	·030 + ·034 + ·070 + ·032 useful mean ·032	18 Exit Hypophrygian 13 Vent	Ex. H. 1 (O O O) ● ● ● ● ● 18/18 (16) (15) 13 12 11 9 8
38	·580 ·016 = Δ ·004 = *de* ·600	·016	·016	·600	·110	·470	·030 + ·033 + ·098 + ·034 useful mean ·033	18 Exit Hypophrygian	Ex. H. 1 (O O O) ● ● ● ● ● 18/18 14 13 12 10 9

TABLE

RECORD OF

of the Notched Flutes of the

Presented by

No. and Class of Flute	Length from Exit f = fipple	Length from Vent to Exit	Diameter (Δ)	Fipple Width or Notch	Δ of Finger-holes	I.D.	M.D.	Note of Fundamental Exit or Vent
No. 1 Class IIIA	·395 emb. f. ·386	·062 Exit	·018 emb. ·017 Exit	·007	·007	·037	9	$\frac{B}{256}$
No. 2 Class IA	·345 ·340 fipple	Exit to Vent ·064	·011 All. = ·019	·006	·008	·036	9	$\frac{B\ 12}{256}$
No. 3 Class IIA	·330 from f. ·320	·276 Vent to Exit ·054	·013	·004	·006	·037	9	$\frac{B\ 23}{256}$
No. 4 Class IIA	·337 from Notch ·330	·276 Vent to Exit ·054	·015 emb. ·016 Exit	·005	·006	·041 (mean)	9	$\frac{B\ 12}{256}$
No. 5 Class IIA	·341 ·332 f.	·279 Vent to Exit ·053	·016 emb. ·017 Exit	·004	·007	·042	9	$\frac{B}{256}$
No. 6 Class IIA	·330 ·323 f.	·271 Vent to Exit ·053	·014 emb. ·015 Exit	·005	·007	·042	8	$\frac{B}{256}$
No. 7 Class IIA	·351 ·346 f.	·280 Vent to Exit ·066	·014 emb. ·016 Exit	·006	·007	·039 mean	9	$\frac{B}{256}$
No. 8 Class IIIA	·284 ·279 f.	·226 Vent to Exit ·053	·011 emb. ·013 Exit	·006	·007	·026	10	$\frac{D\ 20}{512}$

PERFORMANCE

Acholi Tribe of the S. Sudan

Dr. A. N. Tucker

Modal Sequence	
Hypophrygian 9/9 8 7 6 5 *Cents* *204°* *231°* *267°* *316°*	All. at Exit ·386 − ·033 = $\frac{·353}{·037}$ = 9(+·020) ·018 Δ ·011 Δ—fipple ·037 × 9 = ·333 ·004 *de* ·033
Hypophrygian 9/9 8 7 6 5 as above	All. at Exit $\frac{·340}{·036}$ = 9(+16 for Allowance) ·011 Δ ·005 Δ—fipple ·036 × 9 = ·324 ·003 *de* ·019
as above	All. at Exit ·320 + ·025 = $\frac{·345}{·037}$ = 9(+·012 for All.) ·013 Δ ·009 Δ—f. ·003 *de* ·025
as above	All. at Exit ·041 × 9 = ·369 ·016 Δ ·330 + ·030 = $\frac{·360}{·041}$ = 9(−·009) ·011 Δ—f. ·003 *de* ·030
as above	All. at Exit ·332 + ·033 = $\frac{·365}{·042}$ = 9(−·013) ·017 Δ ·013 Δ—f. ·042 × 9 = ·378 ·003 *de* ·033
Hypodorian 8/8 7 6 5 4	All. at Exit ·042 × 8 = ·336 ·015 Δ ·323 + ·028 = $\frac{·351}{·042}$ = 8(+·015 for All.) ·010 Δ—f. ·003 ·028
Hypophrygian 9/9 8 7 6 5	All. at Exit ·039 × 9 = ·351 ·015 Δ ·009 Δ—f. ·346 + ·027 = $\frac{·373}{·039}$ = 9(+·022 for All.) ·003 *de* ·027
Hypolydian 10/10 9 8 7 6	All. at Exit ·026 × 10 = ·260 ·011 Δ ·005 Δ—f. ·279 − ·019 = $\frac{·260}{·026}$ = 10 ·003 *de* ·019

Six Flutes from Bali

Presented by Mr. Soekawati of Batavia

Flutes from Bali, presented by Miss Elsie Hamilton (1937)

No. of Flute	Length, Exit to Emb.	Diameter of Bore	Fipple	δ of Finger-holes	Diameter Allowance at Exit	Position of Hole 1	Modal Determinant	Increment of Distance	Finger-holes	
Bali 1, No. 20	·370	·023 *de* ·002	·007 × ·005	·007	·043		11 Exit	·033	7	See Chap. x. 7 equidistant finger-holes Pélog scale
Bali 2, No. 21	·680 + ·034 All. ·714 eff. length	·018	·007 × ·005	·0085	·034	·484 from emb. ·196 from Exit	Vent 18 (Exit 23)	·031 and ·0535	6	Two groups of three fingerholes with double spacing between the groups indicating the boring for Sléndro Scale
Bali 3, No. 22	·680 + ·034 All. ·714	·018	·007 × ·005	·008	·034	·489 from emb. ·191 from Exit	Vent 18 (Exit 23)	·031 and ·055	6	idem
Bali 4, No. 23	·797 Exit to C. of fipple + ·046 All. with *de*	·024	·007 × ·005	·008	·046 with *de*	·574 from emb. ·220 from Exit = 6 I.D.	Exit 22 Dorian Vent 16	·036 and ·075	6	Modal Sequence with ratios Exit H. 1 2 3 4 5 6 22/22 16 15 14 12 11 10 Modal interval 11/8 — Sléndro or pentatonic (Modal) from Vent
Bali 5, No. 24	·835 Exit to C. of fipple	·026	·008 × ·005	·08	·049	·605 from emb. ·230 from Exit	Exit + All. 22 Dorian Vent 16 i.e. ·605 + ·045 All. 650	·039 to ·040 and ·080	6	idem

Bali 6, No. 25	·894 Exit to C. fipple	·024	·008 × ·005	·01	·046	·637 from emb. ·253 from Exit (6 I.D.)	22 Exit 16 Vent	·039 to ·041 and ·071	6	idem

Four Flutes from Bali

Presented by Miss Elsie Hamilton. Theoretical Scale and ratios arrived too late to be tested

Bali 7, No. 26	·257 Exit to C. fipple	·015	·005			V. ·213 from emb. ·044 from Exit	11 Exit	·023 mean regular	6 equi- distant Pélog	Exit H. 1 2 3 4 5 11/11 10 9 8 7 Octave Exit note $\frac{E\,19}{512}$ Scale
Bali, (No. 8), No. 27	·275 Exit to C. fipple	·015	·006			V. ·218 from emb. ·057 from Exit	11 Exit	·024 ; ·024 ; ·024 ; ·023 ; ·022	6 equi- distant Pélog	idem Exit note $\frac{D\,20}{512}$ Scale
Bali 9, No. 28	·310 Exit to C. fipple	·016	·006		·026	V. ·234 from emb. ·076 from Exit	Phrygian 12 Exit + All. Class II	·028 ; ·0275 ; ·0275 ; ·027 ; ·026	6 equi- distant Pélog	Theoretical Modal Scale M.D. 12 Exit H. 1 2 3 4 5 12/12 10 9 8 7 6 5 Scale Exit note $\frac{B\,12}{256}$ H. 1 $\frac{D\,10}{512}$
Bali 10, No. 29	·370 Exit to C. fipple	·016	·006			V. 296 from emb. ·074 from Exit	Lydian 13 Exit Vent 11 Dorian	·028 mean	6 equi- distant Pélog	Theoretical Modal Scale M.D. 13 from Exit Exit H. 1 2 3 4 5 6 13/13 11 10 9 8 7 6 Scale Exit note $\frac{G\,14}{256}$

The evidence of the Bali flutes with equidistant fingerholes exhibits a definite modal basis. These 10 flutes indicate the use of Pélog Scales in Nos. 20, 26, 27, 28, 29; and of Sléndro in the remaining 5 which are bored with equidistant fingerholes but in two groups of 3 holes separated by a double-spaced increment denoting the Sléndro type of Modal Harmonia.

TABLE XIV. The Silver Pipes of Ur*: Measurements and Modal Scale

From Material and Full-size Photograph, by courtesy of Dr. L. Legrain (Univ. Mus. of Philadelphia) Facsimile of pipe 'B' from the measurements by K. S.

Pipe 'B' (the straight one) consists of 3 fragments; four fingerholes are preserved in the two lower portions, a fact of paramount importance to Modality; the fingerholes are practically equidistant. (See Chapter IX, Conclusion).

MEASUREMENTS

Total length in original specimen, computed by Dr. L. Legrain at ·270

Length of Fragment (A) emb. at which end about 3 cm. are missing = ·074 + ·003 = ·077

Length of Fragment (B) middle, curved, containing two fingerholes and indications of a loss by corrosion at lower end from the highest point of one fracture to the other = ·081

Length of Fragment (C) lower portion containing one complete fingerhole and indications of a second; the exit is complete and defined; from exit = ·059

The lengths have been carefully measured with fine pointed compasses down the centre of the pipe, allowance being made for convexity caused by twisting, corrosion, &c., taking the mean between convex and concave lengths

L. from C. Hole 4 on middle joint to emb. = ·270 − ·121 = ·149 of which only ·103 (viz. A = ·077 + B. H. 4 to fracture ·026) is accounted for, so that ·046 of the pipe is missing

Δ ·004

δ 1st (lowest) ·0035 × ·004. (2) c. ·0035 × ?. (3) ·0025 × ·0055. (4) ·0035 × ·004, irregular

FINGERHOLES, POSITION AND INCREMENTS OF DISTANCE

Total length of silver pipe 'B'	·270 ;	Exit to C. H. 1	= ·030
C. Hole 1 fr. emb.	·240 ;	H. 1 to C. H. 2	= ·029
C. Hole 2 fr. emb.	·211 ;	C. H. 2 to C. H. 3	= ·031
C. Hole 3 fr. emb.	·180 ;	C. H. 3 to C. H. 4	= ·0305
C. Hole 4 fr. emb.	·1495 ;		4)·1205
			·0301

Increment of distance (mean) ·0311
useful means ·030 or ·031

The resonator of the pipe $= \frac{\cdot 270}{\cdot 030} =$ M.D. 9 without mouthpiece

Mp. extrusion at ·120 gives M.D. 13 Lydian Harmonia
Mp. extrusion at ·090 gives M.D. 12 Phrygian Harmonia
Mp. extrusion at ·060 gives M.D. 11 Dorian Harmonia

The Hypolydian, determinant 10, would not be practicable at an extrusion of only ·030 with either the D-R. or with the B-R. mouthpiece, more especially in a resonator of a small calibre of only ·004

MOUTHPIECES AND PERFORMANCE

B-R. Z. Mp. T.L. ·042; T.W. ·0045; Δ ·004; Straw L. ·095

Dorian 11. Harmonia plays on fundamental $C = 128$ v.p.s. with piano in perfect tune, powerful sonorous notes, ratios of Dorian Spondaic, viz.

C	d	e	f♯	g♯
11	10	9	8	7

Tested 5/1/38 and 12/3/38.

Phrygian 12

*B-R. Mp. **Elgin** F 16* at extrusion ·103.
Elgin *F* 16. T.L. ·037; T.W. ·0025; Δ ·003; Straw L. ·130.

Plays Phrygian Pentachord on $\frac{F\ 16}{64}$ in tune; fine resonant tone.

Minor 3rd, perfect 4th and 5th, all in perfect tune.

Tested 5/1/38 and 12/3/38.

Dorian 11 on $C = 128$

B-R. Mp. '*Ur*' (***purple seal***), fine wheat.
T.L. ·035; T.W. ·0025; Δ ·004; Straw L. ·135.

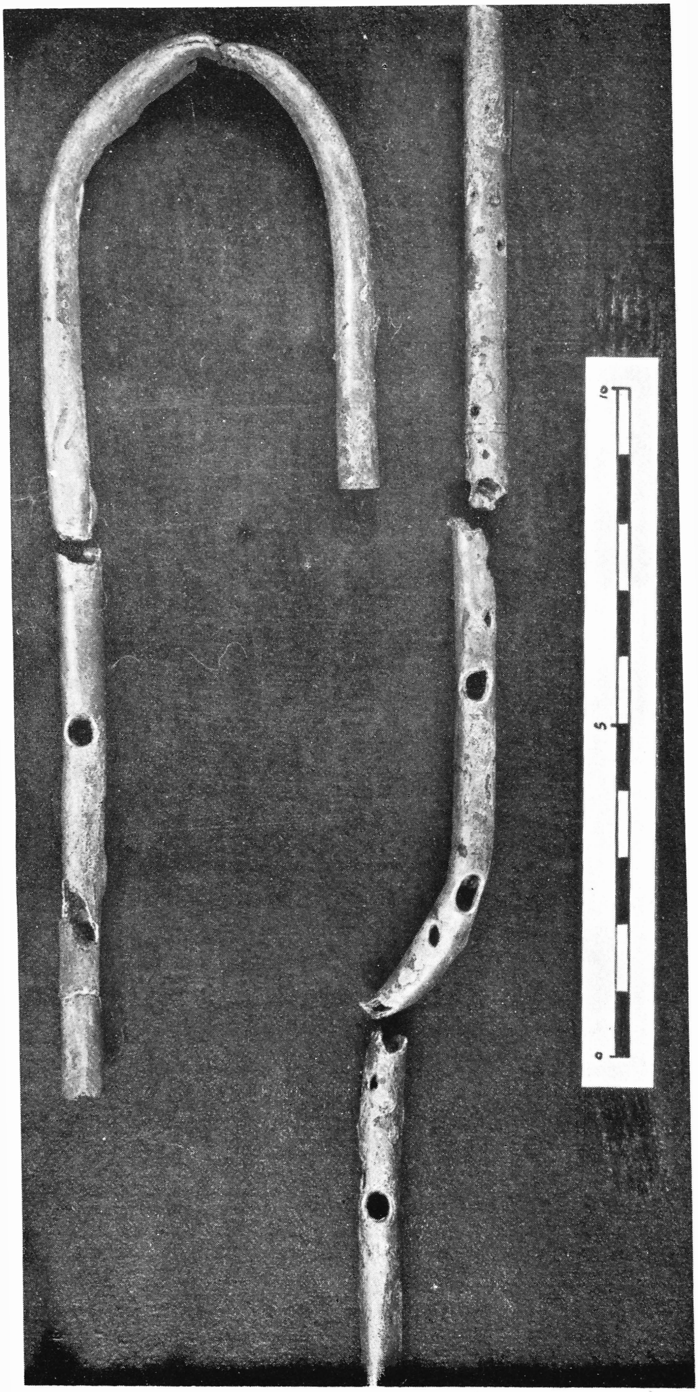
Plays note norm. $\frac{E\ 18}{128}$ plays mp. alone *E* 18, *F* 16, *G* 14, *A* 13 by more breath.

Plays *C* 128 fundamental in Aulos the Dorian Pentachord M.D. 11 as above.

Tested 12/3/38.

* These unique relics were discovered during the excavations at Ur conducted by Sir Leonard Woolley. They were first reported in *Ur Excavations*, Vol. ii, *The Royal Cemetery* (1934), pp. 258–9.

PLATE 18



THE SILVER PIPES OF UR

From the Royal Cemetery, 2700 B.C.

University Museum, Philadelphia. By courtesy of Dr. Legrain

APPENDIX I

A BRIEF OUTLINE OF THE MODAL BASIS OF THE SCHEME OF GREEK NOTATION

The Older Accepted Interpretation by Bellermann. The System of Notation based upon Modality. The Priority of the Vocal Notation. The Main Features of the Vocal Notation. Important Facts indicated by the System of Notation. Mese as the Modal Nucleus of the System of Notation in each Tonos. Tests as a Challenge to the Accepted Interpretation of Bellermann. Test I. Test II. Test III. Test IV. Test V

THE OLDER ACCEPTED INTERPRETATION BY BELLERMANN

SINCE this appendix is only a brief exposition of the main features of the modal basis of the systems of Greek musical notation, acquaintance with the symbols used in the Vocal and Instrumental Notations must be presumed. Both systems are set forth in von Jan's *Musici Scriptores Graeci*, pp. 368–406 (with the text of Alypius) and in Macran's Aristoxenus, pp. 46–61.[1] It must also be taken for granted that there is some knowledge of the main hypotheses upon which Dr. Friedrich Bellermann [2] founded his interpretation of the Tables of Alypius. At the outset it must be stated that a careful analysis of his material, tested in the light of the hypotheses, values and correspondences he himself adopted, yields the following surprising results:

(1) Out of the 75 tetrachords contained in the 15 Tonoi, only six are found to be perfect 4ths; most of the others work out as tritones.

(2) The tetrachordal unit, which on Bellermann's theory should be identical in structure for all five tetrachords of the P.I.S., can be traced only in the notation of the Lydian and Hypolydian Tonoi.

(3) The scheme of Alypius does not yield a single example of the ditonal scale applied in all five tetrachords of the P.I.S.

(4) Moreover, an analysis of the notation of the 15 Tonoi allows only six ditonal tetrachords among the 75.[3] Since the publication of Bellermann's work,

[1] They may be very conveniently consulted, more especially for Test V, as arranged by Macran. There is in the vocal notation of the Hypoionian a misprint in the symbol for Hypate Hypaton which should be ⊔ not ⊓ and also in the Hypolydian for Nete Hyperbolaion, which should be Ⴗ (eta).

[2] *Die Tonleitern und Musiknoten der Griechen* (Berlin, 1847). Similar results were reached by C. Fortlage in his *Das Mus. System der Griechen*, 1847.

[3] These four facts, based upon the equivalence suggested and adopted by Bellermann himself, may be ascertained and confirmed by anyone who will take the trouble to get acquainted with Bellermann's arguments and hypotheses, no sooner laid down than hedged round with qualifications and contradictions necessitated by the endeavour to force the ditonal scale, based upon a single tetrachordal unit, into a framework devised for the Aulos-Harmoniai, with their varied series of intervals. Bellermann's naïve reasoning and approach to the subject throws the onus of inconsistencies and ambiguities upon the Greeks, whose lack of intelligence, displayed in evolving a system of notation that fits in so badly with our keyboard scale, he deplores. Cf. *op. cit.*, pp. 45, 47, 53, &c.

other theories of Greek musical notation have been expounded, notably by Hugo Riemann and Curt Sachs ; but none of them has yet ousted the earlier one, and most of them are based fundamentally on the assumption of a ditonal scale. In the later part of this Appendix the theory of Bellermann is subjected to a number of tests which will make its inadequacy plain to all.

THE SYSTEM OF NOTATION BASED UPON MODALITY

This being so, it would seem that there is need for a new interpretation consistent with the historical development of the Greek musical system through the modality of the Aulos-Harmoniai. The assembling of all the symbols used in the 15 Tonoi into a standard scale is meaningless ; for each symbol involves a ratio which is not fixed but differs according to the Tonos in which the symbol is used. The whole scheme, as found in the Tables of Alypius, is concerned solely with the Harmoniai, as the following facts indicate :

(1) In the seven ancient Tonoi, treated in the P.I.S. as curtailed Modes, the octaves containing the Homonym Species are all noted within the same range of the alphabet from **Ω** to **Γ** (viz. in 22 letters), and Omega falls on the Tonic of the species. The letter proper to the octave of *Arche*, functioning as Mese in the Tonos, is found in place on the correct degree—duly recorded by Alypius—which it occupies in the Harmonia and Species.

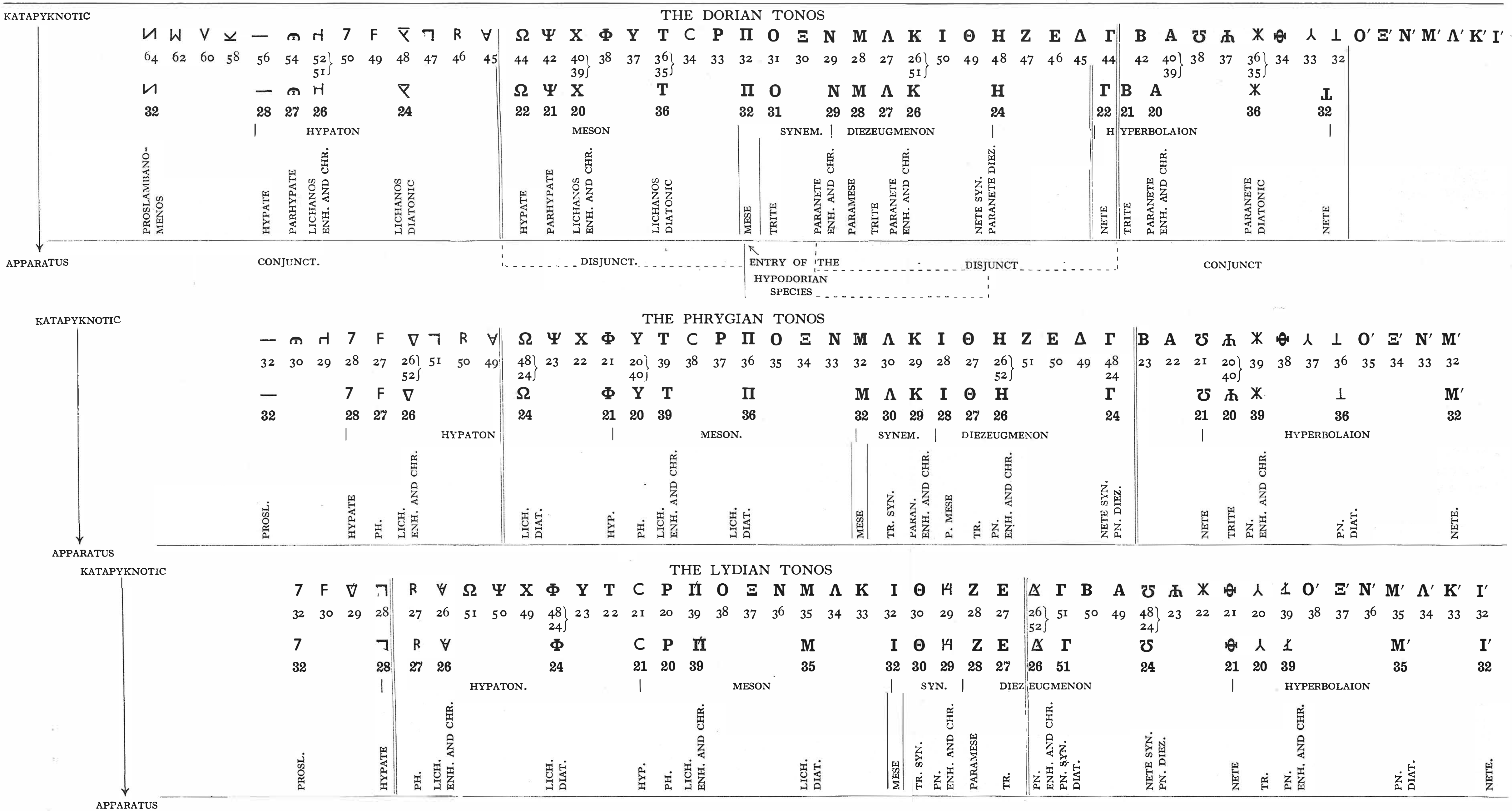
(2) Through the Chromatic Pyknon of the Synemmenon tetrachord in the seven ancient Tonoi, the Greek Ionian alphabet unrolls in groups of triplet letters beginning with the Hypodorian Tonos on **Ω Ψ X** and ending with **Γ B A** in the Hyperphrygian Tonos, the octave of the Hypodorian. There is indeed a break in the sequence at the Dorian Tonos, the reason for which will be explained later.

(3) The full range of symbols from **Ω** to **Γ** has been allotted in the Dorian Tonos to the octave from Hypate Meson to Nete Diezeugmenon, which forms the nucleus of the P.I.S., and in which, in this Tonos alone, the two nomenclatures Kata Thesin and Kata Dynamin are identical ; and thus the Dorian Tonos constitutes the prototype of the species.

THE PRIORITY OF THE VOCAL NOTATION

After these preliminaries we may discuss in some detail the principles on which the allocation of the letters to the Tonoi has been carried out. As our exposition develops, it will become evident that just as vocal music was accorded the first place in the Art of Music in Ancient Greece, so did Vocal Notation precede the instrumental in the first conception of the scheme which is ascribed to Pythagoras by Aristides Quintilianus (p. 28M.) and to the Harmonists by Aristoxenus in one of his polemics against that body (pp. 39–40M., and Macran, transl., pp. 194–5). The question of the priority in age, imputed to the instrumental notation, advanced by Westphal, Gevaert, Maurice Emmanuel and others, was based mainly upon the form of certain of the instrumental symbols which these writers refer to archaic alphabets. This argument, even if it were based upon indisputable facts, could carry no weight unless it could also be shown that the instrumental notation had been constructed out of such materials upon some logical principle, operating through the musical system of the Tonoi, and demonstrating by the order of the symbols some definite relationship to the evolution of the Greek musical system. No such claim, however, can be made for the Instrumental Notation. As for the resemblance of some of the symbols to the forms of more ancient letters, we should not forget that imitation of the archaic in the arts was a well-known device in Greece as the word ἀρχαΐζω testifies ; nor would the use of archaic symbols be an inconvenient convention if, as I believe, the more normal alphabet had already been allocated to a

Fig. 101.—The Katapyknotic Apparatus in Operation in Three Tonoi



The alternative ratio numbers bracketed do not signify that both are found in use in that particular Tonos, but that they occur in other Tonoi, since there is the same range of symbols allotted in all the Tonoi to the Homonym Species or Harmoniai, which by virtue of their characteristic M.D. differ to some extent in the genera. Ratio 51 is used in the Lydian Tonos for Paranete Synemmenon: this ratio number suggests an endeavour to compromise between 30/26 (15/13 augmented 2nd) and the minor 3rd; $\frac{60}{51} \times \frac{13}{15} = \frac{52}{51}$ and $\frac{60}{51} \times \frac{5}{6} = \frac{50}{51}$, a difference of *33·6 cents* and *34·27 cents* respectively. The ┌ in this case seems to be an attempt to provide for that note the same rise in pitch as the diacritical accent gives to it in the Enharmonic-Chromatic genera of that Tonos.

vocal notation. Instrumental Notation with its groups of letters in three positions, normal for the barypyknon, recumbent for the mesopyknon, and reversed for the oxypyknon, e.g. E ш Ǝ, suggests an emphasis on the Enharmonic-Chromatic pyknon, and is perhaps reminiscent of the skill attributed to Pronomus, the Theban, who could produce three dieses from each hole of his Aulos. We may assume that this was accomplished by partially covering a fingerhole in three progressive movements of his finger (see Chap. ii and Chap. vii, Bhārātā). As it is my firmly grounded opinion that inherent in Vocal Notation—by virtue of its recognized alphabetical sequence—a solution is to be found for the many points at issue, the remainder of this appendix will be confined to a discussion of the symbols of Vocal Notation.

THE MAIN FEATURES OF THE VOCAL NOTATION

In so far as the seven original Modes are concerned, the system of notation does not bear traces of gradual aggregation ; it probably emanated as a whole from the brain of a Harmonist who was versed in the genesis of the Harmoniai and in the practical operation of the modal principle.

Modality operates in Notation through the allocation of the letters from **A** to **Ω** by means of what I shall call the *Katapyknotic Apparatus*. The mechanism of the scheme is as follows. Every succession of two letters in alphabetical order represents one diesis, but since the dieses vary in magnitude according to their position in the scale, and to the modal genesis, a number (which forms one element of a modal ratio)[1] peculiar to each Tonos can be assigned to every letter. Not every letter, of course, is used in every Tonos, because there are always some undivided intervals of greater magnitude than a diesis. It is clear that in the original scheme the allotment of letters to the Harmoniai provided, with rigorous accuracy, for all contingencies ; for there are even now in the latest form of the scheme as presented by Alypius very few exceptions to the rule, that for every letter symbol there is a corresponding modal ratio number (or note) in each Tonos. Letters and ratios occur in their own inevitable sequence.

In spite of a few slight irregularities and modifications brought about by time and evolution, sufficient indications remain to suggest that originally the scheme was regular. This, for instance, is the way in which the Katapyknotic Apparatus works out in the Dorian, Phrygian and Lydian Tonoi.

The diagram displays a perfect correlation of alphabetical symbols (as described by Alypius for each step in the P.I.S. in all the Tonoi) and of the ratio numbers allotted by the author as a suggestion, based upon strong presumptive evidence.

It will be noticed that at certain points in the Katapyknotic Apparatus, two numbers, one double the other, are assigned to a single symbol—the lower value belongs to the Chromatic genesis, its double to the Enharmonic—further, that the arithmetical sequence to the left follows one of these and the sequence to the right follows the other. This is a simple and legitimate device by which it becomes possible to use integers even for the intermediate notes of the Enharmonic or Chromatic genus. Thus, for example, in the Phrygian and Lydian Tonoi, the Enharmonic Lichanos Meson indicated by 39 (respectively **T** and **Π**) is a diesis above the Parhypate.

The numbers, then, placed under the symbols, are those of the arithmetical

[1] The number here signifies the number of a note in the modal sequence. The division of a given length by the Modal Determinant results in a series of equal segments, each bearing a number indicating its position and value in the ensuing sequence, which is expressed in full by a proper fraction having the M.D. as constant denominator and the order number of the segment as numerator. See fuller statement under Abbreviations and Formulae.

sequences of the Enharmonic or Chromatic genesis, not of the Homonym Harmonia of the Tonos, but of Proslambanomenos or Mese, the Modal Determinant of which is 64 or 32 respectively in all the Tonoi.

The Apparatus of the Dorian Tonos appears to be less regular than that of Phrygian or Lydian. At three points alternative numbers or notes are assigned to a single symbol, and the sequence of numbers to the right follows the lower of the two, e.g. $\underbrace{52, 51}$, 50, &c., on Hypaton Lichanos Enharmonic and again on its octave in Diezeugmenon; this may indicate merely that at some time both notes were in use. An alternative explanation of this irregularity is that it was due to the intrusion into the progression of the seven original Tonoi, of the Ionian Tonos, as lower Phrygian, whereby the Dorian Mese, which should be indicated by **O** was shifted one diesis lower to **Π**, so that the Synemmenon Pyknon is found noted as **TO.N** instead of **OΞN** which has been allotted to the Ionian Tonos.

This hypothesis is strengthened by the fact that if the Dorian Mese **O** be restored as 32, and the sequence to the left be continued, the 35 as alternative on the Diatonic Lichanos Meson now falls into line. Moreover, continuing to the right from the Dorian Mese **O** as 32, the Synemmenon Pyknon is now seen to consist of the same numbers 32 = **O**, 30 = **Ξ** and 29 = **N** as in the other two Tonoi.

Such a Katapyknotic Apparatus provided elasticity for the notation of compositions, permitted modulation into species, and also the use of delicate shades of intonation. By way of illustration, we may mention the First Delphic Hymn (see Chap. ix, p. 25) noted in the Phrygian Tonos, but composed in the Hypolydian Species. Two symbols foreign to the Phrygian Tonos are introduced towards the end of the Hymn: **O** and **B**, which can immediately be traced by means of the Katapyknotic Apparatus. To **O** falls the ratio 35, and to **B** ratio 23, as implied in Alypius. The introduction of these two extra notes tends to create a Dorian atmosphere, as already suggested in Chapter ix.

If the inner structure of the P.I.S. by dieses be carefully examined it is found that the norm. at the time of Alypius was 9 dieses or 10 covering letters to the tetrachord; 3 dieses or 4 covering letters for the interval of disjunction making an invariable total of 21 dieses and 22 letters to the octave.

This and other flagrant breaches in the original order suggest modifications introduced at the time, perhaps, when the P.I.S. had assumed a Phrygian modality. This transformation was not brought about by altering the Notation or the ratios, but merely by the inclusion of Proslambanomenos as the beginning of the lowest of the tetrachords, a change of modality which marks the 4th stage in the development of the P.I.S. (see Chap. iv).

IMPORTANT FACTS INDICATED BY THE SYSTEM OF NOTATION

The following conclusions can be drawn from an examination of the scheme of Notation:

(1) The Tonoi, as revealed by the Tables of Alypius, do not invariably comprise tetrachords identical in structure within the same octave.

(2) A striking fact is revealed by a study of Notation (as exhibited by the Tables of Alypius); a pair of consecutive symbols—the same for the three genera—has invariably been allotted to the first two notes of each of the 75 tetrachords of the 15 Tonoi. The genus, therefore, can only be determined in Notation by the Lichanos or the Paranete.[1]

[1] This strange treatment of the genera seems to imply either a reminiscent influence of the tetrachords ascribed to Archytas by Ptolemy (ii, 14) or to the dying out in practical music of the feeling for the Enharmonic genus, since a diesis of that

The modal ratios assigned in this work to the degrees of the P.I.S. in each Tonos throw some light upon this difficult point. For instance, in the Hypaton and Diezeugmenon Pykna of the 7 ancient Tonoi, the ratios 28, 27, 26, might be taken to indicate the Enharmonic Genus although, strictly speaking, the Enharmonic octave of genesis begins on 32. In the Ionian Tonos these tetrachords have **KI.H** to which we must assign the modal ratios 28, 27, 25, or 51, since **Θ** is not used. We thus have a mixed Pyknon which starts as Enharmonic and ends as Chromatic.

In the Dorian Tonos, the Meson Pyknon **ΩΨX** with ratio numbers 22, 21, 20, is clearly Chromatic, while the Synemmenon Pyknon **ΠO.N** = 32, 31, 29, affords another example of mixed genera, viz. Enharmonic + Chromatic.[1] With the suggested restoration of the Dorian Mese **O**, it has been shown that this Synemmenon Pyknon becomes **O Ξ N** of ratios 32, 30, 29, in conformity with those of the Phrygian and Lydian in Fig. 101, i.e. Chromatic + Enharmonic. In the actual modal progression of the Dorian Tonos, either the genera tend to merge into one another, or the same octave contains one Pyknon of each genus,[2] so that at times it is difficult to pronounce upon the genus, the students mentioned by Aristoxenus (35M., tr. Macr., p. 191) who failed to answer the question, 'At what point does the Enharmonic begin to pass into the Chromatic?' have all our sympathy.

The Mesai of the Harmoniai (indicated by the number 32 or 16) are represented by their individual symbols, as described by Alypius, duly placed upon the characteristic modal degree of the scale, counted from the Tonic bearing the ratio of the Modal Determinant. The scheme for the allocation of symbols thus brings about the perfect correlation of the letters with the ratios at Mese, by means of the progression of the Synemmenon Pykna shown in Fig. 102. The progression, starting from each Mese in turn, is therefore emphatically inspired by Modality.

The provision of 21 dieses to the octave is in itself remarkable and may indicate that three dieses are seated upon each of the seven degrees of the scale, regardless of the magnitude of the intervals between the degrees.

Let the Katapyknotic Apparatus help to work this out. The first note in every group of three dieses is the Tonic of one of the species.

44 43 42 41 ; 40 39 38 37 ; 36 35 34 33 ; 32 31 30 29 ;

22 3 DORIAN — 20 3 HYPOLYDIAN — 18 3 HYPOPHRYGIAN — 16 3 HYPODORIAN

56 55 54 53 ; 52 51 50 49 ; 48 47 46 45 ; 44

28 3 — 26 3 — 24 3 — 22

14 MIXOLYDIAN — 13 LYDIAN — 12 PHRYGIAN — 11 DORIAN

genus is unsuitable for use as a Diatonic Parhypate or Trite. Which genus has exercised the predominant influence in Notation? The clue must probably be referred to modal exigencies: e.g. 28, 27, 26, conditioned by the adoption of 27 (instead of 26) as the revised Tonic of the Lydian Species; while 32, 30, 29, implies deference to the Diatonic tetrachord of the Hypodorian species 16, 15, 13, 12, used as Synemmenon, in which the pair of consecutive symbols has been definitely allotted to the Diatonic genus, leaving the Chromatic Enharmonic Pyknon to follow suit with 32, 30, 29.

[1] The interval 31/29 of *115·4 cents* exceeds the just semitone 16/15 = *112 cents* and the apotome = *114 cents*.

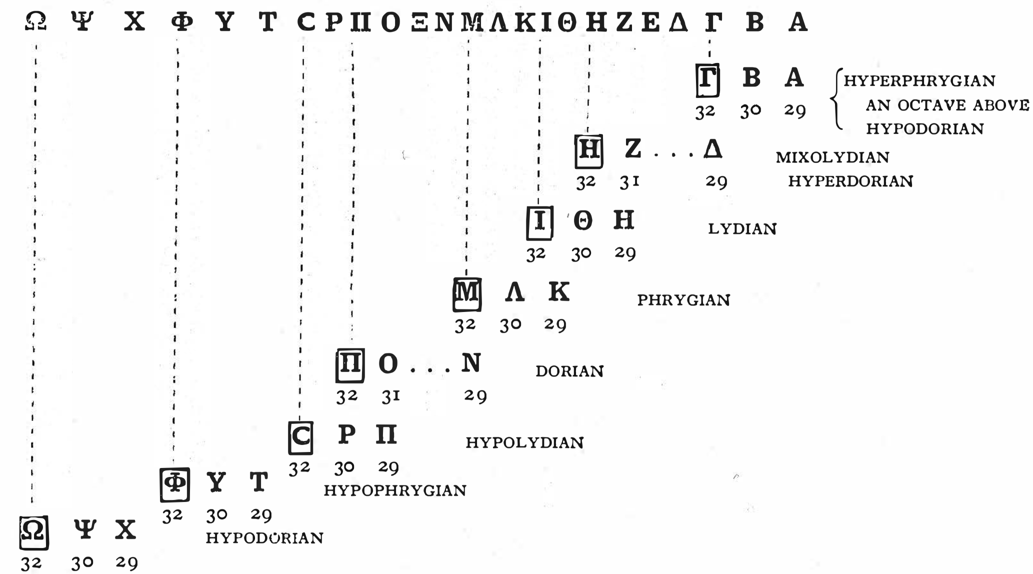
[2] Hypaton and Diezeugmenon are Enharmonic (28, 27, 26), Meson and Hyperbolaion are Chromatic (22, 21, 20), Synemmenon mixed (32, 31, 29) Enharmonic + Chromatic.

FIG. 102.—Nucleus of the Scheme of Notation according to the Tables of Alypius

The Enharmonic Pykna of the Seven Harmoniai in the Synemmenon

The Greek Alphabet in Descending Order

The Synemmenon Pyknon of the Seven Original Harmoniai taken in the Enharmonic Genus. (The Mesai are in squares).



It is obvious that the selection of letters for the Dorian and Mixolydian Tonoi destroys the uniformity of the system. The Pyknon for the Dorian Tonos should clearly be **ΟΞΝ**, and for the Mixolydian **ΖΕΔ**. The modification was probably made when the remaining 8 Tonoi of the Tables of Alypius were interpolated or added.

MESE AS THE MODAL NUCLEUS OF THE SYSTEM OF NOTATION IN EACH TONOS

It has already been shown how Modality has been emphasized in the Notation of the P.I.S. by marking the Tonic of the species of the Harmonia—the Lydian and Hypolydian excepted [1]—with the sign **Ω** as token that the end of the genesis from Arche has been reached at that point; i.e. at the fundamental note common to all the Harmoniai: Omega is the beginning and the end as absolute fundamental, in which all the Archai—as Harmonics—have their being, and the end to which each genesis inevitably comes; on a monochord, for instance, the descending progression from Arche can go no further than the string sounding as a whole, at the beginning and at the end.

Thus does every stage in our investigation of this amazing concept of the Greek genius, bring us back to the inherent potency of the Harmonia, justifying to the full Aristotle's encomium (see Chap. v).

It will have been noticed, further, in Fig. 102, that the break in the progression at the Dorian Pyknon already signalized, has a repercussion at the Mixolydian Synemmenon Pyknon. The intrusion of the Aeolian Tonos at this point again upsets the order, for the Mixolydian Pyknon should by right be **Z E Δ**. The exception presented by the Lydian and Hypolydian Tonoi has already been discussed,[1] but it may be recalled that in the Tables of Alypius the substitution of 27 for 26 as Parhypate Hypaton, the Modal Determinant of the Lydian Harmonia, is implied, 26 being indicated by the next symbol as Oxypyknon in the Hypaton tetrachord thus: ⅂ ꓤ ∀ (28 27 26) and in the Diatonic ⅂ ꓤ Φ C (28 27 24 21). It is a remarkable fact that in the 21 letters from **Ω** to ⌈ lie the sole means of differentiation within the modes or species, when used as Tonoi. In order to determine the precise amount of these differences, reference must be made to the Tonos in question.

When it is remembered that the Synemmenon bears the ratios of the first tetrachord of the Hypodorian Harmonia, it is seen that this forms another link in the unification of the seven Modes into a co-ordinated system. The Hypodorian tetrachord is thus thrust through the modal disjunct octave at Mese, the Hypodorian's own Tonic and Mese, coinciding with that of the P.I.S., and thus providing an alternative conjunction. But when the disjunct octave, passing from Mese through Paramese, is in use, the Hypodorian species disappears from the Tonos, and the Mese is thus dispossessed of its species. The so-called Hypodorian of the theorists based upon Proslambanomenos is but a bastard, lacking the ratios proper to the Harmonia: yet it is known from the theorists [2] that the Hypodorian or ancient Aeolian species, extended from Mese, through Synemmenon and Hyperbolaion; it therefore had its tone of disjunction between these two tetrachords through the omission of Diezeugmenon.

During the period when the Tonos was merely the Dorian species taken in turn in all the other Modes, the succession of ratios was necessarily the same in all the Tonoi. The Tables of Alypius, however, while implying these pre-existing conditions, show clearly that the feeling for independent modality had grown strong as the Art of music developed, and had left its impress upon the scheme of Notation. The ratios of the P.I.S., no longer uniform, are seen to vary with the Tonos, usually showing affinities within those of groups bearing the same common

[1] See Chap. iv, pp. 146, 149 and 163; Chap. v, p. 201; Chap. vii, pp. 280 and 281; Chap. ix, p. 389.

[2] e.g. Ps-Eucl., *Intr. Harm.*, p. 16M.

FIG. 103.—Ratio 26 in the Tonoi, traced by means of its incidence as Chromatic Paranete Diezeugmenon

	Synem.				Diezeugmenon			
	Mese	Trite	Paranete	Nete	Paranete	Trite	Paranete	Nete
Dorian	32	31	29	24	28	27	$\boxed{26}$	24
Chrom.	Π	O	N	H	M	Λ	$\boxed{K}$	Γ
	32	31	$\boxed{26}$	24	28	27	24	22
Diat.	Π	O	$\boxed{K}$	H	M	Λ	H	Γ

	Mese	Trite Syn.	Paranete Syn. Chromatic	Paramese	Trite Diez.	Chr. PN. Diez.	PN. Syn. Diatonic			Nete Syn.			Nete Diez.	
Hypolydian	32	30	29	28	27	$\boxed{26}$	(52)			24			21	
Chromatic	C	P	Π	O	Ξ	$\boxed{N}$	.	.	.	I	.	.	Z	
	32	30		28	27		51			48			21	
Diatonic	C	P	.	O	Ξ		M	.	.	I	.	.	Z	
						52				24				
Katapyknotic Ratios	32	30	29	28	27	26	51	50	49	48	23	22	21	
Alphabetical Sequence	C	P	Π	O	Ξ	$\boxed{N}$	M	Λ	K	I	Θ	H	Z	E

See Chapter IV, *re* Lydian 27 or 29

tribal name. (Turn also to Chap. iv, Fig. 34, ‘The Tonos as curtailed Mode’ and to Chap. v, Fig. 37, ‘The Seven Harmoniai within the Octave F to F’.)

TESTS AS A CHALLENGE TO THE ACCEPTED INTERPRETATION OF BELLERMANN

No analytical study of the Greek System of Notation can fail to bring to light certain contradictions and inconsistencies between the testimony of Alypius and Bellermann's interpretation. Many symbols of notation are in direct conflict with that interpretation. One of the glaring difficulties in Bellermann's theory is that points of pitch which he represents as identical and which should, therefore, have the same symbol, not only have different symbols in the tables of Alypius, but in many instances the symbol which we should expect for a given pitch is found actually in use for some other functional note. It is proposed to introduce here a few test cases, partly as a challenge to Bellermann's theory, and partly as examples of the correlation that exists between modal ratios and the allocation of the symbols.

TEST I

Paramese and PN.S. Chromatic are both supposed to be at an interval of one tone from Mese in each Tonos: the same symbol for these two notes only occurs in 4 out of the 15 Tonoi, viz. in the Hypoionian, and in the three Tonoi of the Aeolian group, while 11 Tonoi are ranged against the implication of Bellermann. A comparison of the example from the Aeolian Synemmenon tetrachord with those from non-conforming Tonoi—the formulae of which differ among themselves—reveals the working of the Katapyknotic Apparatus.

The Formula for the Aeolian Group

	Mese		Tr. S.		PN. S. Chr.		N.S.	PM.
Aeolian	**K**		**I**		**H**		**A**	**H**
Modal Ratios	32		30		28		24	28
Cents		*112*		*119·4*		*267*		

In the Hyperaeolian, Hypoaeolian and Hypoionian (which of course have a different set of symbols), a similar formula of ratios accounts for the identity of PN.S. Chromatic and Paramese.

Formulae of Non-conforming Tonoi

	Synem.							PM.
Hypodorian	**Ω**		**Ψ**		**X**		**Π**	**Φ**
Modal Ratios	32		30		29		24	28
Cents		*112*		*58·7*		*327·5*		

Six other Tonoi: Hypophrygian, Hypolydian, Phrygian, Lydian, Hyperphrygian, Hyperlydian, have a similar formula of ratios which accounts for the different symbols allotted to PN.S. Chromatic and Paramese.

	Synem.							PM.
Ionian	**O**		**Ξ**		**N**		**Z**	**K**
Modal Ratios	32		31		30		24	28
Cents		*55*		*56·7*		*386*		

Hyperionian has a similar formula of ratios and similar divergence of symbol.

	Synem.				PM.
Dorian	Π	Ο	Ν	Η	Κ
Modal Ratios	32	31	29	24	´28
Cents		*55*	*115·4*	*327·5*	

TEST II

The older interpretation professes to present a single standard scale, the ditonal, in 15 different keys, the Proslambanomenoi of which rise successively by a leimma (or apotome) from the Hypodorian Tonos; therefore, the Mese of the higher of two consecutive Tonoi should be of the same pitch as the Trite Synemmenon of the lower Tonos, and these two notes should be represented by the same symbol. Only 6 of the 14 pairs of Tonoi fulfil these conditions. The pairs are:

6 CONFORMING PAIRS	8 NON-CONFORMING PAIRS
Hyperlydian / Hyperaeolian	Hyperaeolian / Hyperphrygian
Hyperionian / Mixolydian	Hyperphrygian / Hyperionian
Lydian / Aeolian	Mixolydian / Lydian
Ionian / Dorian	Aeolian / Phrygian
Hypolydian / Hypoaeolian	Phrygian / Ionian
Hypophrygian / Hypoionian	Dorian / Hypolydian
	Hypoaeolian / Hypophrygian
	Hypoionian / Hypodorian

Moreover, an analagous identity in symbols might be expected in every tetrachord; but even in the 6 conforming pairs of Tonoi, complete identity in all 5 tetrachords exists only in 4 of the pairs. It is to be noted that in the 6 conforming Tonoi, one of each pair belongs either to the Aeolian or to the Ionian group.

A further divergence from the older interpretation is revealed by 6 of the non-conforming pairs, in which Mese's symbol in the higher Tonos is found in the lower, but allocated to PN.S. Chromatic, supposed to be a semitone higher than Trite. These six pairs of Tonoi are:

The Hyperaeolian and Hyperphrygian
Mixolydian and Lydian
Aeolian and Phrygian
Dorian and Hypolydian
Hypoaeolian and Hypophrygian
Hypoionian and Hypodorian

The modal explanation of the lack of identity revealed by this test is of a twofold nature, (1) the progression of the Proslambanomenoi is not by a uniform step, but between each pair of Tonoi by an interval fixed by the Modal Determinants of the two adjacent Tonoi[1] 11/10 between the Dorian and the Hypolydian; 14/13

[1] It will be remembered that Ptolemy devotes a chapter (Lib. ii, Chap. xi) to the thesis that 'The Tonoi ought not to be increased by the semitone method'.

between Mixolydian and Lydian, and so on. We must remember that members of the newer groups were interposed between the 7 original Tonoi. The second cause is the genus of the Synemmenon Pyknon, definitely Enharmonic in some Tonoi, and Chromatic in others.[1]

The other tests, only the purport of which can be given, owing to exigencies of space, will be seen to present no insuperable difficulties when judged in the light of the Modal System of the Harmonia, since the solution lies with the ratios of several functional notes each in its own Tonos.

TEST III

In the Diatonic genus, Trite Diezeugmenon should, according to the older interpretation, correspond in pitch with Paranete Synemmenon, both being at an interval of a minor 3rd above Mese (ditonal 32/37 = *294 cents*). Actually, however, different symbols are used to express these two functional notes in no less than 9 of the 15 Tonoi: once again, the conforming Tonoi belong to the Aeolian and Ionian groups. But the non-conforming Tonoi include the Homonyms of the 7 original Tonoi; and in addition, the octaves of the two lowest of these, of M.D. 16 and 18, are also included in the Hyper groups.

Tests IV and V both turn again on the mistaken conception that the progression of the Proslambanomenoi is by means of a constant step of a semitone or leimma.

TEST IV

This test reveals the fact that in the Diatonic Genus, Lichanos Meson, one tone above the PH. Meson, which should be identical with the PH. Meson of the Tonos next but one higher, is actually only expressed by the same symbol in 5 pairs of Tonoi (of the Aeolian and Ionian groups).

TEST V

This test is a valuable one and yields extraordinary results. As the Proslambanomenos of each Tonos is supposed to start higher (whether by semitone or any other interval) than its predecessor, the same functional notes ought never to bear the same symbol in two successive Tonoi, if the accepted interpretation of Notation be correct. And yet in numerous instances a single symbol does indicate the same functional note in 2 successive Tonoi.

The bearing of the apparent anomalies in Notation expressed by this Test upon our claim that this system was designed originally for the Harmoniai, as they had evolved in the P.I.S., must now occupy our attention.

Test v affords traces of no less than 7 different structures among the Enharmonic and Chromatic genera of the 15 Tonoi; and therefore, if the older interpretation were correct, not one instance of identity of symbol in these 7 differentiated structures should exist. Such correspondences of symbol have in themselves no portentous significance. These seven formulae traced in the system of the 15 Tonoi, however, yield absolute proof that the scale to which Notation applies could by no possibility consist of tetrachords identical in structure, and five in

[1] Four Tonoi have, in all 5 tetrachords, a notation indicative of the Chromatic genus, and 7 Tonoi of the Enharmonic; while the following 4 have Pykna of both genera in the same octave:

Dorian	in 3	tetrachords,	Enh.	Pykna.	in 2,	Chromatic
Mixolydian	in 3	,,	,,	,,	in 2,	,,
Ionian	in 1	,,	,,	,,	in 4,	,,
Hyperionian	in 3	,,	,,	,,	in 2,	,,

number in the P.I.S. For these correspondences, affecting the intonation of certain intervals, do not occur in all 5 tetrachords of the same pair of Tonoi ; they all involve—with only three exceptions—the Aeolian and Ionian groups. In some Tonoi the correspondence is in the Hypaton tetrachord and its octave the Diezeugmenon, implying an octaval structure, since the 2nd tetrachord of the octave does not follow suit (see, for instance, the Lichanos Hypaton T of the Hyperphrygian and of the Hyperionian). In other examples, all 5 tetrachords are affected in the Oxypykna (ex. the ∀ and Π and the Δ and ⊥ in Lydian and Aeolian), and in the Hyperlydian and Hyperaeolian ; in the Hypolydian and Hypoaeolian ; and in the Hypophrygian and Hypoionian pairs. In all these anomalies between pairs of Tonoi, the mystery is solved at once on examining the ratios of the correspondences in question, which will be found to differ. The proportional relation of the Modal Determinants of the pair of Tonoi, operating through Mese and Proslambanomenos, must also be taken into consideration, in the following manner. Chromatic Paranete Synemmenon, for example, has the symbol N allotted to it in both Dorian and Ionian (the higher of the two), and the explanation is simple : the Dorian genesis turns upon number **22** ; and the Ionian upon 23, so that the ratios of the Dorian sequence are lower by 23/22 than those of the Ionian having the same functional and numerical value. The Ionian Mese is thus higher by a small semitone than the Dorian Mese ; but in the Ionian Tonos, Chromatic Paranete Synemmenon has ratio 30, whereas the Dorian bears ratio **29** ; thus the higher pitch of the Ionian Tonos and the lower ratio of its Paranete tend to keep the balance, so that the same symbol serves for both functional notes. The absolute pitch indicated by the vibration frequencies is for the Dorian Paranete Synemmenon $133\frac{15}{29}$; and for the Ionian Paranete Synemmenon $134\frac{14}{15}$ v.p.s., a difference of approximately $1\frac{1}{2}$ v.p.s. in the octave below middle *C*.

This brief outline includes but a small part of what there is to be said on the subject of Greek Musical Notation.[1] The modal interpretation, of which the main principles only have been adumbrated here, provides a working basis which admits of many tests of a positive nature, and falls into line with the evidence already submitted in the foregoing chapters.

[1] Among other important evidence, for which space is lacking, there are the contributions made by Aristides Quintilianus, Bacchius and Gaudentius ; the discussion of the relation between the principles underlying Vocal and Instrumental Notation ; the evidence afforded by the MSS. which is of an extraordinarily interesting nature ; and the notation of the Fragments of Greek Music (see Chap. ix).

APPENDIX II

THE ORIGIN OF THE ECCLESIASTICAL MODES: THEIR RELATION TO THE MODAL SYSTEM OF THE HARMONIAI

Recapitulation of Stages in the Development of the P.I.S. The Modality of the P.I.S. transformed from Dorian into Phrygian. The Rise of a New System of Conjunct Modal Scales: The Birth of the Plagal Modes. Two Examples of the New Independent Conjunct Modal Scales based upon Proslambanomenos. The Octoechos in use in the Greek Church in the Fourth Century A.D. Confirmation from Arabian Sources. Ishāq's Majra through *Wosta* identified as Phrygian Modal Species. Confirmation of the Origin of the Ecclesiastical Modes from the Writings of Early Medieval Theorists

IT is now proposed to turn the light of the System of the Harmoniai upon the vexed question of the origin of the Ecclesiastical Modes, and to offer an explanation of their derivation from the classical modes of Ancient Greece. We are not concerned here to sketch the rise and development of the music of the early Greek Church, nor to state the rival theories that have held the field at different times. The main points indicated in this brief outline are:

(1) The origin of the Ecclesiastical Modes in the fourth and latest development of the P.I.S. (see Chap. iv).
(2) Kinship of the Harmoniai and the Octoechos of the Greek Church of Asia Minor and Northern Africa.
(3) Confirmation through Arabian sources.
(4) Confirmation through the treatises of early medieval writers.

RECAPITULATION OF STAGES IN THE DEVELOPMENT OF THE P.I.S.

In order to trace the structural development of the P.I.S. which I have described as the Fourth Stage, we must recall what had already been accomplished in Stage ii. I consider that *Stage ii* came about on the Kithara through the grouping of the modal species, one by one, round the Dorian Harmonia by practical musicians thus:

DORIAN (M.D. 11)
MESON AND DIEZ.

Phryg. (M.D. 12)	Hypolydian (M.D. 20)
Lich. Hyp.	Parh. Meson
Lydian (M.D. 13)	Hypophr. (M.D. 18)
Parh. Hyp.	Lich. Meson
Mixolydian (M.D. 14)	Hypodor. (M.D. 16)
Hyp. Hypaton	Mese Synem. } Hyperbol. }

To the theorists, this new scale, accommodating all the seven Harmoniai, involved besides an extension of the nomenclature, two fundamental changes. Firstly, the scale now began on Hypate Hypaton, its structure was changed from disjunct, (Meson + Diezeugmenon) to conjunct (Hypaton + Meson to Paramese) whereby Hypate Hypaton became the starting-point of the Lesser Complete System (later expanded into the Greater Complete System). The second change was a momentous one from the Dorian to the Mixolydian species.

The new scale was seen to comprise two conjunct octaves, and this transformation of the Greek System eventually led to *Stage iii*. The third development occurred when Proslambanomenos, the added note, while standing outside the system of the tetrachords, was admitted as lower octave of Mese to complete the disdiapason. Proslambanomenos then came to be regarded as leading to a tone of disjunction, analogous to the tone between Mese and Paramese. But Hypate Hypaton was still recognized in some obscure sense as the beginning of the modal system with the octave from Hypate Hypaton as first species.

THE MODALITY OF THE P.I.S. TRANSFORMED FROM DORIAN INTO PHRYGIAN

Stage iv was reached when Proslambanomenos was eventually drawn into the system of the tetrachords, as implied by Ptolemy in the exposition of the Shades of the Genera (Lib. ii, c. 15) in the form of Modes, distinguished as '*Apo Meses*'. This innovation was entirely non-modal, however, and created a pseudo-Hypodorian species, since the interval between Proslambanomenos and Hypate Hypaton in the P.I.S. is always a tone, whereas from the Hypodorian tonic to the 2nd degree in the Harmonia and modal species, there is the just semitone 16/15.

We now pass on to consider a happening in the domain of Modality which was fraught with far-reaching consequences. The newly found independence of the original Harmoniai developed into new conjunct Modes (κατὰ συναφήν). At some time, as yet undefined, and in a region deeply imbued with Phrygian influences (and traditional), the ***Phrygian conjunct Mode*** developed and flourished; and gaining ascendancy over all the other Modes in the Hellenistic world, it became firmly established in musical composition, as instanced by the Nome of Athene.[1] The theorists then discovered what had taken place; and by them the change may have been chronicled as the passing of the classical P.I.S., or as the advent of a new system embodying the Phrygian modality.

Through the incidence of its M.D. 24 upon Lichanos Hypaton, the Phrygian Mode in its conjunct form was driven to use the old Proslambanomenos in an entirely new capacity. Rescued from its isolation outside the tetrachords, ***Proslambanomenos***[2] was now admitted into the tetrachordal structure itself, to form the Tonic and starting-point for the conjunct modal octave. Proslambanomenos as octave of Mese bore the ratio 32 which is the M.D. of the Hypodorian Harmonia. The P.I.S. has, thus, now become a Phrygian system based upon Proslambanomenos.

This transformation is an ideological and rhythmical one involving neither change of ratios nor of Notation; it is merely a change of mode and incidentally of pivot.

This figure displays the Phrygian Harmonia in conjunct form, in full possession of the P.I.S., and tuned to the Hypolydian Tonos. This new conjunct Phrygian scale exhibits all the most significant features of the Ecclesiastical Modes at the time of their first appearance in the theoretical writings of the Western World.

From Lichanos Hypaton to Proslambanomenos is a 4th, and from the same note to Mese, a 5th. The importance assigned to the dominant in the Ambrosian and Gregorian Modes is seen to be a reflection of the characteristic Phrygian modal interval on the Tonic, expressed by ratio 24/16. The 3rd degree above the Tonic

[1] Plut., *de Mus.*, Chap. xxxiii (ed. Weil and Rein., §§ 367–87; and cf. Intro., p. xviii).

[2] See Hucbald, *De Harm. Inst.*, Gerbert, *Script. Eccl.* i, pp. 112*b* sqq., where the new function of Proslambanomenos is indicated.

FIG. 104.—The Origin of the Ecclesiastical Modes in the Fourth Development of the Perfect Immutable System (in the Hypolydian Tonos) in the Phrygian Modality

		HYPATON			MESON						DIEZEUGMENON			HYPERBOLAION			
	Proslamb.	*Hypate.*	*Parhyp.*	*Lichanos*	*Hypate.*	*Parhyp.*	*Lichanos*	*MESE*	*Trite Synem.*	*Paramese*	*Trite.*	*Paranete*	*Nete.*	*Trite*	*Paranete*	*Nete.*	
Modal Ratios of the P.I.S.	**32**	**28**	**27***	**24**	**22 / 21**	**20**	**18**	$\boxed{16}$	**15**	**14**	**27***	**24**	**21***	**20**	**18**	$\boxed{16}$	
	A	***B♯***	***C***	***D***	***E***	***F***	***G***	$\boxed{A}$	***B♭***	***B♮***	***C***	***D***	***E***	***F***	***G***	***A***	
Auth. i			Tonic	*D*	*E*	*F*	*G*	*A*		*B♯*	*C*	*D*					PHRYGIAN
Plag. i	Tonic *A*	*B♯*	*C*	*D*	*E*	*F*	*G*	*A*									
	32	28	27	24	22	20	18	$\boxed{16}$		28 / 14	27	24					
Auth. ii				Tonic	*E*	*F*	*G*	*A*		*B♯*	*C*	*D*	*E*				DORIAN
Plag. ii		Tonic *B♯*	*C*	*D*	*E*	*F*	*G*	*A*		*B♯*							
		28	27	24	21 / 22	20	18	$\boxed{16}$		28	27	24	21 / 22				
Auth. iii					Tonic	*F*	*G*	*A*	*B♭*	*B♯*	*C*	*D*	*E*	*F*			HYPOLYDIAN
Plag. iii			Tonic *C*	*D*	*E*	*F*	*G*	*A*	*B♭*	*B♯*	*C*						
			27	24	21 / 22	20	18	$\boxed{16}$	15	28	27	24	21	20			
Auth. iv						Tonic	*G*	*A*	*B♭*		*C♯*	*D*	*E*	*F*	*G*		HYPOPHRYGIAN
Plag. iv				Tonic *D*	*E*	*F*	*G*	*A*	*B♭*		*C♯*	*D*					
				24	21 / 22	20	18	$\boxed{16}$	15		13	12	11	10	9		

Note.—The ratios are those of the P.I.S., modified as implied by the allocation of the Symbols in Alypius; the asterisks denote the characteristic ratios which have been modified; whereas in practice the modal 26 and 22 were probably used by lovers of the Modes in spite of the revision by the Theorists.
The encircled numerals denote the Tonic of the Species, the squared the Keynote of the Harmonia or Species.

forms with it the just minor 3rd 24/20. When the scale is continued upwards for an octave from Lichanos Hypaton through Diezeugmenon to Paranete (of ratio number 24 or 12), the Authentos Protos on *D* lies before us, complete and correct in structure, viz. a 5th + a 4th, and exact as to ratio in the modal form still in use in several Greek Churches in Asia Minor.[1] The Plagios Protos likewise appears from Proslambanomenos to Mese, duly composed of the 5th superimposed upon the 4th, and having Mese as true final on the 8th degree, and thus justifying the tonality of the Ecclesiastical Modes as handed down through the early Middle Ages.

In this way the mysterious origin of the Plagal Modes is revealed as the independent conjunct modal octave, spreading out below the Authentic, overlapping it in its upper 5th, and retaining kinship with it through the common Modal Genesis; this fact is apparently recognized in the earliest known nomenclature, e.g. Authentos Protos, Plagios Protos. The lower plagal tetrachord is, of course, merely a transposition an octave lower of the upper tetrachord of the Authentic.

In relation to the Modal System, it is the Phrygian Mode which is the prototype of the early Modes of the Greek Church. The momentous change thus effected through the 4th stage of the P.I.S. signifies a new orientation; the Dorian Mode was displaced in favour of the Phrygian, within which other Modes, authentic and plagal, were taken as species of the Hypolydian Tonos.

THE RISE OF A NEW SYSTEM OF CONJUNCT MODAL SCALES: THE BIRTH OF THE PLAGAL MODES

The attention of the theorists had been focused upon the conjunct octave, as a prelude to a new series of conjunct modal species. This is no new feature, since it formed part of the original P.I.S.; it merely provides a new Onomasia Kata Thesin, Phrygian in modality. The division of the Modes of the Greek Church into authentic and plagal is a natural outcome of this development.

The fundamental change is a modal one, and it establishes modality upon a new footing: for the modal species become modes in their own right, and the nomenclature of the P.I.S. is open to all on equal terms. The species are henceforth emancipated from the domination of the Dorian Harmonia.

This epoch-making change raises the question whether any evidence exists of similar modal independence; in other words, of the use in other Modes also, by musicians, of this form of conjunct scale based upon Proslambanomenos?

TWO EXAMPLES OF THE NEW INDEPENDENT CONJUNCT MODAL SCALES BASED UPON PROSLAMBANOMENOS

The answer is in the affirmative, and evidence is forthcoming of two modes so used:

In the first proposition of the Harmonic Canon of Florence (Chap. v) in which an aliquot division—announced but not carried through—by Modal Determinant 28 'based upon Proslambanomenos' created the conjunct form of the Hypolydian species; but still within the Dorian modality.

PROSL.	H.H.	PH.H.	L.H.	H.M.	PH.M.	L.M.	MESE.
28	26	24	22	20	18	16	14

or in the new Phrygian modality of a conjunct Dorian Species

28	26	24	22	20	18	16	14

[1] According to the testimony of Dr. Joh. Tzetzes, pp. 77 and *passim.*, 'Über die Altgriechische Musik in der Griechischen Kirche' (München, 1874).

Which of these two interpretations—both legitimate with a Modal Determinant 28, based upon Proslambanomenos—was originally intended, remains problematical, for reasons given in Chapter v. By a further use of the same Canon, a second proposition was launched and a fully described division by M.D. 24 was also based upon Proslambanomenos. As this stands in the Canon, the ratios proclaim the scale to be the conjunct form of the Hypophrygian Harmonia, as in No. 1 below. But if the sequence were merely noted in the *A* keyboard scale stated without ratios, it would, as one of the Ecclesiastical Modes, apply equally well to the Protos Subjugalis described by the eleventh-century Abbot Berno [1] of Reichenau, or to the Plagal of the 4th Authentic, transposed a 4th down to *D*. When accompanied by the modal ratios, as in No. 2 below, the meaning is unmistakable.

ALIQUOT DIVISION FROM THE HARMONIC CANON OF FLORENCE BY M.D. 24

	PROSL.	H.H.	PH.H.	L.H.	H.M.	PH.M.	L.M.	MESE
No. 1	24	22 / 21	20	18	16	15	13	12

CONJUNCT FORM OF THE HYPOPHRYGIAN HARMONIA OR FOURTH PLAGAL

K. S. =	*a*	*b*	*c*	*d*	*e*	*f*	*g*♯	*a*

COMPARED WITH

No. 2	(32	28	26	24	22	20	18	16)
	a	*b*	*c*	*d*	*e*	*f*	*g*	*a*

THE FIRST PLAGAL MODE

N.B.—This figure shows how easy it was to confuse the first and fourth plagals in staff notation.

The conjunct form of the Hypophrygian Harmonia is revealed by the ratios of the aliquot division, based upon Proslambanomenos, as an instance of the independent use in other modes of the conjunct forms of the Harmoniai based upon Proslambanomenos.

No. 1 may besides be regarded as the prototype of the 4th Plagal of the Greek Church, and also of the minor Mode of the West.

THE OCTOECHOS IN USE IN THE GREEK CHURCH IN THE FOURTH CENTURY A.D.

In a survey of the immense field of research into which the quest for the origin of the Ecclesiastical Modes leads, the paucity of data dealing with the early centuries of our Era becomes immediately apparent. In the domain of musical notation between the Christian Hymn,[2] *c.* third century A.D., and the earliest Greek MSS. in Neums of the tenth and eleventh centuries, data are lacking. And yet from the wealth of material available for research such as Chronicles, historical accounts of the Greek Church, Exegesis, Decretals [3] and Acts, it is known that

[1] Cf. ' Bernonis Prologus in Tonarium ' (xi C), *ap.* W. Brambach, *Das Tonsystem und die Tonarten d. Christl. Abendlandes im Mittelalter* (Leipzig, 1881), p. 43, § 7, where Berno (or an interpolator) describes the descent from Lichanos Hypaton to Proslambanomenos by tone, semitone, tone as the conjunct limb of the Protos subjugalis or Tonus Secundus. See also Martin Gerbert, *Script. Eccl.*, ii, p. 69*b*.

[2] *Oxyrth. Pap.*, xv, 1786. The hymn is noted in the vocal Greek notation of the Hypolydian Tonos and composed in the Hypophrygian mode.

[3] See F. A. Gevaert, *Les Origines du Chant Liturgique de l'Église Latine* (Gand, 1890).

the Greek Modes (whether the classical Tropoi, or the Hellenistic Echoi evolved directly from them) were in use in the early Greek Church of Hellenistic Asia and Egypt. As evidence we may cite the incident recorded concerning S. Pambo, the Egyptian Abbot, by Gerbert [1] from a fourth-century Greek MS. in the Hofbibliothek in Vienna, a German translation of which is given by Oskar Fleischer.[2] From this document it is learnt that a novice, returning from a visit to the Church of St. Mark in Alexandria, relates that the Troparia were being sung in that Church. S. Pambo, however, peremptorily refused his request to allow chants and Modes (*ἄσματα καὶ ἤχους*) to be sung in the more austere services of the monastery. Dom Jeannin [3] quotes a similar instance from the fourth century A.D. of a mention of the Octoechos in the days of St. Ephraim : when a monk, conversing with Abbot Silvanus of Sinai, said, ' Since I have become a hermit I have chanted the order of the Office and the Hours, and have sung the Hymns of the Octoechos (*καὶ τὰ τῆς ὀκτωήχου ψάλλω*). In another case, a hermit in the desert of Nitria, who when receiving a visit from Paul of Cappadocia, sang to him (*'τὰ τροπάρια καὶ κανόνας ψάλλειν καὶ ἤχους μελίζειν.'* [4]) A 4th incident is related by Fleischer on the authority of Cardinal Pitra [5] concerning a visit by the Abbots Johannes and Sophronius to the aged hermit Nilus on Mt. Sinai, who belonged to the old school and did not permit the singing of Songs and Tropes after Evensong, but allowed only the Doxology and the Kyrie Eleison.

Anthymus and Timokles, composers of Troparia, who flourished during the reign of the Emperor Leo the Great (457–74 A.D.), are cited by Cedrenus,[6] eleventh century, and Theodorus.[7] According to a twelfth-century writer, Zonaras,[8] the Troparia were Greek songs in the ancient Harmoniai, and were used in the Office of the Greek Church in conjunction with the Tropes and the Hirmus. Zonaras mentions the Modes by their classical name Harmonia, not Echos: *Εἱρμὸς δὲ λέγεται ὡς ἀκολουθίαν τινα καὶ τάξιν μέλους καὶ ἁρμονίας διδοὺς τοῖς μετ' αὐτοῦ τροπαρίοις.*

A Hirmus is defined as giving a sequence and order of melos and Harmonia to the Troparia that follow after it. ' Indeed, these receive their rhythm according to the Melos of the Hirmoi and they are tuned and sung to them, and they follow the Harmonia of that Melos, so that the Hirmos makes the Troparion dependent upon its Melos and binds it to itself.'

These data establish the use of the Echoi and of the forms of Hymnody known as Tropes and Troparia, based upon the Harmoniai in a specified Church in Alexandria in the fourth century A.D. and elsewhere in the services of the Greek Church.

[1] *Script. Eccl.*, i, pp. 1–4, Greek and Latin extract from xiii c. Greek codex Bibl. Caesar. Vindob.

[2] See also Oskar Fleischer, *Neumen Studien*, Vol. i and ii *passim*, and Vol. ii, pp. 52–3 ; A. J. Vincent, *Notices et Extraits*, p. 6, Note 1*b*, who also cites Gerb., *De Cantu*, 1, p. 207.

[3] *Mélodies Liturgiques Syriennes* (Paris, 1924), p. 93. See also Nau, *Plémophories de Jean de Maimouma* (Majuma) in *P.O.*, viii, p. 170–80.

[4] Dom Pitra, *Hymnographie de l'Église grecque*, p. 43.

[5] *Juris Ecclesiastici Graecorum historia et Monumenta*, Tom. 1, p. 220.

[6] Georgius Cedrenus : Migne, Vol. 1, p. 612, line 16 (349*c*).

[7] Migne, Vol. 86, Pt. 1, p. 174, § 567. See also Du Fresne and Du Cange, *Gloss. ad Script. Mediae et Infime Graecitatis*, s.v., *τροπάριον* and also Fleischer, *op. cit.*, ii, p. 53 and p. 119.

[8] *Canones Anastasimos Damasceni Intr.* See also Nicolaus Rayaeus, ' de Akolouthia ', in *Acta Sanct. Junii*, Bd. ii, pp. xv sqq. (Antwerp, 1698). See also Fleischer, ii, p. 119, who quotes two passages from Zonaras in Greek.

CONFIRMATION FROM ARABIAN SOURCES

Our investigation now provides an important piece of evidence of a circumstantial nature on the origin of the Ecclesiastical Modes which centres round the figure of the Arabian musician Ishāq-Ibn-Ibrahīm-al-Mauṣili (Mosul, 767–850 A.D.), whose reputation as a theorist rests mainly upon his introduction of a new accordance for the lute, which has already been fully described,[1] and still more upon his revision of the Classification of Songs according to Rhythm and Modality. It will not be possible to show the connexion of these data with the Modes of the Greek Church without assuming a familiarity with the names of the frets of the lute, which may be identified on Fig. 55 (Ch. vii).

Ishāq's classification by Modes is expressed through the courses (MĀJĀRĪ, sing. MĀJRA) on the lute, passing either through *Wosta*, the middle finger fret, a minor 3rd above the open string, or else through *Binṣir*, the ring-finger at a major 3rd above the open string (= MOTLĀQ). By following Ishāq's instructions with care, it is revealed that these courses correspond with the species of the Greek Modes, Harmoniai or Echoi. This important evidence is derived from Ishāq's enthusiastic Chronicler, Al-Ispāhāni.[2]

ISHĀQ'S MĀJRA THROUGH *WOSTA* IDENTIFIED AS PHRYGIAN MODAL SPECIES

Numbers 4, 5, 6, 7, selected from Ishāq's Classification by MĀJĀRĪ (Fig. 105) display the complete Octoechos of the Greek Church, consisting of 8 modal species, indicated by Ishāq but not so named. Following his directions, the 8 species are obtained by taking the P.I.S. in its 4th stage in the conjunct form of the Phrygian Mode. The 1st tetrachord (the Hypaton in the original Onomasia Kata Thesin) is based upon Proslambanomenos, or according to the phraseology of the Arabs, borrowed from the lute, in the MĀJRA of *Wosta on* BAMM (lowest string). Taking the lute accordance of Ishāq (Fig. 55, Chap. vii), tuned from the MOTLĀQ (open string) of Matna as I'MĀD or A'MĀD, the fundamental or arche (here hypothetically equated in pitch with our *A*), corresponding with Mese or with Proslambanomenos, in the Hypolydian Tonos, the species are thus obtained ; if the frets are followed step by step in Fig. 105 as indicated, an exact succession of the four Authentic and four Plagal Modes is obtained in the form and order in which they are first traced in the West.

We are thus confronted with the remarkable fact that an early ninth-century classification of Arabian Songs, according to explicit directions, exhibits sequences identical in structure with the early Modes of the Greek Church. Al-Ispāhāni (tenth century) has, moreover, accepted these 8 Modes ' which did not have fanciful names like those of Persia and Greece, but were named after the fingers ' (i.e. frets).[3] The question which arises is obviously : From what source did Ishāq derive his knowledge of the MĀJĀRĪ or species ? Ishāq, himself, disclaimed any knowledge of the Greek theorists, but it is probable that he had become familiar in practice with the modal sequences during the years he spent in Bagdad with his father.

Al-Ispāhāni, moreover, mentions instances of Arabian lutenists and singers

[1] Ch. vii, and Fig. 53.

[2] *Abul Faraj Ali Ibn Hossein Al-Ispāhāni,* tenth century A.D. ; see Kosegarten's Latin translation, *Alii Ispāhānensis liber Cantilenarum Kitab Al-Aghāni* (Gripesvoldiae, 1840) ; see also J. Rouanet, ' La Musique Arabe ', *ap.* A. Lavignac, *Encyc. de la Musique,* p. 2694.

[3] Kosegarten, *op. cit.*, pp. 179 sqq.

FIG. 105.—Origin of the Ecclesiastical Modes confirmed from Arabian Sources, The Octoechos on the Lute of Ishāq (*cf.* Figs. 55 and 61, Chap. vii, Ishāq's Lute)

The MĀJRA of *Wosta* = Phrygian Species

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4. Through MOTLĀQ in the MĀJRA through *Wosta*

ON MATLĀT AND MATNĀ	$\underbrace{D\;\;E\;\;F\;\;G\;\;A}\,\underbrace{B\;\;C\;\;D}$	Authentos Protos
ON BAMM AND MATLĀT	$\underbrace{A\;\;B\;\;C\;\;D}\,\underbrace{E\;\;F\;\;G\;\;A}$	
	Plagios Protos	

The divisions by 4ths and 5ths are marked by MOTLĀQ

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5. Through SĀBBĀBĀ in the MĀJRA through *Wosta*

ON MATLĀT AND MATNĀ	$\underbrace{E\;\;F\;\;G\;\;A\;\;B}\,\underbrace{C\;\;D\;\;E}$	Authentos Deuteros
ON BAMM AND MATLĀT	$\underbrace{B\;\;C\;\;D\;\;E}\,\underbrace{F\;\;G\;\;A\;\;B}$	Plagios Deuteros

The divisions by 4ths and 5ths are marked by *Sābbābā*

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6. Through *Wosta* in its own MĀJRA

ON MATLĀT AND MATNĀ	$\underbrace{F\;\;G\;\;A\;\;B\;\;C}\,\underbrace{D\;\;E\;\;F}$	Authentos Tritos

ON BAMM AND MATLĀT — *C D E F G A B C* — Plagios Tritos

The divisions by 4ths and 5ths are marked by *Wosta*

7.* Through KHINṢIR in the MĀJRA through *Wosta*

ON MATLĀT, MATNĀ AND ZIR — *G A B C D E F G* — Authentos Tetartos

ON BAMM, MATLĀT AND MATNĀ — *D E F G A B C D* — Plagios Tetartos

The divisions by 4ths and 5ths are marked by *Khinṣir*

* *Added by K. S. from a quotation by Al Ispāhāni, op. cit., Rouanet, p. 2702b.*

N.B.—MĀJRA = Species ; MOTLĀQ = Open String ; *Sābbābā* = 1st fret one tone above the open string ; *Wosta* = Middle finger fret at minor 3rd on the open string ; *Binṣir* = ring finger fret, a major 3rd on the open string ; *Khinṣir* = little finger fret, a 4th on open string.

Names of the strings : BAMM bass = *A* ; MATLĀT = *D* string ; MATNĀ = *A* string ; ZIR = *D* string. By following the MĀJRA OF BINṢIR = Lydian Species, the Species are obtained in a different key and in a different order (see Figs. 55 and 61).

Thus the four Authentic Modes, each with its plagal, are obtained as in the Octoechos and in the same octaves, by following the instructions of the Arabian Lutenist, Ishāq al-Mauṣili (eighth century) given in his book of Songs. From Ispāhāni's account, it seems that Ishāq derived his inspiration from the Christian monks.

who modified their modes through contact with the Christian monks [1] (belonging to the Syrian monasteries).

The exact correspondence exhibited in Fig. 105, between Ishāq's courses 4 to 7 and the four Authentic and four Plagal Modes of the early Greek Church certainly suggests a common origin, viz. the Ancient Greek Modal System of the Harmoniai, traced in this Appendix through the four stages in the development of the P.I.S. It is, therefore, reasonable to suggest that Ishāq was one of the above-mentioned lutenists who learnt something of value by contact with the Christian monks, and that he was indebted to them for the means of codifying the courses on the lute, which in their order as species and in their modality coincides with those of the Octoechos, and of the Ecclesiastical Modes, ***known already in the West in the eighth century*** (see further on Alcuin) at the court of Charlemagne as authentic and plagal, and described in the next century by Hucbald, placed on the correct degrees in the P.I.S.

From these Arabian sources one other interesting point arises which bears on the development of music in the West. Was there a significant relation between the later Western conception of Mode, as major or minor, and the Arabian classification by Ishāq of species passing alternatively on the frets of the lute through *Wosta*, the minor 3rd, or through *Binṣir*, the major 3rd? Ishāq's Classification would have the prior claim.

CONFIRMATION OF THE ORIGIN OF THE ECCLESIASTICAL MODES FROM THE WRITINGS OF EARLY MEDIEVAL THEORISTS

A brief reference to the evidence afforded by early medieval treatises may now be added.

Alcuin (eighth century) [2] gives the Greek names of the four Tonoi and adds 'the names in use amongst us', i.e. in Latin as used at the court of Charlemagne and in the schools of music established by him. Hucbald [3] (ninth to tenth centuries) states that from Lichanos Hypaton, Hypate Meson, Parhypate Meson and Lichanos Meson, 'Quatuor modis vel Tropis quos nunc Tonos dicunt, hic est protus . . . Lichanos Hypaton scilicet autentum protum et plagium eiusdem, id est primum et secundum; hypatemeson autentum deuterum et plagium eius, id est tertium et quartum, &c.' Further on he alludes to the newer system with its orientation from Proslambanomenos (p. 119*b*). 'Usque ad has enim metam inchoandi declinant: hae sunt Proslambanomenos ad Lichanos Hypaton, Hypate Hypaton ad Hypate Meson; sed id raro; parypate [*sic*] hypaton ad parypate meson, &c.'

The eleventh-century description by the Abbot Berno of Reichenau in his *Prologus in Tonarium* has already been mentioned (see *ante*).

[1] Koseg, pp. 199–200 sqq. [2] Gerbert, *Script. Eccl.*, i, p. 26.
[3] Gerb., ibid., p. 119*a*.

APPENDIX III

A NEW LANGUAGE OF MUSIC: POSSIBILITIES OF THE ANCIENT MODES FOR USE IN MODERN COMPOSITION

THE Art of Music seems to be at the crossways seeking a new language of Music since none of the attempts to infuse some new element into Music has so far met with any general measure of success, while some of the innovations have led to bitter controversy. The most important of these departures from the older dialect and forms, *atonality* with its variants, still has many adherents, but scores no decisive victory. On the other hand, the quarter-tone and 7th-tone excursions into indefinite microtonal intonation may appear logical enough on paper, determined by logarithms on a basis of equal temperament; but, in practice, they are merely conjectural; for as far as I know, no method has yet been discovered for converting the graphically measured values into actual sounds of demonstrable vibration frequencies, as may so easily be accomplished by ratios on a monochord. The composer may know what he intends, but has no means of expressing this so that he may make it known to his executants. Meanwhile, inspiration and genius stand by unconcerned.

The Harmonia, as it is presented in this work, forms a new language of Music possessing many distinct advantages, e.g. it is established upon a positive, demonstrable basis, which, given a modal monochord having a rule accurately marked, and numbered segment by segment, can be used by a child even. The notes of scale or intervals are easily obtained by means of a movable bridge (having a knife edge) placed under the string, the effect being to cut off the segments to the left of the bridge, held in position by the left hand, while the length of string comprised in the remaining segments is plucked or bowed by the right hand. The corresponding note in exact intonation may then be produced.

A monochord rule bearing an aliquot division by the Modal Determinant of each Harmonia forms an efficient guide for the piano tuner in giving the modern instrument the accordance of one of the ancient Harmoniai: my choice is the Dorian of Modal Determinant 22, with eleven notes to the octave of ratios 22/22 21/22 20/22 19/22 18/22 17/22 16/22 15/22 14/22 13/22 12/22 11/22.

Many musicians demur when introduced to the ratios of this modally tuned piano, for they consider that a scale containing intervals such as 12/11, 11/10, 15/13, forms a proposition impossible to bring into operation in practical music. That is an objection that comes naturally enough from modern musicians born and brought up in the atmosphere of our major and minor scales, with their more or less false relations, but this is an entirely individual matter, for many there are who react immediately in delight on hearing this new language of music.

It is time now to introduce the subject of the work of Miss Elsie Hamilton, hailing from Adelaide (South Australia), who studied composition with André Gédalge in Paris for five years. In the latter part of 1916, she became acquainted with this new language of music and at once adopted it because, she averred, it provided a natural basis, which she felt was lacking in the modern system. She mastered the new intonation in a few weeks and began at once to compose in it.[1] Her chief difficulty at first was to realize the necessity for abandoning the intricate elaborations of composition, which although quite in place in the well-worn language of modern music, were not adapted to first steps in the practice of intervals so strangely related to each other and to their tonic, for to lose one's bearings in the matter of tonality is an inevitable experience for both executant and listener. It was here that the path of the pioneer began to exhibit its thorns during the rehearsal of compositions, a painful task which could only be shouldered by the composer.

As yet Elsie Hamilton is the only composer who has had the courage and we might perhaps add the freedom, to grapple with the intonation of the new scales, for although many musicians and composers have displayed great interest and enthusiasm, the economic question bars the way.

Attention may now be turned to certain practical questions concerned with composition for which I select the following from the composer's notes on the Harmonic System she has devised. Since all the eleven notes of the Dorian Harmonia are proportionally related to each other and to the Tonic, they may all be used melodically, harmonically and contrapuntally together. In fact, harmonies which would result in cacophony on a piano, normally tuned in equal temperament, produce instead, on the modally tuned piano, a delightfully stimulating and arresting effect, entirely devoid of beats. To each individual composer the choice lies open between (1) a standard scale to be extended at will in both directions, and used

[1] The following is a brief account of the early activities of the courageous pioneer, some of which may be mentioned here:

(1) First demonstration given at the request of Dr. York Trotter in Princes Street in 1917.

(2) A second demonstration of the composer's works was given at Steinway Hall in 1917. Amongst other items a Septet for Violins (1st and 2nd), Viola, 'Cello, Flute, Oboe and Horn, played by members of the Queen's Hall and London Symphony Orchestras.

(3) A Trio for Oboe, Viola and Pianoforte (in which the Greek Dorian scale was approximated to the intervals of the piano) played by McDonagh, Waldo Warner and the Composer at an L.S.Q. concert at Aeolian Hall in 1918. This approximation to the ordinary piano intonation proved a great success, for apart from the exact intonation these ancient Modes possess a characteristic Ethos of which a novel semblance is obtainable even in the approximation.

(4) In 1919 three crowded performances were given at Etlinger Hall, Paddington, of the drama of 'Sensa' (a play of ancient Egypt by Mabel Collins and Maud Hoffman), with incidental music in the Greek Modes by Elsie Hamilton for harps, flutes, oboe and voices.

(5) In 1924 'Agave', a mystical mime by Eva Papp, with incidental music by Elsie Hamilton, was given three times in Madame Matton-Painparé's studio, by a chamber orchestra of string quartet, flutes, oboe cor anglais, harps and kitharas.

The difficulties in the way of such performances will be realized, when it is stated that not once, even at the final rehearsal, were all executants able to attend. Nevertheless, the reception was enthusiastic.

(6) *The Seven Scorpions of Ysit*, by Terence Gray, incidental music by Elsie Hamilton, choreography by Ninette de Valois, was also given at the Court Theatre in 1929, oboe and cor anglais by McDonagh, harp by Miss McDonagh, chanting by the composer.

In 1935 Elsie Hamilton introduced the new language of Music in Germany. At Stuttgart, a small chamber orchestra has been trained to play in the Greek Modes, and performances have been given there and at Freiburg-in-Breisgau.

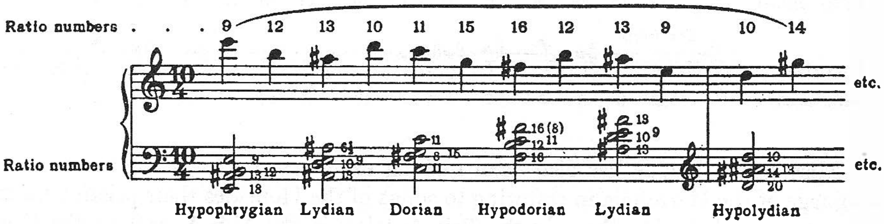
with utmost freedom, and (2) a modal compass used consciously in accordance with the modal characteristics of the seven species of the Harmonia with the object of capturing some of the charm and individual *Ethos* of these ancient Modes.

The unity of the system of the Harmoniai—used as species, for the present, in view of technical and practical exigencies—reveals itself as the harmonic system develops, for each of the notes of the diatonic Harmonia is the Tonic of one of the modal species, so that the basis of any chord, if emphasized by its vocal or instrumental treatment, may evoke a reminiscent association, or even actually interpose a dominating or suggestive savour belonging to another Mode.

' In fact ', the composer says, ' the two systems, the modern and the modal, represent two distinct musical worlds, each quite complete in itself, and which only prove inimical one to the other if one tries to compare them by holding both in the mind at the same time instead of allowing each to work upon one through its own inner logicality.

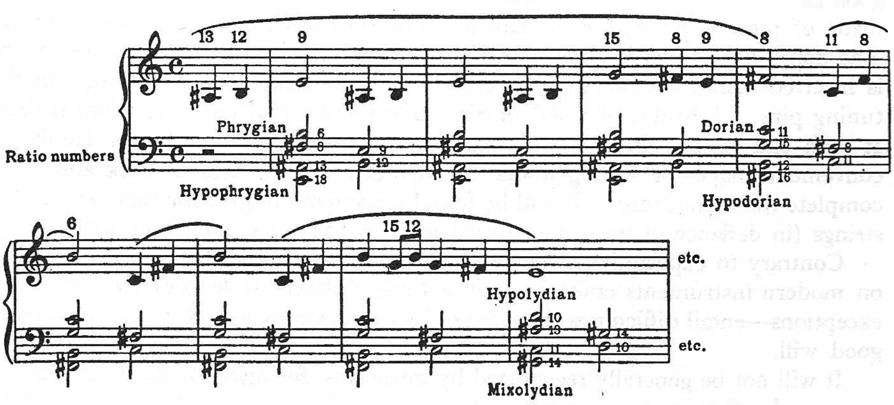
' Although the common chord is also to be found for instance on the ratios 12, 10, 8 and on 15, 12, 10, of the Harmonia, it cannot be obtained on any and every note of the scale. It is evident, therefore, that a new harmonic system is required. Attempts in this direction have been made by building up chords on the fixed notes of the modal tetrachords, for example in Fig. 106.

FIG. 106.—' Sunrise ' (from ' Agave ').



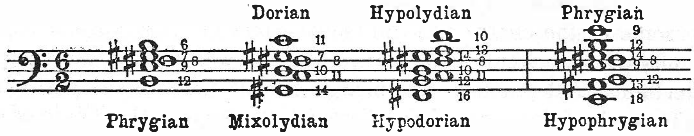
' *Secondly,* by forming a chord from the two dissimilar tetrachords of two related Harmoniai, and by resolving it by the tetrachord they have in common (*see Fig. 107*).

FIG. 107.—Funeral March (from ' Agave ').



' *Thirdly,* by sounding together chords taken from various modes which have features in common, e.g. *in Fig. 108*.

FIG. 108.—‘ Sunrise ’ (from ‘ Agave ’).



‘ Another interesting feature in the Harmoniai is the unique possibility they offer of varying the Ethos or psychological character of one and the same melody by playing it in different modes. The changes in experience brought about in so simple a manner are far beyond what is attainable in our modern scale by merely playing the same melody in different Keys ’ (*see Fig. 109*).

FIG. 109.—From ‘ Agave ’, with allowances for pianoforte approximation. Species of Dorian Mode (*C* 22).

Miss Isabel Dodds has also been making excellent use for some years of the language of the Harmonia in restoring to songs of the Hebrides their pristine modal intonation, and singing them in the British Isles, in the U.S.A. and on the Continent of Europe, with accompaniments on her Celtic harp tuned to the Harmonia (see Chap. ix).

The first desideratum for a study of this new language of music is a monochord, the making of which presents no difficulty. The following dimensions have given good results. The monochord consists of a long narrow box 1·120 m. in length, made of pine or birch, for resonance. The soundboard has a width of ·06 m. The depth of the box is likewise ·06 m. ; a solid block of beech ·07 × ·06 × ·06 is inserted inside the box at each end to take the tension of the strings on the tuning pins. A bridge of wood in triangular section (with a knife edge) is fixed at each end at ·045 from the tuning pin. A movable bridge with a handle of convenient shape for sliding under the strings ; sound-holes in back and front complete the monochord. It will be found convenient to give the monochord two strings (in defiance of its name), tuned to *C* of 128 v.p.s. and *F* of 176 v.p.s.

Contrary to expectations, the production of those somewhat strange intervals on modern instruments embodying an entirely different scale does not—with few exceptions—entail difficulties that cannot be overcome by expert musicianship and good will.

It will not be generally recognized by musicians, for instance, how small is the margin of difference between most of the ratios of intervals, which are used daily in our major and minor scales, and the intervals of the Harmoniai ; nor how easily the executant himself can bring about the necessary modifications on his instrument.

In fact, most of the instruments of the orchestra can reproduce the intonation of the Greek Modes by slight manipulations, such as cross-fingering on certain wind-instruments. This, of course, is merely a counsel of expediency, for use until instruments are constructed specially for the new language of Music.

As matters stand at present, cylindrical keyless flutes may be plotted and bored specifically for the harmonia required (hence references to the 'Sensa flute'), oboe and cor anglais prove by no means intractable; trombones, slide trumpets and horns are amenable to reason. Clarinets can be coaxed. It is not, of course, claimed that this can be accomplished without a certain loss in tone colour. But as soon as structural changes in the instruments are compatible with economic exigencies, there will be a definite gain in beauty of tone, produced by dependence upon proportional impulse instead of upon empirical dalliance with misunderstood factors.

Ω — A

INDEX

LIST OF ABBREVIATIONS

All.	= allowance	**Harm.**	= harmonia
B-R. mp.	= beating-reed mouthpiece	**Hornb.**	= Erich. M. von Hornbostel
Chr.	= chromatic	**Ho.**	= hypo. as hypodorian
Conj.	= conjunct	**Inct.**	= increment
Cr-f.	= cross-fingering	**I.D.**	= increment of distance
Disj.	= disjunct	**M.D.**	= modal determinant
D-R. mp.	= double-reed mouthpiece	**Mp.**	= mouthpiece
Δ	= diameter	**P.I.S.**	= Perfect Immutable System
Δ All.	= diameter allowance	**Prosl.**	= Proslambanomenos
Eff. L.	= effective length	**sd-w.**	= sound-wave (½ or ¼ or 1)
Extr.	= extrusion (of Mp.)	**sd-w.l.**	= sound-wave length (½ or ¼ or 1)
Fl.	= flute	**v.f.**	= vibration frequency
F.M.	= folk music	**v.l.**	= vibrating length
F.T.	= folk tune		
G.C.S.	= Greater Complete System		

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TABLE XV OF REFERENCE. RATIOS AND v.f.s. OF THE 7 HARMONIAI ON $C = 128$; $F = 176$ AND $G = 192$ v.p.s.

(The M.D. and ratio in squares.)

Dorian Harmonia

C	F	G v.f.s
128 22/22	112 16/22	124·2 17/22
	129·06 15/22	
134 21/22		132 16/22
140·8 20/22	138·28 14/22	148·8 15/22
148·2 19/22	148·9 13/22	
		150·8 14/22
156·4 18/22		
	161·3 12/22	162·4 13/22
165·6 17/22		
176 16/22	176 22/22	176 12/22
	184·38 21/22	
187·7 15/22	193·6 20/22	192 22/22
201·1 14/22		
	203·8 19/22	201·1 21/22
216·6 13/22	215·1 18/22	211·2 20/22
	227·6 17/22	222·2 19/22
234·6 12/22		234·6 18/22
	242 16/22	

Phrygian Harmonia

C	F	G v.f.s.
128 24/24	124·2 17/24	121·2 19/24
		128 18/24
133·56 23/24	132 16/24	135·5 17/24
139·63 22/24	140·8 15/24	
146·28 21/24		144 16/24
	150·8 14/24	
153·6 20/24		153·6 15/24
161·68 19/24	162·4 13/24	164·5 14/24
170·6 18/24		
	176 24/24	177·2 13/24
180·7 17/24	183·6 23/24	
186 (33/48)		
192 16/24	192 22/24	192 24/24
	200·18 21/24	200·2 23/24
204·8 15/24		
		209·4 22/24
219·4 14/24	211·2 20/24	219·4 21/24
	222·2 19/24	
		230·4 20/24
236·39 13/24	234·6 18/24	
	248·4 17/24	

Lydian Harmonia

C	F	G v.f.s
128 26/26		124·8 20/26
	120·4 19/26	
	127 18/26	131·3 19/26
133·12 25/26	134·6 17/26	
138·6 24/26		138·6 18/26
144·7 23/26	143 16/26	146·8 17/26
151·27 22/26	152·5 15/26	
158·47 21/26		156 16/26
	163·4 14/26	
166·4 20/26		166·4 15/26
175·15 19/26	176 (13/26) (26/26)	
		178·2 14/26
	183 25/26	
184·3 18/26		
	190·6 24/26	192 26/26
195·76 17/26	198·94 23/26	
		199·6 25/26
208 16/26	208 22/26	208 24/26
	217·8 21/26	217 23/26
221·86 15/26	228·8 20/26	226·8 22/26
237·7 14/26		237·6 21/26

Mixolydian Harmonia

C	F	G v.f.s
128 28/28	123·2 20/28	122·18 22/28
	129·7 19/28	128 21/28
132·7 27/28		134·4 20/28
137·8 26/28	136·8 18/28	
143·36 25/28	145 17/28	141·4 19/28
149·3 24/28		149·3 18/28
	154 16/28	
155·9 23/28		158·1 17/28
162·9 22/28	164·2 15/28	
170·6 21/28		168 16/28
	176 28/28	
179·2 20/28		179·2 15/28
	182·4 27/28	
188·6 19/28	189·5 26/28	
		192 28/28
	197·1 25/28	
199·1 18/28		199 27/28
	205·3 24/28	
210·8 17/28		207·6 26/28
	214·2 23/28	215 25/28
224 16/28	224 22/28	224 24/28
	234·6 21/28	233·6 23/28

Hypodorian Harmonia

C	F	G v.f.s
128 32/32	128 22/32	118·1 26/32
		122·88 25/32
	134·1 21/32	128 24/32
136·5 15/32 30/		133·5 23/32
	140·8 20/32	139·6 22/32
141·2 29/32		
146·3 28/32		146·2 21/32
	148·2 19/32	
151·7 27/32		153·6 20/32
157·5 26/32	156·4 18/32	
		161·6 19/32
163·8 25/32	165·6 17/32	
170·6 24/32		170·6 18/32
178 23/32	176 32/32	
		180·7 17/32
182 22/32	181·66 31/32	
	187·7 30/32	
		192 32/32
195 21/32	194·2 29/32	
	201 28/32	198·2 31/32
204 20/32		204·8 30/32
	208·3 27/32	211·8 29/32
215·58 19/32	216·6 26/32	
		219·4 28/32
227·55 18/32	225·2 25/32	227·4 27/32
	234·6 24/32	
241 17/32	244·4 23/32	

Hypophrygian Harmonia

C	F	G v.f.s
	126·7 25/36	119·1 29/36
128 36/36		
		123·4 28/36
131·6 35/36	132 24/36	128 27/36
		132·9 26/36
135·5 34/36		
	137·7 23/36	
139·6 33/36		138·2 25/36
144 32/36	144 22/36	144 24/36
148·64 31/36		
	150·8 21/36	150·2 23/36
153·6 30/36		
158·9 29/36	158·4 20/36	157·09 22/36
164·6 28/36	166·7 19/36	164·5 21/36
170·6 27/36		172·8 20/36
177·2 26/36	176 36/36	
	181 35/36	181·9 19/36
184·3 25/36		
	186·34 34/36	
192 24/36	192 33/36	192 36/36
	198 32/36	197·4 35/36
200·34 23/36		
	204·4 31/36	203·2 34/36
209·45 22/36	211·2 30/36	209·4 33/36
	218·4 29/36	216 32/36
219·4 21/36		
	226·2 28/36	222·6 31/36
230·4 20/36	234·6 27/36	230·4 30/36
242·5 19/36	243·6 26/36	

Hypolydian Harmonia

C	F	G v.f.s
128 40/40	121·39 29/40	116·3 33/40
	125·7 28/40	120 32/40
		123/8 31/40
131·28	130·3 27/40	
39/40		128 30/40
134·7 38/40	135·3 26/40	132 29/40
142 36/40	140·8 25/40	137 28/40
146·28 35/40	146·6 24/40	142·2 27/40
150·58 34/40		147·7 26/40
155 33/36	153 23/40	153·6 25/40
160 32/40	160 22/40	160 24/40
165·16 31/40	167·6 21/40	167 23/40
170·6 30/40		
176 29/40	176 40/40	174·5 22/40
182·8 28/40	180·5 39/40	182·8 21/40
	185·26 38/40	
189·6 27/40	190·2 37/40	192 40/40
196·9 26/40	195·4 36/40	196·9 39/40
204·8 25/40	291 35/40	202 38/40
213·3 24/40	207 34/40	207·5 37/40
	213·8 33/40	213·2 36/40
222·6 23/40	220 32/40	
232·7 22/40	227·1 31/40	219·4 35/40
		225·8 34/40
243·8 21/40	234·6 30/40	

K. SCHLESINGER

THE GREEK AULOS